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Copula Approach to Performance Evaluation of Manufacturing System

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Abstract

The present paper aimed at examining the performance of multi-station manufacturing system consisting of processors, conveyors and units arranged in series in terms of availability, reliability, Mean Time To failure (MTTF), sensitivity and expected profit under general and copula repairs for rectifying partial and complete failure. Supplementary variable technique and Laplace transforms are used to establish and resolve the differential equations associated with transition diagram, which are essential to this research. The numerical validation of explicit expressions for system availability, reliability, MTTF, MTTF sensitivity, and profit function is performed and presented in the form of tables and graphs. From the tables and graphs, it is clear that copula repair is a better repair policy for system's performance enhancement. The findings of this research are thought to be valuable for analyzing performance and determining the best system design and feasible maintenance strategies that may be used in the future to improve system performance, production output as well as revenue mobilization.

Keywords: Availability, Copula, Evaluation, Operation, Performance, Priority, Profit, Reliability.

1. Introduction

Manufacturing systems today are highly automated and comprise a number of interconnected machines. These interconnected machines are prone to failure, which has an impact on the system's reliability and availability, as well as on the revenue generated. Consumers nowadays demand complete assurance that the goods produced by the manufacturing systems are of high qualities and will continue to function. The design and manufacturing stages are critical in ensuring the reliability of goods produced.

The tendency that a system will work satisfactorily under specified working conditions for a given period of time is known as reliability. Reliability is one of the most crucial performance indices. If the system is unreliable, it will not be able to complete all of the manufacturing tasks or meet the production targets. Several research articles on reliability analysis of various systems have been studied. Similarly, the reliability of manufacturing systems is being researched, with several studies being presented.

Aggarwal et al. [1] investigated the profitability of a system with two units in cold standby. Preventive maintenance is performed on the system on a regular basis. In the event of a failure, repair facilities are set aside. The revenue, maintenance costs, and server costs are used to calculate profit using a stochastic process. Alazzam and Tashtoush [2] developed models for reliability modelling and analysis of lead-free solder due to aging time and temperature using adaptive neuro fuzzy inference system. Aly et al. [3] looked into the use of reliability, availability and maintainability to improve the operational performance of oil and gas industry system.Batra and Malhotra[4] analyzed the availability and reliability for two identical unit cold standby PCB manufacturing system with failure due arrival of faults. Models of availability and reliability are evaluated using the semi Markov and regenerative methods. Chang et al. [5] investigated the dependability of a network with multi-state manufacturing and multiple production lines (MSMN-MPL) when taking into account a joint buffer station.

Chen et al. [6] suggested a mission reliability evaluation for multistate manufacturing systems based on operational data. Chopra and Ram[7] conducted a study of the system's availability and reliability in a parallel network with two distinct units under copula. The availability of crank-case production systems in the automobile industry is described by Garg et al. [8]. The units under investigation can fail in either a regular working or partial failure state. Gulati et al. [9] investigated the reliability of a complex system with three subsystems in series when they failed completely or partially. Gahlot et al. [10] examined the dependability of a complex system made up of two subsystems, subsystem-1 and subsystem-2, in a series configuration under the policies of 2-out-of-3: F and 1-out-of-2: G; copula and general repair policies; and full and partial failure types. Gahlot et al. [11] investigated the performance of a complex system with two subsystems in sequence under the 2-out-of-3: G and 1-out-of-2: G policies. A human operator is attached to the system to keep it up and running, and the system fails completely due to human error.

He et al. [12] developed mission reliability models for multi-station production systems. He[13] proposed cost-oriented predictive maintenance models for mission reliability. Kadyan and Kumar [14] analyzed the availability and profit of feeding system consisting of six subsystems using supplementary variable technique. [15] investigated the reliability measures of a system comprised of two non-identical warm standby units attended by a single server under a first come, first served maintenance policy. The profit of the system is investigated using a semi-Markov process and regenerative methods. Kumar et al. [16] explore the profit of a system made up of two non-identical warm standby units under a first come, first served maintenance policy with preventive maintenance before failure. For manufacturing systems with serial arrangement, Lu and Zhou[17] suggest condition-based maintenance for dependability and quality. Lado and Singh [18] looked at the cost of a serial system with a human operator. Mehta et al. [19] developed models for availability analysis and enhancement of butter oil production system.

Failure is unavoidable in any man-made system. As a result, it can happen at any time during system operations. It can be a partial/minor failure or a total/complete failure. When a system experiences partial failure, its performance is reduced, and when it experiences complete failure, the system's operations are halted. When a system fails partially, it is repaired using general repair, while when it fails completely, it is repaired using the Copula approach Nelson [20]. Niwas and Garg[21] investigated the performance of an industrial system under a free warranty policy. A mathematical model for optimizing system efficiency and failure rate is proposed using the Markov process, resulting in high system profit and availability. Ram and Goyal [22] developed a stochastic model that included repair impact, failure modes, time trend variation, and coverage factor. Sha [23] investigated the working unit dependency using Clayton copula functions and Farlie-Gumbel-Morgenstern established models for parallel-series and series-parallel. Sanusi and Yusuf [24] studied the resilience of a dispersed data center network topology with three components. Singh and Ayagi [25] investigate a complicated system with a serial arrangement that includes two subsystems that are vulnerable to human failure and are monitored by a human operator.

Singh and Ayagi [26] investigated the dependability of complex systems controlled by a human operator, with three units operating under the super-priority, priority, and ordinary policies, as well as a preemptive resume repair policy. Tyagi et al. [27] developed stochastic model for behavioral analysis of a multi-state system consisting of two non-identical units by incorporating the concept of coverage factor and two types of repair facilities. Temraz [28] created models for analyzing availability and reliability in a parallel system with independent and identical units. The cost of component repair is reduced by using the Lagrange multiplier.Ye et al. [29] looked examined the reliability of a repairable machine while it was subjected to shocks and degradation from low-quality feedstock. In order to investigate the relationships between the inspection process, product quality, and machine failures, Ye et al. [30] developed a new model for competing failure. The dependability characteristics of a computer network system comprised of load balancers, distributed database servers, and a centralized server arranged as a series parallel system with three subsystems are discussed in Yusuf et al. [31]. Yusuf and Ismail[32] created availability models for parallel systems involving group and individual unit replacement in the event of partial or complete failure. The effect of replacing individual and groups of units with new and similar ones on system availability is investigated. Models development and evaluation of performance manufacturing system with serial arrangement that takes into account rework and product polymorphism is captured by Zhang et al. [33].

The aforementioned researchers provided excellent work on reliability, availability and profit analysis of complicated repairable systems using a copula technique, claiming that their operations improved the performance of the repairable systems. Still, a new sort of model with a justified and satisfactory evaluation is required. In addition, a few research articles on performance analysis of manufacturing systems using the copula approach have been published based on the literature study. Also, it is erroneous to judge the performance of production systems based on their components. Keeping the above facts in view, this paper dealt with performance analysis of manufacturing system producing two types of product; A and B. The system has two units, conveyors and processors as shown in Figure 1.

This paper is structured as follows: Section 1 contains the introduction as well as a brief review of the literature. Notations, assumptions, and system description are found in Section 2, whereas model formulation and solutions are found in Section 3.Section 4 discusses system analysis for specific scenarios, while the results were discussed in Section 5 and Section 6 brought the work to a close with references.

2. Notations, Assumptions, and Model description

2.1. Notations

t: variable representing time.

s: representing variable of Laplace transform

η_1 : stand for rate of failure of processor 1 and 2

η_2 : stand for rate of failure of conveyor 1

η_3 : stand for rate of failure of unit 1 and 2

η_4 : stand for rate of failure of conveyor 2

η_5 : stand for rate of failure of unit 3 and 4

$h(y_1)$: stand for rate of repair by general repair of processor 1 and 2

$h(y_2)$: stand for rate of repair by general repair of conveyor 2

$h(y_3)$: stand for rate of repair by general repair of unit 1 and 2

$h(y_4)$: stand for rate of repair by general repair of conveyor 2

$h(y_5)$: stand for rate of repair by general repair of unit 3 and 4

$m_0(y_1)$: stand for rate of repair by copula of processor 1 and 2

$m_0(y_2)$: stand for rate of repair by copula of conveyor 2

$m_0(y_3)$: stand for rate of repair by copula of unit 1 and 2

$m_0(y_4)$: stand for rate of repair by copula of conveyor 2

$m_0(y_5)$: stand for rate of repair by copula of unit 3 and 4

$P_i(t)$: stand for chance of the system sojourning in S_i state at instants for $i = 0,1,2,3,...,15$.

$\overline{P}(s)$: stand for Laplace transformation of state transition probability $P(t)$.

$P_i(y_1,t)$: stand for chance of the system sojourning in S_i with y_1 variable of repairand variable time t.

$P_i(y_2,t)$: stand for chance of the system sojourning in S_i with y_2 variable of repair and variable time t.

$P_i(y_3,t)$: stand for chance of the system sojourning in S_i with y_3 variable of repair and variable time t.

$P_i(y_4,t)$: stand for chance of the system sojourning in S_i with y_4 variable of repair and variable time t.

$P_i(y_5,t)$: stand for chance of the system sojourning in S_i with y_5 variable of repair and variable time t.

$E_p(t)$: Expected profit during the time interval $[0,t)$

Z_1, Z_2 : Revenue and service cost per unit time, respectively.

$m_0(x)$: The expression of joint probability according to Gumbel-Hougaard family Copula definition is given as:

$$c_\theta(\mu_1(x), \mu_2(x)) = \exp\left(x^\theta + \{\log\phi(x)\}^{\frac{1}{\theta}}\right)$$

, $1 \le \theta \le \infty$.Where $\mu_1 = \phi(x)$ and $\mu_2 = e^x$.

2.2. Assumptions

1. Units are presumed to be up and running the start.
2. Any unit failure results in sufficient system performance.
3. Any unit of any subsystem that fails while in operation or in the failure state can be restored.
4. Failure rates are taken to be constant and distributed exponentially.
5. Fully failed states can be restored using the Gumbel-Hougaard Family Copula, whereas partially failed states can be restored using universal/general distribution.

2.3. Model description

The system under consideration in this paper is a manufacturing system producing two types of product; A and B. The system consists of two units, conveyors and processors as depicted in Figure 1 below. The processors received items to be processed, and then channel them to the conveyor 1 for the process in either unit 1 and 2 or to the conveyor 2 for onward process in unit 3 and for usage. It is assumed that product A is of higher priority over product B. It is also assumed that unit 3 and 4 can manufacture product B only while unit 1 and 2 can manufacture product A, and product B only at the failure of unit 3 and 4. When the processors received items meant for making product A, the items will be processed in processor 1 and 2 and channeled to conveyor1, which will be forwarded to unit 1 and 2 to manufacture product A, while items meant for product B are processed by processor 1 and 2 and channeled to unit 3 and 4 through conveyor 2.

3. Model formulation and Solutions

3.1. Model formulation

For system modeling and analysis, reliability models were created using the supplementary variable technique and Laplace transforms. The differential equations were generated from the transition diagram using a probabilistic approach. These equations were then solved using initial and boundary conditions to obtain the steady state probabilities that serve as the foundation for the formulation of performance models.

The steps in getting the solutions of the state probabilities $P_k(s)$ for the formulation of the models involve

1. Derivation of the partial differential equations from Figure 1
2. Derivation of the boundary conditions of the states other than initial state
3. Taking the Laplace transformation of (a) and (b) above
4. Solving (c) to obtain the state probabilities $P_k(s)$

The following partial differential equations are obtained via Figure 1:

$$\left(\frac{\delta}{\delta t} + 2\eta_1 + \eta_2 + 2\eta_3 + \eta_4 + 2\eta_5\right)P_0(t) = \int_0^\infty h(y_1)P_1(y_1,t)dy_1 + \int_0^\infty h(y_5)P_2(y_5,t)dy_5 + \int_0^\infty m_0(y_5)P_3(y_5,t)dy_5$$
$$\int_0^\infty h(y_3)P_5(y_3,t)dy_3 + \int_0^\infty m_0(y_1)P_{12}(y_1,t)dy_1 + \int_0^\infty m_0(y_3)P_{13}(y_3,t)dy_3 + \int_0^\infty m_0(y_2)P_{14}(y_2,t)dy_2 +$$
$$\int_0^\infty m_0(y_4)P_{15}(y_4,t)dy_4 \quad (1)$$

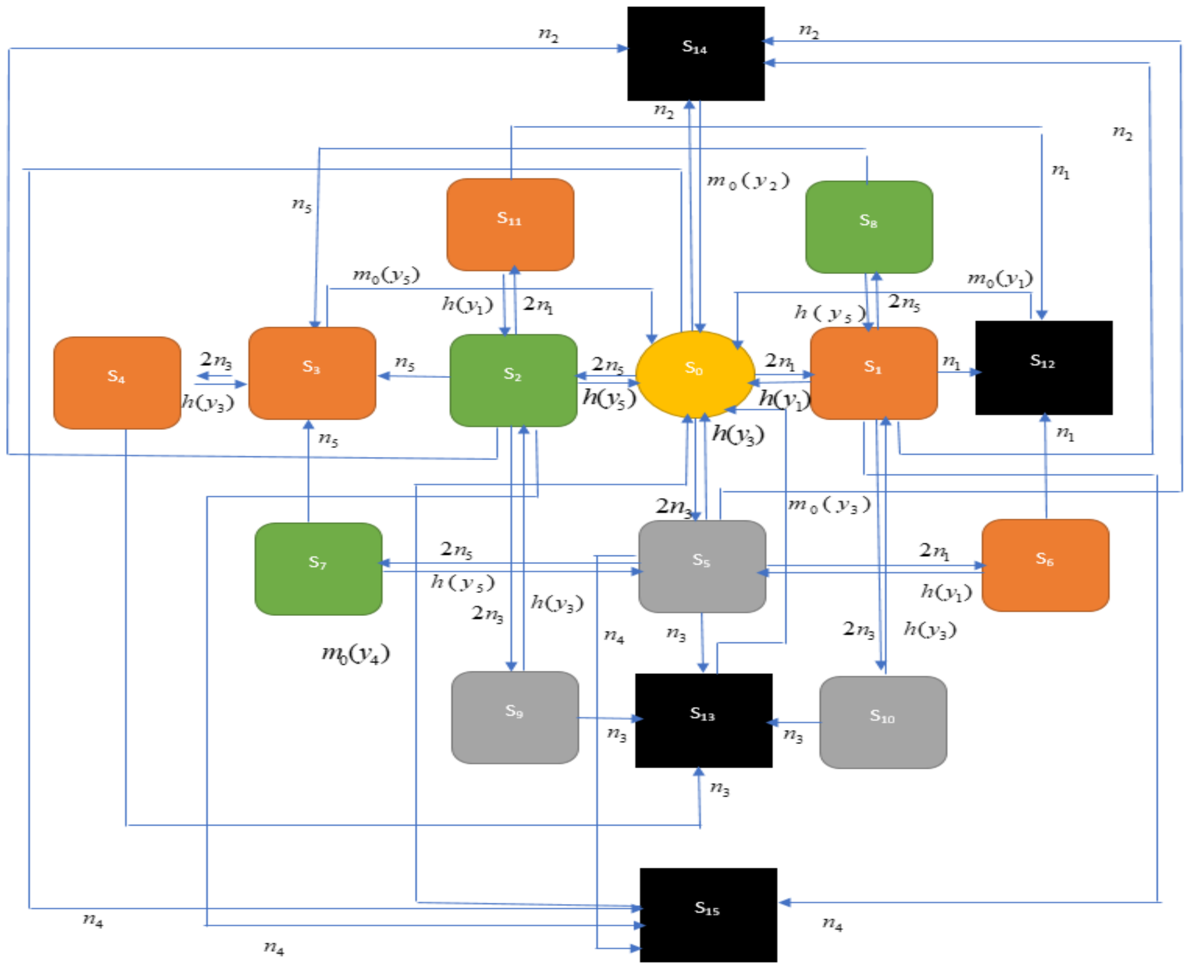

Figure 1. Diagram of transition of the model

S_0: Initial state. The subsystems and the system are in perfect state. The system is up and running.

S_1: Processor 1 has failed; processor 2 is up. The system is up and running.

S_2: Unit 3 has failed on the course of manufacturing product B; unit 4 has continued with manufacture of product B. The system is up and running.

S_3: Unit 3 and 4 have both failed on the course of manufacturing product B; unit 1 continue with manufacture of product B. The system is up and running.

S_4: Previously Unit 3 and 4 have both failed on the course of manufacturing product B; unit 1 has failed on the course of manufacturing product B, unit 2 has continue with manufacture of product B. The system is up and running.

S_5: Unit 1 has failed on the course of manufacturing product A; unit 2 continue with manufacture of product A. The system is up and running.

S_6: Previously unit 1 has failed on the course of manufacturing product A and is followed by the failure of processor I, processor 2 and unit 2 continue with manufacture of product A. The system is up and running.

S_7: Unit 3 has failed on the course of manufacturing product B and unit 4 has continued with manufacture of product B; unit 1 has failed on the course of manufacturing product A and unit 2 continue with manufacture of product A. The system is up and running.

S_8: Processor 1 has failed previously and is followed by failure of unit 3 on the course of manufacturing product B, processor 2 and unit 4 are up. The system is up and running.

S_9: Unit 3 has failed on the course of manufacturing product B, and unit 4 has continued with manufacture of product B; unit 1 has failed on the course of manufacturing product A; unit 2 continue with manufacture of product A. The system is up and running.

S_{10}: Processor 1 has failed; processor 2 is up, unit 1 has failed on the course of manufacturing product A; unit 2 has continued with manufacture of product A. The system is up and running.

S_{11}: Unit 3 has failed on the course of manufacturing product B previously and is followed by failure of processor I; unit 4 has continued with manufacture of product B, processor 2 is up. The system is up and running.

S_{12}: Processor 1 and 2 have both failed. The system is down.

S_{13}: Unit 1 and 2 have both failed. The system is down.

S_{14}: Conveyor1 has failed. The system is down.

S_{15}: Conveyor 2 has failed. The system is down.

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_1}+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5+h(y_1)\right)P_1(y_1,t)=0 \quad (2)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_5}+2\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5+h(y_5)\right)P_2(y_5,t)=0 \quad (3)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_5}+2\eta_5+m_0(y_5)\right)P_3(y_5,t)=0 \quad (4)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_3}+\eta_3+h(y_3)\right)P_4(y_3,t)=0 \quad (5)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_3}+2\eta_1+\eta_2+\eta_3+\eta_4+2\eta_5+h(y_3)\right)P_5(y_3,t)=0 \quad (6)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_1}+\eta_1+h(y_1)\right)P_6(y_1,t)=0 \quad (7)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_5}+\eta_5+h(y_5)\right)P_7(y_5,t)=0 \quad (8)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_5}+\eta_5+h(y_5)\right)P_8(y_5,t)=0 \quad (9)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_3}+\eta_3+h(y_3)\right)P_9(y_3,t)=0 \quad (10)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_3}+\eta_3+h(y_3)\right)P_{10}(y_3,t)=0 \quad (11)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_1}+\eta_1+h(y_1)\right)P_{11}(y_1,t)=0 \quad (12)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_1}+m_0(y_1)\right)P_{12}(y_1,t)=0 \quad (13)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_3}+m_0(y_3)\right)P_{13}(y_3,t)=0 \quad (14)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_2}+m_0(y_2)\right)P_{14}(y_2,t)=0 \quad (15)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y_4}+m_0(y_4)\right)P_{15}(y_4,t)=0 \quad (16)$$

Conditions of boundary

$$P_1(0,t)=2\eta_1P_0(t) \quad (17)$$

$$P_2(0,t)=2\eta_5P_0(t) \quad (18)$$

$$P_3(0,t)=2\eta_5^2(1+2\eta_1+2\eta_2)P_0(t) \quad (19)$$

$$P_4(0,t)=4\eta_3\eta_5^2(1+2\eta_1+2\eta_2)P_0(t) \quad (20)$$

$$P_5(0,t)=2\eta_3P_0(t) \quad (21)$$

$$P_6(0,t)=4\eta_1\eta_3P_0(t) \quad (22)$$

$$P_7(0,t)=4\eta_3\eta_5P_0(t) \quad (23)$$

$$P_8(0,t)=4\eta_1\eta_5P_0(t) \quad (24)$$

$$P_9(0,t)=4\eta_3\eta_5P_0(t) \quad (25)$$

$$P_{10}(0,t)=4\eta_1\eta_3P_0(t) \quad (26)$$

$$P_{11}(0,t)=4\eta_1\eta_5P_0(t) \quad (27)$$

$$P_{12}(0,t)=2\eta_1^2(1+2\eta_3)P_0(t) \quad (28)$$

$$P_{13}(0,t)=2\eta_3^2\left\{1+2\eta_1+2\eta_5+2\eta_5^2\left(1+2\eta_1+2\eta_3\right)\right\}P_0(t) \tag{29}$$

$$P_{14}(0,t)=\eta_2\left(1+2\eta_1+2\eta_3+2\eta_5\right)P_0(t) \tag{30}$$

$$P_{15}(0,t)=\eta_4\left(1+2\eta_1+2\eta_3+2\eta_5\right)P_0(t) \tag{31}$$

Applying (b) to (d) to equations (1) to (31) to derive the state probabilities below:

$$\overline{P}_0(s)=\frac{1}{\Delta(s)} \tag{32}$$

$$\overline{P}_1(s)=\frac{2\eta_1}{\Delta(s)}\left[\frac{1-\overline{s_h}\left(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5\right)}{\left(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5\right)}\right] \tag{33}$$

$$\overline{P}_2(s)=\frac{2\eta_5}{\Delta(s)}\left[\frac{1-\overline{s_h}\left(s+2\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5\right)}{\left(s+2\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5\right)}\right] \tag{34}$$

$$\overline{P}_3(s)=\frac{2\eta_5^2\left(1+2\eta_1+2\eta_5\right)}{\Delta(s)}\left[\frac{1-\overline{s}_{m_0}\left(s+2\eta_3\right)}{\left(s+2\eta_3\right)}\right] \tag{35}$$

$$\overline{P}_4(s)=\frac{4\eta_3\eta_5^2\left(1+2\eta_1+2\eta_2\right)}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_3\right)}{\left(s+\eta_3\right)}\right] \tag{36}$$

$$\overline{P}_5(s)=\frac{2\eta_3}{\Delta(s)}\left[\frac{1-\overline{s_h}\left(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5\right)}{\left(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5\right)}\right] \tag{37}$$

$$\overline{P}_6(s)=\frac{4\eta_1\eta_3}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_1\right)}{\left(s+\eta_1\right)}\right] \tag{38}$$

$$\overline{P}_7(s)=\frac{4\eta_3\eta_5}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_5\right)}{\left(s+\eta_5\right)}\right] \tag{39}$$

$$\overline{P}_8(s)=\frac{4\eta_1\eta_5}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_5\right)}{\left(s+\eta_5\right)}\right] \tag{40}$$

$$\overline{P}_9(s)=\frac{4\eta_3\eta_5}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_3\right)}{\left(s+\eta_3\right)}\right] \tag{41}$$

$$\overline{P}_{10}(s)=\frac{4\eta_1\eta_3}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_3\right)}{\left(s+\eta_3\right)}\right] \tag{42}$$

$$\overline{P}_{11}(s)=\frac{4\eta_1\eta_5}{\Delta(s)}\left[\frac{1-\overline{s}_h\left(s+\eta_1\right)}{\left(s+\eta_1\right)}\right] \tag{43}$$

$$\overline{P}_{12}(s)=\frac{2\eta_1^2\left(1+2\eta_3\right)}{\Delta(s)}\left[\frac{1-\overline{s}_{m_0}(s)}{(s)}\right] \tag{44}$$

$$\overline{P}_{13}(s)=\frac{2\eta_3\left\{1+2\eta_5+2\eta_5^2\left(1+2\eta_1+2\eta_3\right)\right\}}{\Delta(s)}\left[\frac{1-\overline{s}_{m_0}(s)}{s}\right] \tag{45}$$

$$\overline{P}_{14}(s)=\frac{\eta_2(1+2\eta_1+2\eta_3+2\eta_5)}{\Delta(s)}\left[\frac{1-\overline{s}_{m_0}(s)}{s}\right] \tag{46}$$

$$\overline{P}_{15}(s)=\frac{\eta_4(1+2\eta_1+2\eta_3+2\eta_5)}{\Delta(s)}\left[\frac{1-\overline{s}_{m_0}(s)}{s}\right] \tag{47}$$

Where,

$$\Delta(s)=\left\{\begin{array}{l}(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5)-2\eta_1\overline{s_h}(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5)\\ -2\eta_5\overline{s_h}(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)-2\eta_3\overline{s_h}(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5)\\ 2\eta_5^2(1+2\eta_1+2\eta_3)\overline{s}_{m_0}(s+2\eta_3)-\eta_2(1+2\eta_1+2\eta_3+2\eta_5)\overline{s}_{m_0}(s)-\\ 2\eta_1^2(1+2\eta_3)\overline{s}_{m_0}(s)-2\eta_3^2\{1+2\eta_1+2\eta_5+2\eta_5^2(1+2\eta_1+2\eta_3)\}\overline{s}_{m_0}(s)-\\ \eta_2(1+2\eta_1+2\eta_3+2\eta_5)\overline{s}_{m_0}(s)\end{array}\right\}$$

Using the state probabilities of operation states in Figure 1, the chance that the system is up and running is,

$$\overline{P}_{up}(s)=\sum_{j=0}^{11}\overline{P}_j(s)$$

$$\overline{P}_{up}(s)=\frac{1}{\Delta(s)}\left\{\begin{array}{l}1+2\eta_1\dfrac{1-\overline{s_h}(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5)}{(s+\eta_1+\eta_2+2\eta_3+\eta_4+2\eta_5)}+2\eta_5\dfrac{1-\overline{s_h}(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}{(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}\\ 2\eta_5^2(1+2\eta_1+2\eta_5)\dfrac{1-\overline{s}_{m_0}(s+2\eta_3)}{(s+2\eta_3)}+4\eta_3\eta_5^2(1+2\eta_1+2\eta_2)\dfrac{1-\overline{s}_h(s+\eta_3)}{(s+\eta_3)}+\\ 2\eta_3\dfrac{1-\overline{s_h}(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}{(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}+4\eta_5(\eta_1+\eta_3)\dfrac{1-\overline{s}_h(s+\eta_5)}{(s+\eta_5)}+4\eta_1\eta_5\dfrac{1-\overline{s}_h(s+\eta_5)}{(s+\eta_5)}\\ +4\eta_3\eta_5\dfrac{1-\overline{s}_h(s+\eta_3)}{(s+\eta_3)}+4\eta_1\eta_3\dfrac{1-\overline{s}_h(s+\eta_3)}{(s+\eta_3)}+4\eta_1\eta_5\dfrac{1-\overline{s}_h(s+\eta_1)}{(s+\eta_1)}\end{array}\right\} \tag{48}$$

4. Model analysis for specific cases

In order to gain a comprehensive understanding of this study. This section shows numerical simulations of the models.

4.1. System Availability

In the availability model for both copula and general repairs, the following scenarios are taken into account for consistency.

Fixed $s_{m_0}(s)=\overline{s}_{\exp\left[x^\theta+\{\log\varphi(x)\}^\theta\right]^{1/\theta}}(s)=\dfrac{\exp\left[x^\theta+\{\log\varphi(x)\}^\theta\right]^{1/\theta}}{s+\exp\left[x^\theta+\{\log\varphi(x)\}^\theta\right]^{1/\theta}}$, $\overline{s}_{m_0}(s)=\dfrac{h}{s+h}$ and taking failure rates at various values, such as:

Case 1: $\eta_5=0.05, \eta_1=0.01, \eta_3=0.03, \eta_2=0.02$ and $\eta_4=0.04, m_0=h=x=y=1, h(y_1)=h(y_2)=h(y_3)=h(y_4)=h(y_5)=m_0(y_1)=m_0(y_2)=m_0(y_3)=m_0(y_4)=1$

Case 2: $\eta_5=0, \eta_1=0.01, \eta_3=0.03, \eta_2=0.02$ and $\eta_4=0.04, m_0=h=x=y=1, h(y_1)=h(y_2)=h(y_3)=h(y_4)=h(y_5)=m_0(y_1)=m_0(y_2)=m_0(y_3)=m_0(y_4)=1$

Case 3: $\eta_5 = 0, \eta_1 = 0.01, \ \eta_3 = 0.03, \ \eta_2 = 0.02$ and $\eta_4 = 0, m_0 = h = x = y = 1, \ h(y_1) = h(y_2) = h(y_3) = h(y_4) = h(y_5) = m_0(y_1) = m_0(y_2) = m_0(y_3) = m_0(y_4) = 1$

Case 4: $\eta_5 = 0, \eta_1 = 0.01, \ \eta_3 = 0, \ \eta_2 = 0.02$ and $\eta_4 = 0 \ m_0 = h = x = y = 1, \ h(y_1) = h(y_2) = h(y_3) = h(y_4) = h(y_5) = m_0(y_1) = m_0(y_2) = m_0(y_3) = m_0(y_4) = 1$

4.1.1. System Availability via Copula repair

We derive the following explicit equations for copula repair by substituting each of the scenarios considered in equation (48) and using inverse Laplace transformation as:

Case 1

$$P_{up}(t) = \left\{ \begin{array}{l} -0.002996924489e^{-1.05t} - 0,001648902716e^{-1.01t} - \\ 0.003506172167e^{-1.03t} - 0.02804402347e^{-2.799541089t} - \\ 0.03219732828e^{-1.348312856t} - 0.0001279546981e^{-1.201426001t} \\ -0.0009275277378e^{-1.062398069t} + 1.013486002e^{-0.01035423720t} \end{array} \right\} \tag{49}$$

Case 2

$$P_{up}(t) = \left\{ \begin{array}{l} -0.0006884534335e^{-1.01t} - 0.0006161332316e^{-1.03t} + \\ 0.02486438232e^{-2.788179765t} - 0.009705610644e^{-1.18500683t} - \\ 0.0001441966181e^{-1.123828940t} + 0.9862900113e^{-0.001284464747t} \end{array} \right\} \tag{50}$$

Case 3

$$P_{up}(t) = \left\{ \begin{array}{l} -0.0005460429236e^{-1.01t} + 0.008985774245e^{2.743070575t} - \\ 0.006754225850e^{-1.149523258t} - 0.00007252051143e^{-1.083917205t} + \\ 0.9988153713e^{-0.001788962241t} - 0.0004283559453e^{-1.03t} \end{array} \right\} \tag{51}$$

Case 4

$$P_{up}(t) = \left\{ \begin{array}{l} 0.0007603256047e^{-2.739136591t} - 0.0007715044997e^{-1.049180434t} + \\ 0.9931682484e^{-0.00001702574081t} \end{array} \right\} \tag{52}$$

For $t \in [0, 20]$ in the above equations i.e., equation (49)-(52), the result is given below when the repair is done by copula distribution.

Table 1. Availability with passage of time in various scenariosvia copula repair

Time	Case 1	Case 2	Case 3	Case 4
0	1.00000	1.00000	1.00000	1.00000
2	0.98948	0.98276	0.99447	0.99314
4	0.97208	0.98113	0.99161	0.99322
6	0.95241	0.97871	0.98814	0.99327
8	0.93291	0.97621	0.98462	0.99330
10	0.91380	0.97370	0.98111	0.99334
12	0.89507	0.97120	0.97760	0.99337
14	0.87672	0.96871	0.97411	0.99341
16	0.85875	0.96623	0.97063	0.99344
18	0.84115	0.96375	096716	0.99347
20	0.82391	0.96128	0.96371	0.99351

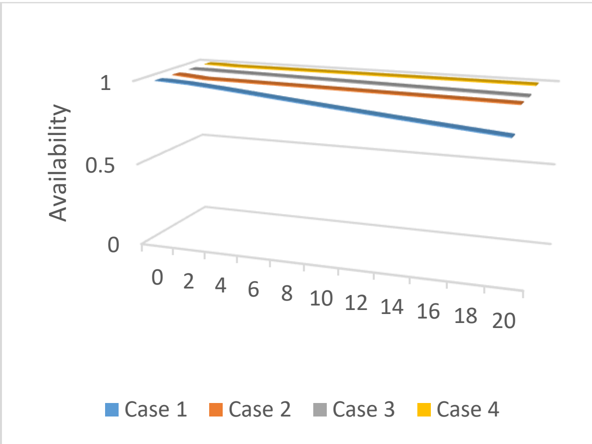

Figure 2. Availability with respect to time in various scenarios via copula repair

4.1.2. System availability via General repair

In a similar way, we obtain expressions for system availability for General repair by substituting each of the cases considered in equation (48) and using Laplace transformation as:

Case 1

$$P_{up}(t)=\begin{Bmatrix} -0.007341858038e^{-1.05t}+0.0005840154227e^{-1.01t}- \\ 0.002741497978e^{-1.391811163t}+0.0001221344453e^{-1.226515584} \\ +0.0001758949705e^{-1.201730597t}+0.002167622614e^{-1.065184534t} \\ 0.01485750988e^{-1.034805818t}+0.9736558977e^{-0.009952303992t}+ \\ 0.01852028107e^{-1.03t} \end{Bmatrix} \quad (53)$$

Case 2

$$P_{up}(t)=\begin{Bmatrix} 0.0002377116294e^{-1.01t}+0.003090659341e^{-1.03t}+ \\ 0.02149851512e^{-1.219142592t}+0.0005490604380e^{-1.124556391t} \\ +0.02528881487e^{-1.035064605t}+0.9493352385e^{-0.001236412652t} \end{Bmatrix} \quad (54)$$

Case 3

$$P_{up}(t)=\begin{Bmatrix} 0.005180660394e^{-1.163096569t}+0.0002835218455e^{-1.084281633t}+ \\ 0.004191164857e^{-1.010857358t}+0.9850928524e^{-0.001764439983t}+ \\ 0.005858375000e^{-1.01t}+0.0006075744279e^{-1.03t} \end{Bmatrix} \quad (55)$$

Case 4

$$P_{up}(t)=\begin{Bmatrix} 0.01102912438e^{-1.059657927t}+0.008243990789e^{-1.010358885t}+ \\ 0.9807268847e^{-0.000016812455554t} \end{Bmatrix} \quad (56)$$

When the general distribution is used, the system's availability is shown in table 2 and figure 4 using varying numbers of time t as:0,2,4,6,8,10,12,14,15,16,18,20 in equation (53)-(56).

Table 2. Availability with respect to time in various scenarios via general repair

Time	Case 1	Case 2	Case 3	Case 4
0	1.00000	1.00000	1.00000	1.00000
2	0.95800	0.95254	0.98342	0.98318
4	0.93612	0.94528	0.97838	0.98111
6	0.91728	0.94239	0.97475	0.98086
8	0.89915	0.93910	0.97129	0.98086
10	0.88142	0.93767	0.96786	0.98089
12	0.86405	0.93535	0.96445	0.98092
14	0.84702	0.93304	0.96106	0.98096
16	0.83033	0.93074	0.95767	0.98099
18	0.81396	0.92844	0.95430	0.98102
20	0.79792	0.92615	0.95094	0.98106

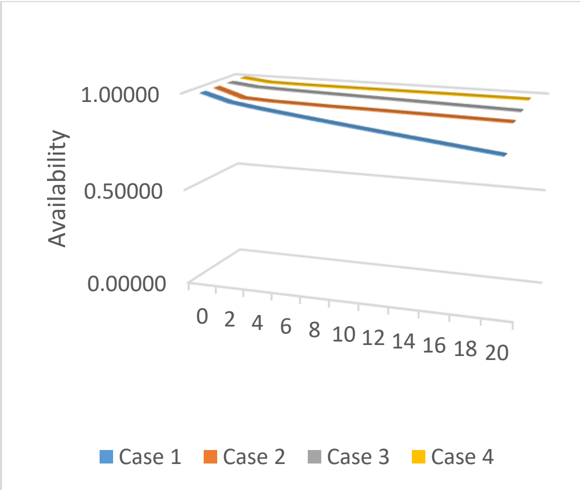

Figure 3. Availability with respect to passage of time in various scenarios via general repair

4.2. Reliability analysis

In reliability analysis, vanishing repairs to zero and considering $\eta_5 = 0.05,\ \eta_1 = 0.01,\ \eta_3 = 0.03,\ \eta_2 = 0.02$ and $\eta_4 = 0.04,$ in (48) and applying Laplace transformation, we obtain reliability equation as:

$$R(t) = \begin{Bmatrix} -5.121846878e^{-0.2400000000t} + 2e^{-0.2400000000t} + 0.1391304348e^{-0.1000000000t} \\ 2e^{-0.2300000000t} + 0.4210526316e^{-0.5000000000t} + 2e^{-0.1900000000t} + \\ 0.3000000000e^{-0.6000000000t} + 0.3582857143e^{-0.3000000000t} \end{Bmatrix} \tag{57}$$

Table 3 and the corresponding figure 5 represent system reliability when $t \in [0, 20]$ are used in equation (89).

Table 3. System reliability with respect to time

Time	0	2	4	6	8	10	12	14	16	18	20
$R(t)$	1.00000	0.88716	0.73789	0.59175	0.46472	0.36106	0.27960	0.21713	0.16993	0.13460	0.10826

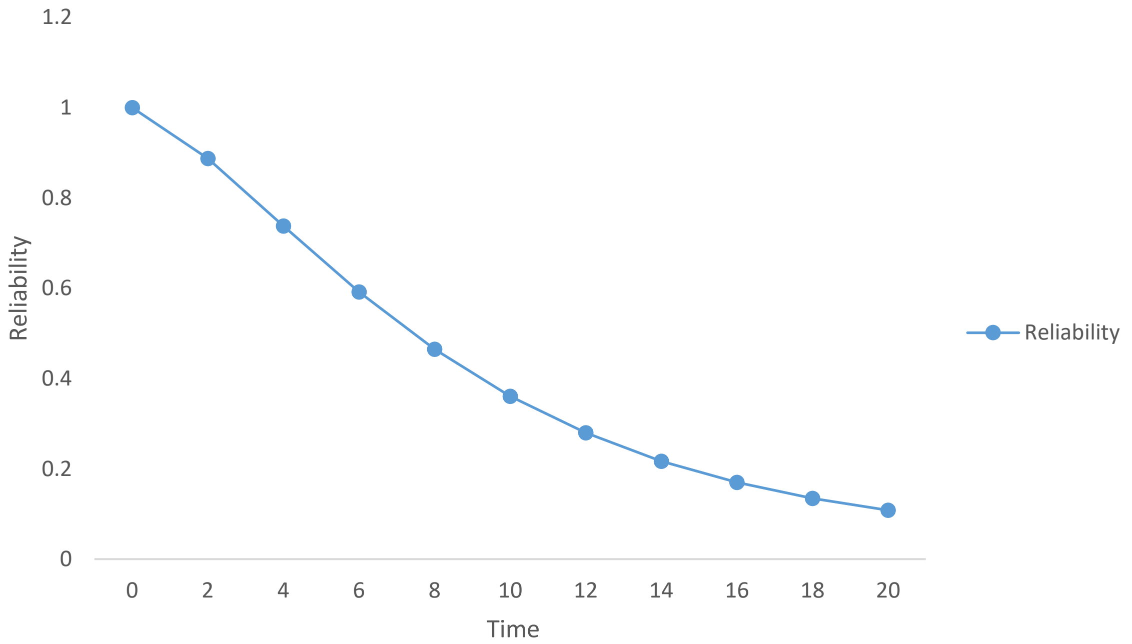

Figure 4. System reliability with respect to passage of time

4.3. Mean Time To Failure (MTTF) analysis

Supposing all repairs vanished to zero. When *s*tends to zero, the MTTF can be evaluated using limit as follows:

$$MTTF = \lim_{s \to 0} \overline{P}_{up}(s) = \frac{1}{\Delta(s)} \begin{Bmatrix} 1 + \dfrac{2\eta_1}{\eta_1 + \eta_2 + 2\eta_3 + \eta_4 + 2\eta_5} + \dfrac{2\eta_5}{2\eta_1 + \eta_2 + 2\eta_3 + \eta_4 + 2\eta_5} + \\ \dfrac{2\eta_3}{2\eta_1 + \eta_2 + 2\eta_3 + \eta_4 + \eta_5} + \dfrac{\eta_5^2(1 + 2\eta_1 + 2\eta_3)}{\eta_3} + \\ 4\eta_5^2(1 + 2\eta_1 + 2\eta_2) + 2(6\eta_1 + 2\eta_3 + 5\eta_5) \end{Bmatrix} \tag{58}$$

Table 4 and the associated figure 6 display the MTTF when the values of failure rates are fixed at $\eta_5 = 0.05,$ $\eta_1 = 0.01,\ \eta_3 = 0.03,\ \eta_2 = 0.02$ and $\eta_4 = 0.04,$ and each $\eta_j \in [0.01, 0.10]$

Table 4. MTTF variation with respect to failure rate

η_j	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
0.01	11.33244	12.03159	13.53796	13.71016	13.32648
0.02	10.79382	11.33244	12.12079	12.81847	12.57653
0.03	10.35470	10.70744	11.33244	12.03159	12.03717
0.04	9.98772	10.14562	10.76760	11.33244	11.63629
0.05	9.67470	9.63803	10.32443	10.70744	11.33244
0.06	9.16433	9.17735	9.95955	10.14562	11.10008
0.07	9.40318	8.75746	9.64973	9.63803	10.92253
0.08	8.95173	8.37329	9.38086	9.17735	10.78836
0.09	8.76064	8.02054	9.14368	8.75746	10.68942
0.10	8.58742	8.69557	8.93176	8.37329	10.61967

Table 5. System sensitivity against failure rate

η_j	$\frac{\delta(MTTF)}{\delta\eta_1}$	$\frac{\delta(MTTF)}{\delta\eta_2}$	$\frac{\delta(MTTF)}{\delta\eta_3}$	$\frac{\delta(MTTF)}{\delta\eta_4}$	$\frac{\delta(MTTF)}{\delta\eta_5}$
0.01	-60.09123	-74.04882	-222.90246	-95.08696	-89.25306
0.02	-48.32948	-66.00476	-97.38191	-83.61018	-62.88985
0.03	-39.94090	-59.17734	-65.04694	-74.04882	-46.13418
0.04	-33.75326	-53.33617	-49.40803	-66.00476	-34.70971
0.05	-29.05445	-48.30249	-39.90348	-59.17734	-26.47453
0.06	-25.39490	-43.93590	-33.44354	-53.33617	-20.26999
0.07	-22.48115	-40.12500	-28.74873	-48.30249	-15.42769
0.08	-20.11592	-36.78050	-25.17691	-43.93590	-11.54038
0.09	-18.16312	-33.83019	-22.36553	-40.12500	-8.34792
0.10	-16.52660	-31.21521	-20.09321	-36.78050	-5.67717

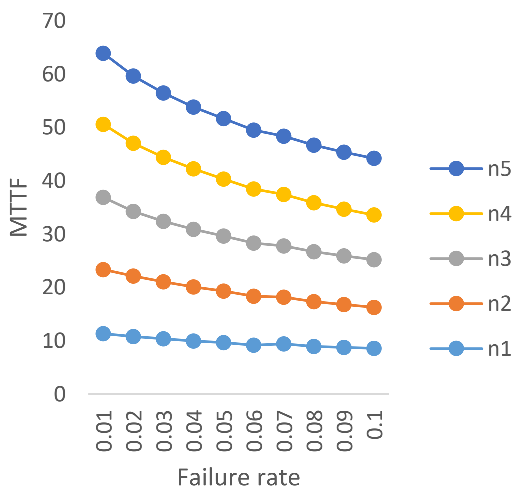

Figure 5. MTTF variation against η_j

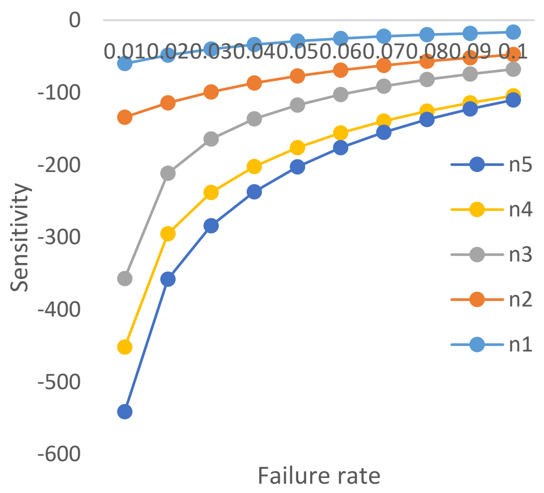

Figure 6. Sensitivity versus η_j

4.4. Sensitivity analysis

To calculate system sensitivity when the values of failure rates are fixed at $\eta_5 = 0.05,\ \eta_1 = 0.01$, $\eta_3 = 0.03$, $\eta_2 = 0.02$ and $\eta_4 = 0.04$, each $\eta_j \in [0.01, 0.10]$, the result is

4.5. Profit analysis

The expected profit for $t \geq 0$ is

$$E_p(t) = Z_1 \int_0^t P_{up}(t)\,dt - Z_2 t \tag{59}$$

4.5.1. Expected profit when the repair policy is obeyed by copula distribution

Assuming $\eta_5 = 0.05,\ \eta_1 = 0.01$, $\eta_3 = 0.03$, $\eta_2 = 0.02$ and $\eta_4 = 0.04$, one can obtain equation (92)by combining equation (79) and equation (91) as:

$$E_p(t) = Z_1 \left\{ \begin{array}{l} -0.003404050648e^{-1.0030000000t} - 0.001001736448e^{-2.799541089t} + \\ 0.02387971615e^{-1.3488312856t} + 0.0001021107079e^{-1.226267748t} \\ +0.0001065023547e^{-1.201426001t} + 0.0008730510388e^{1.062398069t} \\ -97.88128110e^{-0.01035423720t} + 0.002854213799e^{-1.05000000t} + \\ 0.001632576947e^{-1.01000000t} + 97.85844624 \end{array} \right\} - Z_2 t \tag{60}$$

Table 6 and figure 8 are when $t \in [0,20]$ and by applying the inverse Laplace transform to equation (92) with $Z_2 \in [0.01, 0.06]$ and $Z_1 = 1$ respectively.

Table 6. Expected profit for repair policy obeyed by copula distribution

Time	$E_P(t)$ $Z_2 = 0.06$	$E_P(t)$ $Z_2 = 0.05$	$E_P(t)$ $Z_2 = 0.04$	$E_P(t)$ $Z_2 = 0.03$	$E_P(t)$ $Z_2 = 0.02$	$E_P(t)$ $Z_2 = 0.01$
0	0	0	0	0	0	0
2	1.86599	1.88599	1.90599	1.92599	1.94599	1.96599
4	3.70855	3.74855	3.78855	3.82855	3.86855	3.90855
6	5.51307	5.57307	5.63307	5.69307	5.75307	5.81307
8	7.27834	7.35834	7.43834	7.51834	7.59834	7.67834
10	9.00498	9.10498	9.20498	9.30498	9.40498	9.50498
12	10.69378	10.81378	10.93378	11.05378	11.17378	11.29378
14	12.34551	12.48551	12.62551	12.76551	12.90551	13.04551
16	13.96093	14.12093	14.28093	14.44093	14.60093	14.76093
18	15.54078	15.72078	15.90078	16.08078	16.26078	16.44078
20	17.08578	17.28578	17.48578	17.68578	17.88578	18.08578

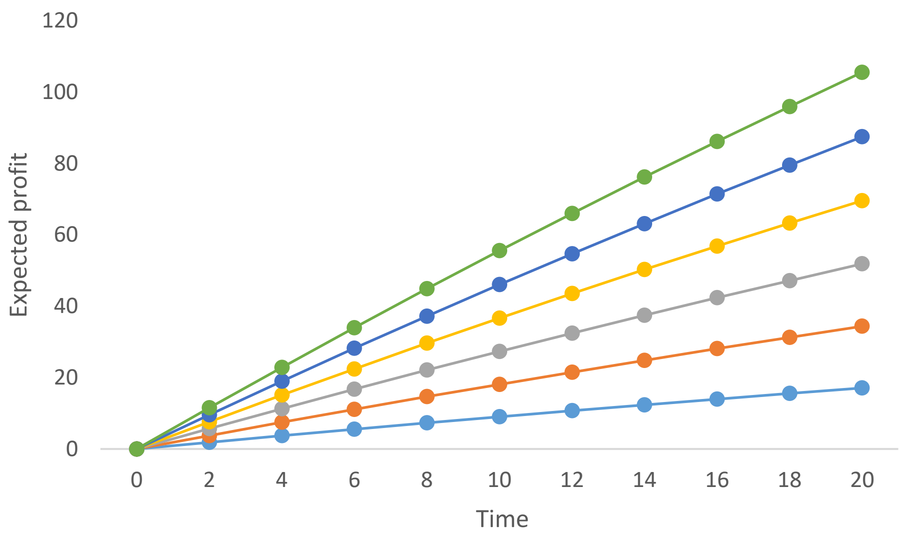

Figure 7. Expected profit for repair policy obeyed by copula distribution versus time t

4.5.2. Expression for Expected profit when the repair policy is obeyed by general distribution

Assuming $\eta_5 = 0.05$, $\eta_1 = 0.01$, $\eta_3 = 0.03$, $\eta_2 = 0.02$ and $\eta_4 = 0.04$, one can getprofit function for general repair by using equation (79) in equation (91) as:

$$E_p(t) = Z_1 \left\{ \begin{array}{l} -0.01798085541e^{-1.0030000000t} + 0.001969734150e^{-1.391811163t} - \\ 0.00009957838848e^{-1.226515584t} - 0.0001463680553e^{-1.201730597t} \\ -0.002034973795e^{-1.065184534t} - 0.0143577575e^{-1.034805818t} \\ -97.832221036e^{-0.009952303992t} + 0.00692245750e^{-1.05000000t} + \\ 0.0005782330918e^{-1.01000000t} + 97.85844624 \end{array} \right\} - Z_2 t \qquad (61)$$

Table 6 and figure 8 are when $t \in [0,20]$ and by applying the inverse Laplace transform to equation (93) with $Z_2 \in [0.01, 0.06]$ and $Z_1 = 1$ respectively.

Table 7. Expected profit for repair policy obeyed by general distribution versus time t

Time	$E_P(t)$ $Z_2 = 0.06$	$E_P(t)$ $Z_2 = 0.05$	$E_P(t)$ $Z_2 = 0.04$	$E_P(t)$ $Z_2 = 0.03$	$E_P(t)$ $Z_2 = 0.02$	$E_P(t)$ $Z_2 = 0.01$
0	0	0	0	0	0	0
2	1.83083	1.85083	1.87083	1.89083	1.91083	1.93083
4	3.60391	3.64391	3.68391	3.72391	3.76391	3.80391
6	5.33711	5.39711	5.45711	5.51711	5.57711	5.63711
8	7.03346	7.11346	7.19346	7.27346	7.35346	7.43346
10	8.69397	8.79397	8.89397	8.99397	9.09397	9.19397
12	10.31938	10.43938	10.55938	10.67938	10.79938	10.91938
14	11.91040	12.05040	12.19040	12.33040	12.47040	12.61040
16	13.46769	13.62769	13.78769	13.94769	14.10769	14.26769
18	14.99193	15.17193	15.35193	15.53193	15.71193	15.89193
20	16.48376	16.68376	16.88376	17.08376	17.28376	17.48376

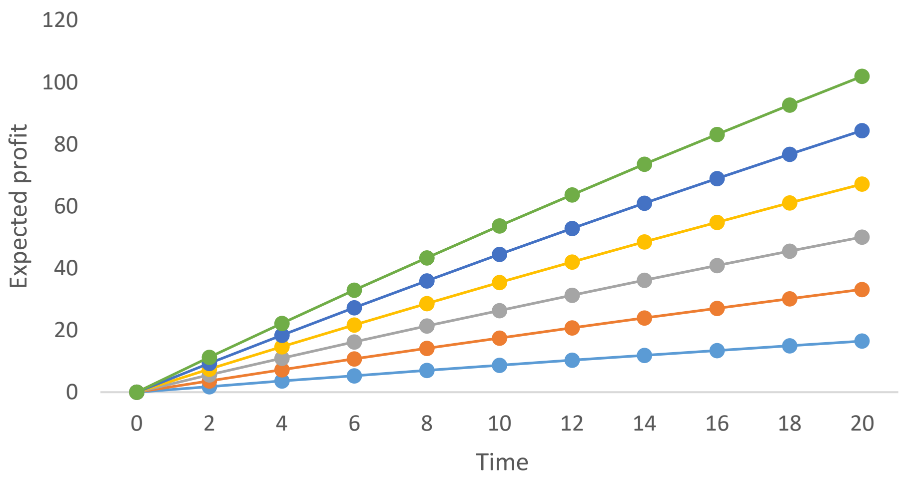

Figure 8. Expected profit for repair policy obeyed by general distribution versus time t

5. Results discussion

The decision-making process for performance evaluation of the model under consideration is carried out based on Tables 1-7 and Figures 2-8. First and foremost, failure rates must be determined, preferably ones with the lowest risk of error.

Figure 2 and Table 1 show the system's availability when repair policy is followed by copula which reduces and eventually stabilizes as time passes in all the scenarios studied. This analysis demonstrates that the system's performance can be simply deduced at any point in time. Table 1 and figure 2 also revealed that the failure rates with the highest availability are the failure rates of processor 1 and 2 and conveyor 1, respectively. So, it is required to start enhancing the performance of the system from these subsystems. Similar trend is observed in Figure 3 and table 2 when the repair is done via general distribution. The system's availability for copula repair appears to be greater than its availability for general repair, implying that copula repair is preferable to general repair. For the system under copula and general repair, four different cases (1 to 4) are considered to see the impact of change in failure rate of unit in some subsystems on system effectiveness and performance optimization. As can be seen in tables and figures where case 4 has the highest availability due to nonfailure of unit 1 and 2, conveyor 1 and 2, unit 3 and 4. This will allow the plant management to lay emphasis on regular inspection, condition base maintenance, both online and offline preventive maintenance to avoid sudden system failure that can be detrimental to the system, costly, low production output, inadequate product quality as well as less revenue mobilization.

When the repairs vanished to zero, and the failure rates are set as: $\eta_5 = 0.05$, $\eta_1 = 0.01$, $\eta_3 = 0.03$, $\eta_2 = 0.02$ and $\eta_4 = 0.04$, Figure 4 and Table 3for system reliability. The reliability of various failure rates degrades drastically over time, as shown in this table 3 and figure 4. This analysis illustrates the consequences of failing to restore the system. It is widely believed that the higher the maintenance, the greater the reliability is. The table and figure show that system reliability decreases dramatically with time, from maximum level 1.0000 to 0.10826. This enables the plant management to choose the best time to perform PM. According to the table and figure, it is worthwhile to perform preventive maintenance on the system at regular intervals [4, 6].

Table 4 and figure 5 show the diversity of critical reliability measures viz, mean time to failure. With rising values of $\eta_j,\ j=1,2,3,4,5$, it is discovered that the MTTF decreases. This implies that the system MTTF can be enhanced on decreasing the values of $\eta_j,\ j=1,2,3,4,5$. According to table 4 and figure 5, the MTTF is higher at the start of system operation and gradually decreases as the system's life span begins to deteriorate with increasing unit/subsystem failure rates. However, when compared to the MTTF of other units/subsystems, the conveyor 2 has the lowest value of 8.37329 because the failure rate is 0.10. This clearly demonstrates the disparity in MTTF values. The problem can be avoided by implementing various maintenance strategies such as the use of fault tolerant units, applying the k-out-of-n: G policy to the system's units, regular inspection, and preventive maintenance prior to system failure.

The information on sensitivity analysis explored in this study is presented in table 5 and figure 6. It is a way of telling the outcome of a choice based on a set of variables. The negative signs in this table and figure suggest that when the value of the system parameters (failure rates) increases, the performance indices decrease.

The change in the profit with passage of time t for $Z_2 \in [0.01, 0.06]$ when Z_1 is fixed at 1for repair policy of copula as shown in Figure 7 and Table 6. Figure 7 and Table 6 show that increasing the duration(time) and decreasing the service rate/cost will likely raise the estimated profit. Table 7 and Figure 8 show the same outcome with the repair policy of general distribution. When comparing the two procedures: repair policy of general and copula distributions, it appears that the predicted profit is larger when the repair policy follows copula distribution, and becomes lower when the repair follows general distribution. In both circumstances, the predicted profit is highest when the service cost is lowest, and lowest when the service cost is highest. This analysis will assist the analysts to set the budget for the system's smooth operation in advance based on the system's usability.

6. Conclusion

The availability, reliability, maintenance strategy/technique, and revenue generated are some of the factors that influence the development of any process sector. So, in order to get the most out of operating production systems, they must be meticulously maintained so that the rate of failure and repair is kept to a minimum. In this manner, various maintenance policies or tactics can be planned to improve system strength and performance. As a result of the preceding, this paper presents evaluation of performance of a series-parallel manufacturing system through copula characteristics. The inclusion of copula characteristics has increased the application of the developed model to a wider range of performance analysis of repairable systems operating under repair policy of copula and general repair. System MTTF, availability, cost function, sensitivity, and reliability explicit expressions are established and statistically validated. Based on the availability analysis, the failure rates priority for each subsystem are identified. Through the cost function analysis, it has been discovered that higher cost of services entailed lower system profit and vice versa. On the basis of availability and cost analysis, repair policy of copula distribution enhanced system profit and availability than repair policy of general distribution. These are the contributions of this study. This work can be extended by incorporating offline and online preventive maintenance.

Conflicts of interests

The authors declare that there is no conflicts of interest with regard to this manuscript

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