JJMIE

Volume 19, Number 3, Sep. 2025
ISSN 1995-6665
Pages 557– 570
https://doi.org/10.59038/jjmie/190308
Editor-in-Chief: Prof. Ali M. Jawarneh

Jordan Journal of Mechanical and Industrial Engineering

Application of CUSUM Control Charts in Maintenance Activities

Ghada Shwaheen, Mohammad M. Hamasha*

Department of Industrial Engineering, Faculty of Engineering, The Hashemite University, Zarqa, 13133, Jordan

Received 24 Apr 2025 Accepted 28 Jul 2025

Abstract

Statistical Process Control (SPC) is a collection of useful and effective methods for problem-solving that aim to stabilize processes, increase their capacity, and ensure quality assurance, which is a key factor in how customers choose products and services. This research focuses on the application of SPC techniques in the maintenance of oil-free air compressor in Alyoum for Food Industry manufacturing plant. The aim of this study is to develop control charts for pressure, temperature, and time between failures (TBF) of the compressor, using both Shewhart and cumulative sum (CUSUM) methods and to investigate whether CUSUM control charts can be an effective tool for monitoring the pressure and temperature levels compared with Shewhart control charts, and predicting potential equipment failures before they occur by monitoring a particular performance indicator. The study uses data collected over 100 days to construct the control charts and evaluate their effectiveness in detecting changes in the process. The results show that the CUSUM method is more effective in detecting small shifts in the process mean compared to the Shewhart method. The average run length (ARL) is used as a performance measure for the CUSUM method, and it is found that increasing the value of the decision interval (h) increases the ARL. The study concludes that the CUSUM method is more suitable for monitoring the compressor process due to its ability to detect small shifts in the process mean. The study recommends further research on the application of modified CUSUM charts and other SPC techniques for monitoring the compressor process.

Keywords: Statistical Process Control (SPC); Shewhart Control Charts; Control Limits; Cumulative Sum (CUSUM); Average Run Length (ARL); Maintenance Activities.

1. Introduction

Quality is a key driver of customer satisfaction in today's competitive business environment. Perfect goods and services with few variations are needed to reach the target level of quality. For the efficient continuous monitoring of quality characteristics, statistical process control quality control approaches like control charts, have assumed a greater significance. Statistical Process Control (SPC) serves as a set of effective and practical techniques to address issues by achieving stability and improving the process capacity via reducing variability to accomplish quality assurance, a critical component in the customer decision-making process about products and services [1]. Statistical control charts are commonly employed to get stability in SPC. It also ensures products and process quality control [2]. The control chart, which demonstrates when a process varies and calls for corrective action depending on a consecutive sample, is the most well-known and significant statistical tool for SPC. A control chart often depicts quality traits examined in numerous samples [3]. By using a control chart, it is possible to systematically reduce the variability in a quality attribute of a good or service, which is represented by the monitoring variable. To bring the process under statistical control, it is used to identify and eliminate variances from special reasons. According to their design structure, control charts can be divided into two groups: memoryless and with memory, or time weighted. Once their control structure, which consists of the statistical plot and other decision rules, is dependent just on the latest observation, standard Shewhart charts are the memoryless variety. The most widely used memory-type control charts, such cumulative sum (CUSUM) and exponential weighted moving average (EWMA), are made so that its statistical plot uses both past and present observations, making them more sensitive to tiny and moderate changes in the process parameters of interest than Shewhart control charts.

A crucial instrument for preventing products and processes from deviating from the required level is SPC. A production system's effectiveness depends heavily on uninterrupted equipment and process operation, as reduced equipment performance can negatively impact the final product's quality. To improve quality level and decrease breakdowns and process variances, a thorough maintenance policy is important. This context makes it clear that quality and maintenance are tied to one another. The researchers have created integrated economic models employing the SPC and maintenance concepts to lower the overall cost of quality and maintenance because of the relationship between quality and maintenance. The two

* Corresponding author e-mail: mhamasha@hu.edu.jo.

most important instruments for managing production processes are maintenance management and statistical process control. There has been substantial research on the traditional methods of establishing the best maintenance and quality control plans [4].

Much research and the experience of practitioners agree that machines can deteriorate to a less desirable working state before completely breaking down. This machine's subpar functionality could result in greater rejection and operating costs, higher failure rates, and ultimately higher repair/replacement costs. Industry practices suggest that the quality control and maintenance strategies are optimized independently of one another, despite the apparent inter - dependence between quality control and maintenance strategy. However, in recent years the academic community has demonstrated a greater interest in researching the connection between maintenance and process quality [5-8]. Ben-Daya and Duffuaa emphasized how crucial it is to include process quality control in PM. [9] used PM time and an X control chart to track how a system's failure rate rose as more units were produced in order to find the best design parameters. By combining a PM action with an X control chart, [10-12] sought to reduce the rising hazard rate caused by the deterioration of the process during the in-control period.

In order to monitor the production process and aid in the identification and elimination of assignable causes, practitioners frequently utilize control charts, an effective tool for statistical process control. Since [13] first suggested using control charts with an economic design strategy to maintain process control, this topic has become crucial in the field of quality assurance. A hot topic in the dependability field is also optimizing maintenance strategy. Although it is necessary and reasonable to study the two issues separately, they typically have relevance and influence on one another. To lower the machine failure rate and lower product variation, planned/preventive maintenance must be performed as part of statistical process control. Similar to preventative maintenance, corrective maintenance must be carried out to return an out-of-control condition to an in-control state. Doing so will affect the machine's failure mode, which will ultimately result in a decrease in quality [14] and change the need for process control. Numerous researchers have concentrated on the integrated control chart and maintenance model, which is more practical in practice, as a result of the close relationship between quality and maintenance.

To assess the evolution of a quality control factor with product characteristic deviation, PM processes are put into place. As a result, by lowering the deviation from a target value, PM methods can enhance the quality of the final product. The goal of this research is to create a common model that would concurrently incorporate quality assurance, maintenance plan, and common optimization. This paper primarily offers a generalized model for enhancing process control tactics, maintenance practices, and ways for identifying equipment and process defects. The control chart is further employed in the control and assessment process to ascertain the real operational state. When a part of the machine fault with decreased quality is discovered, a corrective maintenance (CM) action is then carried out to restore the control condition. Thus, the suggested approach should have two key benefits: I) the removal of quality costs associated with an uncontrolled operation brought on by either machine wear or outside factors, II) Enhanced process control, and III) increased machine reliability by shielding it from failures.

In manufacturing plants, equipment downtime due to unexpected failures can result in significant production losses and maintenance costs. Therefore, it is important to have a proactive approach to equipment maintenance that can detect potential equipment failures before they occur. The pressure and temperature levels of rotating machinery can be used as an indicator of potential equipment failures. However, the traditional control charts like Shewhart control charts used to monitor these levels are often not effective in detecting small changes in these parameters' levels, which can lead to missed opportunities for maintenance. Therefore, the research problem is to investigate whether CUSUM control charts can be an effective tool for monitoring the pressure, temperature, and Time Between Failures (TBF) levels of oil-free air compressor in Alyoum for Food Industry when it deviates from the target value and predicting potential equipment failures before they occur. Therefore, this paper aims to compare the performance of CUSUM control charts and Shewhart control charts in detecting small shifts in the pressure, temperature, and TBF levels of oil-free air compressor in Alyoum for Food Industry when it deviates from the target value to take a corrective action and to compute average run length (ARL) in a CUSUM control chart and Shewhart control charts to provide additional evidence to support the contribution of using CUSUM control charts and select the actual shift in levels in practical situations.

Recent studies have demonstrated the importance of integrating advanced materials and modeling techniques in industrial applications to enhance performance and sustainability. For instance, Jawarneh et al. investigated the transient behavior of non-toxic natural and hybrid desiccant composite materials for efficient water extraction from atmospheric air, highlighting the potential of environmentally friendly designs in industrial systems [15]. Similarly, Akash et al. explored solar-assisted evaporator heat pump systems, showing how localized climate conditions influence thermal performance [16]. Complementary to such engineering advancements, guidelines for structured technical writing and electronic article preparation remain crucial, as emphasized by Strunk and White [17] and by Mettam and Adams in their comprehensive work on electronic publishing standards [18].

2. Literature Review

Variations are possible in the production procedures. The two primary categories of these variances include general cause variation and special cause variation. Common cause variation always exists, regardless of how well the process is planned and how diligently it is maintained. This variance is comparatively modest in size, uncontrollable, and caused by numerous little, inescapable factors. If there is just common cause variation, a process is under statistical control. This variance is a natural byproduct of the procedure. The process is referred to as being out of control if there are other sources of variation that do not fall under the category of common causes. The special (or assignable) causes related to the equipment, operators, materials, etc. may be the source of one or more of the additional variations. Through the reduction of variability, SPC is a group of strong tools that can be used to maintain and enhance process performance. Data is gathered, organized, analyzed, and interpreted in order to sustain the process at its current level or improve it to a better level of quality. Histograms, check sheets, Pareto charts, cause and effect diagrams, defect concentration diagrams, scatter diagrams, and control charts are some of the tools used in SPC, an approach that may be used to reduce variance in any process. The SPC toolkit is the official name for this collection. The most crucial tool for determining if a process is under control is the control chart.

2.1. Control Charts

To maintain the measurement of the quality attributes of the product produced between two limits known as the upper control limit (UCL) and lower control limit (LCL), control charts are frequently employed as a statistical tool for online process control. The center line (CL) designates the target value of the process location. Shewhart invented this tool in the year 1931. Since then, it has been utilized all over the world to regulate the process's statistical and financial performance. In statistical control charts, the overall focus is on maintaining the statistical constants of the chart, such as type I error probability (α), the chart's power ($1-\beta$), and so forth. In contrast, in an economic design of control chart, the process is targeted to reduce overall loss from the process to maximize profit. There are several different forms of control charts in the literature, including mean and range (X-bar and R), moving average, EWMA, CUSUM, and others. Charts such as the control chart for the number of defects (P chart), the control chart for the number of defects (C chart), the control chart for the proportion of defectives (np chart), etc. are also developed for attribute data. There are also several non-parametric charts, like sign charts and signed rank charts, among others. Each of these graphs has unique statistical characteristics. A formula for the loss (gain) per unit of time or per unit of output is achieved in the economic design of control charts, and it is optimized regarding the design parameters of sample size (n), sampling interval (h), and control limit multiplier (k) in terms of sigma units.

2.1.1. X-bar Control Charts

X-bar control charts are used by Duncan [13] to regulate the average value of a production process. In the meantime, Duncan also put forth the cost model, which considers the costs of sampling, inspection, evaluation, and charting as well as the costs of looking for an assignable reason when an out-of-control state arises [19]. Since then, numerous studies for the best economic design of the three control chart parameters based on [13] model have been carried out (e.g., [20-22]). Various control charts also contain some literature on related subjects (e.g., [23-24]). The main drawback of this approach is that it ignores the statistical performance of control charts, which could lead to an excessive number of nonconforming products and false alarms when used to design control charts to monitor the manufacturing process [25].

Saniga in 1989 first considered an economic design of control charts with statistical limits and then suggested the economic statistical design of the joint x-bar and R control charts for normal data because the results of an economic design of control charts may result in poor statistical properties (e.g., low power; high Type I error). Economic statistical designs seek to reduce the anticipated total cost per unit of time as well as Type I error and power, which are dependent on the requirements of the designer. Economic statistical designs typically cost more than economic designs because they include additional statistical constraints (such as minimum power and maximum Type I error values) [26].

2.1.2. CUSUM Control Charts

A popular monitoring tool for enhancing the quality of industrial and medical operations is the CUSUM control chart [27]. This scheme was first presented by Page (1963) as an alternative to the standard Shewhart control chart. Comparatively speaking to the standard Shewhart control chart, the CUSUM chart statistic aggregates past and present data of the process, which increases sensitivity to identify minor and moderate adjustments. Setting up the control limit is necessary for designing a CUSUM control chart, and it is frequently believed that the known in-control parameters will be used. However, since this presumption is unfounded, the CUSUM chart is applied in two stages. Phase I involves estimating the unknown parameters using random observations gathered from a steady process. To track and identify process changes in Phase II, the CUSUM chart is built using the estimations from the prior data. Small shifts can be addressed by the CUSUM charts quite effectively. This research suggests three additional CUSUM chart approaches to further strengthen this capability by minimizing the ARL1 for fixed ARL0.

2.2. Quality Control and Maintenance

Control charts have been routinely used in businesses to track process and product deviations for many years. Because a well-constructed quality control chart may aid in spotting any unusual behavior in the process and aid in the beginning of a restoration effort. Like this, several maintenance procedures are employed to maintain the functionality of machinery and equipment. Because a proper preventive maintenance (PM) strategy increases machine performance in terms of lower production costs and improved product quality, it also lowers the likelihood of machine failure. Considering this, quality control and maintenance management are important tools in industrial practice every day. However, studies have revealed a connection between process quality and equipment maintenance, and concomitant consideration of these two policies can be more effective and lucrative [8].

Although these integrated models are becoming more and more common among scholars today, Tagaras' development of an integrating cost model for the joint optimization of process control and maintenance is where it all began [28]. Rahim [9], who came after him, jointly identified the best settings for an X-bar control chart and the amount of time for preventive maintenance for a manufacturing system with a growing failure rate. Starting with the studies of Banerjee and Rahim [29], they generalized Duncan's (Duncan, 1956) approach for the economic design of X-bar control charts.

The Markovian group has examined routine maintenance of mechanical equipment as well as product quality control at various times, with restricted vision, and with impaired production systems. To establish how to coordinate SPC and planned maintenance operations to lower overall costs, Lindermand [7] provided a generic analysis model. Panagiotidou and Tagaras [30-31] developed an economic model to maximize ideal and incomplete maintenance procedures, for example, two operating conditions of equipment such as in-control and out-of-control states and in SPC. These two quality-related economic models were then introduced to make the best use of PM procedures. A technique for creating an ideal PM control chart was put forth by Chiu and Huang [32] and uses a set sample interval and risk increase rate. In addition, they believe that after PM, the production system will become a good new state.

3. Proposed Methodology

The primary objective of this study is to investigate whether CUSUM control charts can be an effective tool for monitoring the pressure and temperature levels of oil-free air compressor in Alyoum for Food Industry. Alyoum for Food Industry is a dairy plant located in Jordan-Zarqa that produces yogurt and beverages. The plant's success is attributed to its efficient production processes and the use of high-quality equipment. One of the most critical pieces of equipment in the plant is the oil-free air compressor. This equipment plays a critical role in several key operations, including the operation of valves in boilers and reverse osmosis (RO) systems and the pasteurization process.

Figure 1 depicts the diagram of the methods used to conduct the current research. This study aims to evaluate the effectiveness of CUSUM control charts in monitoring the pressure and temperature of the oil-free air compressor at Alyoum for Food Industry. Alyoum for Food Industry is a dairy plant located in Jordan-Zarqa that produces yogurt and beverages. To conduct reliability analysis, Shewhart and CUSUM control charts information on the pressure, temperature, and TBF of oil-free air compressor was gathered over 100 days period. To determine whether pressure, temperature, and TBF data are independently and identically distributed (iid), trend tests and serial correlation tests were run. Then, we utilize Arena (input analyzer) tool to identify the parameters and pressure, temperature, and TBF data that best suit the distribution. Then the reliability of oil-free air compressor was calculated, and finally optimum CUSUM control charts were developed by investigate whether CUSUM control charts can be an effective tool for monitoring the process in different shifts and limits to select best parameters based on the ARL.

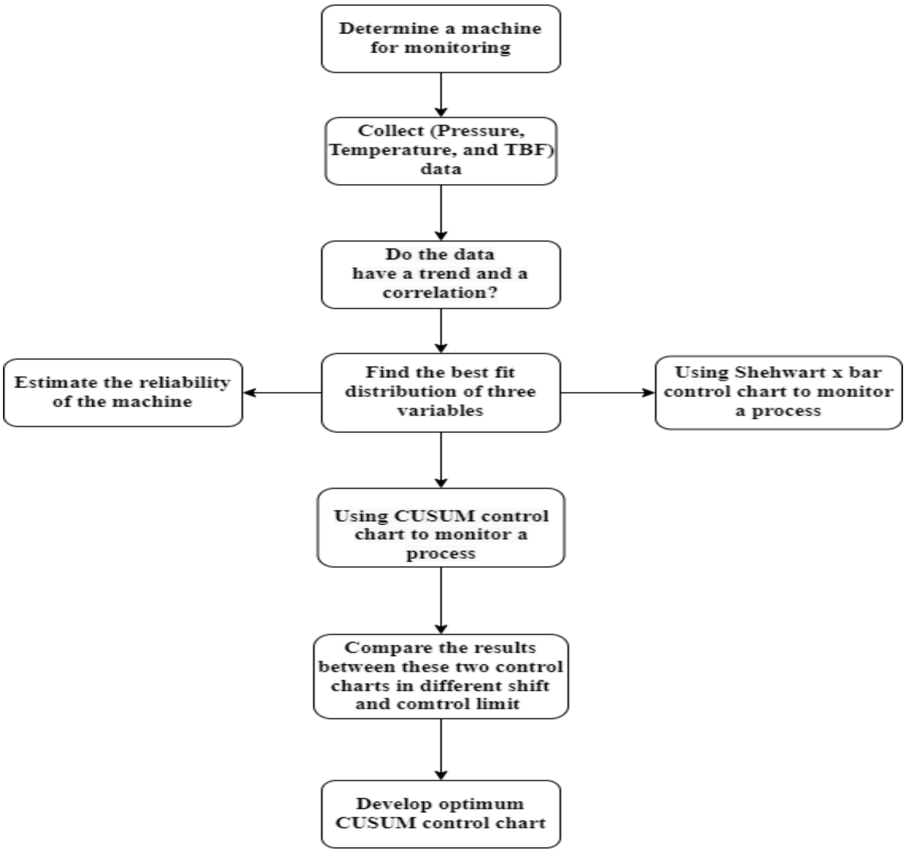

Figure 1. Overview of the Research Process for Applying CUSUM Charts in Maintenance of Oil-Free Compressors.

3.1. Data Collection Instrument

This Study combines primary and secondary data for our investigation. Interviews, production process observation, and equipment monitoring were used to gather primary data. Direct questions were directed at the relevant company stakeholders during the interview process. Interruptions in the operation of the oil-free air compressor were observed during the production process.In a study on the use of CUSUM control charts in monitoring the variables levels of rotating machinery, the primary data would be the pressure, temperature, and TBF measurements collected from the selected machinery using appropriate sensors and data collection methods. The secondary data in this research could include previous studies and literature on the topic, such as review articles, research papers, and industry reports, which provide insights into the use of CUSUM control charts and other control chart methods in machine condition monitoring. Secondary data can also include historical data on the maintenance practices and downtime of the selected machinery, which can help to contextualize the study findings and provide insights into the impact of maintenance practices on machinery performance.

3.2. Overview of X-bar and CUSUM charting

The Shewhart-type chart, which only takes into account data from the last point displayed on the chart [33-34] is the most popular statistical control chart and is frequently used in monitoring processes. Other control chart types may occasionally be used in place of or in addition to Shewhart type control charts and offer benefits. The traditional CUSUM and the EWMA memory-type control charts mentioned in this study fall into this category. These charts, in contrast to Shewhart-type charts, blend the most recent data with earlier data to detect tiny and moderate changes in process parameters with a significantly lower.

3.2.1. Traditional Shewhart chart (x- bar chart)

The control chart for individual measures, Xi, those data are in the individual observations, is the most basic Shewhart-type control chart. It is impracticable to attempt to combine these data in any way with the intention of producing charts of the rational subgroups in this case. In two different scenarios the Xi chart based on known parameter values and the Xi chart based on assumed or unknown parameter values this chart might be helpful for tracking the position of a process. Assuming the mean (μ) and standard deviation (σ) of the process are known, the control limits are set to 3± σ x for n=1 and 3±σ X for n>1 where n X =, provided the observations of the quality characteristic are reasonable and assumed to follow a normal distribution as an appropriate model with no correlation.

The conditions should be examined, just like with statistical methods in general, with the assumption of normality for an Xi chart being crucial. The respective estimators can, however, be easily produced based on the samples taken from a process that appears to be in control if the process parameters for the mean and standard deviation are unknown. A set of recent historical data for the process would be used to determine $\bar{x}$, which is the typical estimator for μ. Although the estimator μ has no clear option, this is not the case when choosing how to estimate σ. It is preferable to employ two assessors, one to analyze the historical data set and the other to keep track of Phase I and Phase II of the procedure. This recommendation is being made because one estimator works better for Phase I and the other for Phase II.

ARL, which represents the run length distribution's mean, is a measure frequently used to assess the effectiveness of a control chart. $ARL0$ is described for the in-control process as:

$$ARL_0 = \frac{1}{\alpha} \tag{1}$$

where α is the probability of type-I error. For the out-of-control process, the mean shift is detected by using the formula:

$$ARL_1 = \frac{1}{(1-\beta)} \tag{2}$$

where β is the probability of type-II error.

The probability of in-control Pin for the existing control chart for the case of known process mean m is calculated as follows:

$$P_{in} = P(LCL2 \leq \bar{x} \leq UCL2) \tag{3}$$

$$P_{in} = \phi(A_{22}d_2\sqrt{n}) - \phi(-A_{22}d_2\sqrt{n}) \tag{4}$$

where ϕ(.) is the cumulative standard normal distribution function.

The probability of out-of-control $Pout$ for the existing control chart is calculated as follows:

$$P_{out} = P(\bar{x} \geq \text{UCL1}) + P(\bar{x} \leq \text{LCL1}) \tag{5}$$

$$P_{out} = 1 - \phi(A_{21}d_2\sqrt{n}) - \phi(-A_{21}d_2\sqrt{n}) \tag{6}$$

3.2.2. CUSUM

As an alternative to the Shewhart-type chart for the quick identification of tiny and moderate changes in location and/or dispersion of a process using independent and normally distributed observations, Page (1963) suggested the classical CUSUM control chart. CUSUM control charts come in a variety of formats, although the tabular version is typically the most popular. In this method, deviations from each observation's nominal value $\mu0$ that are higher than the nominal value with the statistic $Ci+$ and deviations from $\mu0$ that are lower than the nominal value with the statistic $Ci-$ are both accumulated.

For monitoring a process' mean, initially defined as $Ci+ = Ci- = 0$, the statistics $Ci+$ and $Ci-$, also known as upper and lower one-sided CUSUM, are calculated as follows [35]:

$$Ci^+ = \max(0, C_{i-1}^+ + Xi - \mu_0 - K) \tag{7}$$

$$Ci^- = \max(0, (\mu_0 - K) - Xi + C_{i-1}^-) \tag{8}$$

where Xi (i=1, 2,...) is a set of independent observations with normal distribution, Xi ~ N(μ,σ^2), and $\mu_0$the nominal value and σ is standard deviation. If one wants the statistics Ci^+ and Ci^- for the sample means, $\bar{x}$, should be used in place of Xi in Equations above, and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, should be used in place of in the following equations. The process is deemed out of control if the value of Ci^+ or Ci^- exceeds the decision interval H = h*σ. Five times the process' standard deviation, or H=5σ, is a reasonable value for H. In above equations, the reference value K is often set at half the change's magnitude (measured in standard deviation units), or K = k*σ = (δ/2) *σ = |$\mu_1 - \mu_0$| / 2. The CUSUM chart's parameters are the numbers k (reference value) and h (standardized decision interval). The (k, h) pair's selection is crucial because it significantly affects the ARL performance of this chart.

ARL, or typical number of samples needed to detect an out-of-control instance or generate a false alarm, can be used to evaluate the performance of a control chart. The in-control ARL (ARL0) is utilized for the false alarm rate, but the out-of-control ARL (ARL1) is frequently employed as an indicator of the power (or effectiveness) of the control scheme. The evaluation of typical run lengths is one of the main challenges in the economic design of CUSUM charts. The current situation is as follows. An N (μ,σ^2) standard distribution with samples of size n is used to operate a CUSUM chart with reference value (K) and decision interval (h).

The CUSUM chart's ARL is calculated by [36]. This approximation has been beneficial [35] have explained its numerical solution, and Alwan has further modified and applied it [37]. The value for Siegmund's approximation of the ARL is given as:

$$ARL = \frac{\exp(-2\Delta b)+2\Delta b-1}{2\Delta^2} \tag{9}$$

For Δ≠ 0, where $\Delta = \delta - k$ for the upper one-sided CUSUM Ci^+, $\Delta = -\delta - k$ for the upper one-sided CUSUM Ci^-, and $b = h + 1.166$. For Δ = 0, can use ARL = b^2.

The quantity δ represents the magnitude of quality shift, for which the ARL is to be calculated. Therefore, if δ = 0, we would calculate ARL_0^+, ARL_0^- as following:

$$ARL_0^+ = ARL_0^- = \frac{\exp(2kb)-2kb-1}{2k^2} \tag{10}$$

Whereas if $\delta \neq 0$, we would calculate ARL_1^+, ARL_1^- as following:

$$ARL_1^+ = \frac{\exp(2(\delta-k)b)+2(\delta-k)b-1}{2(\delta-k)^2} \tag{11}$$

$$ARL_1^- = \frac{\exp(2(-\delta-k)b)+2(-\delta-k)b-1}{2(-\delta-k)^2} \tag{12}$$

To calculate the ARL of the two-sided CUSUM from the ARLs of the two-sided statistics, ARL_1^+, ARL_1^-, we use:

$$\frac{1}{ARL} = \frac{1}{ARL_1^+} + \frac{1}{ARL_1^-} \tag{13}$$

3.3. Construct Data Set of Pressure, Temperature, and TBF

To construct a data set of pressure, temperature, and TBF of a compressor over a period of 100 days, we need to follow a systematic process that involves several steps. The first step is to gather the data on the set values of pressure and temperature for the compressor. This data should be collected at regular intervals over the 100-day period and should be recorded in a spreadsheet or other suitable format that facilitates analysis and visualization. The next step is to calculate the TBF for the compressor. The TBF is the amount of time that elapses between each failure of the compressor. To calculate the TBF, we need to record the date and time of each failure of the compressor during the 100-day period. We can then calculate the time between each failure by subtracting the date and time of the previous failure from the date and time of the current failure.

Once we have gathered the data on pressure, temperature, and TBF, we can organize it in a spreadsheet or other suitable format that facilitates analysis and visualization. We can then use this data to construct control charts to monitor the performance of the compressor over time. By monitoring the control charts over the 100-day period. So, detecting any trends or patterns in the data that may indicate changes in the performance of the compressor. If any points fall outside of the control limits, further investigation may be needed to identify the cause of the deviation and take appropriate corrective action.

3.3.1. Construct X-bar and CUSUM Control Charts of Pressure, Temperature, and TBF

Constructing X-bar control charts for a set of data on pressure, temperature, and TBF for a compressor can help us monitor the performance of the compressor over time and identify any issues that may require attention. By using the X-bar chart to detect and respond to process variation, we can improve the reliability and efficiency of the compressor and reduce the likelihood of downtime or failure.

If the X-bar control charts for pressure and temperature data are both in control, but the X-bar control chart for the TBF data has one point out of control, it could indicate a potential issue with the reliability of the compressor. It is important to investigate the out-of-control point and determine if it is a legitimate signal of process variation or simply a random fluctuation.

The first step is to identify the point on the TBF chart that is out of control which determines if there are any special causes of variation that may have affected the performance of the compressor. This could include changes in operating conditions, maintenance activities, or other factors that may have impacted on the TBF data. If we identify a special cause of variation, we can take corrective action to address the issue and prevent it from occurring in the future. This may involve adjusting the operating conditions, conducting maintenance or repairs, or implementing other changes to the process or equipment.

To construct CUSUM control charts for pressure, temperature, and TBF, we need to follow certain steps. First, calculate the target value for each parameter based on historical data. Then, choose the value of h (the CUSUM increment) and k (the CUSUM threshold) based on the desired level of sensitivity and the false alarm rate. Once the target values and the values of h and k were determined, we can begin to calculate the CUSUM values for each parameter. The CUSUM value for each observation is calculated as the difference between the observed value and the target value, minus h. If the result is negative, it is set to zero. Then sum the CUSUM values for each observation to get the cumulative sum.

Next, plot the cumulative sums for each parameter on a CUSUM control chart, with the horizontal axis representing the observation number and the vertical axis representing the cumulative sum. We also plot the upper and lower CUSUM control limits, which are determined by the values of h and k. To interpret the CUSUM chart, we look for any shifts or trends in the cumulative sums. If the cumulative sum exceeds the upper control limit or falls below the lower control limit, it indicates that a shift or trend has occurred, and that action may be needed to investigate and correct the issue.

The values of h and k in CUSUM control charts depend on the specific application and the desired level of sensitivity for detecting shifts in the process mean.

Generally, a larger h value results in a slower detection of small shifts, but a reduced likelihood of false alarms. Conversely, a smaller h value allows for more rapid detection of small shifts but may result in more false alarms. The k value is used to set the reference value for the CUSUM chart and should be chosen based on historical data or process specifications. Typically, the k value is set to half of the specification limits, or the target value of the process mean.

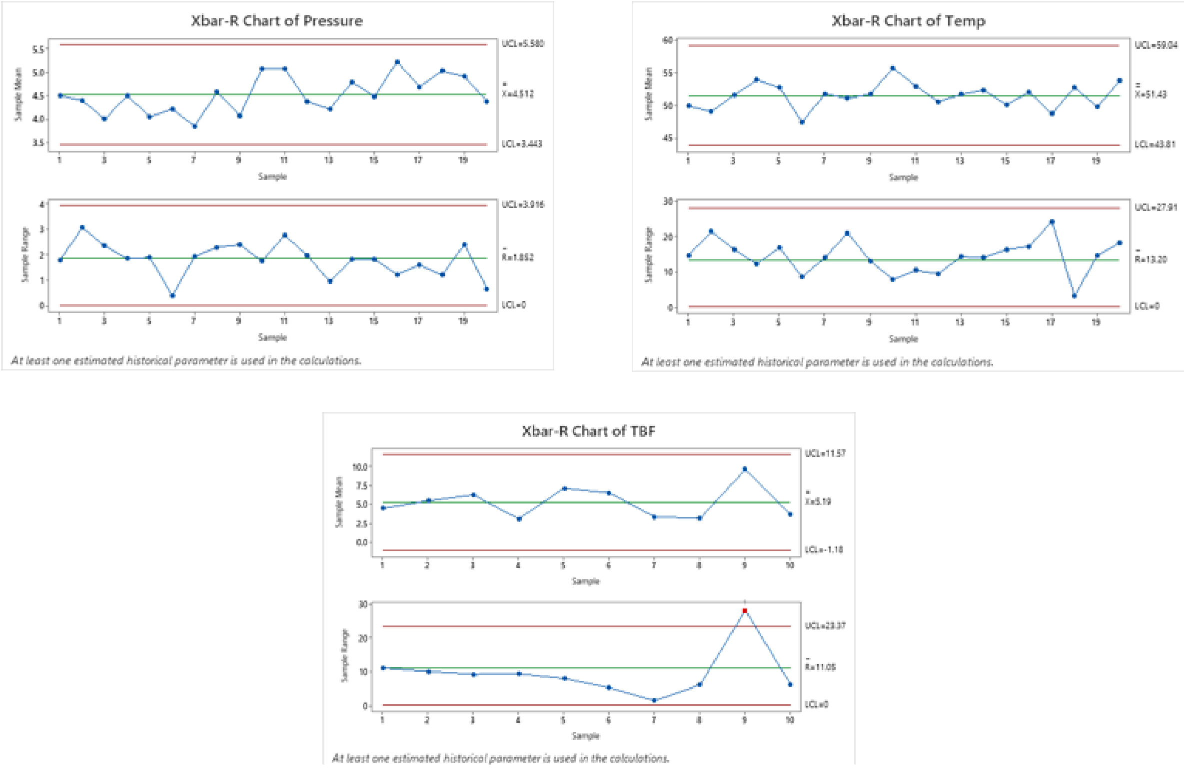

Figure 2. Xbar-R Control Chart of pressure, temp, and TBF

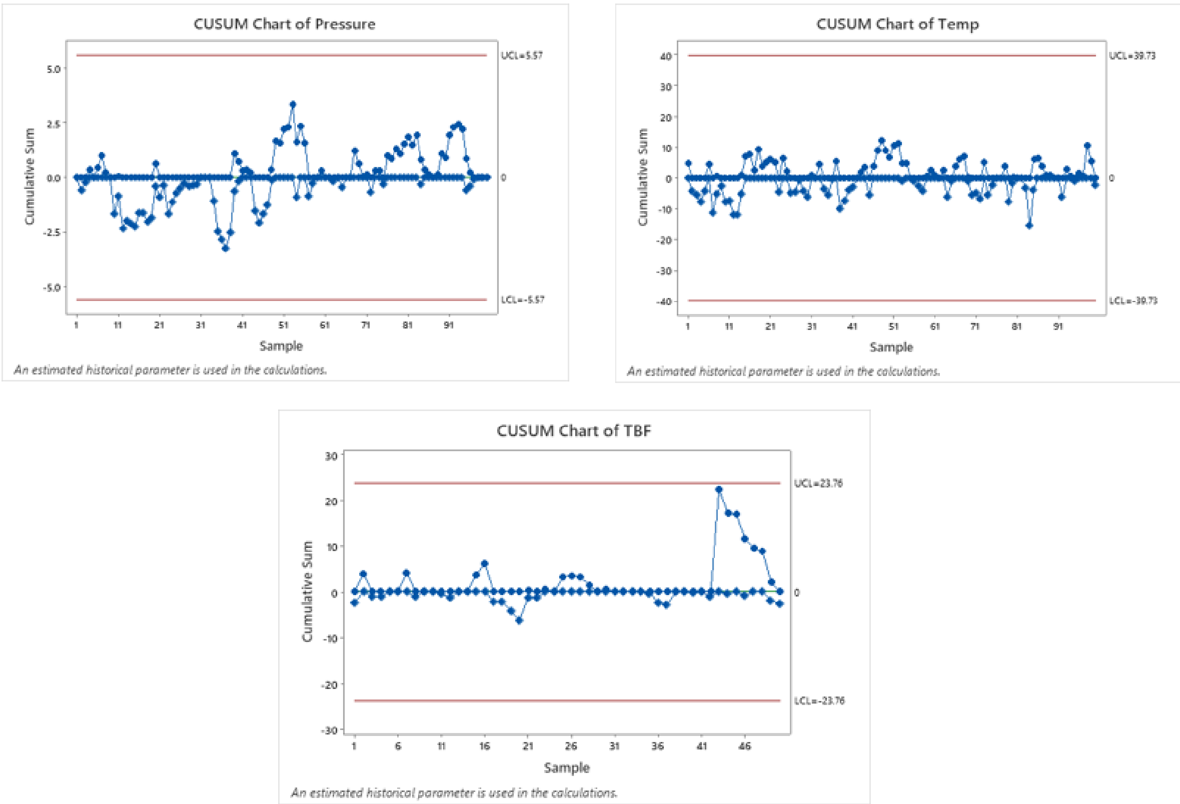

Figure 3. CUSUM Control Chart of pressure, temp, and TBF

3.4. Construct CUSUM Control Charts of Pressure with 0.1,0.2,0.3,0.5, and 1σ shift in Mean.

If there is a shift of 0.1 in the mean of the process, it may indicate a change in the underlying process conditions. To monitor this change, CUSUM control charts can be used. To construct the CUSUM chart, first the values of h and k must be determined based on the desired detection sensitivity and the estimated standard deviation of the process. Once the values of h and k are determined, the CUSUM chart can be constructed by calculating the cumulative sum of deviations from the target mean for each sample. If the cumulative sum exceeds the control limits, it indicates that a significant shift has occurred in the mean of the process.

In this case, if there is a shift of 0.1 in the mean of the process, it may be detected by the CUSUM control chart. The chart will show an upward trend in the cumulative sum of deviations from the target mean for each sample, indicating that the process has shifted from its previous level. The CUSUM chart can be used in conjunction with other control charts, such as X-bar charts, to provide a comprehensive monitoring system for the process. By using the CUSUM chart to detect shifts in the mean of the process, timely corrective action can be taken to prevent the occurrence of defects and ensure that the process is operating at its desired level of performance.

To plot a CUSUM control chart for pressure, temperature, and TBF with a shift in the mean of 0.1, you will need to follow these steps:

1. Calculate the mean and standard deviation for each variable based on the historical data that shifted 0.1 in mean and using excel sheet construct the formula and the parameters of CUSUM chart and 10 runs were made of shifted data to get accurate ARL.
2. Determine the desired level of sensitivity (h) and the target shift size (k) for the chart.
3. Calculate the CUSUM values for each data point using the following formula:

$$Ci^{+} = \max(0, C_{i-1}^{+} + Xi - \mu_0 - K) \tag{14}$$

$$Ci^{-} = \max(0, (\mu_0 - K) - Xi + C_{i-1}^{-}) \tag{15}$$

Where Ci is the CUSUM value at time i, Xi is the observation at time i, μ_0 is the mean value of the historical data, and k is the target shift size.

4. Plot the CUSUM values on a control chart with the time on the x-axis and the CUSUM values on the y-axis.
5. Add horizontal lines at h and -h to indicate the control limits, for pressure and temp h=7 and for TBF h=5.
6. Monitor the chart for any points that exceed the control limits, which would indicate a significant shift in the process mean.

Note that the specific values of h and k will depend on the desired level of sensitivity and the characteristics of the process being monitored. You may need to adjust these values and monitor the chart over time to ensure that it is providing effective control.

ARL is an important metric used to evaluate the performance of a CUSUM control chart. It represents the average number of observations that will be collected before the chart signals an out-of-control condition. In the case of the pressure CUSUM chart, we can calculate the ARL using the following procedure:

1. Define the in-control state: To calculate the ARL, we first need to define the in-control state of the pressure CUSUM chart. This can be done by collecting a set of in-control data and estimating the mean and standard deviation of the pressure measurements.
2. Determine the decision interval: The decision interval for the pressure CUSUM chart is determined by choosing appropriate values for the h and k parameters. In this case, let's assume that we have chosen h = 7 and k = 0.05.
3. Calculate the ARL: The ARL is calculated as the expected number of observations that must be collected before the cumulative sum of deviations exceeds the decision interval. In other words, it is the expected time until the chart signals an out-of-control condition. The ARL can be calculated using simulation methods, such as Monte Carlo simulation or by formula below.

By calculating the ARL of the pressure CUSUM chart, we can evaluate its ability to detect out-of-control conditions in a timely manner. A lower ARL indicates that the chart is more sensitive to shifts in the mean of the pressure measurements, while a higher ARL indicates that the chart is less sensitive.

The value for [37] approximation of the ARL is given as:

$$ARL = \frac{\exp(-2\Delta b)+2\Delta b-1}{2\Delta^2} \tag{16}$$

For Δ≠ 0, where $\Delta = \delta - k$ for the upper one-sided CUSUM Ci^{+}, $\Delta = -\delta - k$ for the upper one-sided CUSUM Ci^{-}, and $b = h + 1.166$. For Δ = 0, can use ARL = b^2.

The quantity δ represents the magnitude of quality shift, for which the ARL is to be calculated. Therefore, if δ = 0, we would calculate ARL_0^{+}, ARL_0^{-} as following:

$$ARL_0^{+} = ARL_0^{-} = \frac{\exp(2kb)-2kb-1}{2k^2} \tag{17}$$

Whereas if $\delta \neq 0$, we would calculate ARL_1^{+}, ARL_1^{-} as following:

$$ARL_1^{+} = \frac{\exp(2(\delta-k)b)+2(\delta-k)b-1}{2(\delta-k)^2} \tag{18}$$

$$ARL_1^{-} = \frac{\exp(2(-\delta-k)b)+2(-\delta-k)b-1}{2(-\delta-k)^2} \tag{19}$$

To calculate the ARL of the two-sided CUSUM from the ARLs of the two-sided statistics, ARL_1^{+}, ARL_1^{-}, we use:

$$\frac{1}{ARL} = \frac{1}{ARL_1^{+}} + \frac{1}{ARL_1^{-}} \tag{20}$$

	A	B	C	D	E	F	G	H	I	J
1	Mean	4.5117		5.57						
2	STD	0.7962		-5.57			shift	0.1		
3										
4	pressure	Pressure-M	Pressure + 0.1	Pressure-M	Ci+	Pressure-0.1	Pressure-M	Ci-	Mean	STD
5	5.71333	1.2016281	5.813328094	1.301628094	1.30163	5.61332809	-1.101628094	-1.101628094	4.6117	0.7962
6	5.58236	1.0706568	5.682356762	1.170656762	2.47228	5.48235676	-0.970656762	-2.072284856		
7	5.4654	0.9537004	5.56540041	1.05370041	3.52599	5.36540041	-0.85370041	-2.925985266		
8	3.23573	-1.275973	3.335727402	-1.175972598	2.35001	3.1357274	1.375972598	-1.550012669		
9	4.35247	-0.159227	4.452473178	-0.059226822	2.29079	4.25247318	0.259226822	-1.290785847	4.6543	0.88
10	3.63118	-0.880521	3.731178544	-0.780521456	1.51026	3.53117854	0.980521456	-0.310264391		
11	5.04969	0.5379917	5.149691703	0.637991703	2.14826	4.9496917	-0.437991703	-0.748256094		
12	3.74559	-0.766109	3.845591489	-0.666108511	1.48215	3.64559149	0.866108511	0		
13	5.14494	0.6332357	5.244935715	0.733235715	2.21538	5.04493571	-0.533235715	-0.533235715	4.6422	0.7566
14	4.72609	0.2143882	4.826088232	0.314388232	2.52977	4.62608823	-0.114388232	-0.647623947		
15	4.75284	0.241137	4.852836962	0.341136962	2.87091	4.65283696	-0.141136962	-0.788760908		
16	4.09465	-0.417051	4.194648918	-0.317051082	2.55386	3.99464892	0.517051082	-0.271709826		
17	5.10951	0.5978064	5.209506352	0.697806352	3.25166	5.00950635	-0.497806352	-0.769516178	4.6143	0.8718
18	3.69984	-0.811864	3.799835715	-0.711864285	2.5398	3.59983572	0.911864285	0		
19	3.66485	-0.846853	3.764847463	-0.746852537	1.79295	3.56484746	0.946852537	0		
20	2.37677	-2.134929	2.476771208	-2.034928792	0	2.27677121	2.234928792	0		
21	2.07854	-2.433164	2.178535627	-2.333164373	0	1.97853563	2.533164373	0	4.6796	0.7573
22	4.06991	-0.441793	4.169907194	-0.341792806	0	3.96990719	0.541792806	0		
23	4.2704	-0.241296	4.370403511	-0.141296489	0	4.17040351	0.341296489	0		
24	3.85506	-0.656636	3.955063876	-0.556636124	0	3.75506388	0.756636124	0		
25	3.64365	-0.868053	3.743647071	-0.768052929	0	3.54364707	0.968052929	0	4.6037	0.8419
26	4.8942	0.382496	4.994195976	0.482495976	0.4825	4.79419598	-0.282495976	-0.282495976		

	K	L	M	N	O	P	Q	R	S	T	U	V	W
1													
2													
3													
4													
5	Ci+	0.42149	0	0.08519	0.95561	0.86603	1.82119	2.87531	2.59195	2.72191	0.74161	1.2725	
6		1	1	1	1	1	1	1	1	1	1	1	
7	Ci-	-0.22149	0	0	-0.67041	-0.38083	-1.13599	-1.99011	-1.50676	-1.43672	0	-0.3309	
8		1	1	1	1	1	1	1	1	1	1	1	
9	Ci+	0	0	0.77014	1.33716	2.15575	2.11242	1.77976	1.86201	2.3518	3.858	4.3972	5.8894
10		1	1	1	1	1	1	1	1	1	1	1	
11	Ci-	0	0	-0.57014	-0.93716	-1.55575	-1.31242	-0.77976	-0.66201	-0.9518	-2.258	-2.5972	-3.8894
12		1	1	1	1	1	1	1	1	1	1	1	
13	Ci+	0.36312	1.29352	0.30742	1.34661	1.3246	0.86675	1.90191	1.82043	2.47581	1.59594	2.43662	4.01
14		1	1	1	1	1	1	1	1	1	1	1	
15	Ci-	-0.16312	-0.89352	0	-0.83919	-0.61718	0	-0.83516	-0.55368	-1.00907	0	-0.64067	-2.0170
16		1	1	1	1	1	1	1	1	1	1	1	
17	Ci+	0.75237	1.90676	1.24818	0.35084	0.84666	1.22088	1.78334	2.26894	2.15572	2.64369	1.98616	2.0523
18		1	1	1	1	1	1	1	1	1	1	1	
19	Ci-	-0.55237	-1.50676	-0.64818	0	-0.29582	-0.47004	-0.8325	-1.11811	-0.80488	-1.09285	-0.23532	-0.1014
20		1	1	1	1	1	1	1	1	1	1	1	
21	Ci+	1.79066	1.82442	1.85348	2.45935	3.07008	2.61275	4.11387	4.5927	4.82903	3.75544	3.84443	3.2403
22		1	1	1	1	1	1	1	1	1	1	1	
23	Ci-	-1.59066	-1.42442	-1.25348	-1.65935	-2.07008	-1.41275	-2.71387	-2.9927	-3.02903	-1.75544	-1.64443	-0.8403
24		1	1	1	1	1	1	1	1	1	1	1	
25	Ci+	0.98847	0.36172	0.75797	1.54563	2.0636	3.2646	3.87245	4.56401	3.83074	4.42766	4.25912	5.170
26		1	1	1	1	1	1	1	1	1	1	1	

Figure 4. Construct the formula and the parameters pressure to plot CUSUM by excel.

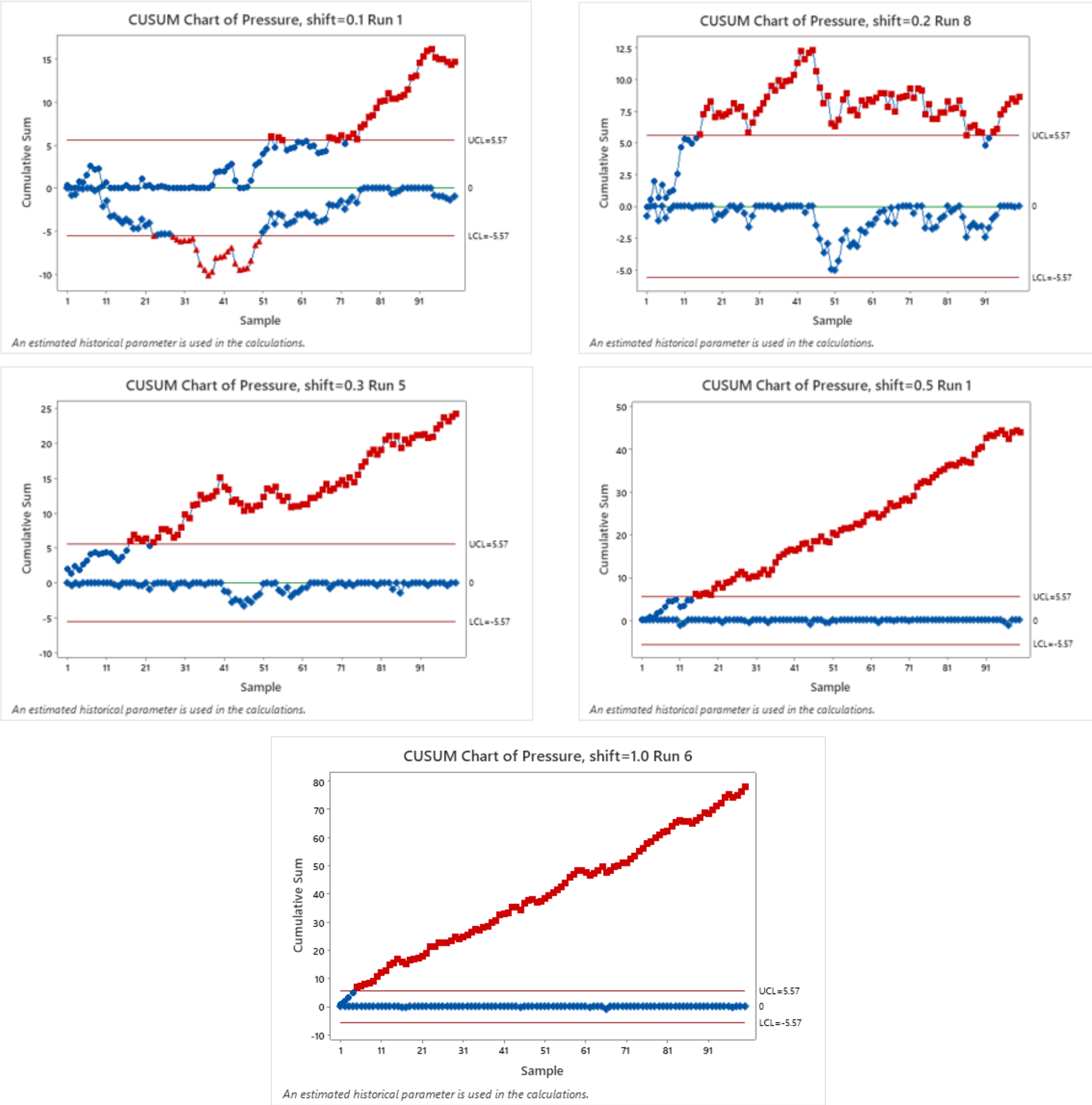

Figure 5. CUSUM charts for pressure with shift 0.1,0.2,0.3,0.5,1.0.

Table 1. CUSUM ARL for Different Shifts and h for pressure data

Shift in Mean (Multiple of σ)	h = 4	h = 5	h =7
0	169.04	469.11	3510
0.1	26.4	28.4	36
0.2	23.16	24.5	25.7
0.3	15.8	21.5	23.3
0.5	13.1	14	15.7
1	2.8	3.5	4.7

Table 2. Shewhart ARL for Different shifts for pressure data

Shift in Mean (Multiple of Sigma)	Shewhart ARL (Two-Sided, Zero-State)
0	370.40
0.1	306.75
0.2	136.2
0.3	64.9
0.5	18.24
1	2.37

3.5. Construct CUSUM Control Charts of for Temp with 0.1,0.2,0.3,0.5, and 1 σ shift in Mean

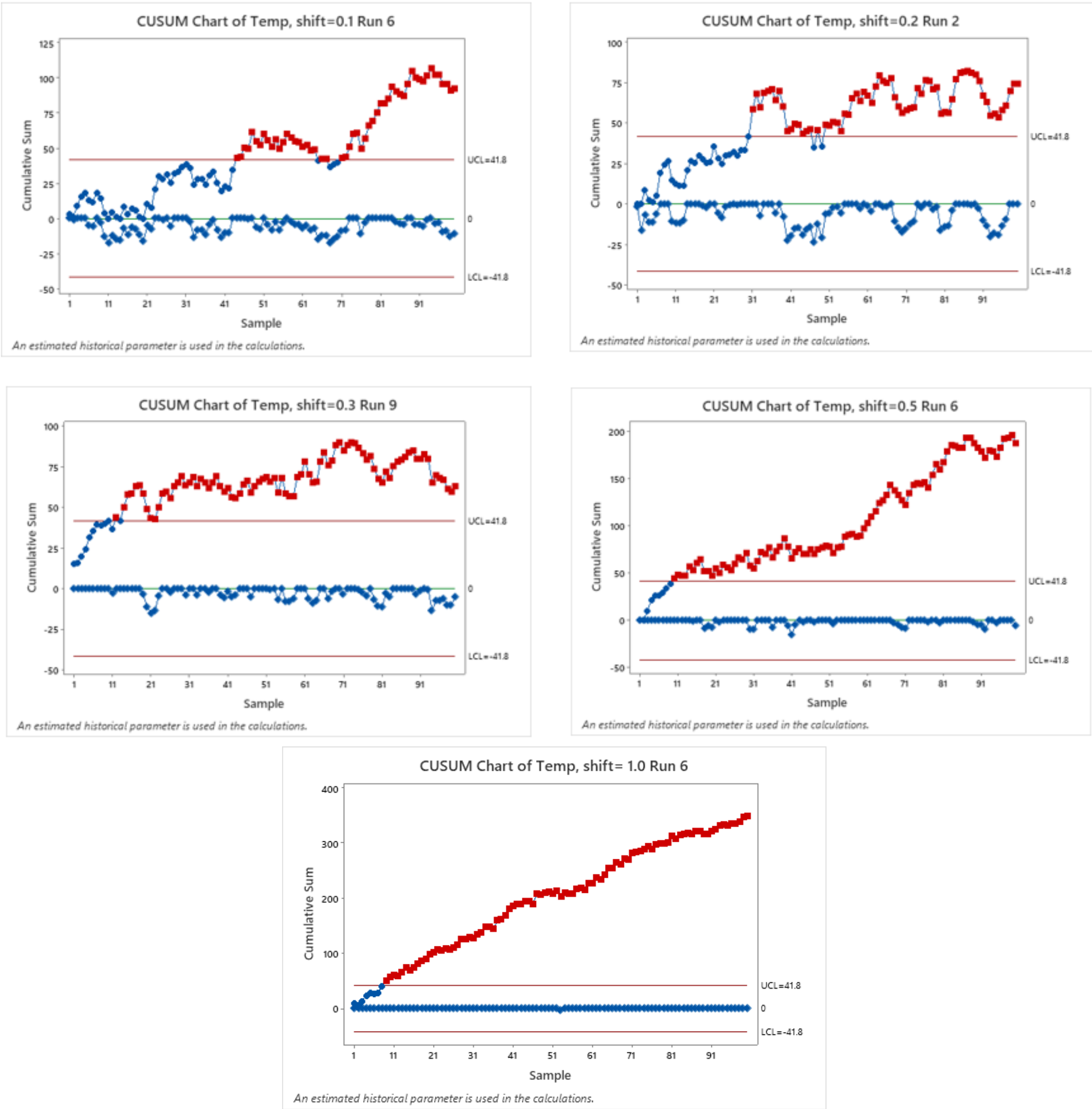

Figure 6. CUSUM charts for Temp with shift 0.1,0.2,0.3,0.5,1.0.

Table 3. CUSUM ARL for Different Shifts and h for Temp data

Shift in Mean (Multiple of σ)	h = 4	h = 5	h =7
0	169.04	469.11	3510
0.1	25.4	26.3	33.4
0.2	16.2	23.16	25
0.3	15.8	21.5	23.5
0.5	13.1	14.7	16.1
1	4.92	7.4	8.3

Table 4. Shewhart ARL for Different shifts for Temp data

Shift in Mean (Multiple of Sigma)	Shewhart ARL (Two-Sided, Zero-State)
0	370.40
0.1	357
0.2	185
0.3	101
0.5	32
1	4.4

3.6. Construct CUSUM Control Charts of for TBF with 0.1,0.2,0.3,0.5, and 1 σ shift in Mean.

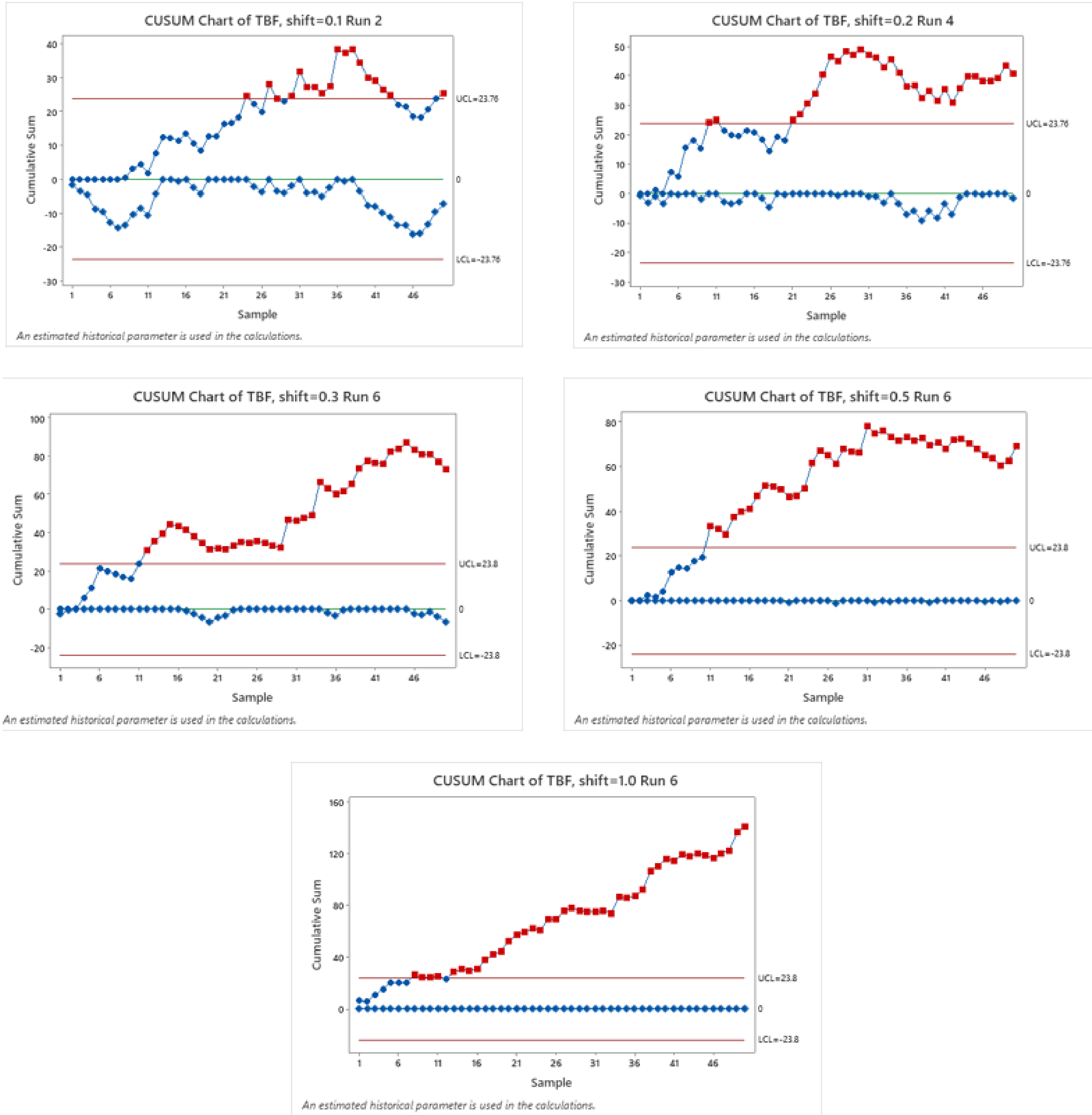

Figure 7. CUSUM charts for TBF with shift 0.1,0.2,0.3,0.5,1.0.

Table 5. CUSUM ARL for Different Shifts and h for TBF data

Shift in Mean (Multiple of σ)	h = 3	h = 4	h =5
0	59	169	469
0.1	20.7	22.4	25.3
0.2	13.3	14	15.8
0.3	10.7	11.4	13.1
0.5	6.5	7.1	8.7
1	3.5	4.4	5.8

Table 6. Shewhart ARL for Different shifts for Temp data

Shift in Mean (Multiple of Sigma)	Shewhart ARL (Two-Sided, Zero-State)
0	370.40
0.1	357
0.2	185
0.3	101
0.5	32
1	4.4

After constructing CUSUM control charts for pressure, temperature, and TBF for the compressor, ARL was calculated. ARL represents the average number of samples needed to detect a shift in the process mean. The ARL value depends on the shift in the mean, the sample size, and the value of h. In this study, ARL was calculated for different values of shift in mean, different values of h. The ARL values were calculated using mathematical formulas based on the Weibull distribution for the TBF data, and normal distribution for pressure and temperature data. Minitab software was used to calculate the ARL values. The ARL results showed that as the shift in mean increased, the ARL value decreased, which means that the chart becomes more sensitive to detecting the shift. The ARL value also decreased as the value of h decreased, which indicates that decreasing the value of h results in a more sensitive chart.

Overall, the ARL results demonstrated that the CUSUM control charts were effective in detecting shifts in the process mean for pressure, temperature, and TBF data in different scenarios of shift and h. The ARL values can be used as a guide to select appropriate values of h for specific process control applications.

4. Conclusion

CUSUM control charts can be used in maintenance applications to track an item's performance and notice any behavioral changes that might point to a problem or require maintenance. The CUSUM control chart can be used in a maintenance application to track a specific performance indicator, like the frequency of a spinning machine's vibrations or the furnace's temperature. The maintenance crew may watch the trend of the performance indicator and spot any changes in its behavior by using the CUSUM chart, which depicts the cumulative sum of the deviations from a target value over time.

When the CUSUM chart detects a significant deviation from the target value, it can trigger an alarm or alert the maintenance team to investigate the cause of the deviation. By taking timely corrective action, the maintenance team can prevent equipment failures and avoid costly downtime. The use of CUSUM control charts in maintenance applications is a proactive approach to equipment maintenance, allowing maintenance teams to detect potential issues before they cause equipment failure. This approach can lead to reduced maintenance costs, improved equipment reliability, and increased productivity. This research compared the performance of CUSUM control charts and Shewhart control charts in monitoring the pressure, Temperature, and TBF levels of compressor machinery. They found that the CUSUM control chart was more effective in detecting small shifts in pressure, Temperature, and TBF levels compared to the Shewhart control chart. The CUSUM chart was able to detect small shifts that the Shewhart chart missed, which allowed maintenance teams to take corrective action before equipment failures occurred. Therefore, the study suggests that CUSUM control charts may be a more effective tool for monitoring the vibration levels of rotating machinery in a manufacturing plant compared to Shewhart control charts.

Shewhart control charts are effective at detecting large shifts in process parameters, but they may not be able to detect small shifts that occur over a longer period of time. CUSUM control charts, on the other hand, are designed to detect small shifts in process parameters over time. Shewhart control charts are relatively simple to use and interpret, but they may not be as effective in detecting small shifts as CUSUM control charts. CUSUM control charts can be more complex to use and interpret, but they can be more sensitive to small shifts. Shewhart control charts require data to be normally distributed, whereas CUSUM control charts do not have this requirement. This makes CUSUM control charts more flexible and applicable to a wider range of data.

The ARL results obtained for different shifts and h values for pressure, temp, and TBF were analyzed to evaluate the performance of the CUSUM control charts. The shifts considered were 0.1, 0.2, 0.3, 0.5, and 1.0, while the h values varied from 4 to 7 for pressure and temp data but h values varied for TBF data from 3 to 5. The results showed that the ARL decreased as the shift in mean increased for all variables. This was expected, as larger shifts result in faster detection of out-of-control conditions. However, the ARL also decreased as the h value decreased, indicating that small h values resulted in more sensitive CUSUM charts.

Overall, the results demonstrate the effectiveness of the CUSUM control charts in detecting shifts in the mean for pressure, temp, and TBF. The optimal h value depends on the sensitivity required and the acceptable ARL. The findings provide valuable insights into the selection of appropriate h values for different variables and shifts. The optimal h value and chart type depend on the specific needs and requirements of the process being monitored. However, based on the ARL calculations, it is possible to determine which h values and chart types are more effective at detecting shifts in the mean. For the pressure and temperature data, the ARL values for the CUSUM charts with h values of 4, 5, and 7 were calculated. The results showed that the ARL values decreased as the h value decreased, indicating that smaller h values are more sensitive in detecting shifts in the mean. However, the

ARL values for h=4 were found to be the lowest, indicating that this value may be the most effective in detecting shifts in the mean for the pressure and temperature data and for TBF data the ARL values for the CUSUM charts with h values of 3, 4, and 5 were calculated. The results showed that the ARL values decreased as the h value decreased, indicating that smaller h values are more sensitive in detecting shifts in the mean. However, the ARL values for h=3 were found to be the lowest, indicating that this value may be the most effective in detecting shifts in the target value for TBF data.

In terms of chart type, the CUSUM chart was found to be more effective at detecting smaller shifts in the mean compared to the Shewhart chart. Therefore, for the pressure and temperature data, the CUSUM chart with an h value of 4 may be the optimal choice. It is important to note that the optimal h value and chart type may vary depending on the specific process being monitored and the desired level of sensitivity and false alarm rate. It is recommended to conduct further analysis and simulations to determine the most effective chart type and h value for a specific application.

References

[1] D. C. Montgomery, "The economic design of control charts: A review and literature survey", Journal of Quality Technology, Vol. 12, No. 2, 1980, pp. 75–87, https://doi.org/10.1080/ 00224065.1 980.11980940

[2] S. P. G. F. Lampreia, J. A. M. Dias, V. M. Vairinhos, P. I. S. Barbosa, & J. F. G. Requeijo, "Condition monitoring based on modified CUSUM and EWMA control charts", Journal of Quality in Maintenance Engineering, Vol. 24, No. 1, 2018, pp. 119–132, https://doi.org/10.1108/JQME-08-2013-0057

[3] S. A. Mingoti, "Using geostatistics to estimate the variability of autocorrelated processes", Vol. 2, No. 1, 2005, pp. 39–56.

[4] A. Garg, & S. G. Deshmukh, "Maintenance management: Literature review and directions", Journal of Quality in Maintenance Engineering, Vol. 12, No. 3, 2006, pp. 205–238, https://doi.org/10.1108/13552510610685075

[5] M. Ben-Daya, & S. O. Duffuaa, "Maintenance and quality: The missing link", Journal of Quality in Maintenance Engineering, Vol. 1, No. 1, 1995, pp. 20–26, https://doi.org/ 10.1108/13552519510083110

[6] C. R. Cassady, R. O. Bowden, L. Liew, & E. A. Pohl, "Combining preventive maintenance and statistical process control: A preliminary investigation", IIE Transactions (Institute of Industrial Engineers), Vol. 32, No. 6, 2000, pp. 471–478, https://doi.org/10.1080/07408170008963924

[7] K. Linderman, K. E. McKone-Sweet, & J. C. Anderson, "An integrated systems approach to process control and maintenance", European Journal of Operational Research, Vol. 164, No. 2, 2005, pp. 324–340, https://doi.org/10.1016/ j.ejor.2003.11.026

[8] D. Pandey, M. S. Kulkarni, & P. Vrat, "Joint consideration of production scheduling, maintenance and quality policies: A review and conceptual framework", International Journal of Advanced Operations Management, Vol. 2, No. 1/2, 2010, pp. 1, https://doi.org/10.1504/ijaom.2010.034583

[9] M. A. Rahim, "Economic design of $\bar{X}$ control charts assuming Weibull in-control times", Journal of Quality Technology, Vol. 25, No. 4, 1993, pp. 296–305, https://doi.org/10.1080 /00224065.1993.11979475

[10] M. A. Rahim, "Joint determination of production quantity, inspection schedule, and control chart design", IIE Transactions (Institute of Industrial Engineers), Vol. 26, No. 6, 1994, pp. 2–11, https://doi.org/10.1080/07408179408 966632

[11] M. Ben-Daya, "Integrated production maintenance and quality model for imperfect processes", IIE Transactions (Institute of Industrial Engineers), Vol. 31, No. 6, 1999, pp. 491–501, https://doi.org/10.1080/07408179908969852

[12] M. Ben-Daya, & M. A. Rahim, "Effect of maintenance on the economic design of $\bar{x}$-control chart", European Journal of Operational Research, Vol. 120, No. 1, 2000, pp. 131–143, https://doi.org/10.1016/S0377-2217(98)00379-8

[13] A. J. Duncan, "Charts used to maintain current control of a process", Journal of the American Statistical Association, Vol. 51, No. 274, 1956, pp. 228–242.

[14] H. Wang, "A survey of maintenance policies of deteriorating systems", European Journal of Operational Research, Vol. 139, No. 3, 2002, pp. 469–489, https://doi.org/10.1016/S0377 -2217(01)00197-7

[15] A. M. Jawarneh, F. M. AL-Oqla, & A. Abu Jadoo, "Transient behavior of non-toxic natural and hybrid multi-layer desiccant composite materials for water extraction from atmospheric air", Environmental Science and Pollution Research, Vol. 28, No. 33, 2021, pp. 45609–45618, https://doi.org/10.1007/s11356-021-13995-3

[16] W. Strunk Jr, & E. B. White, The elements of style, 3rd ed., Macmillan, New York, 1979.

[17] B. Akash, S. Odeh, & S. Nijmeh, "Modeling of solar-assisted double-tube evaporator heat pump system under local climate conditions", Proceedings of the 5th Jordanian International Mechanical Engineering Conference, Amman, Jordan, 2004.

[18] G. R. Mettam, & L. B. Adams, "How to prepare an electronic version of your article", in: B. S. Jones, & R. Z. Smith (Eds.), Introduction to the electronic age, E-Publishing Inc, New York, 1999, pp. 281–304.

[19] C.-Y. Chou, C.-H. Chen, & H.-R. Liu, "Economic-statistical design of X¥ charts for non-normal data by considering quality loss", Journal of Applied Statistics, Vol. 27, No. 8, 2000, pp. 939–951, https://doi.org/10.1080/ 026647600501 73274

[20] I. N. Gibra, "Economically optimal determination of the parameters of $\bar{X}$-control chart", Management Science, Vol. 17, No. 9, 1971, pp. 635–646, https://doi.org/10.1287/ mnsc.17.9.635

[21] A. L. Goel, S. C. Jain, & S. M. Wu, "An algorithm for the determination of the economic design of -charts based on Duncan's model", Journal of the American Statistical Association, Vol. 63, No. 321, 1968, pp. 304–320, https://doi. org/ 10.1080/01621459.1968.11009244

[22] H. A. Knappenberger, & A. H. E. Grandage, "Minimum cost quality control tests", AIIE Transactions, Vol. 1, No. 1, 1969, pp. 24–32, https://doi.org/10.1080/05695556908974410

[23] C. Ho, & K. E. Case, "Economic design of control charts: A literature review for 1981-1991", Journal of Quality Technology, Vol. 26, No. 1, 1994, pp. 39–53, https://doi.org/10.1080/00224065.1994.11979497

[24] L. C. Vance, "Bibliography of statistical quality control chart techniques, 1970-1980", Journal of Quality Technology, Vol. 15, No. 2, 1983, pp. 59–62, https://doi.org/10.1080/ 00224065.1983.11978845

[25] W. H. Woodall, "The statistical design of cusum charts", Quality Engineering, Vol. 5, No. 4, 1985, pp. 559–570, https://doi.org/10.1080/08982119308918998

[26] E. M. Saniga, "Economic statistical control-chart designs with an application to x and r charts", Technometrics, Vol. 31, No. 3, 1989, pp. 313–320, https://doi.org/ 10.1080 /00401706.1989.10488554

[27] M. R. Abujiya, M. Riaz, & M. H. Lee, "Enhanced cumulative sum charts for monitoring process dispersion", PLoS ONE, Vol. 10, No. 4, 2015, pp. 1–22, https://doi.org/10.1371/ journal. pone .0124520

[28] G. Tagaras, "An integrated cost model for the joint optimization of process control and maintenance", Journal of the Operational Research Society, Vol. 39, No. 8, 1988, pp. 757–766, https://doi.org/10.1057/jors.1988.131

[29] P. K. Banerjee, & M. A. Rahim, "Economic design of x-control charts under Weibull shock models", Technometrics, Vol. 30, No. 4, 1988, pp. 407–414, https://doi.org/10.1080/00401706.1988.10488435

[30] S. Panagiotidou, & G. Tagaras, "Evaluation of maintenance policies for equipment subject to quality shifts and failures", International Journal of Production Research, Vol. 46, No. 20, 2008, pp. 5761–5779, https://doi.org/10.1080/00207540601182260

[31] S. Panagiotidou, & G. Tagaras, "Optimal preventive maintenance for equipment with two quality states and general failure time distributions", European Journal of Operational Research, Vol. 180, No. 1, 2007, pp. 329–353, https://doi.org/10.1016/j.ejor.2006.04.014

[32] H. N. Chiu, & B. S. Huang, "The economic design of x-control charts under a preventive maintenance policy", International Journal of Quality and Reliability Management, Vol. 13, No. 1, 1996, pp. 61–71, https://doi.org/10.1108/02656719610108323

[33] N. Abbas, M. Riaz, & R. J. M. M. Does, "An EWMA-type control chart for monitoring the process mean using auxiliary information", Communications in Statistics - Theory and Methods, Vol. 43, No. 16, 2014, pp. 3485–3498, https://doi.org/10.1080/03610926.2012.700368

[34] M. Klein, "Two alternatives to the Shewhart $\bar{X}$ control chart", Journal of Quality Technology, Vol. 32, No. 4, 2000, pp. 427–431, https://doi.org/10.1080/00224065.2000.11980028

[35] W. H. Woodall, & B. M. Adams, "The statistical design of cusum charts", Quality Engineering, Vol. 5, No. 4, 1993, pp. 559–570, https://doi.org/10.1080/08982119308918998

[36] M. Pollak, & D. Siegmund, "A diffusion process and its applications to detecting a change in the drift of Brownian motion", Biometrika, Vol. 72, No. 2, 1985, pp. 267–280.

[37] L. C. Alwan, "Designing an effective exponential CUSUM chart without the use of nomographs", Communications in Statistics - Theory and Methods, Vol. 29, No. 12, 2000, pp. 2879–2893, https://doi.org/10.1080/03610920008832642