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A Numerical Study on Deterministic Inventory Model for Deteriorating Items with Selling Price Dependent Demand and Variable Cycle Length

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Abstract

In the present paper, an inventory replenishment model for deteriorating items is developed with the assumptions that demand is a function of selling price and the cycle length of successive replenishments is a variable in the planning period. It is assumed that the cycle length in each cycle decreases in Arithmetic Progression. Shortages are allowed and are completely backlogged. The instantaneous state of inventory with shortages is derived. The total cost function of the horizon is obtained with suitable costs. The optimal pricing and ordering policies of the model are derived. The objective is to determine a replenishment policy that minimizes the total inventory cost. The model is illustrated with some numerical results. The sensitivity of the model with respect to the parameters and cost is also discussed. This model includes some of the earlier models as particular cases.

Keywords: *Perishable Inventory, Cycle Length, Demand Rate and Cost Function, Optimal Ordering Policies and Sensitivity Analysis*.

1. Introduction

Inventory control deals with the determination of the optimal stock levels of items to meet future demand. According to Nadoor [3], starting from the development of the first lot size inventory model in 1912, a wide variety of models have been developed for inventory control with various assumptions. In order to analyze the practical situations arising at places, like business, production, material handling, resource sharing etc., inventory models are essential. The nature of the inventory model varies depending upon the items under consideration. In general, the items can be classified as deteriorating and non-deteriorating. In deteriorating items, the life time of the commodity is finite and it is lost after a certain period of time. Inventory of deteriorating items was first developed and analysed by Within [1], who considered the deterioration of fashion goods at the end of a prescribed storage period. Ghare and Schrader [2] extended the classical EOQ formula with exponential decay of inventory due to deterioration, developing a mathematical model of inventory of deteriorating items. Dave and Patel [6] developed the first deteriorating inventory model with a linear trend in demand. They considered demand as a linear function of time. Goyal and Giri [15] explained the recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and other various conditions or constraints. Ouyang *et al.* [16] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Alamri and Balkhi [17] studied the effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Dye and Ouyang [18] found an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assumed that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases.

Ajanta Roy [19] studied a model in which the deterioration rate is time proportional, demand rate is function of selling price and holding cost is time dependent. Much work has been reported in deteriorating inventory models developed by many researchers, like Goyal *et al.* [13], Haipingxu *et al.* [10], Nahmias [7], and Sachan [8]. For modeling the inventory system, the prominent factors are demand and replenishment of items. Datta and Pal [12], Dave [9], Donaldson [4], Gioswami and Chaudhury [11] developed deterministic lot size

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inventory models with shortages and a linear trend in demand. In many of these models, they assumed that the cycle length, i.e., the time between two replenishments, is fixed or constant. In several inventory systems, the cycle length is to be made as a variable in order to have optimal operating policies. For example, in case of the production of edible oils, food products etc., the cycle length is to be reduced gradually in the planning period (Horizon). Nahamias [7] reviewed the perishable inventory models. Bhunia and Maiti [14] and Dave and Patel [6] developed inventory models with shortages with the assumptions that the successive replenishment cycles were diminished by constant amounts of time without considering the deterioration of items. Haipingxu *et al.* [10] and Sachan [8] developed the inventory models for deteriorating items with time dependent demand. Mondal *et al.* [20] investigated the finite replenishment inventory models of a single product with imperfect production process. In this process, a certain fraction or a random number of produced items are defective. Skouri *et al.* [21] studied inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Chung and Huang [22] studied ordering policy with permissible delay in payments to show the convexity of total annual variable cost function. Shah and Mishra [23] studied an EOQ model when units in inventory deteriorate at a constant rate and demand is stock dependent. The salvage value is associated to deteriorated units.

Wou [24] developed an inventory model with a stochastic demand. Jaggi Chandra and Priyanka Verma [25] developed and analyzed a two-warehouse inventory model for deteriorating items with linear trend in demand and shortages under inflationary conditions. Uma Maheswara Rao *et al.* [26] developed and analyzed a production inventory model for deteriorating items by assuming that the demand is a function of both on-hand inventory and time. It is also assumed that the lifetime of the commodity is random and follows a Weibull distribution. A case study is carried out to determine the production schedules in a pickle manufacturing industry.

Hung [27] made a continuous review of inventory models under time value of money and crashable lead time. Lin [28] analyzed inventory models with managerial policy independent of demand. Lin *et al.* [29] studied an inventory model with ramp type demand under stock dependent consumption rate. Roy and Chaudhuri [30] developed and analyzed an EPLS model for a variable production rate with stock price sensitive demand and deterioration. Khana *et al.* [31] developed a model which investigates an Economic Order Quantity (EOQ) model over a finite time horizon for an item with a quadratic time dependent demand by considering shortages in inventory under permissible delay in payments. They derived the model under three different circumstances depending on the time of occurrence of shortages, credit period, and cycle time. Karmakark and Dutta Choudhury [32] gave an inventory model with ramp-type demand for deteriorating items with partial backlogging and time-varying holding cost. Bhunia *et al.* [33] made an attempt to develop two inventory models for deteriorating items with a variable demand dependent on the selling price and the frequency of the advertisement of items. In the first model, shortages are not allowed, whereas in the second, they are allowed and partially backlogged with a variable rate dependent on the duration of the waiting time up to the arrival of the next lot. In both models, the deterioration rate follows a three-parameter Weibull distribution and the transportation cost is considered for explicitly replenishing the order quantity. Vipin Kumar *et al.* [34] studied a two-Warehouse partial backlogging inventory model for deteriorating items with ramp type demand. Srinivasa Rao *et al.* [35] developed and analyzed an EOQ model for deteriorating items with permissible delay in payments under inflation. They assumed that the demand is a function of both time and selling price. Further, they assumed that the lifetime of the commodity is random and follows a generalized Pareto distribution. Bhunia *et al.* [36] developed a paper which deals with a deterministic inventory model for the linear trend in demand under inflationary conditions with different rates of deterioration in two separate warehouses (owned and rented warehouses). Goel and Aggarwal [5] considered perishable inventory models with a selling price dependent demand. In many research papers, the researchers assumed that the cycle length is constant for successive replenishments; [10-13,15-17], etc. are examples for this type of models, except the works of Bhunia and Maiti [14] in which they developed an inventory model for deteriorating items with infinite rate of replenishment and time dependent linearly increasing demand over a finite time horizon. Shortages are allowed and are fully backlogged. The model is formulated by assuming that the successive replenishment cycle lengths are in arithmetic progression.

However, very few studies reported regarding the inventory models for deteriorating items having a variable cycle length of successive replenishments with selling price dependent demand, which are more useful in analyzing the inventory situation of deteriorating items. In the present paper, we develop and analyze an inventory model for deteriorating items having selling price dependent demand with variable cycle lengths for successive replenishments. Using the total cost function, the optimal selling price and cycle lengths are derived and the sensitivity of the parameters are analyzed.

2. Assumptions and Notations

We have considered an inventory model with the following assumptions and notations:

1. Replenishment is instantaneous.
2. The system operates for a prescribed period of H units of time (planning Horizon) inventory level is zero at times $t = 0$ and $t = H$
3. The demand rate at any instant *"t"* is a linear function of the selling price s and is of the form $\lambda(s) = a + b.s$ where $a > 0, b < 0$.
4. Lead time is zero.
5. Shortages are allowed and are fully backlogged. Shortages are not allowed in the final cycle.
6. T_i is the total time that elapses up to and including the i^{th} cycle (i = 1,2, ..., m), where, m denotes the total number of replenishments to be made during the prescribed time horizon H. Hence $T_0 = 0, T_m = H$
7. t_i is the time at which the inventory in the i^{th} cycle reaches zero ($i = l,2,...,m - l$).

8. T is the length of the first replenishment cycle and w is the rate of reduction of the successive cycle lengths.
9. The on hand inventory deteriorates at a constant rate of θ $(0 < \theta < 1)$ per unit time and there is neither repair nor replacement of the deteriorated inventory during H.
10. The inventory holding cost C_1 per unit per unit time, the shortage cost C_2 per unit time, the unit cost C and the replenishment cost (ordering cost) C_3 per replenishment are known and constant during the planning time horizon H.

3. The Inventory Model

The schematic diagram of the inventory model is given in Figure 1:

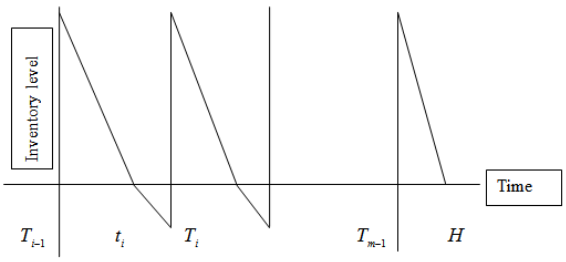


Figure 1. The schematic diagram of the inventory model

$I_i(t)$ denote the amount of inventory at time t, during the i^{th} cycle $T_{i-1} \le t \le t_i (i=1,2.....,m)$ The rate of change in inventory at time t during the i^{th} cycle is due to deterioration which amounts to $\theta.I_i(t)$ and demand rate $\lambda(s)=a+b.s$. Therefore, the differential equation governing the system during i^{th} cycle is:

$$\frac{d}{dt} I_i(t) + \theta I_i(t) = -\lambda(s);\ T_{i-1} \le t \le t_i \left(i=1,2.....,m\right) \quad (1)$$

The rate of change in inventory at time t during the cycle $t_i \le t \le T_i$ $i = 1,2, ..., (m-1))$ is due to unfulfilled demand as a consequence of backlogged shortages. Therefore, the differential equation governing the system during i^{th} cycle is:

$$\frac{d}{dt} I_i(t) = -\lambda(s);\ t_i < t < T_i \ \left(i=1,2.....,m-1\right) \quad (2)$$

with the initial conditions $I_i(t) = 0$ at $t = t_i$ and $\lambda(s) = (a+bs)$; where $a > 0, b < 0$.

Solving the equation (1) we get:

$$I_i(t) = \left(\frac{a+bs}{\theta}\right)\left(e^{\theta\left(t_i - 1\right)} - 1\right), \quad (3)$$

$T_{i-1} \le t \le t_i;\ i=1,2.....,m$

From equation (2) we get:

$$I_i(t) = (a+bs)\left(t_i - t\right); \quad (4)$$

$t_i \le t < T_i, \left(i=1,2........,\ m-1\right)$

The $(i+1)^{th}$ replenishment time T_i can be expressed as:

$$T_i = iT - i(i-1)\frac{w}{2},\ i=0,1,2......,m-1 \quad (5)$$

The length of the i^{th} cycle is

$$T_i - T_{i-1} = T - (i-1)w \quad i = 1, 2, ..m \quad (6)$$

Hence $$T = (m-1)\frac{w}{2} + \frac{H}{m} \quad (7)$$

The total cost of the system during the planning horizon H is:

$$K(m,t_i,T_i) = m.C_3 + \sum_{i-1}^{m-1}\left[\left(C_1 + C\theta\right)\int_{T_{i-1}}^{t_i} I_i(t)dt + C_2\int_{t_i}^{T_i} I_i(t)dt\right] + \left(C_1 + C\theta\right)\int_{T_{m-1}}^{H} I_m(t)dt + C_2\sum_{i=1}^{m}\int_{t_i}^{T_i} I_i(t)\,dt \quad (8)$$

$$= m.C_3 + \sum_{i-1}^{m-1}\left[\left(C_1 + C\theta\right)\int_{T_{i-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta\left(t_i - t\right)} - 1\right)dt\right] + \left(C_1 + C\theta\right)\int_{T_{m-1}}^{H}\frac{a+bs}{\theta}\left(e^{\theta(H-1)} - 1\right)dt + C_2\sum_{i=1}^{m}\int_{t_i}^{T_i}(a+bs)(t_i - t)dt$$

To ensure the convexity of the total cost function formulate the Hessian matrix and the determinant of the said matrix is observed as positive. That is:

$$\begin{vmatrix} \frac{\partial^2 K}{\partial t_i^2} & \frac{\partial^2 K}{\partial t_i\,\partial T} \\ \frac{\partial^2 K}{\partial t_i\,\partial T} & \frac{\partial^2 K}{\partial T_i^2} \end{vmatrix} > 0$$

Hence, the parameters and costs are assumed such that the Hessian matrix associated with the decision variables t_i and T_i is a positive definite.

For a fixed m, the corresponding optimal values of t_i are the solutions of the system of $(m-1)$ equations

$$\frac{\partial K(m,t_i,T_i)}{\partial t_i} = 0;\ (i=1,2,.....m-1)$$

For a fixed i,

$$K(m.t_i,T_i) = m.C_3 + \left(C_1 + C\theta\right)\int_{T_{i-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta\left(t_i - t\right)} - 1\right)dt$$

$$+C_2\int_{t_i}^{T_i}(a+bs)(t_i-t)dt+(C_1+C\theta)\int_{T_{m-1}}^{H}\frac{a+bs}{\theta}\left(e^{\theta(H-t)}-1\right)dt$$

This implies

$$\frac{\partial K(m,t_i,T_i)}{\partial t_i}=\frac{\partial}{\partial t_i}\left[\begin{array}{l}(C_1+C\theta)\int_{T_{i-1}}^{t_i}\left(\frac{a+bs}{\theta}\right)\left(e^{\theta(t_i-t)}\theta\right)dt+\\ C_2\int_{t_i}^{T_i}(a+bs)dt\end{array}\right]$$

$$=\frac{(C_1+C\theta)(a+bs)}{\theta}\left(1-e^{\theta(t_i-T_{i-1})}\right)+C_2(a+bs)(T_i-t_i)$$

Using the power series expansion of and neglecting higher powers of θ,, we get:

$$\frac{\partial K(m,t_i,T_i)}{\partial t_i}=\frac{(C_1+C\theta)(a+bs)}{\theta}\left[\theta(T_{i-1}-t_i)\right]$$
$$+C_2(a+bs)(T_i-t_i)$$

Solving $\frac{\partial K(m,t_i,T_i)}{\partial t_i}=0$ we get

$$t_i=\frac{T_i+V.T_{i-1}}{1+V},\ i=1,2.....,m-1 \tag{9}$$

Where $V=\frac{C_1+C.\theta}{C_2}$

Using equation (8), neglecting the terms of θ^3 and higher powers of θ, we obtain

$$K(m,t_i,T_i)=m.C_3+\frac{(C_1+C\theta)(a+bs)}{6}\left\{3(H-T_{m-1})^2+\theta(H-T_{m-1})^3\right\}$$
$$+\sum_{i-1}^{m-1}\left[\begin{array}{l}\frac{(C_1+C\theta)(a+bs)}{6}\left\{3(t_i-T_{i-1})^2+\theta(t_i-T_{i-1})^3\right\}\\ -C_2(a+bs)\frac{(t_i-T_i)^2}{2}\end{array}\right] \tag{10}$$

Using equation (9) in (10) and substituting:

$$T_i=i\ T-i\ (i-1)\frac{w}{2}\ \text{ and }\ T_i-T_{i-1}=T-(i-1)w \tag{11}$$

$$K(m,t_i,T_i)=m.C_3+\frac{(C_1+C\theta)(a+bs)}{6(1+V)^2}\sum_{i-1}^{m-1}3(1-V)\left(T^2-2T(i-1)w+w^2(i-1)^2\right)$$
$$+\frac{(C_1+C\theta)(a+bs)}{6(1+V)^3}\sum_{i-1}^{m-1}\theta\left(T^3-(i-1)^3w^3-3(i-1)^2+3T(i-1)w^2\right)$$
$$+\frac{(C_1+C\theta)(a+bs)}{6}\left\{3\left(H-(m-1)T+(m-1)(m-2)\frac{w}{2}\right)^2+\theta\left(H-(m-1)T+(m-1)(m-2)\frac{w}{2}\right)^3\right\} \tag{12}$$

Substituting $T=(m-1)\frac{w}{2}+\frac{H}{m}$ from the equation (7) in the equation (12), the cost function K reduced to a function of three variables m, s and w only of which m is a discrete, s and w are continuous variables. Let it be $\overline{K}(m.s.w)$.

For given value m_0 (>1) of m, the optimal value of w and s are obtained by minimizing the total cost i.e.

$$\frac{d\overline{K}}{dw}(\mathrm{m_0},\mathrm{s},\mathrm{w})=0 \tag{13}$$

$$\frac{d\overline{K}}{ds}(\mathrm{m_0},\mathrm{s},\mathrm{w})=0 \tag{14}$$

Solving the equations (13) and (14) by using the numerical methods, the optimal value of w (say, $w(m_0)$) and s (say, s^*) can be obtained. The corresponding optimal value of $\overline{K}\left(m_o,s^*,w(m_o)\right)=K^*(m_0)$, which can be calculated from equation (12). Putting $m_0 = 2, 3, 4$ ... we can calculate $K^*(2)$, $K^*(3)$ and so on.

For m = 1, the system reduces to a single period with finite time horizon. In such case the total cost for the period H is fixed and is:

$$K^*(1)=C_3+\frac{(C_1+C\theta)(a+bs^*)}{6}\left(3H^2+\theta H^3\right) \tag{15}$$

The values of $K^*(l)$, $K^*(2)$, $K^*(3)$ are the optimal costs and the corresponding values of m_0 $(=m^*)$ and w $(=w^*)$are their optimal values. The optimal values of T $(=T^*)$ and T_i $(=T_i^*,\ i=1,2,\ldots m-1)$ can be obtained from equations (7) and (5), respectively.

As we have the total cost of the system for fixed m, it is to be noted that

$$K(s,t_i,T_i)=m.C_3+\sum_{i=1}^{m-1}(C_1+C\theta)\int_{T_{i-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta(t_i-t)}-1\right)dt$$
$$+C_2\int_{t_i}^{T_i}(a+bs)(t_i-t)dt+(C_1+C\theta)\int_{T_{m-1}}^{H}\left(\frac{a+bs}{\theta}\right)\left(e^{\theta(H-t)}-1\right)dt \tag{16}$$

(Since total cost is a function of selling price "s", it is denoted by $K(s,t_i,T_i)$ Total Revenue = $s.\lambda(s).H$

$$s(a+bs)H \tag{17}$$

The total profit function is:

$$P(s,t_i,T_i) = s\,(a+bs)H - \left\{ m.C_3 + \sum_{i=1}^{m-1}\left(C_1 + C\theta\right)\int_{T_{m-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta(H-t)} - 1\right)dt + C_2\int_{t_i}^{T_i}\left(a+bs\right)\left(t_i - t\right)dt\right] \quad (18)$$

$$+\left(C_1 + C\theta\right)\int_{T_{m-1}}^{H}\left(\frac{a+bs}{\theta}\right)\left(e^{\theta(H-t)} - 1\right)dt\Bigg\}$$

Regarding the concavity of the Profit function one can verify that $\begin{vmatrix} \dfrac{\partial^2 P}{\partial t_i^{\,2}} & \dfrac{\partial^2 P}{\partial t_i\,\partial T_i} \\ \dfrac{\partial^2 P}{\partial t_i\,\partial T_i} & \dfrac{\partial^2 P}{\partial T_i^{\,2}} \end{vmatrix} < 0$ Which means that the Hessian Matrix is a negative definite.

4. Numerical Illustration

As an illustration of the above model consider the values of the parameters as:

a = 25, b = -1, c = 2, $c_1 = 0.1$, $c_2 = 5$, $c_3 = 9$, *H = 12.*

Substituting these values in equations (14) and (15) and solving the equations iteratively by using "MAT CAD", we obtained the optimum values of selling price (*s**), optimum *w**. Optimum cycle length (T_1*, T_2*, T_3*) for various values of θ and given in the following Table 1.

Table 1. Optimal values of the parameters of the model with shortages and with fixed selling price *a*=25, *b*=-1, *H*=12

θ	M	C_1	C_2	C_3	C	s	w*	T_1*	Q_1*	T_2*	Q_2*	T_3*	Q_3*	T_4*	Q_4*	K*	P*
0.01	3	0.1	5	9	2	3	1.1434	5.1434	278.1074	9.1434	320.4827	12	342.8228	-	-	189.1761	602.8239
0.01	3	0.1	5	9	2	4	1.7499	5.7499	268.3892	9.7499	305.8548	12	314.5645	-	-	90.535	701.465
0.01	3	0.1	5	8	2	5	2.5	6.5	248.1172	10.5	293.1746	12	310.4464	-	-	50.5625	741.4299
0.01	3	0.1	5	9	2	3	1.1434	5.1434	278.1074	9.1434	420.4827	12	442.8228	-	-	89.1761	702.8239
0.02	2	0.1	5	9	2	3	11.25	11.625	567.7659	12	272.3085	-	-	-	-	76.0333	715.9667
0.02	3	0.1	5	9	2	3	1.7391	5.7391	266.2339	9.7391	380.3554	12	395.8698	-	-	91.4879	700.5103
0.02	4	0.1	5	9	2	3	0.0076	3.0114	136.2772	6.0152	202.1746	9.014	27.6425	12	337.1866	91.5429	700.4571
0.02	3	0.1	5	9	2	3	1.7391	5.7391	266.2339	9.7371	380.3554	12	395.8698	-	-	81. 4897	710.5103
0.02	3	0.1	5	10	2	3	1.7391	5.7391	266.2339	9.7371	380.3554	12	395.8698	-	-	84.4897	707.5103
0.02	3	0.1	5	11	2	3	1.7391	5.7391	266.2339	9.7371	380.3554	12	395.8698	-	-	87. 4897	704.5103
0.02	3	0.1	5	9	2	3	1.7391	5.7391	266.2339	9.7371	315.1127	12	317.0637	-	-	81. 4897	710.5103
0.02	3	0.1	5	9	3	3	1.8719	5.8119	279.7668	8.8119	316.9802	12	323.6075	-	-	118.7213	673.2787
0.02	3	0.1	5	9	4	3	1.8726	5.8726	283.4184	9.8726	380.3554	12	395.8698	-	-	127.7704	664.2296
0.1	3	0.1	5	9	2	3	1.8494	5.8494	264.5525	9.8494	308.0438	12	312.2836	-	-	95.2885	696.7115
0.2	3	0.2	5	9	2	3	0.6795	4.6795	324.0051	8.6795	343.7388	12	375.7254	-	-	342.8275	449.1725
0.2	3	0.5	5	9	2	3	1.3263	5.3263	282.7075	9.3263	335.0714	12	345.4114	-	-	521.8641	270.1359
0.2	3	0.1	2	9	2	3	0.4589	4.4589	252.1787	8.4589	319.1518	12	377.2152	-	-	252.8018	539.1982

For fixed values of *m*, C_1, C_2, C_3, *C*, *s*, *H* and θ the optimal values of *w**, T_1*$(i=1,2,\ldots,.m)$, Q_1*$(i=1,2,\ldots.,m)$, *K** and *P** are computed and presented in the Table 1. From Table 1, it is observed that the optimal ordering quantities, optimal cycle lengths and optimal total profit are significantly affected by the parameters and cost. It is observed that as the rate of deterioration increases the optimal value of the reduction in successive cycle length *w** increases when the other parameters and costs remain fixed and, hence, the optimal cycle length of the first cycle increases. It is also observed that the optimal ordering quantity decreases as "θ" increases. However, the total cost increases and profit decreases when "θ" increases for the fixed values of the other parameters and costs. It is also observed that as the number of cycles (orders) increases, the optimal values of *w** decrease and the optimal ordering quantity per a cycle also decreases, when the other costs and parameters remain fixed. It is also observed that as the number of orders increases, the total cost increases and, hence, the profit decreases when the other parameters are fixed. Since the demand is dependent on the selling price and we assume that the selling price increases, the demand decreases.

As the cost per a unit increases, the optimal value of *w* increases and, hence, the optimal ordering quantity of the first cycle increases since T_1* increases when the other parameters and costs remain fixed. However, in the second and third cycles, the optimal ordering quantities decrease since their cycle lengths decrease when the cost per a unit increases.

The profit decreases when the cost per unit increases for the fixed values of the other parameters and costs. If the holding cost C_1 increases the optimal value of successive reduction of the cycle length increases when the other parameters and costs are fixed. The phenomenon has a vital influence on the ordering quantities; the optimal ordering quantity of the first cycle increases when C_1 increases because the first cycle length increases. The total profit decreases as the holding cost increases for fixed values of the other parameters and costs when the holding cost increases. The replenishment cost has no influence on the optimal value of the reduction of the cycle length and, hence, the optimal ordering quantities and cycle lengths are not affected by the changes in C_3, when the number of orders are fixed. But the total cost of the planning period increases, hence the reduction in the total profit when C_3 increases. It is also observed that the shortages have a vital influence on the reduction of the cycle length, when the other parameters and costs are fixed. The optimal value of w^* increases as the penalty cost (shortage cost) increases. The optimal ordering quantities also increase in each cycle. However, there is a decline in the profits when C_2 increases for fixed values of the parameters and costs. There is an increase in the optimal ordering quantities of the second and third cycle even though cycle lengths are less than the earlier cycle lengths because of fulfilling the backlogged demand in the earlier cycle. This may reduce the loss due to the deterioration and holding cost but increase the penalty cost. Hence, the optimal strategy for the inventory system under consideration is to choose the optimal ordering quantities and the optimal cycle lengths for the given values of the number of cycles, rate of deterioration, holding cost, penalty cost, cost per a unit, replenishment cost and selling price which maximizes profit.

5. Particular Cases

Case (i): For $m = 1$, the system reduces to a single period with finite time horizon without shortages. In such a case, the total cost for the period H is fixed and is:

$$K^*(1) = C_3 + \frac{\left(C_1 + C\theta\right)\left(a+bs\right)}{6}\left(3H^2 + \theta H^3\right) \tag{19}$$

and the optimal ordering quantity Q^* is:

$$Q^* = \frac{\left(a+bs\right)}{2}\theta H^2 \tag{20}$$

Hence, this model reduces to the usual inventory model for deteriorating items with fixed cycle length.

Case (ii) :

If the rate of deterioration $\theta \to 0$ in the above model, we obtain:

$$t_i = \frac{T_i + V.T_{i-1}}{1+V} \; i = 1,2,3.....,m\,.$$

Where $$V = \frac{C_1}{C_2} \tag{21}$$

The total cost function for the entire horizon H can be obtained as:

$$K(m,w) = m.C_3 + \frac{C_1\left(a+bs\right)}{6\left(1+V\right)^2}\sum_{i=1}^{m-1} 3\left(1+V\right) * \left(\left(m\text{-}1\right)\frac{w}{2} + \frac{H}{m} - \left(i\text{-}1\right)w\right)^2 + \frac{C_1\left(a+bs\right)}{6} * \left\{\left(H\text{-}\left(m\text{-}1\right)\left(\left(m\text{-}1\right)\frac{w}{2} + \frac{H}{m}\right) + \left(m\text{-}1\right)\left(m\text{-}2\right) + \frac{w}{2}\right)^2\right\} \tag{22}$$

This gives the optimal total cost for the inventory model for non-deteriorating items with variable cycle lengths. The optimal ordering quantities are:

$$Q_i^* = t_i^* + t_{i-1}^*\left(a+bs\right)\;;\; i = 1,2,3,...,m. \tag{23}$$

6. Sensitivity Analysis

A sensitivity analysis was carried out to explore the effect on the optimal policies by varying the value of each parameter at a time and all parameters together. The results obtained by changing parameters by -15%, -10%. -5%, +5% +10% and +15% are exhibited in Table 1(a) and Figure 2.

The values of the total cost K varies from 130.011 to 166.177 and the total profit varies from 693.103 to 693.366 for 15% under estimation and over estimation of all parameters under consideration.

7. Optimal Pricing and Ordering Policies under Variable Selling Price

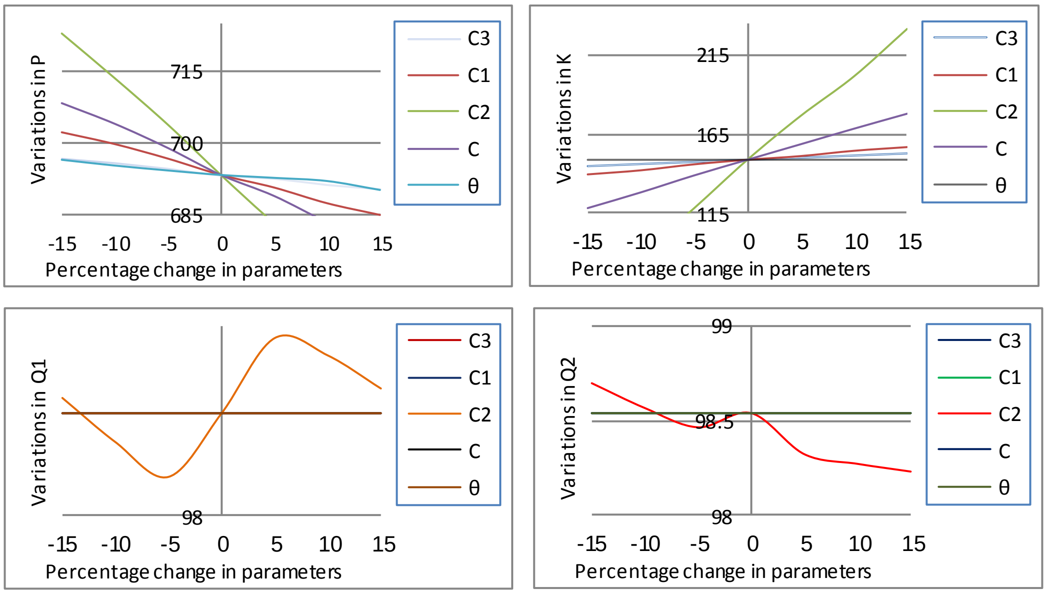
In this section, we obtain the optimal pricing and ordering policies of the inventory system under a variable selling price. In the previous section, we considered the selling price "***s***" as fixed. However, in many situations the selling price is variable and can be fixed by developing an optimal pricing policy. To obtain the optimal selling price along with the optimal ordering quantity, we maximize the total profit of the inventory system with respect to the selling price and the time at which shortages occur in each cycle (i.e. $\boldsymbol{t_i}$, $\boldsymbol{i = 1, \ldots, m\text{-}1}$).

From equation (18) we have the total profit function as:

$$P(s,t_i,T_i) = s\;(a+bs)\;H - \left\{m.C_3 + \sum_{i=1}^{m-1}\left(C_1 + C\theta\right) * \int_{T_{m-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta\left(H-t\right)} - 1\right)dt + C_2\int_{t_i}^{T_i}\left(a+bs\right)\left(t_i - t\right)dt\right] + \left(C_1 + C\theta\right)\int_{T_{m-1}}^{H}\left(\frac{a+bs}{\theta}\right)\left(e^{\theta\left(H-t\right)} - 1\right)dt\right\} \tag{24}$$

Table 1(a) Sensitivity of the model with fixed selling price and having Shortages

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
C_3	K	144.691	146.041	147.391	148.741	150.091	151.441	152.791
	Q_1	98.542	98.542	98.542	98.542	98.542	98.542	98.542
	Q_2	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	Q_3	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	P	696.71	695.80	694.62	693.17	692.53	691.21	690.32
C_1	K	139.449	142.041	145.839	148.741	151.091	154.441	156.791
	Q_1	98.542	98.542	98.542	98.542	98.542	98.542	98.542
	Q_2	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	Q_3	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	P	702.171	699.680	696.62	693.17	690.253	687.21	684.732
C_2	K	58.373	88.51	117.975	148.741	176.615	201.697	231.908
	Q_1	98.623	98.389	98.205	98.542	98.94	98.848	98.673
	Q_2	98.597	98.417	98.276	98.541	98.074	98.001	98.942
	Q_3	98.700	98.568	98.466	98.541	98.323	98.272	98.231
	P	723.02	713.58	703.78	693.17	683.01	673.91	663.98
C	K	118.024	128.041	138.789	148.741	158.601	168.471	178.009
	Q_1	98.542	98.542	98.542	98.542	98.542	98.542	98.542
	Q_2	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	Q_3	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	P	708.317	703.960	698.862	693.17	688.753	683.21	678.572
θ	K	148.711	148.721	148.731	148.741	148.751	148.761	148.771
	Q_1	98.542	98.542	98.542	98.542	98.542	98.542	98.542
	Q_2	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	Q_3	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	P	696.366	695.149	694.114	693.17	692.258	691.891	690.005
All Parameters	K	130.011	136.921	142.731	148.741	154.651	160.376	166.177
	Q_1	98.542	98.542	98.542	98.542	98.542	98.542	98.542
	Q_2	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	Q_3	98.541	98.541	98.541	98.541	98.541	98.541	98.541
	P	693.366	693.249	693.199	693.17	693.158	693.11	693.103



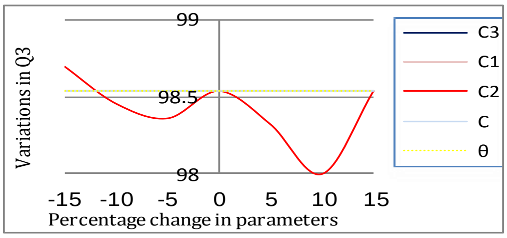


Figure 2. Graphical representation of the sensitivity with respect to the parameters of the model with fixed selling price and with shortages when *a = 25, b= -1, H= 12.*

To find the optimal value of $\boldsymbol{t_i}$, we maximise the function $P(s,t_i,T_i)$ with respect to t_i

i.e $\frac{\partial}{\partial t_i}\left(P\left(s,t_i,T_i\right)\right)=0$ and

$$\frac{\partial^2}{\partial t_i}\left(P\left(s,t_i,T_i\right)\right)<0$$

Now $\frac{\partial}{\partial t_i}\left(P\left(s,t_i,T_i\right)\right)=0$ implies

$$\frac{\partial}{\partial t_i}\left(K\left(s,t_i,T_i\right)\right)=0$$

We have $t_i=\frac{T_i+V.T_{i-1}}{1+V}$

where $V=\frac{C_1+C\theta}{C_2}$

Hence, the total profit function $P\ (s,\ t_i,\ T_i)$ has maximum value when $t_i=\frac{T_i+V.T_{i-1}}{1+V}$ Substituting this value of $\boldsymbol{t_i}$ in equation (24), the total profit function will become a function of the variables "*s*" and T_i.

We have $T_i-T_{i-1}=T-(i-1)w$, and

$$T=\left(m\text{-}1\right)\frac{w}{2}+\frac{H}{m}$$

Substituting these values in the equation (24), the total profit function becomes a function of the variables "*s*" and "*w*" only. Hence we denote the profit function by *P(w,s)*

$$\begin{aligned} P\left(w,s\right) &= s(a+bs).Hm.C_3-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6\left(1+V\right)^2}\sum_{i=1}^{m\text{-}1}3\left(1+V\right)\left(T-\left(i-1\right)w\right)^2-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6\left(1+V\right)^2} \\ &* \sum_{i=1}^{m\text{-}1}3\left(1+V\right)\left(T-\left(i-1\right)w\right)^2-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6} \\ &* \left\{3\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)^2+\theta\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)^3\right\} \end{aligned} \tag{25}$$

where $T=\left(m-1\right)\frac{w}{2}+\frac{H}{m}$. To find the optimal values of $\boldsymbol{w}$ and $\boldsymbol{s}$, equate the first order partial derivatives of *P(w,s)* with respect to $\boldsymbol{w}$ and $\boldsymbol{s}$ to zero and $\frac{\partial}{\partial s}\left(P\left(w,s\right)\right)=0$ implies

$$\begin{aligned} & a.H+2\ b\ s\ H-\frac{\left(C_1+C\theta\right)b}{2\left(1+V\right)}\sum_{i=1}^{m\text{-}1}\left(T\left(i\text{-}1\right)-w\right)^2-\frac{\left(C_1+C\theta\right)b\theta}{6\left(1+V\right)^3}\sum_{i=1}^{m\text{-}1}\left(T\left(i\text{-}1\right)-w\right)^3 \\ & -\frac{\left(C_1+C\theta\right)b}{6}\left\{3\left(H-\left(m\text{-}1\right)T+\left(m\text{-}1\right)\left(m\text{-}2\right)\frac{w}{2}\right)^2+\theta\left(H-\left(m\text{-}1\right)T+\left(m\text{-}1\right)\left(m\text{-}2\right)\frac{w}{2}\right)^3\right\}=0 \end{aligned} \tag{26}$$

$\frac{\partial}{\partial s}\left(P\left(w,s\right)\right)=0$ implies

$$\begin{aligned} & \frac{\left(C_1+C\,\theta\right)\left(a+b\,s\right)}{2\left(1+V\right)^3}\sum_{i\text{-}1}^{m-1}2\left(T\text{-}\left(i\text{-}1\right)w\right)\left(\frac{m-1}{2}-\left(i\text{-}1\right)\right)-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{2\left(1+V\right)^3}\sum_{i\text{-}1}^{m-1}3\left(T\text{-}\left(i\text{-}1\right)w\right)\left(\frac{m-1}{2}-\left(i\text{-}1\right)\right) \\ & -\frac{\left(C_1+C\,\theta\right)\left(a+bs\right)}{6}\left\{6\left(H\text{-}\left(m\text{-}1\right)T+\left(m\text{-}1\right)\left(m-2\right)\frac{w}{2}\right)\left(-\left(\mathrm{m}-1\right)\frac{\left(\mathrm{m}-1\right)\left(\mathrm{m}-2\right)}{2}\right)\right. \\ & \left.+3\theta\left(H\text{-}\left(m\text{-}1\right)T+\left(m\text{-}1\right)\left(m-2\right)\frac{w}{2}\right)^2\left(\left(\mathrm{m}-1\right)\frac{\left(\mathrm{m}-1\right)}{2}+\frac{\left(\mathrm{m}-1\right)\left(\mathrm{m}-2\right)}{2}\right)\right\}=0 \end{aligned} \tag{27}$$

Where, $T = (m-1)\frac{w}{2} + \frac{H}{m}$ Implies $\frac{dT}{dw} = \frac{m-1}{2}$

Solving the equations (26) and (27) we get the optimal values w^* of w and s^* of s respectively. Substituting the values of w^* and s^* in (25) we get the optimal value of the profit function $P(w,s)$ as $P^*(w^*, s^*)$.

For various values of the parameters m, θ, C_1, C_2, C_3, C and H the optimal values of the selling price s and the rate of reduction in successive cycle periods w are computed by solving equations (26) and (27) iteratively using the Newton Raphson's Method and are given in Table (2).

Table 2. Optimal values of the parameters of the model with variable selling price and with shortages $a = 25$, $b= -1$, $H= 12$

θ	m	C_1	C_2	C_3	C	s*	w*	T_1*	Q_1*	T_2*	Q_2*	T_3*	Q_3*	$T4^*$	Q_4*	K*	P*
0.01	3	0.1	5	9	2	12.628	1.138	5.138	148.81	9.1385	163.244	12	175.5811	-	-	65.2393	1839.5616
0.01	4	0.1	5	9	2	12.623	0.150	3.225	81.05	6.4000	119.699	9.22	150.0848	12	183.7	66.5438	1828.2746
0.02	3	0.1	5	9	2	12.716	1.140	5.140	151.63	9.1406	170.562	12	181.2659	-	-	73.5213	1821.8624
0.02	3	0.2	5	9	2	13.126	0.972	4.972	164.90	8.9727	184.324	12	195.7792	-	-	205.496	1664.79
0.02	3	0.5	5	9	2	13.336	0.614	4.614	143.31	8.6142	180.602	12	190.6658	-	-	261.026	1505.58
0.02	3	0.1	2	9	2	12.927	0.459	4.459	138.38	8.4590	175.137	12	206.9962	-	-	150.843	1721.9648
0.02	3	0.1	6	9	2	13.163	1.943	5.943	174.91	9.9430	197.309	12	211.9344	-	-	215.552	1654.1618
0.02	3	0.1	5	9	2	12.716	1.140	5.140	151.63	9.1406	170.562	12	181.2659	-	-	73.5213	1821.8624
0.02	3	0.1	5	10	2	12.716	1.140	5.140	151.63	9.1406	170.562	12	181.2659	-	-	73.5213	1821.8624
0.02	3	0.1	5	11	2	12.716	1.140	5.140	151.63	9.1406	170.562	12	181.2659	-	-	73.5213	1821.8624
0.02	3	0.1	5	9	2	12.668	1.276	5.276	127.53	9.2769	156.674	12	171.6926	-	-	76.8838	1877.0003
0.02	3	0.1	5	9	3	12.668	1.714	5.714	155.69	9.7146	175.116	12	177.4824	-	-	82.6161	1861.9589
0.02	3	0.1	5	9	4	13.438	1.75	5.75	168.29	9.75	176.192	12	178.2755	-	-	87.4324	1737.7755
0.1	3	0.1	5	9	2	12.807	1.147	5. 479	154.29	9. 4790	177.669	12	189.9128	-	-	116.916	1756.9043
0.2	3	0.1	5	9	2	12.716	1.140	5.140	151.63	9.1406	170.562	12	181.2659	-	-	73.5213	1821.8624

From Table 2, we observe that the selling price is much influenced by the values of the parameters and costs. As the number of the cycles increases, the optimal value of the selling price decreases when the other parameters and costs are fixed. Even though the optimal selling price decreases as the number of the cycles increases, the optimal total profit increases. It is also observed that as the decay rate (i.e., rate of deterioration) increases, the optimal value of the selling price also increases and the total profit decreases, when the other parameters and costs are fixed. This phenomenon is very close to the realistic situation with the perishable inventory system, since the rate of deterioration increases, the wastage is more, and the burden is to be balanced between the customer and the seller. It is also observed that as the shortage cost increases the optimal value of selling price increases and the total profit decreases, when the other parameters and costs are fixed. There is no influence of the ordering cost on the optimal value of the selling price. However, the total profit decreases when the other parameters and costs are fixed. As the cost per a unit increases, the optimal value of the selling price increases to maintain the profits at a maximum level. Hence, by the suitable choice of the parameters and costs for the commodity under consideration, one can have the optimal values of the selling price and the ordering quantities for each cycle.

8. Sensitivity Analysis

A sensitivity analysis was carried out to explore the effect on the optimal policies by varying the value of each parameter at a time and all parameters together. The results obtained by changing the parameters by -15%, -10%. -5%, +5% +10% and +15% are tabulated in Table 2(a) and Figure 3.

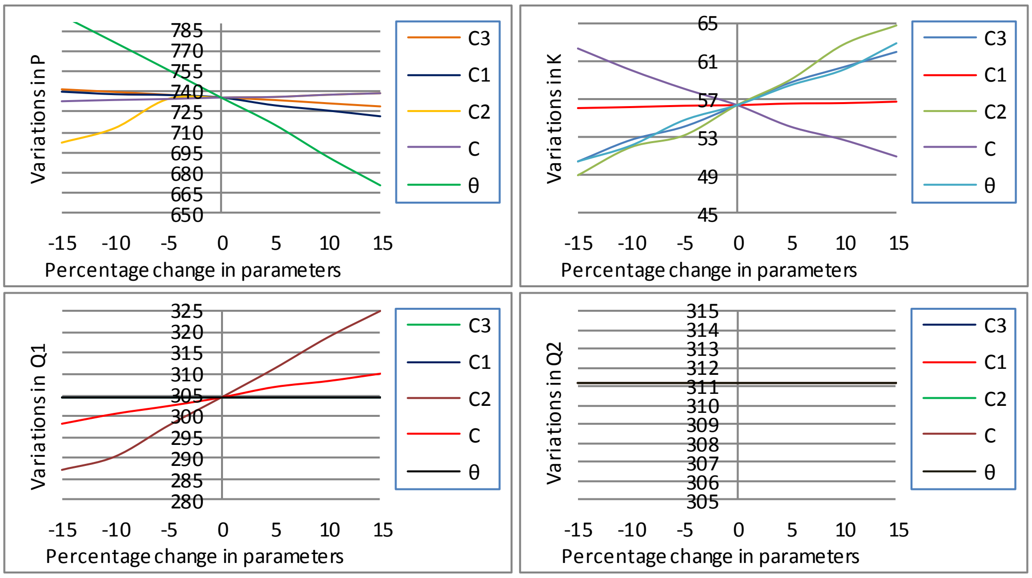
The values of the total cost K varies from 38.474 to 74.105 and the total profit varies from 435.423 to 735.736 for 15% under estimation and over estimation of all parameters under consideration.

9. Inventory Model With-out Shortages

In this section, we consider that the shortages are not allowed. When we assume that shortages are not allowed, it is not necessary to have a backlog fulfillment. Then, the parameter t_i becomes T_i $(i = 1, 2, \ldots, m-1)$ and the shortage cost C_2 is to be considered as $C_2 \rightarrow \infty$. Substituting these values in the corresponding equations given in the total cost function becomes:

Table 2(a). Sensitivity of the model with variable selling price and having Shortages

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
C_3	K	50.461	52.741	54.147	56.424	58.801	60.441	62.051
	Q_1	304.420	304.420	304.420	304.420	304.420	304.420	304.420
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	319.711	319.711	319.711	319.711	319.711	319.711	319.711
	P	741.67	739.690	737.62	735.576	733.753	731.321	729.009
C_1	K	56.109	56.216	56.356	56.424	56.591	56.637	56.791
	Q_1	304.420	304.420	304.420	304.420	304.420	304.420	304.420
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	319.711	319.711	319.711	319.711	319.711	319.711	319.711
	P	702.171	699.680	696.62	735.576	690.253	687.21	684.732
C_2	K	139.449	142.041	145.839	56.424	151.091	154.441	156.791
	Q_1	287.115	290.300	297.669	304.420	311.324	318.742	315.130
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	319.711	319.711	319.711	319.711	319.711	319.711	319.711
	P	702.171	699.680	696.62	735.576	690.253	687.21	684.732
C	K	62.414	60.128	58.158	56.424	54.156	52.751	50.989
	Q_1	298.140	300.520	302.371	304.420	306.92	308.327	310.114
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	288.866	298.676	309.48	319.711	329.711	339.675	349.661
	P	732.817	733.790	734.486	735.576	736.012	737.621	738.752
θ	K	50.471	52.148	54.831	56.424	58.521	60.161	62.968
	Q_1	304.420	304.420	304.420	304.420	304.420	304.420	304.420
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	319.711	319.711	319.711	319.711	319.711	319.711	319.711
	P	796.347	776.349	756.114	735.576	715.692	691.891	670.545
All Parameters	K	38.474	44.797	50.731	56.424	62.567	68.676	74.105
	Q_1	304.420	304.420	304.420	304.420	304.420	304.420	304.420
	Q_2	311.198	311.198	311.198	311.198	311.198	311.198	311.198
	Q_3	319.711	319.711	319.711	319.711	319.711	319.711	319.711
	P	735.736	735.684	735.634	735.576	735.524	735.470	435.423



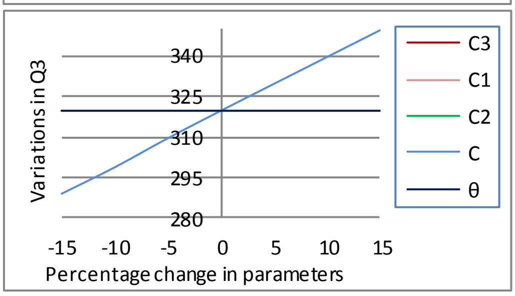


Figure 3. Graphical representation of the sensitivity with respect to the parameters of the model with variable selling price and with shortages when *a = 25, b= -1, H= 12*.

$$\begin{aligned}\overline{K}(m,w) &= m.C_3 + \frac{(C_1+C\theta)(a+bs)}{6}\sum_{i-1}^{m-1} 3\left((m-1)\frac{w}{2}+\frac{H}{m}-(i\text{-}1)w\right)^2 \\ &+ \frac{(C_1+C\theta)(a+bs)}{6}\sum_{i-1}^{m-1} 3\left((m-1)\frac{w}{2}+\frac{H}{m}-(i\text{-}1)w\right)^3 \\ &- \frac{(C_1+C.\theta)(a+bs)}{6}\left\{3\left(H\text{-}(m\text{-}1)\left((m\text{-}1)\frac{w}{2}+\frac{H}{m}\right)+(m\text{-}1)(m-2)\frac{w}{2}\right)^2\right. \\ &\left.+\theta\left(H\text{-}(m\text{-}1)\left((m\text{-}1)\frac{w}{2}+\frac{H}{m}\right)+(m\text{-}1)(m-2)\frac{w}{2}\right)^3\right\}\end{aligned} \tag{28}$$

for fixed m the optimal value w^* *of* w can be obtained by minimizing the cost function:

$$\begin{aligned}\frac{d\overline{K}(w)}{dw} &= \frac{(C_1+C\theta)(a+bs)}{6}\sum_{i-1}^{m-1}\left\{\left((m-1)\frac{w}{2}+\frac{H}{m}-(i\text{-}1)w\right)(m+1\text{-}2i)\right\} \\ &+ \frac{(C_1+C\theta)(a+bs)}{4}\sum_{i-1}^{m-1}\left\{\left((m-1)\frac{w}{2}+\frac{H}{m}-(i\text{-}1)w\right)^2(m+1\text{-}2i)\right\} \\ &+ \frac{(C_1+C\theta)(a+bs)}{6}\left[6\left(H-(m-1)\left((m-1)\frac{w}{2}+\frac{H}{m}\right)+(m-1)(m\text{-}2)\frac{w}{2}\right)\right. \\ &\left.+3\theta\left(H-(m-1)\left((m-1)\frac{w}{2}+\frac{H}{m}\right)+(m-1)(m\text{-}2)\frac{w}{2}\right)^2\right]\end{aligned}$$

The optimal ordering quantity Q_i^* is:

$$\begin{aligned}Q_i^* &= \frac{(a+bs)}{2}.\theta\left(T_i^*-T_{i\text{-}1}^*\right)^2 \\ &+\left(T_i^*-T_{i\text{-}1}^*\right)(a+bs)\end{aligned} \tag{29}$$

For different values of the parameters and costs m, θ, C_1, C_2, C_3, C and s the optimal values of w^* (i.e., the reduction in successive cycle lengths) are computed. The optimal values of the ordering quantities for the i^{th} cycle *(i = 1, 2,..., m)* and cycle lengths are presented in Table 3 along with total cost and profits.

From Table 3, we observe that the consideration of not allowing shortages has a significant effect on the optimal ordering policies of the model. It is also observed that the optimal profit of the model, without shortages, is less when compared with the optimal profits of the model with shortages when all parameters and costs are fixed. It is also observed that the optimal ordering quantities in the first cycle, second cycle and third cycle are more for this model than those of the model with shortages. However, the first cycle length increases; the rate of reduction in each cycle length also increases for this model in comparison with the model with shortages when all the parameters and costs are fixed. This phenomenon clearly indicates that it is better to have the strategy of allowing shortages and fully back-logging than without shortages in order to maximize profits even though there is a penalty cost for allowing shortages.

10. Sensitivity Analysis

A sensitivity analysis was carried out to explore the effect on the optimal policies by varying the value of each parameter at a time and all parameters together. The results obtained by changing parameters by -15%, -10%. -5%, +5% +10% and +15% are tabulated in Table 3(a) and Figure 4**.**

The values of the total cost K varies from 150.871 to 172.111 and the total profit varies from 825.423 to 829.550 for 15% under estimation and over estimation of all parameters under consideration.

Table 3. Optimal values of the parameters of the model with out shortages and with fixed selling price $a = 25$, $b = -1$, $H = 12$

θ	m	C_1	C_3	C	s	w^*	T_1^*	Q_1^*	T_2^*	Q_2^*	T_3^*	Q_3^*	T_4^*	Q_4^*	K^*	P^*
0.01	3	0.1	9	2	3	1.886	5.886	266.643	9.886	294.982	12	297.314	-	-	93.8653	698.1347
0.01	3	0.1	9	2	3	1. 886	5.886	242.402	9.886	280.936	12	283.157	-	-	90.7349	701.2651
0.01	3	0.1	9	2	3	1. 886	5.888	335.258	9.886	340.709	12	345.314	-	-	229.590	562.4099
0.02	3	0.1	9	2	3	2	6	279.84	10	315.040	12	319.760	-	-	112.099	681.6975
0.02	4	0.1	9	2	3	1.586	5.379	248.722	9.172	291.219	11.379	300.919	12	307.82	110.302	681.6975
0.02	3	0.2	9	2	3	0.909	4.909	322.037	8.909	374.030	12	376.400	-	-	405.880	386.1191
0.02	3	0.5	9	2	3	0.065	4.065	251.622	8.065	335.845	12	358.660	-	-	526.714	265.0858
0.02	3	0.1	9	2	3	2	6	279.84	10	315.040	12	319.760	-	-	112.099	679.9007
0.02	3	0.1	9	2	3	2	6	279.84	10	315.040	12	319.760	-	-	115.099	679.9007
0.02	3	0.1	9	2	3	2	6	279.84	10	315.040	12	319.760	-	-	118.099	679.9007
0.02	3	0.1	10	2	3	2	6	279.84	10	315.040	12	319.760	-	-	112.099	679.9007
0.02	3	0.1	11	4	3	1.769	5.769	275.806	9.769	313.480	12	316.364	-	-	118.297	673.7027
0.02	3	0.1	9	2	3	2	6	279.84	10	315.040	12	319.76	-	-	112.099	679.9007
0.1	3	0.1	9	2	3	2.124	6. 20	271.199	10.120	311.198	12	319.012	-	-	56.4240	735.576

Table 3(a). Sensitivity of the model with Fixed selling price Without Shortages

Variation Parameters		Percentage change in parameter						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
C_3	**K**	161.645	162.872	163.991	164.099	165.549	166.365	167.745
	P	833.154	830.895	828.999	827.755	825.675	823.175	821.447
C_1	**K**	163.465	163.689	163.991	164.099	164.579	164.965	165.075
	P	833.565	831.220	829.013	827.755	825.871	821.475	820.755
C	**K**	158.777	160.489	162.888	164.099	165.771	166.115	167.871
	P	830.111	829.544	828.443	827.755	826.471	825.695	824.235
θ	**K**	163.465	163.689	163.991	164.099	164.579	164.965	165.075
	P	824.115	825.730	826.336	827.755	828.681	829.846	830.114
All Parameters	**K**	150.871	154.221	159.002	164.099	167.258	169.996	172.111
	P	829.550	829.007	828.589	827.755	826.158	526.094	825.423

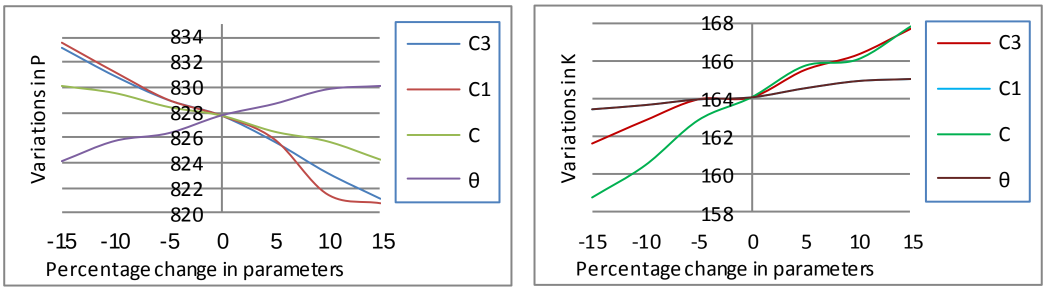


Figure 4. Graphical representation of the sensitivity with respect to the parameters of the model with fixed selling price and without shortages when $a = 25$, $b = -1$, $H = 12$.

Now the Profit function of the model is given by:

$$P(w,s) = a(a+bs)H - m.C_3 - \frac{(C_1 + C.\theta)(a+bs)}{6}\sum_{i=1}^{m-1} 3\left(T-(i-1)w\right)^2 - \frac{(C_1+C\theta)\theta(a+bs)}{6}\sum_{i=1}^{m-1}\left(T-(i-1)w\right)^3$$

$$-\frac{(C_1+C\theta)(a+bs)}{6}\left\{3\left(H-(m-1)T+(m-1)(m-2)\frac{w}{2}\right)^2 + \theta\left(H-(m-1)T+(m-1)(m-2)\frac{w}{2}\right)^3\right\}$$

where $T = (m-1)\frac{w}{2} + \frac{H}{m}$

To find the optimal values of ***w*** and ***s***, equate the first order partial diversities of ***P (w,s)*** with respect to ***w*** and ***s*** to zero than solve the equations $\frac{\partial}{\partial w}\left(P\left(w,s\right)\right)=0$ and $\frac{\partial}{\partial s}\left(P\left(w,s\right)\right)=0$

$\frac{\partial}{\partial w}\left(\mathrm{P}\left(\mathrm{w,s}\right)\right)=0$ implies.

$$\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6}\left\{\sum_{i=1}^{m-1}6\left(T-\left(i-1\right)w\right)\left(\frac{m-1}{2}-\left(i-1\right)\right)-\sum_{i=1}^{m-1}3\left(T-\left(i-1\right)w\right)^2\left(\frac{m-1}{2}-\left(i-1\right)\right)\right\}$$
$$-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6}\left\{6\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)-\left(m-1\right)\frac{m-1}{2}+\frac{\left(m-1\right)\left(m-2\right)}{2}+3.\theta\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)^2\right.$$
$$\left.-\frac{\left(C_1+C\theta\right)\left(a+bs\right)}{6}\left(-\left(m-1\right)\frac{m-1}{2}+\frac{\left(m-1\right)\left(m-2\right)}{2}\right)\right\}=0 \tag{30}$$

Here $T=\left(m-1\right)\frac{w}{2}+\frac{H}{m}\Rightarrow\frac{dT}{dw}=\frac{m-1}{2}$

Again $\frac{\partial}{\partial s}\left(P\left(w,s\right)\right)=0$ implies

$$a.H+2bH-\frac{b\left(C_1+C\theta\right)}{6}\sum_{i=1}^{m-1}3\left(T-\left(i-1\right)w\right)^2\frac{b\left(C_1+C\theta\right)}{6}\sum_{i=1}^{m-1}3\left(T-\left(i-1\right)w\right)^3$$
$$-\left\{3\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)^2-\theta^3\left(H-\left(m-1\right)T+\left(m-1\right)\left(m-2\right)\frac{w}{2}\right)^3\right\}=0 \tag{31}$$

For different values of the parameters and costs, the optimal values of T_1*, T_2*, T_3*, . . . , T_m*, Q_1*, Q_2*, . . . Q_m*, $K*$, $P*$ and optimal selling price $s*$ are computed from the equations using the Newton Raphson's method and given in Table 4.

From Tables 2 and 4, it is observed that allowing shortages has a tremendous effect on the optimal selling price and the operating policies of the system.

11. Sensitivity Analysis

A sensitivity analysis was carried out to explore the effect on the optimal policies by varying the value of each parameter at a time and all parameters together. The results obtained by changing parameters by -15%, -10%. -5%, +5% +10% and +15% are tabulated in Table 4(a) and Figure 5.

The values of the total cost *K* varies from 836.112 to 896.777 and the total profit varies from 1463.870 to 1493.869 for 15% under estimation and over estimation of all parameters under consideration.

12. Conclusion

In the present model, when there is no shortage, it is observed that the net profit decreases when the deterioration parameter decreases and the selling price varies slightly. In the real market, the selling price of an item is the main factor for its demand and it optimizes the net profit. The other important factor for net profit is the replenishment time interval and the retailer's lot size is affected by the demand of the product and the demand of the product is dependent on the selling price of the product. Therefore, in order to optimize the net profit, we either reduce the price of the product or increase the replenishment cycle time. Hence, this model becomes more practicable and very useful in the business organizations dealing with domestic goods especially the perishable products. Also, it is observed that the optimal value of the selling price is more in the model without shortages than that in the model with shortages when the parameters and the costs are fixed. Even though the optimal value of the selling price is less with shortages, the optimal profit increases more than that of the model without shortages. Hence, it is observed that allowing shortages fully backlogging is a better strategy for both the customer and the stock keeper. This coincides with the natural phenomenon of increasing the productivity by allowing shortages even though some penalty is to be paid for back orders.

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Table 4. Optimal values of the parameters of the model with variable selling price and without shortages a = 25, b= -1, H= 12

Θ	m	C_1	C_3	C	s*	w*	T_1*	Q_1*	T_2*	Q_2*	T_3*	Q_3*	T_4*	Q_4*	K*	P*
0.01	3	0.1	9	2	12.625	1.600	5.600	174.038	9.600	184.519	12	189.761	-	-	132.121	1800.400
0.02	2	0.1	9	2	12.721	1.201	6.100	107.526	12	166.035	12	-	-	-	83.1327	1791.281
0.02	3	0.1	9	2	12.860	1.626	5.626	147.648	9.626	170.569	12	178.554	-	-	154.506	1741.316
0.02	4	0.1	9	2	16.622	0.147	3.220	54.7894	6.294	79.2328	9.220	102.448	12	124.425	68.2741	1602.805
0.02	3	0.2	9	2	13.667	0.909	4.909	180.522	8.909	210.996	12	216.671	-	-	76.6083	1598.054
0.02	3	0.5	9	2	14.576	0.909	4.909	167.216	8.909	184.024	12	194.224	-	-	322.099	1438.999
0.02	3	0.1	9	2	12.860	1.626	5.626	147.648	9.626	167.549	12	172.585	-	-	154.506	1718.165
0.02	3	0.1	10	2	12.860	1.626	5.626	147.648	9.626	167.549	12	172.585	-	-	157.506	1718.165
0.02	3	0.1	9	2	12.860	1.626	5.626	147.648	9.626	167.549	12	172.585	-	-	160.605	1718.165
0.02	3	0.1	10	2	12.669	1.5	5.5	146.864	9.5	172.426	12	172.585	-	-	77.5432	1797.125
0.02	3	0.1	11	2	12.698	1.785	5.785	158.819	9.785	177.460	12	180.572	-	-	84.9028	1789.625
0.02	3	0.1	9	2	13.940	1.786	5.786	167.648	9.786	187.549	12	181.245	-	-	154.506	1718.165
0.1	3	0.1	9	2	12.940	1.786	5. 786	143.165	9. 786	167.549	12	172.585	-	-	164.409	1718.165

Table 4(a). Sensitivity of the model with variable selling price and without shortages a = 25, b= -1, H= 12

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
C_3	K	878.158	874.666	870.227	866.232	862.113	858.232	854.787
	P	1466.888	1470.934	1474.222	1478.448	1482.513	1486.666	1490.982
C_1	K	863.123	864.332	865.111	866.232	867.787	868.147	869.824
	P	1481.412	1480.118	1479.404	1478.448	1477.668	1476.874	1475.006
C	K	863.132	864.006	865.418	866.232	867.555	868.999	869.542
	P	1481.209	1480.176	1479.021	1478.448	1478.121	1477.333	1477.078
θ	K	878.158	874.666	870.227	866.232	862.113	858.232	854.787
	P	1466.888	1470.934	1474.222	1478.448	1482.513	1486.666	1490.982
All Parameters	K	836.112	846.437	856.999	866.232	876.487	886.335	896.777
	P	1493.869	1488.555	1483.148	1478.448	1473.682	1468.111	1463.870

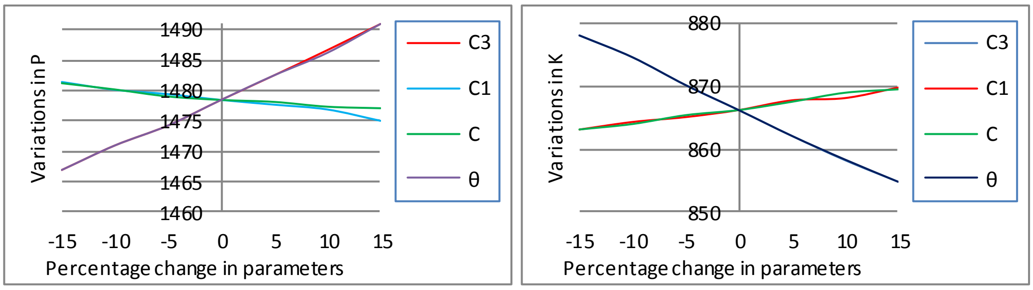

Figure 5. Graphical representation of the sensitivity with respect to the parameters of the model with variable selling price and with-out shortages when *a = 25, b= -1, H= 12.*

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Annexure

In the study of inventory models for deteriorating items, it is observed that the conventional method is to consider invariable cycle time for all cycles in the horizon, but in many situations viz., edible oil & food processing industries, market yards etc. the commodity under consideration may be influenced by seasonality. Due to the influence of season, the cycle lengths decrease and results unequal. To have effective control and monitoring of the inventory system with deteriorating items in particular, it is needed to decrease the cycle length in an arithmetic progression subject to the minimization of the cost.

Thus successive replenishment cycle times can be obtained by

$T_1 = T,\ T_2 = 2T - w,\ T_3 = 3T - 3w$ and in general $T_i = iT - [1+2+3+...+(i-1)]w$

This implies

$$T_i = iT - \frac{i(i-1)}{2}w,\ i = 1,2,...,(m-1)$$

The length of the i^{th} cycle = $T_i - T_{1-1}$ = $T - \frac{i\,(i-1)}{2}w,\ i = 1,2,3,...,(m-1)$

But the total horizon is H which implies

$$\sum_{i=1}^{m}\left[T - (i-1)w\right] = H$$

$$\Rightarrow T + (T-w) + ... + \left[T - (m-1)w\right] = H$$

Therefore $T = (m-1)\frac{w}{2} + \frac{H}{m}$

Inventory model

Let $I_i(t)$ denote the amount of inventory at time t, during the i^{th} cycle $\left(T_{i-1} \le t \le t_i;\ \ i-1,2,3,...,m\right)$. The rate of change in inventory at time t during the i^{th} cycle is due to deterioration which amounts to $\theta I_i(t)$) and demand rate $\lambda(s) = a + bs$. Therefore the differential equation governing the system during i^{th} cycle is

$$\frac{d}{dt}I_i(t) + \theta I_i(t) = -(a+bs);\ T_{i-1} \le t \le t_i\ (i=1,2.....,m) \quad \text{(i)}$$

The rate of change in inventory at time t during the cycle $\left(t_i \le t \le T_i;\ \ i-1,2,3,...,m-1\right)$ is due to unfulfilled demand as a consequence of backlogged shortages.. Theretore the differential equation governing the system during i^{th} cycle is

$$\frac{d}{dt}I_i(t) = -(a+bs);\ t_i < t < T_i\ \ (i=1,2.....,m-1) \quad \text{(ii)}$$

with the initial conditions $I_i(t) = 0$ at $t = t_i$ and $a > 0, b < 0$.

Solution of the equation (i)

Consider the equation (i) as

$$\frac{d}{dt}I_i(t) + \theta I_i(t) = -(a+bs);$$

$$T_{i-1} \le t \le t_i\ (i=1,2.....,m).$$

This is a linear ordinary differential equation of first order and first degree.

Therefore $I_i(t)e^{\int\theta dt} = \int -(a+bs)e^{\int\theta dt}dt$

$$\Rightarrow I_i(t)e^{\theta t} = \int -(a+bs)e^{\theta t}dt$$

$$\Rightarrow I_i(t)e^{\theta t} = -\frac{(a+bs)e^{\theta t}}{\theta} + C_1$$

Using the initial condition $I_i(t) = 0$ at $t = t_i$ we have

$$0 = -\frac{(a+bs)e^{\theta t_i}}{\theta} + C_1$$

Implies

$C_1 = \frac{(a+bs)e^{\theta t_i}}{\theta}$ .Using the value we have

$$\Rightarrow I_i(t)e^{\theta t} = -\frac{(a+bs)e^{\theta t}}{\theta} + \frac{(a+bs)e^{\theta t_i}}{\theta}$$

$$\Rightarrow I_i(t) = \frac{(a+bs)}{\theta}\left(e^{\theta(t_i - t)} - 1\right);$$

$T_{i-1} \le t \le t_i\ (i=1,2.....,m)$ (iii)

Again consider the equation

$$\frac{d}{dt}I_i(t) = -(a+bs);\ t_i < t < T_i\ \ (i=1,2.....,m-1).$$

This is an ordinary differential equation of first order and first degree.

Therefore $I_i(t) = -(a+bs)t + C_2$

Using the initial condition $I_i(t) = 0$ at $t = t_i$ we have

$0 = -(a+bs)t_i + C_2$.

This implies $C_2 = (a+bs)t_i$

Substituting the value of C_2 then

$$I_i(t) = -(a+bs)t + (a+bs)t_i$$

$$\Rightarrow I_i(t) = (a+bs)(t_i - t);\ t_i \le t < T_i,\ (i=1,2........,\ m\text{-}1) \quad \text{(iv)}$$

Calculation of the cost function

To find the total cost function, we consider various costs like Ordering Cost C_3, Holding Cost C_1, Unit Cost or Purchasing Cost $C$and Shortage Cost C_2.

Ordering cost: According to the assumptions of the model Ordering Cost per replenishment is C_3. Hence for all the 'm' cycles the ordering cost = $m.\,C_3$ **… (A)**

Holding cost: To find the holding cost we calculate

Inventory during the i^{th} cycle = $\int_{T_{i-1}}^{t_i} I_i(t)\,dt$ =

$$\int_{T_{i-1}}^{t_i}\frac{(a+bs)}{\theta}\left(e^{\theta(t_i - t)} - 1\right)dt$$

Inventory during the last cycle = $\int_{T_{m-1}}^{H} I_i(t)\,dt$ =

$$\int_{T_{m-11}}^{H}\frac{(a+bs)}{\theta}\left(e^{\theta(H-t)} - 1\right)dt$$

By making use of these equations we have

The holding cost during the i^{th} cycle = C_1

$$\left[\int_{T_{i-1}}^{t_i} \frac{(a+bs)}{\theta}\left(e^{\theta(t_i - t)} - 1\right)dt\right]$$

And the holding cost during the last cycle = C_1

$$\left[\int_{T_{m-1}1}^{H} \frac{(a+bs)}{\theta}\left(e^{\theta(H-t)} - 1\right)dt\right]$$

Hence the total holding cost of the inventory is given by

$$\sum_{i=1}^{m-1} C_1\left[\int_{T_{i-1}}^{t_i} \frac{(a+bs)}{\theta}\left(e^{\theta(t_i - t)} - 1\right)dt\right] +$$

$$\left[\int_{T_{m-1}1}^{H} \frac{(a+bs)}{\theta}\left(e^{\theta(H-t)} - 1\right)dt\right] \quad \text{(B)}$$

Unit cost: To find the unit cost, we calculate the ordering quantity Q_i in the i^{th} cycle. It is given by

Q_i = Deterioration in the i^{th} cycle +Demand in the i^{th} cycle + Backlog demand in the $(i-1)^{st}$ cycle

$$= \theta \int_{T_{i-1}}^{t_i} I_i(t)\,dt + \int_{T_{i-1}}^{t_i}(a+bs)dt + \int_{t_{i-1}}^{T_{i-1}}(a+bs)dt$$

Hence the total unit cost of the inventory is given by

$$\sum_{i=1}^{m} \begin{matrix} \theta \int_{T_{i-1}}^{t_i} I_i(t)\,dt + \int_{T_{i-1}}^{t_i}(a+bs)dt \\ + \int_{t_{i-1}}^{T_{i-1}}(a+bs)dt \end{matrix} \quad \text{(C)}$$

The shortage cost in the i^{th} cycle $C_2 \int_{t_i}^{T_i} I_i(t)\,dt$

$$= C_2 \int_{t_i}^{T_i}(a+bs)\,(t_i - t)\,dt$$

Hence the total shortage cost of the inventory is given by

$$\sum_{i=1}^{m-1} C_2 \int_{t_i}^{T_i}(a+bs)(t_i - t)\,dt \quad \text{(D)}$$

By adding all the costs given in equations (A), (B), (C) and (D) the total Cost function is given by

$$k(m,t_i,T_i) = m.C_3 + \sum_{i-1}^{m-1}\left[(C_1 + C\theta)\int_{T_{i-1}}^{t_i} I_i(t)dt + C_2\int_{t_i}^{T_i} I_i(t)dt\right] +$$

$$(C_1 + C\theta)\int_{T_{m-1}}^{H} I_m(t)dt + C_2\sum_{i=1}^{m}\int_{t_i}^{T_i} I_i(t)\,dt$$

$$= m.C_3 + \sum_{i-1}^{m-1}\left[(C_1 + C\theta)\int_{T_{i-1}}^{t_i}\frac{a+bs}{\theta}\left(e^{\theta(t_i - t)} - 1\right)dt\right] \quad \text{(v)}$$

$$+ (C_1 + C\theta)\int_{T_{m-1}}^{H}\frac{a+bs}{\theta}\left(e^{\theta(H-1)} - 1\right)dt + C_2\sum_{i=1}^{m}\int_{t_i}^{T_i}(a+bs)(t_i - t)dt$$