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Selection of Phase Change Materials for Energy Storage Applications Using BHARAT Decision-Making Methodology

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Abstract

Phase change materials have been studied for many applications aiming at enhancing energy efficiency and economy. Nevertheless, in order to select the right phase change material (PCM) to meet the required standards, a trade-off between competing quantitative and qualitative criteria (also referred to as attributes) is usually required. The PCM selection literature analysis reveals that researchers employed many multi-attribute decision-making (MADM) methods. However, these MADM methods have their merits as well as demerits. Hence, this study offers a simple and efficient MADM method named"Best Holistic Adaptable Ranking of Attributes Technique (BHARAT)" to select the optimal PCM for various energy storage applications. The BHARAT method is illustrated with three case studies. In the first case study, the best PCM for a thermal energy storage unit with a solar box cooker is chosen by considering 5 PCMs and 8 attributes; in the second case study, the best PCM for a ground source heat pump integrated with a phase change thermal storage system is chosen by considering 8 PCMs and 13 attributes; In the third case study, 20 PCMs and 5 attributes are considered in order to determine which PCM is optimal for energy storage and thermal comfort in a vehicle. Comparisons are made between the results of the BHARAT method and those of other popular MADM methods. The BHARAT method's potential has been thoroughly proved and validated by the results of the three PCM selection case studies. It has been demonstrated that the BHARAT methodis straightforward, simple to implement, free of fuzzy logic requirements, provides a logical method for assigning attributes' weights, and is adaptable to PCM selection problems in various scenarios.

Keywords: PCM selection; Multi-attribute decision-making; BHARAT; Total scores.

Abbreviations

AHP	Analytic Hierarchy Process
BHARAT	Best Holistic Adaptable Ranking of Attributes Technique
BWM	Best-Worst Method
C	Cost price
CoCoSo	Combined Compromise Solution
COPRAS	Complex Proportional Assessment
Cpl	Specific heat for liquid-state
Cps	Specific heat for solid-state
CRITIC	CRiteria Importance Through Intercriteria Correlation
EDAS	Evaluation based on Distance from Average Solution
EXPROM2	EXtended PROMETHEE
F	Flammability
GSHP	Ground Source Heat Pump
ITARA	Indifference Threshold-based Attribute Ratio Analysis
k	Thermal conductivity
Ks	Thermal conductivity for solid-state
L	Latent heat of transition
LH	Latent heat of fusion
MADM	Multi-Attribute Decision-Making
MEREC	MEthod based on the Removal Effect of Criteria
MOORA	Multi-objective Optimization Of Ratio Analysis
MULTIMOORA	Multi-objective Optimization Of Ratio Analysis with MULTIplicative form
PCM	Phase Change Material
PCTS	Phase Change Thermal Storage
PROMETHEE	Preference Ranking Organization METHod for Enrichment Evaluations
PS	Phase Separation
R	Recycle
SC	Supercooling
T	Toxicity
TS	Thermal stability
V	Volume change
VIKOR	VIšekriterijumsko KOmpromisno Rangiranje
VP	Vapor Pressure
WASPAS	Weighted Aggregated Sum Product ASsessment

Greek symbols

ρ	Density
ρs	Density for solid-state
ρl	Density for liquid-state

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1. Introduction

The performance of the PCM has a major impact on efficient thermal energy storage. As such, choosing the optimal PCM for a given application is not as simple as it might seem. A number of requirements should be satisfied, including the thermal properties (such as specific heat, thermal stability, latent heat of transition, thermal conductivity, etc.), physical properties (such as volume change, density, vapor pressure, etc.), kinetic properties (such as supercooling, phase separation, etc.), chemical properties (such as recycle, toxicity, flammability, etc.), economic performance (such as cost), and certain managerial considerations. It could be difficult to select the ideal PCM for a given application because there is a vast array of PCM materials with different types and characteristics available. No PCM can have every desirable property, such as a high specific heat and high latent heat for high storage capacity, a proper melting point that falls within the storage system's operating range, sufficient thermal conductivity to allow for proper storage operations, available at low price, etc.

The selection of PCM was based on experience in most research studies, with data or physical availability of the material being taken into consideration on occasion. However, this approach is erroneous. Hence, researchers have developed a precise and dependable methodology for choosing the best PCM for a particular application using a methodology known as multi-attribute decision-making (MADM). Many MADM methods are available in literature and are used for different applications [1-6]. The MADM methods can play a significant role in the advancement of thermal energy storage by assisting in choosing the best PCM from a group of PCMs for the best storage performance.The decision-maker, on the other hand, weighs the attributes according to his/her knowledge and professional judgment about the relevance of attributes for the given application.

For the past ten years, researchers have been using various MADM methods to address PCM selection issues for various applications. Kulish et al. [7] useda PCM selection method, based on computing the Rényi entropy for a set of attributes. Rastogi et al. [8] used the entropy method to get the attributes' objective weights and used the TOPSIS method for optimum PCM selection for heating, ventilation, and air-conditioning applications. Socaciu et al. [9] used the AHP method for PCM selection. Loganathan and Mani [10] used fuzzy AHP method to get the weights and used those weights in TOPSIS, VIKOR (višekriterijumsko kompromisno rangiranje) method, and PROMETHEE (preference ranking organization method for enrichment evaluations) method for PCM selection in an electronic cooling system. However, it may be noted that the fuzzy logic uses various functions and defuzzification methods and different results may be produced. Fuzzy numbers are manipulated in a way that not only complicates the process but also detracts from the original numbers' elegant and straightforward representation of the judgments.It's possible that fuzzifying the inconsistent decisions will make things worse rather than better [11].

Yang et al. [12] used the entropy method to get the objective weights, the AHP method to get the subjective weights, and combined these weights into composite weights to use in the TOPSIS methodfor choosing the best PCM for a thermal storage system combined with a ground source heat pump. Nadeem et al. [13] used the AHP method for ranking the PCMs. Amer et al. [14] employed the AHP method to select the best PCM for solar energy storage. Oluah et al. [15] used the TOPSIS method with the objective weights given by the entropy method to improve the Trombe wall system's performance, including PCMs. In order to select the best PCM for interior building surface applications, Maghsoodi et al. [16] employed the best-worst method (BWM) to get the subjective weights and then combined an interval-valued structural approach with the CoCoSo (combined compromise solution) and MULTIMOORA (multi-objective optimization of ratio analysis with multiplicative form) methods.

Anilkumar et al. [17] used entropy and CRITIC (criteria importance through intercriteria correlation) methods to obtain the attributes' objective weights, AHP method to obtain subjective weights, and then used these weights as well as the combined weights in TOPSIS, MOORA, and EDAS (evaluation based on distance from average solution) methods to choose the best PCM for a solar cooker that incorporates a thermal energy storage device. Das et al. [18] used entropy based objective weights of attributes and the TOPSIS method for PCM selection and passive thermal management. Kumar et al. [19] used TOPSIS method for selection of PCM for thermal management of electronic devices. Mukhamet et al. [20] used TOPSIS method for PCM selection for buildings.

Hamdan et al. [21] used PCMs in experiments to cool photovoltaic (PV) panels in order to increase their efficiencyAfter analyzing the stored data, it was discovered that the PCM-cooled PV panel outperformed the regular panel by 2.6%. Al-Maghalseh [22] provided four-dimensional models in order to simulate a Latent Heat Thermal Energy Storage System. The system consisted of a rectangular container with a horizontal pipe in the middle encircling paraffin wax, which was a PCM with a melting point of 600^0C. The ANSYS/FLUENT simulations yielded data on the distribution of instantaneous temperatures, the dynamics of solidification and melting, and the field of velocities within the storage unit during the melting procedure.

Ababneh et al. [23] presented a novel approach to thermal energy storage in solid materials, such as Li_2SO_4, by utilizing the phase-to-phase change principle. This allowed the material to remain solid at temperatures above 500°C. The heat transfer fluid used in this process was sodium-potassium eutectic alloy NaK. The analysis showed that the solid storage material largely stayed within a small temperature range during the energy storage process, which was best represented as a temperature step wave moving through the storage medium at a nearly constant speed.

Nijmeha et al. [24] evaluated the application of PCMs in the cooling and thermal control of photovoltaic (PV) panels, both technically and economically. The technical analysis was based on experimental testing that was done at Hashemite University in Jordan for a full year on two identical 3.99 kWp PV systems.

AL-Migdady et al. [25] carried out numerical simulations to investigate the cooling behaviour of aluminum foam-integrated PCM-based heat sinks. Keeping the heat flux input constant, the performance was investigated under various operating parameters such as three percentages of metal foam porosity, two PCMs, and three values of convective heat transfer coefficient. The heat sink that was filled with RT35HC showed better cooling performance when compared to one that was based on RT44HC.

Sadiq et al. [26] constructed a latent heat thermal energy storage system of horizontal shell-and-tube. Two cases of paraffin wax with different thermal conductivities were used as PCMs. The effect of thermal conductivity on the thermal performance of thermal energy during the solidification process was investigated experimentally.

Nicolalde et al. [27] used entropy method and a method based on the removal effect of criteria [MEREC] for getting the objective weights of attributes and then used those weights in TOPSIS, VIKOR, and COPRAS (complex proportional assessment) methods and chose saveENRG PCM-HS22P for energy storage related to thermal comfort in a vehicle.Pradeep and Reddy [28] obtained the weights of attributes using the ITARA method and then used those weights in the TOPSIS method for choosing a PCM-based filler for a thermal energy storage system. Akgun et al. [29] used subjective and objective weights of the attributes in MOORA and WASPAS (weighted aggregated sum product assessment) methodsto select carbon-based nanomaterials in PCMs.

Yang et al. [30] used the weights obtained by range analysis in the TOPSIS method for PCM selection for a triple tube heat exchanger unit at different time scales. Gadhave et al. [31] used the entropy method for getting the objective weights, the AHP method for getting the subjective weights, and combined these weights of attributes to use in TOPSIS, VIKOR, and EXPROM2 (a version of PROMETHEE method) to select a PCM for a domestic water heating system. Rao [32] used an effective decision-making method for PCM selection. Ali et al. [33] briefly reviewed the MADM methods used for optimum PCM selection.

The PCM selection literature analysis reveals that many MADM approaches, including TOPSIS, VIKOR, MOORA, MULTIMOORA, COPRAS, WASPAS, PROMETHEE, EXPROM2, EDAS, CoCoSo, and WPM, were employed by the researchers. The AHP method, entropy method, CRITIC method, compromise weights approach, BWM, MEREC, etc., have been used for getting the attributes’ weights, and those weights are utilized in the MADM methods. Additionally, fuzzy scales are employed to translate qualitative attributes into quantitative ones. It is noted that the TOPSIS method is the one that researchers utilized most frequently to choose PCM.

The MADM methods mentioned above are effective in various decision-making situations. However, these methods have merits and demerits[34, 35]. Research should create simple and powerful MADM methods that may provide dependable and effective solutions to difficult PCM selection problems using a wide range of alternative PCMs and attributes. Moreover, the development of such simple and approachable methods allows for quick decision-making and can be utilized in various decision situations. They can also handle qualitative attributes, imprecise data, and decision-makers with different levels of information processing proficiency. The first author of this paper has recently proposed a powerful MADM method named BHARAT [28,29]. This paper attempts to extend the BHARAT method for the best PCM selection for a given energy storage application. Three distinct thermal energy storage case studies employing the BHARAT decision-making method for PCM selection are presented. The proposed method's outcomes are compared with those of other popular MADM methods. The important features of the proposed BHARAT method are given below.

- It is simple to understand, easy to implement, and useful for evaluating the performance of PCMs and, thereby, for choosing the optimum alternative PCM for different energy storage applications.
- It offers a logical way of assigning attributes’ weights and proves that objective weights need not be used.
- It can convert qualitative information about the attributes into quantitative without the need for fuzzy scales.
- It computes the positions of alternatives with respect to the best value of each attribute. This is a more accurate assessment of an alternative's relative position to the best alternative that corresponds to an attribute.
- It provides a general approach to decision-making that may be used for a variety of selection problems with several attributes and alternatives.

The BHARAT methodology is explained in the next section.

2. Multi-attribute decision-making methodology of BHARAT for PCM selection

The steps are described below:

- **Step 1:** Identify the relevant PCM selection attributes *Ai* (*i = 1, 2,... m*), and the alternative PCMs *Bj* (for *j = 1, 2,... n*). The attributes can be either non-beneficial or beneficial. Beneficial attributes should have higher values, whereas non-beneficial attributes should have lower values.
- **Step 2:** The decision-makers evaluation of each attribute's relevance in terms of 1, 2, 3, 4, and so on should be used to order the attributes in order to establish the weights w_i (for i=1, 2,..., m). The proposed BHARAT approach adopts the R-method, which was recently developed [26]. The computation of the attributes' weights is demonstrated below, for example, if the ranks of 1, 2, and 3 are given to three attributes P, Q, and R, the weights are assigned as explained below.

For 3-attributes:

Inverse of inverse of rank 1: 1/ (1/1) = 1.000000

Inverse of sum of inverses of ranks up to 2: 1/(1/1 + 1/2) = 0.666666

Inverse of sum of inverses of ranks up to 3: 1/(1/1 + 1/2 + 1/3) = 0.545454

Grandsum = 1.000000 + 0.666666 + 0.545454 = 2.212121

Hence, the weights of ranks 1, 2, and 3 are 0.45205 (=1.000000/2.212121), 0.30137 (=0.666666/2.212121), and 0.24657 (=0.545454/2.212121), respectively.

As an additional example, suppose the decision-maker assigns the ranks of 1, 2, 3, and 4 to four attributes P, Q, R, and S. In such a case, the weights of the attributes are calculated as follows.

For 4-attributes:

Inverse of inverseof rank 1: 1/ (1/1) = 1.000000

Inverse of sum of inverses of ranks up to 2: 1/(1/1 + 1/2) = 0.666666

Inverse of sum of inverses of ranks up to 3: 1/(1/1 + 1/2 + 1/3) = 0.545454

Inverse of sum of inverses of ranks up to 4: 1/(1/1 + 1/2 + 1/3 + 1/4) = 0.48

Grand sum = 1.000000 + 0.666666 + 0.545454 + 0.48 =2.69212

Hence, the weights of 0.37145 (=1.000000/2.69212), 0.24763 (=0.666666/2.69212), 0.20261 (=0.545454/2.69212), and 0.17829 (=0.48/2.69212) are allocated to ranks 1, 2, 3, and 4, respectively.

Table A of the Appendix shows the 35 ranks of the attributes and the associated weights. Several attributes can be added to this. The weights for any number of ranks can be assigned using Eq. (1) [36].

$$w_i = \frac{\left[1 / \sum_{k=1}^{i} (1/r_k) \right]}{\sum_{i=1}^{mi} \left[1/ \sum_{k=1}^{i} (1/r_k) \right]} \quad (1)$$

w_i = i^{th}attribute’s weight (i = 1, 2, ….., m)

r_k= k^{th}attribute’s rank (k = 1, 2, ….., i)

m = no. of attributes

The decision-maker can directly give the weights to the attributes by using Table A. In cases where two or more attributes are deemed equally important, an average rank will be assigned. For example, suppose the decision-maker gives rank 1 to attribute P out of four attributes. If the decision-maker believes that Q and R are equally significant, then both can be given an average rank of 2.5 or (i.e., (2+3)/2). Rank 4 can be awarded to the attribute S. The attributes P, Q, R, and S are then given the following weights from Table A: 0.37145, 0.22512, 0.22512, and 0.17829, in that order. It should be mentioned that Q and R have an average weight of 0.22512 (i.e., (0.24763/0.20261)/2).

- **Step 3:** For every alternative, calculate the attribute performance V_{ji} (performances can be qualitative or quantitative). Translate the descriptive language used to describe the attributes (the qualitative data) into quantitative data. Use a basic ordinary scale to translate the qualitative attributes data into numerical data rather than a fuzzy scale. Rao [28, 29] showed that fuzzy scales are not necessary because normal basic scales can achieve the same objectives as fuzzy ones. Simple conventional scales can readily replace fuzzy scales created by researchers to address linguistic or qualitative attributes using distinct membership functions. Table 1 shows how an 11-point rating scale can be used to translate a verbal or qualitative attribute into a numerical value.
- **Step 4:** Normalization of the performance measurements of alternatives Vji (where j = 1, 2,... n and i = 1, 2,... m) is necessary. An attribute’s value is normalized by comparing it with the "best" value of that attribute across a range of alternatives. The normalization process is to be done for each attribute to acquire the normalized values. "Best" refers to the highest value available for a beneficial attribute and the lowest value for a non-beneficial attribute. The normalized value (Vji)normalized is Vji/Vi.bestfor a beneficial attribute, and for a non-beneficial attribute,it is Vi.best/Vji, where Vi.best is the best value for the ithattribute.
- **Step 5:** An alternative's total score is equal to ∑ wi*(Vji)normalized which is obtained by multiplying the attribute weights with the normalized attribute valuesof the alternatives.The total scores of alternatives can be computed in this way.
- **Step 6:** Sort the PCMS in descending order of the total score values. The PCM that comes out on top overall for the specific selection problem under investigation is the one that is to be chosen.

Fig. 1 depicts the flowchart of the BHARAT method.

Table 1. Translation of a qualitative attribute on an 11-point scale into a quantitative one [34, 35]

Linguistic or qualitative expression	Fuzzy scale value for a beneficial attribute [6]	Fuzzy scale value for a non-beneficial attribute [6]	Simple scale value for a beneficial attribute	Simple scale value for a non-beneficial attribute
Exceptionally low (or a similar expression)	0.0455	0.9545	0.0	1.0
Extremely low (or a similar expression)	0.1364	0.8636	0.1	0.9
Very low (or a similar expression)	0.2273	0.7727	0.2	0.8
Low (or a similar expression)	0.3182	0.6818	0.3	0.7
Below average (or a similar expression)	0.4091	0.5909	0.4	0.6
Average (or a similar expression)	0.5	0.5	0.5	0.5
Above average (or a similar expression)	0.5909	0.4091	0.6	0.4
High (or a similar expression)	0.6818	0.3182	0.7	0.3
Very high (or a similar expression)	0.7727	0.2273	0.8	0.2
Extremely high (or a similar expression)	0.8636	0.1364	0.9	0.1
Exceptionally high (or a similar expression)	0.9545	0.0455	1.0	0

The effectiveness of the suggested BHARAT method is briefly illustrated by three case studies of PCM selection for different applications in the following section.

3. Applications of BHARATmethod to the case studies of phase change material selection

3.1. Case study 1: PCM selection for a solar box cooker's integrated thermal energy storage unit

Anilkumar et al. [17] presented a case study to select the optimal PCM from the available options for a thermal energy storage (TES) unit integrated into a solar box cooker (SBC). There are two types of SBCs with TES unit designs. The first design type works by utilizing the heat energy from the TES materials positioned below the absorber plate. Rather than a cooker, the second design type integrates a TES unit with a cooking pot. The cooking pot with the integrated TES system, shown in Figure 4, has two concentric cylindrical vessels with PCM filled in the annular space. D_i and $D_0$are the inner and outer diameters of the TEC unit, $d_i$and $d_0$are the inner and outer diameters of the cooking pot, and L is the height of the TES unit.

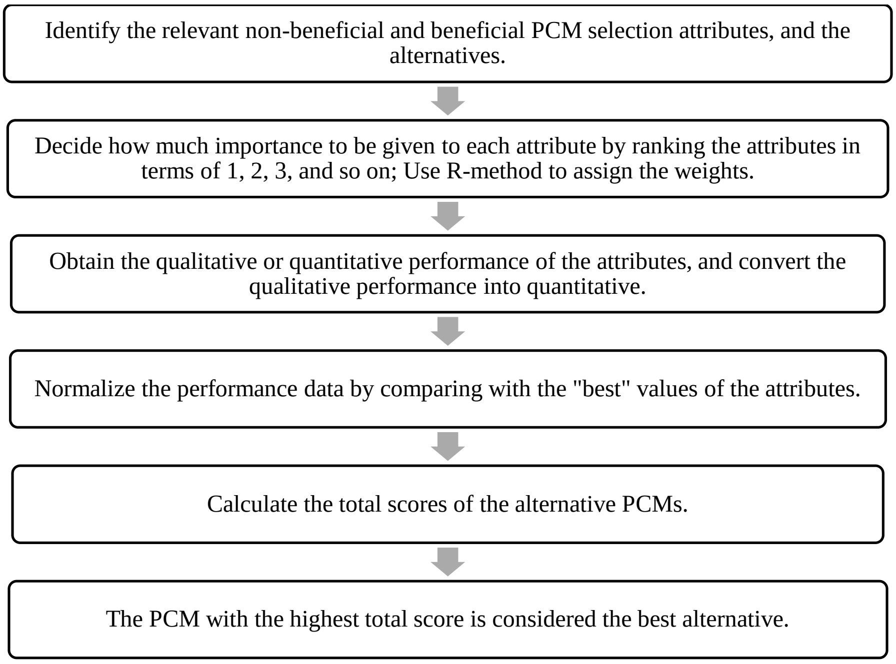

Figure 1. Flowchart of BHARAT methodology.

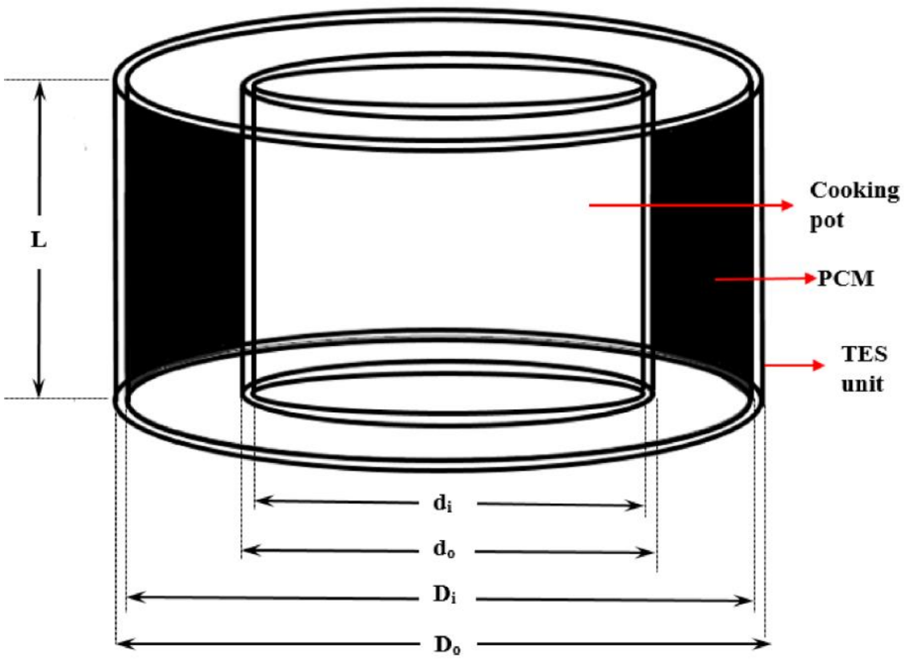

Figure 2. A cooking vessel surrounded by TES unit [11]

The decision-making problem considered 5 alternative PCMs analyzed under 8 attributes. The alternative PCMs were: Acetanilide, Erythritol, Paraffin wax, Magnesium chloride hexahydrate, and Oxalic acid dihydrate. The attributes are the material properties such as latent heat of fusion (LH), density for solid-state (ρs), density for liquid-state (ρl), specific heat for the solid-state (Cps), specific heat for liquid-state (Cpl), thermal conductivity for solid-state (Ks), thermal stability (TS) and cost price (C).

- **Step 1:** The alternative PCMs and the attributes considered are the same as those considered by Anilkumar et al. [17] and the related data is shown in Table 5. The attributes are the material properties such as LH, ρs, ρl, Cps, Cpl, Ks, TS, and C. Except C, all other attributes are beneficial. The thermal stability (TS) and cost (C) are expressed linguistically. The numbers in parentheses in Table 2 indicate the proper quantitative values, which are allocated using a simple 11-point scale, given their nature. Table 1 is used for the translation of TS and C.
- **Step 2:** The ranks 1-8 are given to LH, Ks, ρl, TS, ρs, Cpl, C, and Cps, respectively. Table 3shows the ranks and the weights of the 8 attributes taken from Table A, and the best values of the attributes.
  LH is given a rank of 1, and hence, the weight is assigned as 0.23299 using Table A corresponding to 8 attributes. The attribute ρs is given rank 5, and hence, the weight is assigned from Table A as 0.10204. The remaining attributes are also given weights according to their ranks using Table A.
- **Step 3:** Table 1 is used to convert the qualitative expressions of TS and C into quantitative values without the use of fuzzy logic. Following this assignment, the values given for C can be regarded as beneficial for the purpose of computing the ratios. These values are shown in parentheses in the last rows of Table 2.
- **Step 4:** The "best" PCM for each attribute is used to normalize the data. The attributes' best values are shown in the last row of Table 3. Table 4shows the normalized values of the 8 attributes. This type of normalization makes it evident where the alternatives stand with respect to the attributes' "best" values.
- **Step 5:** Total scores of the PCMs are calculated by multiplying the weights of the attributes listed in Table 3with the associated normalized values **for** the PCMs listed in Table 4.For instance, the total score for Acetanilide is calculated as follows:
  Total score (Acetanilide) = 0.23299*0.6 + 0.10204*0.73333 + 0.12709*0.6375 + 0.08573*1 + 0.09510*0.70922 + 0.15533*0.68213 + 0.11183*0.625 + 0.08986*0.71428 = 0.688868.
  Similarly, the total scores for the other PCMs are computed as, Acetanilide: 0.688868; Erythritol: 0.852197; Paraffin wax: 0.608042; Magnesium chloride hexahydrate: 0.71372; Oxalic acid dihydrate: 0.800279.
- **Step 6:** The PCMs are sorted in the descending order of the total scores.

Erythritol - Oxalic acid dihydrate - Magnesium chloride hexahydrate - Acetanilide - Paraffin wax.

Erythritol which has the highest total score is regarded as the best PCM for this case study 1.

Table 2. Information about the 8 attributes and 5 alternative PCMs of case study [17]

S. No.	Attributes (Properties of PCMs)	Alternative PCMs				
		Acetanilide	Erythritol	Paraffin wax	Magnesium chloride hexahydrate	Oxalic acid dihydrate
1	LH (kJ/kg)	222	339	140	167	370
2	ρs (kg/m^3)	1210	1480	880	1569	1650
3	ρl (kg/m^3)	1020	1300	770	1450	1600
4	Cps (kJ/kg.K)	2	1.38	1.8	1.72	1.62
5	Cpl (kJ/kg.K)	2	2.76	2.4	2.82	1.62
6	Ks (W/m.K)	0.5	0.733	0.21	0.694	0.57
7	TS	A (0.5)	H (0.7)	VH (0.8)	L (0.3)	L (0.3)
8	C	A (0.5)	H (0.3)	L (0.7)	H (0.3)	AA (0.4)

L: low; A: average; AA: above average; H: high; VH: very high

Table 3. Ranks and matching weights for 8 attributes of case study 1

	Attributes							
	LH	ρs	ρl	Cps	Cpl	Ks	TS	Cost
Ranks	1	5	3	8	6	2	4	7
Weights	0.23299	0.10204	0.12709	0.08573	0.09510	0.15533	0.11183	0.08986
Best values	370	1650	1600	2	2.82	0.733	0.8	0.7

Table 4. Normalized valuesof case study 1

S. No.	Attributes (Properties of PMCs)	Normalized values				
		Acetanilide	Erythritol	Paraffin wax	Magnesium chloride hexahydrate	Oxalic acid dihydrate
1	LH (kJ/kg)	0.6	0.91622	0.37838	0.45135	1
2	ρs (kg/m^3)	0.73333	0.89697	0.53333	0.95091	1
3	ρl (kg/m^3)	0.6375	0.8125	0.48125	0.90625	1
4	Cps (kJ/kg.K)	1	0.69	0.9	0.86	0.81
5	Cpl (kJ/kg.K)	0.70922	0.97872	0.85106	1	0.57447
6	Ks (W/m.K)	0.68213	1	0.28649	0.94679	0.77763
7	TS	0.625	0.875	1	0.375	0.375
8	C	0.71428	0.42857	1	0.42857	0.57143

Anilkumar et al. [17]used AHP method, entropy method, and CRITIC method for obtaining the attributes' weightsand finally combined those weights to get the compromised weights of 0.548, 0.056, 0.074, 0.008, 0.021, 0.196, 0.063, and 0.015 for LH, ρs, ρl, Cps, Cpl, Ks, TS, and C respectively. Using the weights obtained by AHP method, entropy method, CRITIC method, and the compromise weights, the authors applied the MADM methods of TOPSIS, EDAS, and MOORA to calculate the scores of PCMs and thereby to select an optimum.However, the compromise weights used by Anilkumar et al. [17] were different from the weights used in BHARAT method. Hence, for a fair comparison, the compromise weights used by Anilkumar et al. [17] are used in the BHARAT method also now, and Table 5 shows the ranking of the alternative PCMs.

All the methods suggested Erythritol as the best choice. It is clear that the BHARAT method also suggested the same ranking of PCMs and proposed Erythritol as the best choice, using the same compromise weights as those used in TOPSIS, EDAS, and MOORA. The BHARAT method involved a simple normalization procedure, and the total scores of PCMs are computed by multiplying the normalized values with the attributes' weights (assigned using Table A or the weights used by Anilkumar et al. [17] for fair comparison purpose).

It may also be seen that when the compromise weights used by Anilkumar et al. [17] are used in BHARAT for fair comparison, it has suggested the same Erythritol is the best choice. It is to be noted here that the TOPSIS, EDAS, and MOORA methods involve too lengthy calculations for normalization, calculating the subjective weights using AHP, calculating the objective weights using entropy method and CRITIC method, then calculating the compromise weights, and then using those weights in the computationally intensive steps of TOPSIS, EDAS, and MOORA methods.In contrast to these methods, the normalization process of the suggested BHARAT method is simple to comprehend. The BHARAT method eliminates the need for a fuzzy scale, unlike the approach of Anilkumar et al. [17], and facilitates the translation of qualitative attributes into quantitative data. The BHARAT method permits the use of attribute weights determined by the decision-maker using experience or intuition, or weights determined by other means, as demonstrated in this case study.

It can be observed from this case study 1 that the BHARAT method has given the same rankings of PCMs as those given by TOPSIS, EDAS, and MOORA when the same compromise weights of attributes are used. Without these compromise weights also, the BHARAT method has its own procedure, which is very simple and straightforward compared to the laborious computations involved in TOPSIS, EDAS, and MOORA methods.

3.2. Case study 2: PCM selection for a phase change thermal storage system combined with a ground source heat pump

Yang et al. [12] presented a case study to select an optimum PCM for a phase change thermal storage (PCTS) system combined with a ground source heat pump (GSHP). A university in Tianjin used a GSHP system in conjunction with a PCTS system to handle the cooling and heating demands of its library. When choosing PCM, the authors took into account the attributes that are thermal, physical, kinetic, chemical, and economical. Fig. 3 shows the GSHP with the PCTS system. The working modes of GSHP are described below.

- Heating supply and storage mode in parallel: The GSHP system runs at its highest demand of the month during the valley electricity pricing period, storing any extra heat in the PCTS device. In Fig. 3, the other valves are closed while the V1, V2, V4, V5, and V7 valves are open in this state.
- Mode of PCTS priority: Heating is provided by the PCTS first (V1, V2, and V5 valves are closed, while the other valves in Fig. 3) and subsequently by the GSHP unit following the completion of the PCTS discharge (V1, V2, and V7) during the peak power pricing period.

The decision-making problem considered 8 alternative PCMs analyzed under 13 attributes. The PCMS considered were: Paraffin wax $C_{31}H_{64}$, Paraffin wax $C_{32}H_{66}$, Paraffin wax $C_{33}H_{68}$, Paraffin wax $C_{34}H_{70}$, Stearic acid $CH_3(CH_2)_{16}$ COOH, Salt hydrate $Ba(OH)_2{\cdot}8H_2O$, Eutectic $LiNO_3$ (14%)-$MgNO_3{\cdot}6H_2O$ (86%), and Eutectic Urea (82%) +$LiNO_3$ (18%). These 8 PCMS are denoted by M1, M2, M3, M4, M5, M6, M7, and M8 respectively. Now, the steps of the proposed BHARAT method are followed to choose the best PCM among the 8 PCMs.

- **Step 1:** Table 6 presents the data, which is the same as that presented by Yang et al. [12]. These are: thermal properties (latent heat of transition (L), thermal conductivity (K), specific heat for solid (Cps), and specific heat for liquid (Cpl)), physical properties (density (ρ), volume change (V), and vapor pressure (VP)), kinetic properties (supercooling (SC) and phase separation (PS)), chemical properties (recycle (R), toxicity (T), and flammability (F)), and economic property (cost PRICE (C)).The attributes V, VP, SC, PS, T, F, and C are non-beneficial. The attributes V, VP, SC, PS, R, T, and F are expressed linguistically. In Table 6, the relevant quantitative values are assigned using 11-point scale, as indicated by the numbers in parentheses.Table 1 is used for the appropriate transformation of R and V, VP, SC, PS, T, and F.

Table 5. Ranks of the 5 PCMs obtained by using different decision-making methods

PCM	Ranks given by different decision-making methods				
	TOPSIS*	EDAS*	MOORA*	BHARAT*	BHARAT**
Erythritol	1	1	1	1	1
Oxalic acid dihydrate	2	2	2	2	2
Magnesium chloride hexahydrate	4	4	4	4	3
Acetanilide	3	3	3	3	4
Paraffin wax	5	5	5	5	5

- **Step 2:** The ranks 1-8 are given to L, K, Cps, F, Cpl, V, T, VP, ρ , R, PS, SC, and C respectively. Table 7 shows the ranks and weights assigned to the 13 attributes using Table A. The best values are also shown in Table 7.

The attribute L is given a rank of 1, and hence, the weight is assigned as 0.16793 using Table A corresponding to 13 attributes. The attribute K is given rank 2, and hence, the weight is assigned from Table A as 0.11195. In a similar manner, based on the ranks, weights are allocated to the remaining attributes from Table A.

- **Step 3:** Without the use of fuzzy logic, the linguistic expressions of the attributes V, VP, SC, PS, R, T, and F are translated to quantitative values using Table 1. These values are shown in the corresponding columns of Table 8 in parentheses. The assigned values for V, VP, SC, PS, T, and F can be considered beneficial for normalization purposes after assigning like this.
- **Step 4:** As seen in Table 10, the "best" PCM for each attribute is used to normalize the data. Table 8 displays the normalized values for each of the 13 attributes.

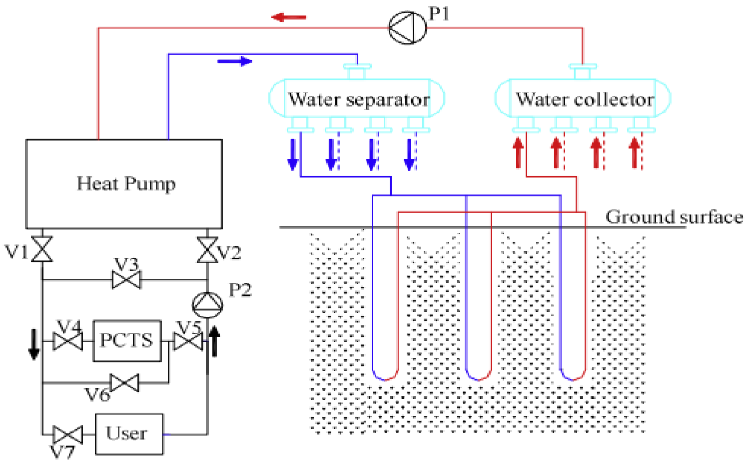

Figure 3. GSHP with PCTS system [12]

Table 6. Information aboutthe 8 alternative PCMs and 13 attributes of case study 2[12]

S. No.	Alternative PCM	Attributes												
		L	K	ρ	Cpl	Cps	C	V	VP	SC	PS	R	T	F
M1	Paraffin wax $C_{31}H_{64}$	242	0.2	808	2	3	4307	BA (0.6)	VL (0.8)	VL (0.8)	VL (0.8)	VH (0.8)	EL (0.9)	H (0.3)
M2	Paraffin wax $C_{32}H_{66}$	266	0.2	809	2	3	4307	BA (0.6)	VL (0.8)	VL (0.8)	VL (0.8)	VH (0.8)	EL (0.9)	H (0.3)
M3	Paraffin wax $C_{33}H_{68}$	256	0.2	810	2	3	4307	BA (0.6)	VL (0.8)	VL (0.8)	VL (0.8)	VH (0.8)	EL (0.9)	H (0.3)
M4	Paraffin wax $C_{34}H_{70}$	269	0.2	811	2	3	4307	BA (0.6)	VL (0.8)	VL (0.8)	VL (0.8)	VH (0.8)	EL (0.9)	H (0.3)
M5	Stearic acid $CH_3(CH_2)_{16}{\cdot}COOH$	210.8	0.172	848	2.2	1.6	3302	BA (0.6)	L (0.7)	VL (0.8)	VL (0.8)	VH (0.8)	EXL (1)	H (0.3)
M6	Salt hydrate $Ba(OH)_2{\cdot}8H_2O$	280	1.26	2180	2.44	1.34	4039	L (0.7)	L (0.7)	VH (0.2)	H (0.3)	L (0.3)	H (0.3)	L (0.7)
M7	Eutectic $LiNO_3$ (14%)-$MgNO_3{\cdot}6H_2O$ (86%)	180	0.7	1713	2.9	2.38	6872	L (0.7)	L (0.7)	H (0.3)	H (0.3)	L (0.3)	H (0.3)	L (0.7)
M8	Eutectic Urea (82%) +$LiNO_3$ (18%)	218	0.85	1438	2.02	1.77	8145	L (0.7)	L (0.7)	A (0.5)	A (0.5)	A (0.5)	A (0.5)	A (0.5)

VL: very low, L: low, BA: below average, A: average, H: high, VH: very high, EL: extremely low, EXL: exceptionally low.

Table 7. Ranks and matching weights for 13 attributes of case study 2

	Attributes												
	L	K	ρ	Cpl	Cps	C	V	VP	SC	PS	R	T	F
Ranks	1	2	9	5	3	13	6	8	12	11	10	7	4
Weights	0.16793	0.11195	0.05936	0.07354	0.0916	0.0528	0.06854	0.06179	0.05411	0.05561	0.05733	0.06476	0.0806
Best values	280	1.26	2180	2.9	3	3302	0.7	0.8	0.8	0.8	0.8	1	0.7

Table 8. Normalized valuesof case study 2

Material	Normalized values												
	L	K	ρ	Cpl	Cps	C	V	VP	SC	PS	R	T	F
M1	0.86428	0.15873	0.37064	0.68965	1	0.76665	0.85714	1	1	1	1	0.9	0.42857
M2	0.95	0.15873	0.37110	0.68965	1	0.76666	0.85714	1	1	1	1	0.9	0.42857
M3	0.91428	0.15873	0.37156	0.68965	1	0.76666	0.85714	1	1	1	1	0.9	0.42857
M4	0.96071	0.15873	0.37202	0.68965	1	0.76666	0.85714	1	1	1	1	0.9	0.42857
M5	0.75286	0.13651	0.38899	0.75862	0.5333	1	0.85714	0.875	1	1	1	1	0.42857
M6	1	1	1	0.84138	0.4466	0.81753	1	0.875	0.25	0.375	0.375	0.3	1
M7	0.64286	0.55555	0.78578	1	0.7933	0.48050	1	0.875	0.375	0.375	0.375	0.3	1
M8	0.77857	0.67460	0.65963	0.69655	0.59	0.40540	1	0.875	0.625	0.625	0.625	0.5	0.71428

- **Step 5:** The weights of the attributes mentioned in Table 7 are multiplied by the corresponding normalized data of the PCMs listed in Table 8 to determine the total scores. For instance, the total score for M1 is calculated as,

Total score (M1) = 0.16793*0.86428 + 0.11195*0.15873 + 0.05936*0.37064 + 0.07354*0.68965 + 0.0916*1 + 0.0528*0.76665 + 0.06854*0.85714 + 0.06179*1 + 0.05411*1 + 0.05561*1 + 0.05733*1 + 0.06476*0.9 + 0.0806*0.42857 = 0.74812.

Similarly, the total scores for the other PCMs are computed and are shown below.

M1: 0.748123; M2: 0.762544; M3: 0.756574; M4: 0.764398; M5: 0.70141; M6: 0.763709; M7: 0.673651; M8: 0.689061.

- **Step 6:** The PCMs are arranged in descending order of the total scores as follows: M6 – M4 – M2 – M3 – M1 – M5 – M8 – M7.The PCM denoted as M6 is regarded as the best PCM for this case study 2.

Yang et al. [12] used the entropy method for getting the objective weights, the AHP method for getting the subjective weights, and combined those weights to obtain the compromised weights of 0.151, 0.146, 0.044, 0.094, 0.135, 0.016, 0.070, 0.052, 0.03, 0.034, 0.043, 0.066, and 0.120 for L, K, ρ , Cpl, Cps, C, V, VP, SC, PS, R, T, and F respectively. Using these compromise weights, the authors applied the TOPSIS method to calculate the scores of PCMs and thereby select an optimum PCM. For example, the ranking of the PCMs using the compromise weights by TOPSIS, are arranged in the diminishing order of their scores as shown below.

TOPSIS rankings: M6 – M8 – M7 – M4 – M2 – M3 – M1 – M5.

TOPSIS method also suggested M6 as the first choice. However, the compromise weights used by Yang et al. [6]were different from the weights used in the BHARAT method. Hence, for a fair comparison, if the compromise weights used by Yang et al. [12] in the TOPSIS method are used in the BHARAT method also, then the PCMs can be arranged asM6 – M4 – M2 – M3 – M7 – M1 – M8 – M5.It is evident that M6 is recommended as the first option by the suggested BHARAT method, employing the same compromise weights as the TOPSIS method. The TOPSIS method recommended M8 as the second option, but BHARAT recommends M4 instead. A review of the values of the attributes corresponding to these PCMs indicates that M4 is better than M8 in 8 attributes (L, Cps, C, VP, SC, PS, R, and T) out of 13 with the summed weightage of 0.527 (i.e., 52.7%). Thus, proposing M4 as the second choice by BHARAT is logical. Similarly, M2, as the third choice by BHARAT compared to M7 of TOPSIS, is more logical.

Once again, the BHARAT method involved a simple normalization procedure, and the total scores of PCMs are computed by multiplying the normalized data with the attributes' weights (assigned using Table A or the compromise weights used by Yang et al. [12] for fair comparison purpose). It may also be seen that when the compromise weights used by Yang et al. [12] are used in the BHARAT method, it suggested the same M6 as the best choice, and the other choices suggested are more logical than those suggested by TOPSIS.

The TOPSIS method used by Yang et al. [12] requires excessively long computations for normalization, the entropy method for objective weight calculation, the AHP method for subjective weight calculation, the compromise weight calculation, and the use of those weights in the remaining computationally demanding steps. Unlike the TOPSIS approach, the recommended BHARAT method's normalizing procedure is simple and easy to understand. In contrast to Yang et al. [12], the BHARAT method eliminates the need for a fuzzy scale and streamlines the process of converting qualitative attributes into quantitative ones. The BHARAT method permits the use of attribute weights determined by the decision-maker using experience or intuition, or weights determined by other means, as demonstrated in this case study.

It can be observed from this case study 2 that the BHARAT has given the rankings of PCMs more meaningfully compared to those given by TOPSIS when the same compromise weights of attributes are used. Furthermore, the computation involved in the BHARAT method is less.

3.3. Case study 3: PCM selection for energy storage for thermal comfort in a vehicle

Nicolalde et al. [27] considered 20 alternative PCMs and 5 attributes for the selection of the right PCM for the vehicle's rooftop. The data is displayed in Table 9.

Now, following the BHARAT methodology, the normalization of the data is done. Of the 5 attributes, Nicolalde et al. [27] considered the density as a non-beneficial attribute for the application considered and hence the normalization is done accordingly and the values are displayed in Table 10.

Nicolalde et al. [27] used the entropy method and MEREC method for computing the weights of attributes and then used these weights in TOPSIS, VIKOR, and COPRAS methods. The weights for the attributes phase change temperature, density, heat of fusion, specific heat capacity, and thermal conductivity are 0.02, 0.16, 0.18, 0.36, and 0.28, respectively, using the MEREC method. For a fair comparison, BHARAT used the same weights, and the rankings of the alternative PCMs are given below (detailed steps are not shown for space reasons).

TOPSIS (with MEREC weights): M8-M7-M20-M2-M1-M3-M5-M6-M16-M14-M15-M19-M10-M12-M13-M14-M17-M11-M9-M18.

COPRAS (with MEREC weights): M8-M7-M20-M2-M1-M3-M5-M6-M16-M14-M15-M19-M10-M12-M13-M14-M17-M11-M9-M18.

VIKOR (with MEREC weights): M8-M7-M20-M2-M1-M5-M3-M16-M6-M4-M19-M15-M10-M12-M14-M11-M13-M17-M9-M18.

BHARAT (with MEREC weights): M8-M7-M20-M15-M4-M5-M2-M1-M6-M19-M3-M16-M10-M12-M13-M14-M11-M17-M9-M18.

All these methods suggest M8 (i.e., savENRG PCM-HS22P) as the best PCM for the application considered, with M7 and M20 as the second and third choices. Following purely the methodology of BHARAT and assigning the rank of 1 to specific heat capacity, 2 to thermal conductivity, 3 to heat of fusion, 4 to density, and 5 to phase change temperature leads to the weights of 0.3195, 0.213, 0.1743, 0.1533, and 0.14 respectively from Table A. Using these weights, BHARAT method gives the following rankings: M8-M7-M20-M15-M4-M9-M16-M10-M5-M2-

M1-M6-M3-M12-M14-M13-M11-M9-M17-M18. It can be observed that BHARATalso suggests M8 as the best PCM and M7 and M20 as the second and third choices.

The results of the three case studies of PCM selection described above have amply demonstrated and validated the potential of the BHARAT method as aMADM method. In all three case studies, the BHARAT method has given the rankings of PCMs more meaningfully compared to those given by the other MADM methods. Furthermore, the computation involved in the BHARAT method is less.The basic linear scales can be used to accomplish the goal of decision-making without the need for fuzzy logic. When making decisions in the actual world, this is really beneficial.

The choice made regarding how to determine weights will greatly affect how the decision turns out. There is no need to use objective weights by ignoring the decision-maker's preferences. The weights generated by the BHARAT method involving the R-method take into account the preferences of the decision-maker. The weights suggested by the R-method are more stable and consistent than those generated by other ranking strategies, such as rank order centroid (ROC) weights, reciprocal weights (RW), equal weights (EW), and rank sum (RS), as Table A demonstrates. For example, in the case of two attributes, the ROC technique provides the attributes with weights of 0.75 and 0.25, which is a relatively steep step. Likewise, the RW approach yields attribute weights of 0.6666 and 0.3333. On the other hand, the recommended approach gives 0.6 and 0.4 weights, which makes more sense.

The BHARAT method has an interesting feature in that the decision-maker can opt to apply attributes' weights based on experience, intuition, or personal choice rather than using the weights specified by the approach. In that scenario, the total scores of the alternatives can be ascertained by using the same procedure as the methodology described.

Table 9. Information of case study 3

PCMs	Attributes				
	Phase change temperature (^{0}C)	Density (kg/ m^3)	Heat of fusion (kJ/ kg)	Specific heat capacity (kJ/ kg.K)	Thermal conductivity (W/m.K)
M1	24	1500	190	2	0.6
M2	25	1500	190	2	0.6
M3	24	1500	180	2	0.6
M4	25	770	385	1.6	0.2
M5	25	1530	180	2.2	0.54
M6	23	1530	175	2.2	0.54
M7	24	1820	185	2.26	1.09
M8	23	1820	185	3.05	1.09
M9	25	650	102	1.6	0.2
M10	25	810	226	2.15	0.18
M11	25	880	179	2	0.2
M12	25	785	150	2.26	0.18
M13	23	785	145	2.22	0.18
M14	24	790	145	2.22	0.18
M15	28	774	243	2.3	0.15
M16	27.45	1496	161.15	2.2	0.53
M17	23	1100	127.2	2.26	0.16
M18	23	1475	155	0.69	0.43
M19	28	769	193	2.22	0.21
M20	24	1710	175	2	1

Table 10. Normalized values of case study 4

PCMs	Normalized data				
	Phase change temperature	Density	Heat of fusion	Specific heat capacity	Thermal conductivity
M1	0.857143	0.433333	0.493506	0.655738	0.550459
M2	0.892857	0.433333	0.493506	0.655738	0.550459
M3	0.857143	0.433333	0.467532	0.655738	0.550459
M4	0.892857	0.844156	1	0.52459	0.183486
M5	0.892857	0.424837	0.467532	0.721311	0.495413
M6	0.821429	0.424837	0.454545	0.721311	0.495413
M7	0.857143	0.357143	0.480519	0.740984	1
M8	0.821429	0.357143	0.480519	1	1
M9	0.892857	1	0.264935	0.52459	0.183486
M10	0.892857	0.802469	0.587013	0.704918	0.165138
M11	0.892857	0.738636	0.464935	0.655738	0.183486
M12	0.892857	0.828025	0.38961	0.740984	0.165138
M13	0.821429	0.828025	0.376623	0.727869	0.165138
M14	0.857143	0.822785	0.376623	0.727869	0.165138
M15	1	0.839793	0.631169	0.754098	0.137615
M16	0.980357	0.434492	0.418571	0.721311	0.486239
M17	0.821429	0.590909	0.33039	0.740984	0.146789
M18	0.821429	0.440678	0.402597	0.22623	0.394495
M19	1	0.845254	0.501299	0.727869	0.192661
M20	0.857143	0.380117	0.454545	0.655738	0.917431

4. Conclusions

A recent area of research that connects energy production and consumption is thermal energy storage. Phase change materials (PCMs) with high energy storage density and isothermal operating characteristics are critical for latent heat storage units. The usage of PCMs is essential to the thermal energy storage system's effective and efficient heat storage. The PCMs have been studied for many applications aimed at enhancing energy efficiency and economy. For the right PCM selection, competing quantitative and qualitative attributes usually need to be compromised. A great deal of researchers select the PCMs based on experience, availability, and cost attributes. However, in addition to these attributes, PCMs in the present work are chosen using a variety of attributes. Based on total scores, this work offers a potential MADM methodology to select the optimal PCM for various energy storage applications.

To demonstrate the potential of the BHARAT methodology, three case studies are presented. The first case study addressed the issue of choosing the best PCM for a thermal energy storage unit integrated into a solar box cooker by taking into account 5 alternative PCMs and 8 attributes and suggested Erythritol as the best choice; the second case study addressed PCM selection for a ground source heat pump with phase change thermal storage system by taking into account 8 alternative PCMs and 13 attributes and suggested Salt hydrate $Ba(OH)_2{\cdot}8H_2O$ as the best choice; the third case study addresses the problem of choosing the best PCM for energy storage for thermal comfort in a vehicle by taking into account 20 alternative PCMs and 5 attributes and suggested savENRG PCM-HS22P as the best choice. The three case studies have sufficiently illustrated the suggested method's potential as a MADM method.

It is worth noting that the ranking does not change when the decision-maker uses fuzzy scales instead of simple linear scales to translate linguistic expressions. This suggests that basic linear scales can be used to accomplish the goal of decision-making without the need for fuzzy logic. This is really beneficial when making decisions in the real world. The proposed decision-making method has an interesting feature in that the decision-maker can opt to apply attributes' weights based on experience, intuition, or personal choice rather than using the weights specified by the approach. In that scenario, the total scores of the alternatives can be ascertained by using the same procedure as the methodology described.

The proposed methodology helps in computing the total score values that assess the alternative PCMs for the given selection problem. It can simultaneously include all possible alternatives as well as both quantitative and qualitative attributes. The proposed methodology employs simple linear scales, which may make it easier for decision-makers to assign numerical values to the qualitative attributes. Each of the three case studies that are provided in this paper explains this fact. The method addresses the PCM selection problem comprehensively and is simple for decision-makers to implement.

The proposed BHARAT method provides a generic logical procedure that can be used for many selection problems involving multiple attributes and alternatives, as well as other problems that arise in different scientific and engineering disciplines. Applications to the selection problems involved in energy and thermal engineering will be attempted by the authors in the near future. The method will also be tested as a pruning method for selecting the best alternative solution from a set of Pareto optimal non-dominated solutions in multi- and many-objective optimization problems of energy and thermal engineering.

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Appendix A

Table A [36]. Various ranks of the attributes and the associated weights.

	Number of attributes													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rank *↓	Associated weights													
1	0.6	0.452054795	0.3714543	0.319480916	0.282626336	0.254847479	0.232999618	0.215269575	0.200531189	0.188044339	0.177300512	0.167937568	0.159689863	0.152357647
2	0.4	0.301369863	0.2476362	0.212987277	0.188417557	0.169898319	0.155333078	0.14351305	0.133687459	0.125362893	0.118200342	0.111958378	0.106459908	0.101571764
3		0.246575342	0.202611436	0.174262318	0.15415982	0.139007716	0.1270907	0.117419768	0.109380649	0.10256964	0.09670937	0.09160231	0.087103561	0.083104171
4			0.178298064	0.15335084	0.135660641	0.12232679	0.111839816	0.103329396	0.096254971	0.090261283	0.085104246	0.080610032	0.076651134	0.07313167
5				0.139918649	0.123777957	0.111612034	0.102043628	0.094278646	0.087823878	0.082355185	0.077649859	0.0735493	0.069937166	0.066725977
6					0.115357688	0.104019379	0.095101885	0.087865133	0.081849465	0.076752791	0.072367556	0.068545946	0.065179536	0.062186795
7						0.098288284	0.089862111	0.083024078	0.077339852	0.072523988	0.068380363	0.06476931	0.061588377	0.058760525
8							0.085729163	0.079205625	0.073782829	0.069188456	0.065235405	0.061790432	0.058755797	0.056058004
9								0.07609473	0.07088492	0.066470997	0.062673207	0.059363539	0.056448093	0.053856259
10									0.068464787	0.064201563	0.060533436	0.057336766	0.054520858	0.052017514
11										0.062268866	0.058711163	0.055610725	0.052879586	0.050451601
12											0.057134539	0.054117359	0.051459562	0.049096778
13												0.052808335	0.050214826	0.047909194
14													0.049111734	0.046856751
15														0.04591535

Table A continued…

	Number of attributes																			
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Rank* ↓	Associated weights																			
1	0.14579	0.13986	0.13448	0.12957	0.12506	0.12091	0.11708	0.11352	0.11021	0.10711	0.10422	0.10149	0.09894	0.09653	0.09425	0.09209	0.09005	0.08811	0.08627	0.08451
2	0.09719	0.09324	0.08965	0.08638	0.08337	0.08061	0.07805	0.07568	0.07347	0.07141	0.06948	0.06767	0.06596	0.06435	0.06284	0.06139	0.06004	0.05874	0.05751	0.05634
3	0.07952	0.07629	0.07335	0.07067	0.06822	0.06595	0.06386	0.06192	0.06011	0.05842	0.05685	0.05536	0.05397	0.05265	0.05141	0.05023	0.04912	0.04806	0.04705	0.04609
4	0.06998	0.06713	0.06455	0.06219	0.06003	0.05804	0.05619	0.05449	0.05289	0.05141	0.05002	0.04872	0.04749	0.04633	0.04524	0.04421	0.04323	0.04229	0.04141	0.04056
5	0.06385	0.06125	0.05889	0.05674	0.05477	0.05295	0.05128	0.04972	0.04827	0.04691	0.04564	0.04445	0.04333	0.04227	0.04127	0.04033	0.03944	0.03859	0.03778	0.03701
6	0.05950	0.05708	0.05488	0.05288	0.05104	0.04935	0.04778	0.04633	0.04498	0.04372	0.04253	0.04142	0.04038	0.0394	0.03847	0.03759	0.03675	0.03596	0.03521	0.03449
7	0.05622	0.05394	0.05186	0.04997	0.04823	0.04663	0.04515	0.04378	0.04250	0.04131	0.04019	0.03914	0.03815	0.03723	0.03635	0.03552	0.03473	0.03398	0.03327	0.03259
8	0.05364	0.05145	0.04948	0.04767	0.04601	0.04448	0.04307	0.04176	0.04054	0.03941	0.03834	0.03734	0.03640	0.03551	0.03467	0.03388	0.03313	0.03242	0.03174	0.03109
9	0.05153	0.04943	0.04753	0.0458	0.04420	0.04274	0.04138	0.04012	0.03895	0.03786	0.03683	0.03587	0.03497	0.03412	0.03331	0.03255	0.03183	0.03114	0.03049	0.02987
10	0.04977	0.04775	0.04591	0.04423	0.04269	0.04128	0.03997	0.03875	0.03762	0.03657	0.03558	0.03465	0.03378	0.03295	0.03218	0.03144	0.03074	0.03008	0.02945	0.02885
11	0.04827	0.04631	0.04453	0.04290	0.04141	0.04004	0.03876	0.03759	0.03649	0.03546	0.03451	0.03361	0.03276	0.03196	0.03121	0.03049	0.02982	0.02917	0.02856	0.02798
12	0.04698	0.04506	0.04333	0.04175	0.04030	0.03896	0.03772	0.03658	0.03551	0.03451	0.03358	0.03270	0.03188	0.03110	0.03037	0.02967	0.02902	0.02839	0.0278	0.02723
13	0.04584	0.04397	0.04228	0.04074	0.03932	0.03802	0.03681	0.03569	0.03465	0.03368	0.03277	0.03191	0.03111	0.03035	0.02963	0.02896	0.02831	0.02770	0.02712	0.02657
14	0.04483	0.04301	0.04135	0.03984	0.03846	0.03718	0.03600	0.03491	0.03389	0.03294	0.03205	0.03121	0.03042	0.02968	0.02898	0.02832	0.02769	0.02709	0.02653	0.02599
15	0.04393	0.04214	0.04052	0.03904	0.03769	0.03644	0.03528	0.03421	0.03321	0.03228	0.03140	0.03058	0.02981	0.02909	0.02840	0.02775	0.02713	0.02655	0.02599	0.02546
16	0.04312	0.04137	0.03977	0.03832	0.03699	0.03576	0.03463	0.03357	0.03259	0.03168	0.03082	0.03002	0.02926	0.02855	0.02788	0.02724	0.02663	0.02606	0.02551	0.02499
17		0.04066	0.03909	0.03767	0.03636	0.03515	0.03403	0.03300	0.03204	0.03114	0.0303	0.02951	0.02876	0.02806	0.02740	0.02677	0.02618	0.02561	0.02508	0.02457
18			0.03847	0.03707	0.03578	0.03459	0.03349	0.03248	0.03153	0.03064	0.02981	0.02904	0.02830	0.02761	0.02696	0.02635	0.02576	0.02521	0.02468	0.02417
19				0.03652	0.03525	0.03408	0.03300	0.03199	0.03106	0.03019	0.02937	0.02860	0.02788	0.02720	0.02656	0.02596	0.02538	0.02483	0.02431	0.02382
20					0.03476	0.03361	0.03254	0.03155	0.03063	0.02977	0.02896	0.02821	0.02750	0.02683	0.02619	0.02559	0.02503	0.02449	0.02397	0.02349
21						0.03317	0.03211	0.03114	0.03023	0.02938	0.02858	0.02784	0.02714	0.02648	0.02585	0.02526	0.02470	0.02417	0.02366	0.02318
22							0.03172	0.03076	0.02986	0.02902	0.02823	0.0275	0.02681	0.02615	0.02554	0.02495	0.0244	0.02387	0.02337	0.02289
23								0.03039	0.02951	0.02868	0.02791	0.02718	0.02649	0.02585	0.02524	0.02466	0.02412	0.02359	0.02310	0.02263
24									0.02918	0.02836	0.0276	0.02688	0.02620	0.02556	0.02496	0.02439	0.02385	0.02333	0.02284	0.02238
25										0.02807	0.02731	0.02659	0.02592	0.02529	0.0247	0.02413	0.02359	0.02309	0.02261	0.02214
26											0.02703	0.02633	0.02567	0.02504	0.02445	0.02389	0.02336	0.02286	0.02238	0.02192
27												0.02608	0.02542	0.02481	0.02422	0.02366	0.02314	0.02264	0.02217	0.02171
28													0.02519	0.02458	0.024	0.02345	0.02293	0.02243	0.02196	0.02152
29														0.02436	0.02379	0.02324	0.02273	0.02224	0.02177	0.02133
30															0.02359	0.02305	0.02254	0.02205	0.02159	0.02115
31																0.02286	0.02236	0.02187	0.02142	0.02098
32																	0.02219	0.02171	0.02125	0.02082
33																		0.02155	0.02109	0.02066
34																			0.02095	0.02052
35																				0.02038