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# Braking Dynamics of Mechanical Systems with Elastic Links: Knitting Technology as Case Study

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## Abstract

The goal of this paper is to develop a research method for analyzing the braking dynamics of mechanical systems with elastic links. Utilizing modern theoretical research methods, a system model based on the theory of dynamics for mechanical systems with elastic links is presented. Furthermore, a method for studying the braking dynamics of mechanical systems has been developed, enabling the assessment of dynamic loads induced by braking. A case study in knitting technology is employed to verify the proposed method. The findings of this study reveal that the braking of a mechanical system induces dynamic loads in its mechanisms that significantly exceed the loads experienced during start-up. This crucial observation must be taken into account during the design of such machines. Based on these theoretical investigations, an algorithm and an engineering method for determining the maximum dynamic loads that arise in a mechanical system during braking have been developed.

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Keywords: Mechanical system with elastic links, dynamics of braking, dynamic loads, dynamic loads caused by braking.

## 1. Introduction

Dynamic load during braking is a vital factor in machine design, necessitating the development of system models that describe this operation and control its variables. The presence of elastic links and multiple masses within the system introduces further complexities. This study employs a knitting machine as a relevant case study, given its braking system incorporating elastic links. Knitting, a globally popular technique for fabric and garment formation accounting for approximately 25% of fabric production, has experienced rapid advancements due to electronic sensors and computers, propelling knitting technology into the era of Industry 4.0 worldwide [1-4 and various digital web sources].

Extensive research has been conducted on the dynamics of knitting technology, as documented in the literature [e.g., 5-21]. These studies have examined various machine components and proposed methods for reducing dynamic loads. However, a notable shortcoming of existing studies on the dynamics of mechanical systems is their frequent omission of elastic links and multi-mass systems. Furthermore, there is a limited number of studies specifically addressing the braking process, which can impact the overall efficiency of these systems.

The dynamic design of the machine frame is crucial for ensuring operational reliability and product quality. The frame must possess sufficient rigidity to withstand the braking forces encountered during the machine's run-down stage [5].

Enhancing machine efficiency can be achieved by mitigating dynamic loads resulting from braking through improved methods for studying braking dynamics and the development of novel, more reliable braking systems [6]. It is well-established that the magnitude of dynamic loads generated during braking significantly exceeds those experienced during start-up. While numerous dynamics studies exist in the open literature, a few examples warrant mention. The influence of needle design on operational durability has been investigated, leading to the proposal of a new needle design [7 and 8]. Other approaches to improve efficiency and reduce dynamic load include the utilization of twin springs [9] and torsion springs [10]. An analysis of stopping reasons identified causes such as yarn breakages, set-off, machine cleaning and fabric roll cutting, yarn joining, needle breakages, and oil-related issues [11]. Notably, none of the aforementioned dynamics studies provide a comprehensive analysis of the entire system during braking. Therefore, the aim of this research is to present this comprehensive picture.

# 2. System model

Figure (1) shows the suggested model in this study that represents a KO type circular knitting machine [12] and this will be the starting point for the system model. In order to establish the model, some old references have

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been used as they contained the basics of the art [13-21]. Some modern management methods are used worldwide. The enclosure of the designs of several machines, especially light industry machines has been treated as in references[187 and18]. It has been concluded that it is desirable to use a four-mass model with a branching of driven masses as a dynamic braking model. To do this, a dynamic model of a circular machine is taken as an example, the design of which corresponds to these parameters.

As known, the maximum dynamic loads in elastic links and, accordingly, in mechanisms occur at the initial moment of braking of the machine [17], that is, at the first stage of braking. Therefore, further studies of the dynamics of braking of the circular knitting machine will be considered for this stage.

Figure (2) shows an example of a dynamic model representing a circular knitting machine of the type of KO. This figure shows the four stages of braking.  $T_B$  is braking torque applied to the rotor of the motor,  $T_3$  is the moment of the resistance forces of knitting mechanism and  $T_4$  is the moment of the forces of resistance of the fabric take-up mechanism.  $J_1$  is the total moment of inertia of the rotor of the motor and the drive pulley of the belt transmission,  $J_2$  is the total moment of inertia of the driven pulley of the belt transmission (train),  $J_3$  is the moment of inertia of the rotating masses of knitting mechanism, and J<sub>4</sub>is the moment of inertia of the rotating masses of the fabric takeup mechanism. C in the figure represents the stiffness where  $C_{12}$  is the stiffness of the belt transmission of the drive mechanism,  $C_{23}$  is the stiffness of the vertical portion of the drive shaft that transmits the movement to mechanism of knitting, and C24is the stiffness of the vertical portion of the drive shaft that transmits the movement to fabric take-up mechanism.

The initial conditions of the first braking stage are as follows: [17 and 18]:

When t = 0

282

$$T_{(12)0} = T_3 + T_4; \dot{T}_{(12)0} = 0; T_{(23)0} = T_3; \dot{T}_{(23)0} = 0; T_{(24)0} = T_4; \dot{T}_{(24)0} = 0$$

The equations of motion of rotating masses of the machine at the first braking stage (Fig. 2. a) are in the form:

$$J_1 \ddot{\varphi}_1 = T_B + T_{12}; J_2 \ddot{\varphi}_2 = -T_{12} + T_{23} + T_{24}; J_3 \ddot{\varphi}_3$$
  
=  $-T_{23} + T_3; J_4 \ddot{\varphi}_4 = -T_{24} + T_4$  (1)

where  $T_{12}$ ,  $T_{23}$ ,  $T_{24}$ , - the moments that arise when braking with respect to elastic links  $C_{12}$ ,  $C_{23}$ ,  $C_{24}$ ;

$$T_{12} = C_{12}(\varphi_1 - \varphi_2); T_{23} = C_{23}(\varphi_2 - \varphi_3); T_{24} = C_{24}(\varphi_2 - \varphi_4)$$
(2)

 $\varphi_1, \varphi_2, \varphi_3, \varphi_4$  – rotation angles of the respective masses.

Substituting the parameters  $\ddot{\varphi}_1$ ,  $\ddot{\varphi}_2$ ,  $\ddot{\varphi}_3$ ,  $\ddot{\varphi}_4$ , derived from equations (1), in the expressions  $\ddot{T}_{12} = C_{12}(\ddot{\varphi}_1 - \ddot{\varphi}_2), \ddot{T}_{23} = C_{23}(\ddot{\varphi}_2 - \ddot{\varphi}_3), \ddot{T}_{24} = C_{24}(\ddot{\varphi}_2 - \ddot{\varphi}_4)$ , we find:

$$\ddot{T}_{12} = \frac{C_{12}}{J_1 J_2} [(J_1 + J_2)T_{12} - J_1 T_{23} - J_1 T_{24} + J_2 T_B]$$
  
$$\ddot{T}_{23} = \frac{C_{23}}{J_2 J_3} [-J_3 T_{12} + (J_2 + J_3)T_{23} + J_3 T_{24} - J_2 T_3] \qquad (3)$$
  
$$\ddot{T}_{24} = \frac{C_{24}}{J_2 J_4} [-J_4 T_{12} + J_4 T_{23} + (J_2 + J_4)T_{24} - J_2 T_4]$$



**Figure 1.** KO type circular knitting machine[Where: 1 Adjustment Motor, 2 Gearbox, 3 Main Electric Motor, 4 Gear, 5 Impeller, 6 Electric Motor, 7 Driven Pulley, 8 Pulley, 9 Gear, 10 Roller, 11, 12 (pulleys) V-belt transmission, 13 Shaft, 14, 15 Bevel Gears, 16, 17 Gears, 18 Handle, 19 Ratchet Gear, 20 Gear].



**Figure 2.** Dynamic model of circular knitting machines of the type of KO [where (a), (b), (c) and (d) are first, second, third and fourth stages of braking respectively].

The solutions of equations (3)  $T_{ij}$  can be represented by the sum of the general solution of homogeneous equations  $Y_{ij}$  and a solution of inhomogeneous equations  $a_{ij}$ :

$$T_{ij} = Y_{ij} + a_{ij} \tag{4}$$

$$(J_1 + J_2)a_{12} - J_1a_{23} - J_1a_{24} - J_2T_B - J_3a_{12} + (J_2 + J_3)a_{23} + J_3a_{24} = J_2T_3 - J_4a_{12} + J_4a_{23} + (J_2 + J_4)a_{24} = J_2T_4$$
(5)

Then [14 and 15]:

$$a_{12} = \frac{\Delta a_{12}}{\Delta a}; \quad a_{23} = \frac{\Delta a_{23}}{\Delta a}; \quad a_{24} = \frac{\Delta a_{24}}{\Delta a} \tag{6}$$

Where:

$$\Delta a = \begin{vmatrix} (J_1 + J_2) & -J_1 & -J_1 \\ -J_3 & (J_2 + J_3) & J_3 \\ -J_4 & J_4 & (J_2 + J_4) \end{vmatrix}$$
(7)

$$\Delta a_{12} = \begin{vmatrix} -J_2 T_B & -J_1 & -J_1 \\ J_2 T_3 & (J_1 + J_2) & J_3 \\ J_2 T_4 & J_4 & (J_2 + J_4) \end{vmatrix}$$
(8)

$$\Delta a_{23} = \begin{vmatrix} (J_1 + J_2) & -J_2 T_B & -J_1 \\ -J_3 & J_2 T_3 & J_3 \\ -J_4 & J_2 T_4 & (J_2 + J_4) \end{vmatrix}$$
(9)

$$\Delta a_{24} = \begin{vmatrix} (J_1 + J_2) & -J_1 & -J_2 T_B \\ -J_3 & (J_2 + J_3) & J_2 T_3 \\ -J_4 & J_4 & J_2 T_4 \end{vmatrix}$$
(10)

Frequency equation describing the free vibrations of the masses of the system is given by [17]:

$$\beta^{6} - \left(C_{12}\frac{J_{1}+J_{2}}{J_{1}J_{2}} + C_{23}\frac{J_{2}+J_{3}}{J_{2}J_{3}} + C_{24}\frac{J_{2}+J_{4}}{J_{2}J_{4}}\right)\beta^{4} + \left(C_{12}C_{24}\frac{J_{1}+J_{2}+J_{4}}{J_{1}J_{2}J_{4}} + C_{12}C_{23}\frac{J_{1}+J_{2}+J_{3}}{J_{1}J_{2}J_{3}} + C_{23}C_{24}\frac{J_{2}+J_{3}+J_{4}}{J_{2}J_{3}J_{4}}\right)\beta^{4} - \left(C_{12}C_{23}C_{24}\frac{J_{1}+J_{2}+J_{3}+J_{4}}{J_{1}J_{2}J_{3}J_{4}}\right) = 0$$
(11)

Solving equation (11) using the Cardan method [18], we find the frequencies of the vibrations of the masses of the mechanical system  $\beta_1, \beta_2$  and  $\beta_3$ .

Then the solution of equation (3) can be written as:

$$\begin{split} T_{12} &= A_{(12)1} \mathrm{cos} \beta_{1t} + A_{(12)2} \mathrm{cos} \beta_{2t} + A_{(12)3} \mathrm{cos} \beta_{3t} + \\ B_{(12)1} \sin \beta_{1t} + B_{(12)2} \sin \beta_{2t} + B_{(12)3} \sin \beta_{3t} + a_{12}, \end{split}$$

 $T_{23} = A_{(23)1} \cos\beta_{1t} + A_{(23)2} \cos\beta_{2t} + A_{(23)3} \cos\beta_{3t} + B_{(23)1} \sin\beta_{1t} + B_{(23)2} \sin\beta_{2t} + B_{(23)3} \sin\beta_{3t} + a_{23},$ (12)

$$T_{24} = A_{(24)1} \cos\beta_{1t} + A_{(24)2} \cos\beta_{2t} + A_{(24)3} \cos\beta_{3t} + B_{(24)1} \sin\beta_{1t} + B_{(24)2} \sin\beta_{2t} + B_{(24)3} \sin\beta_{3t} + a_{24}$$

The constant integrations *A* and *B* were found using the well-known method [14].

Determining the constant *A* at a cyclic frequency  $\beta_1$  based on equations (3) consists of the following system of equations:

$$-(\beta_{12}^{2} + \beta_{1}^{2})A_{(12)1} + \frac{c_{12}}{J_{2}}A_{(23)1} + \frac{c_{12}}{J_{2}}A_{(24)1} = \frac{c_{12}}{J_{1}}T_{B} - \frac{c_{23}}{J_{2}}A_{(12)1} + (\beta_{23}^{2} + \beta_{1}^{2})A_{(23)1} + \frac{c_{23}}{J_{2}}A_{(24)1} = \frac{c_{23}}{J_{3}}T_{3} - \frac{c_{24}}{J_{2}}A_{(12)1} + \frac{c_{24}}{J_{2}}A_{(23)1} + (\beta_{24}^{2} + \beta_{1}^{2})A_{(24)1} = \frac{c_{24}}{J_{4}}T_{4}$$
(13)  
Where:  

$$\beta_{12}^{2} = \frac{c_{12}(J_{1}+J_{2})}{J_{1}J_{2}}; \ \beta_{23}^{2} = \frac{c_{23}(J_{2}+J_{3})}{J_{2}J_{3}}; \ \beta_{24}^{2} = \frac{c_{24}(J_{2}+J_{4})}{J_{2}J_{4}}$$
(14)

The solution of the system of equations (13) is written as:

$$A_{(12)1} = \frac{\Delta A_{(12)1}}{\Delta A_1}; \ A_{(23)1} = \frac{\Delta A_{(23)1}}{\Delta A_1}; \ A_{(24)1} = \frac{\Delta A_{(24)1}}{\Delta A_1} (15)$$

Where:

$$\Delta A_{1} = \begin{vmatrix} -(\beta_{12}^{2} + \beta_{1}^{2}) & \frac{C_{12}}{J_{2}} & \frac{C_{12}}{J_{2}} \\ -\frac{C_{23}}{J_{2}} & (\beta_{23}^{2} + \beta_{1}^{2}) & \frac{C_{23}}{J_{2}} \\ -\frac{C_{24}}{J_{2}} & \frac{C_{24}}{J_{2}} & (\beta_{24}^{2} + \beta_{1}^{2}) \end{vmatrix}$$
(16)

$$\Delta A_{(12)1} = \begin{vmatrix} \frac{c_{12}}{J_1} T_B & \frac{c_{12}}{J_2} & \frac{c_{12}}{J_2} \\ \frac{c_{23}}{J_3} T_3 & (\beta_{23}^2 + \beta_1^2) & \frac{c_{23}}{J_2} \\ \frac{c_{24}}{J_4} T_4 & \frac{c_{24}}{J_2} & (\beta_{24}^2 + \beta_1^2) \end{vmatrix}$$
(17)

$$\Delta A_{(23)1} = \begin{vmatrix} -(\beta_{12}^2 + \beta_1^2) & \frac{C_{12}}{J_1} T_B & \frac{C_{12}}{J_2} \\ -\frac{C_{23}}{J_2} & \frac{C_{23}}{J_3} T_3 & \frac{C_{23}}{J_2} \\ -\frac{C_{24}}{J_2} & \frac{C_{24}}{J_4} T_4 & (\beta_{24}^2 + \beta_1^2) \end{vmatrix}$$
(18)

$$\Delta A_{(24)1} = \begin{vmatrix} -(\beta_{12}^2 + \beta_1^2) & \frac{c_{12}}{J_2} & \frac{c_{12}}{J_1} T_B \\ -\frac{c_{23}}{J_2} & (\beta_{23}^2 + \beta_1^2) & \frac{c_{23}}{J_3} T_3 \\ -\frac{c_{24}}{J_2} & \frac{c_{24}}{J_4} & \frac{c_{24}}{J_4} T_4 \end{vmatrix}$$
(19)

The system of equations to determine the constants *B* at frequency  $\beta_1$  is [14]:

$$-\frac{\sigma_{23}}{J_2}\beta_1 B_{(12)1} + (\beta_{23}^2 + \beta_1^2)\beta_1 B_{(23)1} + \frac{\sigma_{23}}{J_2}\beta_1 B_{(24)1} = \frac{c_{23}}{J_3}\dot{T}_{(23)0},$$
(20)

$$-\frac{C_{24}}{J_2}\beta_1 B_{(12)1} + \frac{C_{24}}{J_2}\beta_1 B_{(23)1} + (\beta_{24}^2 + \beta_1^2)\beta_1 B_{(24)1}$$
$$= \frac{C_{24}}{J_4}\dot{T}_{(24)0}$$

By replacing  $\beta_1^2$  in equations (13), (20) on  $\beta_2^2$  and  $\beta_3^2$  can derive the system of equations to determine the constants *A* and *B* at cyclic frequencies  $\beta_2$  and  $\beta_3$ , respectively.

Using the solutions to the system of equations (20) Cramer's rule [19], we obtain:

$$B_{(12)1} = \frac{\Delta B_{(12)1}}{\Delta B_1}; \ B_{(23)1} = \frac{\Delta B_{(23)1}}{\Delta B_1}; \ B_{(24)1} = \frac{\Delta B_{(24)1}}{\Delta B_1}$$
(21)

Following the same steps above for finding the solution of equation (13), we obtain:

$$\Delta B_{1} = \begin{vmatrix} -(\beta_{12}^{2} + \beta_{1}^{2})\beta_{1} & \frac{c_{12}}{J_{2}}\beta_{1} & \frac{c_{12}}{J_{2}}\beta_{1} \\ -\frac{c_{23}}{J_{2}}\beta_{1} & (\beta_{23}^{2} + \beta_{1}^{2})\beta_{1} & \frac{c_{23}}{J_{2}}\beta_{1} \\ -\frac{c_{24}}{J_{2}}\beta_{1} & \frac{c_{24}}{J_{2}}\beta_{1} & (\beta_{24}^{2} + \beta_{1}^{2})\beta_{1} \end{vmatrix}$$
(22)

$$\Delta B_{(12)1} = \begin{vmatrix} \frac{c_{12}}{J_1} \dot{T}_{(12)0} & \frac{c_{12}}{J_2} \beta_1 & \frac{c_{12}}{J_2} \beta_1 \\ \frac{c_{23}}{J_3} \dot{T}_{(23)0} & (\beta_{23}^2 + \beta_1^2) \beta_1 & \frac{c_{23}}{J_2} \beta_1 \\ \frac{c_{24}}{J_4} \dot{T}_{(24)0} & \frac{c_{24}}{J_2} \beta_1 & (\beta_{24}^2 + \beta_1^2) \beta_1 \end{vmatrix}$$
(23)

$$\Delta B_{(23)1} = \begin{vmatrix} -(\beta_{12}^2 + \beta_1^2)\beta_1 & \frac{c_{12}}{J_1}\dot{T}_{(12)0} & \frac{c_{12}}{J_2}\beta_1 \\ -\frac{c_{23}}{J_2}\beta_1 & \frac{c_{23}}{J_3}\dot{T}_{(23)0} & \frac{c_{23}}{J_2}\beta_1 \\ -\frac{c_{24}}{J_2}\beta_1 & \frac{c_{24}}{J_4}\dot{T}_{(24)0} & (\beta_{24}^2 + \beta_1^2)\beta_1 \end{vmatrix}$$
(24)

$$\Delta B_{(24)1} = \begin{vmatrix} -(\beta_{12}^2 + \beta_1^2)\beta_1 & \frac{c_{12}}{J_2}\beta_1 & \frac{c_{12}}{J_1}\dot{T}_{(12)0} \\ -\frac{c_{23}}{J_2}\beta_1 & (\beta_{23}^2 + \beta_1^2)\beta_1 & \frac{c_{23}}{J_3}\dot{T}_{(23)0} \\ -\frac{c_{24}}{J_2}\beta_1 & \frac{c_{24}}{J_4}\beta_1 & \frac{c_{24}}{J_4}\dot{T}_{(24)0} \end{vmatrix}$$
(25)

Dynamic overloads arising in the elastic links of the mechanical system during braking are determined in accordance with the equations:

$$K_{12} = \frac{T_{12} \max}{T_3 + T_4}, \ K_{23} = \frac{T_{23} \max}{T_3}, \ K_{24} = \frac{T_{24} \max}{T_4}$$
 (26)

where  $K_{12}$ ,  $K_{23}$ ,  $K_{24}$  coefficients of dynamic overloads of elastic links  $C_{12}$ ,  $C_{23}$ ,  $C_{24}$  of the drive and corresponding mechanisms,  $T_{12 max}$ ,  $T_{23 max}$ ,  $T_{24 max}$ maximum torques arising in the elastic links of the machine mechanisms in Fig. 2. during braking.

Dynamic overloads arising in the elastic links of the mechanical system during braking are determined from the equations:

$$K_{12} = \frac{T_{12 \ max}}{T_3 + T_4}; \ K_{23} = \frac{T_{23 \ max}}{T_3}; \ K_{24} = \frac{T_{24 \ max}}{T_4}$$

284

where  $K_{12}$ ,  $K_{23}$ ,  $K_{24}$  are the coefficients of dynamic overloads of elastic links  $C_{12}$ ,  $C_{23}$ ,  $C_{24}$  of the drive and corresponding mechanisms.

 $T_{12 max}, T_{23 max}, T_{24 max}$  - are the maximum torques arising in the elastic links of the machine mechanisms Fig. 1. during braking.

Using Cardan's method, we find the frequency of vibrations of the system mass [20]. For this purpose, the frequency equation (11) reduces to:

$$x^{3} + bx^{2} + cx + d = 0$$
Where:  

$$x = \beta^{2},$$

$$c = \frac{J_{2} + J_{3}}{2} + c = \frac{J_{2} + J_{3}}{2} + c = \frac{J_{2} + J_{4}}{2}$$
(27)

$$b = -C_{12} \frac{J_{22}}{J_1 J_2} + C_{23} \frac{J_{23}}{J_2 J_3} + C_{24} \frac{J_{24}}{J_2 J_4},$$

$$c = C_{12} C_{24} \frac{J_1 + J_2 + J_4}{J_1 J_2 J_4} + C_{12} C_{23} \frac{J_1 + J_2 + J_3}{J_1 J_2 J_3} + C_{23} C_{24} \frac{J_2 + J_3 + J_4}{J_2 J_3 J_4},$$

$$(28)$$

$$u = -c_{12}c_{23}c_{24} \frac{1}{J_1 J_2 J_3 J_4}$$
.   
By substitution of the unknown in equation (27)

$$z = x + \frac{b}{3}$$
(29)

we obtain the above equation:  $z^3 + nz + a = 0$ 

$$z^3 + pz + q = 0$$
(30)  
where:

$$p = \frac{3c - b^2}{3}; \quad q = \frac{2b^3}{27} - \frac{bc}{3} + d \tag{31}$$

The roots of the equation (30) are in the form:  

$$z_1 = n + \gamma; \ z_2 = \varepsilon_1 n + \varepsilon_2 \gamma; \ z_3 = \varepsilon_1 n - \varepsilon_2 \gamma$$
 (32)  
where:

$$n = \sqrt[3]{-\frac{q}{2} + \sqrt{D}}, \ n = \sqrt[3]{-\frac{q}{2} - \sqrt{D}}, \ D = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2, \ \varepsilon_1 - \varepsilon_2 = \frac{(-1\pm i\sqrt{3})}{2}$$
(33)

If D < 0, the solution of equation (30) can be represented in the form:

$$z_1 = -2R\cos\frac{\varphi}{3}; \ z_2 = -2R\cos\left(\frac{\varphi}{3} + \frac{2\pi}{3}\right); \ z_2 = -2R\cos\left(\frac{\varphi}{3} + \frac{4\pi}{3}\right)$$
where:
$$(34)$$

$$\cos\varphi = \frac{q}{2R^3}; \ R = (\operatorname{sign} q) \sqrt{\frac{|p|}{3}}$$
(35)

Using (28), we find the coefficients of the equation (27):

 $b = -2173136.8; c = 6.1517672 \cdot (10)^{11}; d =$  $-4.1038294 \cdot (10)^{16}$ .

We find the coefficients of equation (31) as:

 $p = -9.589978 \cdot (10)^{11}; \quad q = -3.5561648 \cdot (10)^{17}.$ Then, in accordance with (33):

 $D = -1.0497116 \cdot (10)^{33}$ 

The "-" sign indicates that a cubic equation has three real roots. To determine them, we use the dependence (34), predefined the parameters  $R: \varphi$  from (35):

 $R = -565390.07; \ \varphi = 10.323378^{\circ}$ 

Substituting the values of R and  $\varphi$  in equation (34), we have:

$$z_1 = 1128741.3; \ z_2 = -623150.14; \ z_3 = -505591.21$$

The roots of equation (27), according to (29):

 $x_1 = 1853120.2; \ x_2 = 218787.72; \ x_3 = 101228.79$ We find the frequencies of vibrations of the system masses:

$$\beta_1 = \sqrt{x_1} = 1361.2936; \ \beta_2 = \sqrt{x_2} = 467.7475; \ \beta_3 = \sqrt{x_3} = 318.1647$$

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Define the constant integrations A performing the pre-  
calculations (14):  

$$\beta_{12}^2 = 213681.16; \ \beta_{23}^2 = 349942.86; \ \beta_{24}^2$$
  
 $= 1609512.8$   
Substituting the results of calculations and input data  
according to (16) - (19) after transformations we find:  
 $\Delta A_{(23)1} = -140.83468 \cdot (10)^{17}; \ \Delta A_{(24)1}$   
 $= 57.394406 \cdot (10)^{17}$   
Using (15), we find:  
 $A_{(12)1} = -2.897 \text{ N} \cdot \text{m}; \ A_{(23)1} = 0.938 \text{ N} \cdot \text{m}; \ A_{(24)1}$   
 $= -0.356 \text{ N} \cdot \text{m}$   
Replacing in (16) - (19)  $\beta_1^2$  to  $\beta_2^2$ , we find that:  
 $\Delta A_2 = -2.9012822 \cdot (10)^{17}; \ \Delta A_{(12)2}$   
 $= 46.47211 \cdot (10)^{17}$   
We find the constant integration A at the cycle  
frequency  $\beta_2$ :  
 $A_{(12)2} = -16.018 \text{ N} \cdot \text{m}; \ A_{(23)2} = 1.863 \text{ N} \cdot \text{m}; \ A_{(24)2}$   
 $= -8.565 \text{ N} \cdot \text{m}$   
Replacing in (16) - (19)  $\beta_1^2$  to  $\beta_3^2$ , we find that:  
 $\Delta A_3 = -1.2661792 \cdot (10)^{17}; \ \Delta A_{(12)3}$   
 $= 31.018269 \cdot (10)^{17}$   
We find the constant integration A at the cycle  
frequency  $\beta_3$ :  
 $A_{(12)3} = -0.9782194 \cdot (10)^{17}; \ \Delta A_{(24)3}$   
 $= 17.172189 \cdot (10)^{17}$   
We find the constant integration A at the cycle  
frequency  $\beta_3$ :  
 $A_{(12)3} = -24.497 \text{ N} \cdot \text{m}; \ A_{(23)3} = 0.772 \text{ N} \cdot \text{m}; \ A_{(24)3}$   
 $= -13.562 \text{ N} \cdot \text{m}$ 

Finding the constant integration *B*:

Since the braking process is performed with the initial conditions:

 $\dot{T}_{(12)0} = 0; \dot{T}_{(23)0} = 0; \dot{T}_{(24)0} = 0, \text{ analyzing } (21)$  -(25), we conclude that:

$$B_{(12)1} = 0; \ B_{(23)1} = 0; \ B_{(24)1} = 0$$
  
Similarly:

$$B_{(12)2} = 0; B_{(23)2} = 0; B_{(24)2} = 0; B_{(12)3} = 0; B_{(23)3} = 0; B_{(24)3} = 0$$

Substituting the results in equations (12), we have:  $T_{12} = -2.897 \cos 1361.3t$ 

$$-16.018 \cos 467.7t$$

$$-24.497 \cos 318.1t - 46.43$$

$$T_{23} = 0.938 \cos 1361.3t$$

$$-1.863 \cos 467.7t$$

$$+ 0.772 \cos 318.1t - 5.51$$

$$T_{24} = -0.356 \cos 1361.3t$$

$$- 8.565 \cos 467.7t$$

$$- 13.562 \cos 318.1t - 24.34$$
An analysis of equations (12) and the results for

alysis of equations (12) and the resultsshow that the maximum value of the dynamic loads arising during braking in mechanisms of circular machine KO2 can be determined from the conditions:

$$T_{12max} = |A_{(12)1}| + |A_{(12)2}| + |A_{(12)3}| + |a_{12}|;$$
  

$$T_{23max} = |A_{(23)1}| + |A_{(23)2}| + |A_{(23)3}| + |a_{23}|;$$
  

$$T_{24max} = |A_{(24)1}| + |A_{(24)2}| + |A_{(24)3}| + |a_{24}|$$
  
Then:

 $T_{12max} = 89.842 \text{ N} \cdot \text{m}; \ T_{23max} = 9.083 \text{ N} \cdot \text{m}; \ T_{24max}$  $= 46.823 \text{ N} \cdot \text{m}$ 

The coefficients of dynamic overloads of the elastic links of the circular knitting machine mechanisms, using (26), results in:

 $K_{12} = 4.06; K_{23} = 0.51; K_{24} = 10.64.$ 

### 3. Conclusions and future work

A system model for the dynamic load during the braking of a KO-type circular knitting machine is presented. This model incorporates the effects of elastic links. The proposed method for studying the braking dynamics of mechanical systems with elastic links enables the determination of the maximum dynamic loads arising within machine mechanisms during braking. as demonstrated in the preceding section. The coefficients of dynamic overload for the elastic links in the circular knitting machine mechanisms were determined. The study revealed that during the braking of mechanical systems, dynamic loads significantly surpass those encountered during the start-up phase, a crucial factor that must be considered in machine design. The findings of this research hold potential for future applications in the development of novel machines and mechanisms.

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