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Effect of Transverse Steady Magnetic Field on MHD Flow Under Free Convection Conditions in Vertical Microchannels

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Abstract

Comsol software was used to investigate the effect of temperature jump and velocity slip on the hydrodynamic and thermal behavior of MHD flows under free convection conditions between parallel vertical plates and along a vertical plate. Further, the continuum model of fluid was used with Knudssen (Kn) number regime 0.001 <Kn<0.1, with the Maxwell slip velocity being applied along with the Smoluchowski temperature jump boundary on the solid-fluid surface interface.

It was found that the applied transverse magnetic produces Lorentz force tends to retard the flow velocity, which was found to be directly proportional with both the magnetic field number (N) and the Knudsen Number (Kn). This decrease in the flow velocity was recorded in the case of two parallel plates, while Lorentz force was found to decreases the thickness of the velocity boundary layer in the case of a single plate. Also, it was found that the increase in the magnetic field applied and the increase of Kn number lead to a decrease of the skin friction factor, Nusselt (Nu) number and the thickness of the velocity boundary layer. Finally, it was fond that the applied magnetic field will cause an increase in the fluid temperature and hence both the friction coefficient factor and Nusselt number will be decreased by the increase of both Kn number and the magnetic influence number N.

Keywords: *Free convection, Knudsen number, Velocity slip, Lorentz force, Temperature jump, magnetic field.*

1. Introduction

Microfluidics is one field of MEMS that deals with fluid flows in or around micro devices, and it can be found in a wide spectrum of applications. Microvalves, microducts, and micropumps are examples of such small devices involving the flow of liquids and gases. The increasing need of device miniaturization in fluid applications results in the need of a deep understanding of the fundamentals of fluid flow and heat transfer since it has been observed that flows differ from those in macroscopic machines, where the conventional flow models such as Navier -Stokes equations with no-slip boundary conditions on the fluid-solid interface are no more valid.

Magneto Fluid Dynamics (MFD) deals with the mutual interaction between conducting fluid flow and the applied magnetic field and hence such fluids are limited to liquid metals, plasmas, and strong electrolytes. Such interaction was found that the transverse magnetic field ( ) affects the motion of the conducting fluid. This effect has been studied previously at the macro-scale level but not at the micro-scale level. The influence of magnetic field on fluid dynamics and heat transfer in a conductive fluid is of interest in micro devices.

Recently and as a result of the increase interest in micro-flow area research, a significant number of publications on this subject had been published. Gad-el-Hak (1999),(2002) [1],[[2] has published two papers, which presented an excellent state of the art review on the status of fluid flow phenomena related to micro devices. These papers concentrated on the use of MEMS as sensors and actuators for flow diagnosis and control. Furthermore, he concluded that that fluid flows in small devices may not be predicted from conventional flow models such as Navier-Stokes

On the other hand Gad El-Hak (2002) [2] presented a summary of the experiments that have been carried out to investigate the behavior of fluid flow in microchannels, over a large range of Reynolds numbers, geometries and experimental conditions, are presented in Table 6.3 of “The Handbook of MEMS”.

Lauga (2005) [3], has conducted direct analytical simulation to investigate the performance of generic slip

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boundary conditions on the dynamics of the two – dimensional wake behind a circular cylinder.

Yu et al. (1995)[4] investigated a similar study of flows through micro tubes with variable diameters, extremely high Reynolds numbers for microscale conditions, up to 20,000 was obtained from this work

Haddad et al. (2006) [5] studied the fluctuating driving force frequency on basic gaseous micro-flows. It was found that the velocity slip and temperature jump increase with Knudsen number and /or the frequency of the driving force.

The dependency of velocity profiles on Knudsen number; during the transmission from smaller Knudsen number to a higher one was proved by Justyna and Steffen , (2007) [6]. It was also proved the non-dimensional wall velocity increases and the centreline velocity decreases.

In their study, Al-Nimr et al.(2010) [7] investigated the effect of velocity-slip boundary conditions on Jeffery-Hamel flow solutions using the first and the second order velocity slip models, the obtained results were compared with the no-slip mode. It was found that the obtained skin friction coefficient decreases with the Kn number.

Hamdan et al. (2010)[8] investigated the effect of second order velocity-slip/temperature-jump in micro channel flows. Also, they studied the effect of Knudsen number on first and second order velocity slip then compared this to the no slip condition.

Al-Nimr and Hader, (1999) [9] conducted an analytical study on MHD free convection flow in open –ended vertical porous channels, it was found that in under fully developed conditions, the volumetric flow rate reaches its maximum value and any further increase in the channel height will not lead to an increase in the volumetric flow rate. However, Nusselt number reaches its minimum limiting value and remains constant irrespective of any increase in the channel height.

Duwairi and Damseh (2004) [10] applied radiative vertical porous surface to investigate the Magnetohydrodynamic natural convection heat transfer from radiate vertical porous surfaces. They found out that the velocity and the heat transfer rates inside the boundary layer decreases with the magnetic field strength.

Mehmood, and Ali, (2006) [11] investigated the effect of slip condition of an unsteady MHD flow of a viscous fluid in planner channel. They found out that fluid slip at the lower wall caused the velocity at the wall to be increased. Furthermore, it was observed that the Hartmann number, the porosity parameter and the Grashoff number, decrease the slip at the wall, while the effect of the Peclet number is to strengthen the slip.

Muthuraj and Srinivas (2009) [12] conducted a work to study the influence of magnetic field and wall slip conditions on long wavy wall and steady flow between parallel flat wall and ????. It was observed that increased suction parameter tended to decrease in fluid velocity.

Kalita (2012) [13] studied the effect of magnetic field on unsteady free convection MHD flow between two heated vertical plates, one of which is an adiabatic one. They found out that this effect of the magnetic field was of maximum value at an angle of π/2 to the directions of the fluid and this decreases slowly as the angle decreases from π/2 to 0.

Hamdan et al (2015) [14] studied the effect of velocity slip and temperature jump on the hydrodynamic and thermal behaviors of MHD flows in the case of forced convection over flat plate, and in the case of forced convection between two parallel flat plates have been studied. It was found that the applied transverse magnetic produces Lorentz force that acts as an external body force tends to retard the flow velocity. Also, it was found that an increase in both the magnetic field applied and Kn number will decrease the skin friction factor, the Nusselt number, and also decrease the thickness of the velocity boundary layer.

In this work the effect of transverse steady magnetic field on the velocity slip and temperature jump of MHD flow will be studied for two different flow patterns in microchannel by implementing the first order slip/jump models .Those two cases are free convection over flat plate and free convection between two parallel flat plates.

2. Governing Equations

As stated, in this study the continuum model, which is based on continuity and momentum equations together with the energy will be used. Energy conservation principles with the two dimensional steady state assumption will add terms to both the momentum and energy equations due to the imposed magnetic field as shown in the following three equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+g\rho\beta(T-T_\infty)-\sigma B^2 u \tag{2}$$

$$\rho c\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_y^2 u^2 \tag{3}$$

2.1. Free Convection Flow over Flat Plate

The behavior of a free convection flow over a vertical flat plate will be investigated and analyzed under the presence of the transverse magnetic field B and with the assumption of the validity of steady Navier Stokes model with slip velocity and temperature jump on the surface – fluid interface as shown in figure 1.

Consider the following non-dimensional parameters

$$U=\frac{u}{u_\infty},\theta=\frac{T-T_\infty}{T_w-T_\infty},V=\frac{v}{u_\infty},Y=\frac{yu_\infty}{\upsilon},X=\frac{xu_\infty}{\upsilon}$$

$$\vec{B}=B_y\,\vec{j}$$

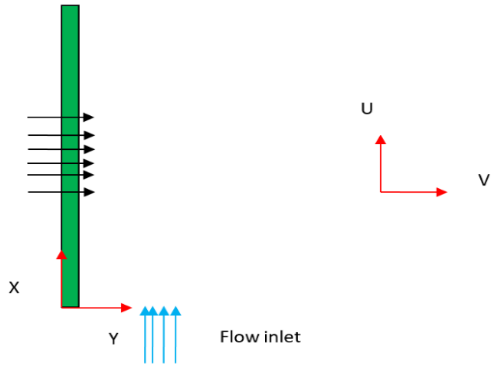

Figure 1: vertical plate setup

Where, U and θ are the non-dimensional velocity and temperature respectively, while u and T are the free stream velocity and temperature respectively, equations (1) ,(2) and (3) become respectively.

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=0 \quad (4)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{k}{\mu c}\frac{\partial^2\theta}{\partial Y^2}+\frac{u_0^2}{c\Delta T}\left(\frac{\partial U}{\partial Y}\right)^2+\frac{\sigma u_0 B^2 x_0}{\rho c\Delta T}U^2 \quad (5)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial Y^2}+E_c\left(\frac{\partial U}{\partial Y}\right)^2+RU^2 \quad (6)$$

Where

N=, $E_c=\frac{u_\infty^2}{c(T_w-T_\infty)}$ and R=$\frac{\sigma B_y{}^2\upsilon}{\rho c(T_w-T_\infty)}$

Represent a system of non-dimensional equations that model the fluid flow over a vertical plate and can be solved simultaneously with the following boundary conditions:

1. At X=0 and any Y
   U=0, V=0, θ=0
2. Y→∞ and any X
   U=0
3. Y=0 and any X

$$U(X,0)=\left(\frac{2-\sigma_u}{\sigma_u}\right)Kn\frac{\partial U}{\partial Y}(X,0)$$

$$\theta(X,0)=1+\left(\frac{2-\sigma_T}{\sigma_T}\right)\left(\frac{2\gamma}{\gamma+1}\right)\frac{1}{Pr}Kn\frac{\partial\theta}{\partial Y}(X,0)$$

The friction factor is given by the formula

$$C_f=\left|\frac{\partial U}{\partial Y}\right|$$

While the Nusselt $Nu=\left|\frac{\partial\theta}{\partial Y}\right|$

2.2. Free Convection Flow Between Two Vertical Plates

The thermal behavior and the hydrodynamic behavior of a free convection flow between two parallel vertical plates will be investigated under the effect of a transverse magnetic field, this will be performed by incorporating the effect of the velocity slip and temperature jump on the fluid –surface interface. This case is presented in figure 2.

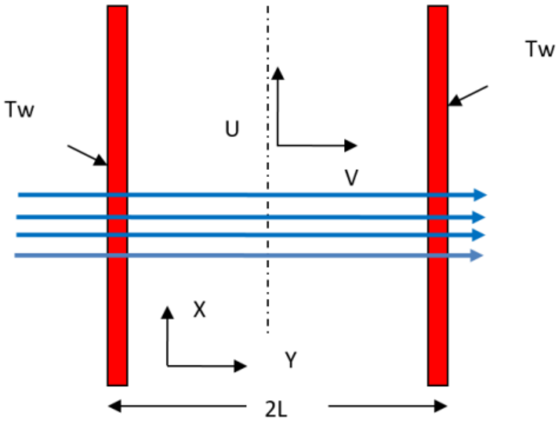

Figure 2: two vertical plates setup The governing equations of the system are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (7)$$

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\mu\frac{\partial^2 u}{\partial y^2}+g\rho\beta(T-T_\infty)-\sigma B^2u \quad (8)$$

$$\rho c\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_y^2u^2 \quad (9)$$

Assuming unidirectional flow, that is:

Applying it to equation (8), the left term will disappear and equation (8) will be reduced to the equation:

$$0=\mu\frac{\partial^2 u}{\partial y^2}+g\rho\beta(T-T_\infty)-\sigma B^2u \quad (10)$$

Also with the assumption that, $T=T(y)$ the energy equation (9) is reduced to:

$$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_y^2u^2 \quad (11)$$

Equations (7), (10) and (11) are combined and solved with the following two boundary conditions:

At y=0

$$\left.\begin{aligned}\frac{\partial u}{\partial y}&=0\\ \frac{\partial T}{\partial y}&=0\end{aligned}\right\} \quad (12)$$

And At y=L

$$\left.\begin{aligned}u(L)&=-\frac{(2-\sigma_u)}{\sigma_u}\lambda\frac{\partial u}{\partial y}\\ T(L)-T_w&=-\frac{(2-\sigma_T)}{\sigma_T}\frac{2\gamma}{\gamma+1}\frac{\lambda}{pr}\frac{\partial T}{\partial y}(L)\end{aligned}\right\} \quad (13)$$

Furthermore, substituting the following dimensionless parameters:

$$\left.\begin{aligned} Y &= \frac{y}{L} \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty} \\ U &= \frac{u}{u_0} \end{aligned}\right\} \quad (14)$$

Into the momentum equation (10) , the following will be obtained:

$$0 = \theta + \frac{\partial^2 U}{\partial Y^2} - NU \quad (15)$$

Also substituting the dimensionless parameters into the energy equation (11) to het:

$$0 = \frac{\partial^2 \theta}{\partial Y^2} + E\left(\frac{\partial U}{\partial Y}\right)^2 + FU^2 \quad (16)$$

Where $E = \frac{(T_w - T_\infty) L^4 \beta^2 \rho^2}{\mu k}$ and

$$F = \frac{\sigma B^2 u_0^2 L^2}{k(T_w - T_\infty)}$$

Equations (15) and (16) are solved using the following boundary conditions

At Y=0

$$\left.\begin{aligned} \frac{\partial U}{\partial Y}(0) &= 0 \\ \frac{\partial \theta}{\partial Y}(0) &= 0 \end{aligned}\right\} \quad (17)$$

At Y=L

$$\left.\begin{aligned} U(1) &= -\frac{(2-\sigma_u)}{\sigma_u} kn \frac{\partial U}{\partial Y}(1) \\ \theta(1) &= 1 - \frac{(2-\sigma_T)}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\frac{kn}{\Pr}\frac{\partial \theta}{\partial Y}(1) \end{aligned}\right\} \quad (18)$$

3. Results and Discussion

In this work the commercial software COMSOL 3.5a is used to solve the above derived equations, this is done in conjunction with both the Multi-physics model and partial differential model. Both the vertical flat plate and the two vertical plates case will be taken individually.

3.1. Vertical flat plate

In this case the behavior of a steady free convection of a conducting fluid flow over a heated vertical flat plate is studied, with the effect of the transverse magnetic field for the slip/no slip, temperature jump and no temperature jump being taken into consideration.

In figure 3, the effect of the transverse magnetic field on the velocity profile within the hydrodynamic boundary layer is shown under the case of no slip (Kn=0). As indicated, the effect of the transverse applied magnetic field is to decrease the velocity profile, this is due to the creation of a retarding body force that acts in the opposite direction of the fluid. Finally, and as noted, this decrease in the flow increases with the value of N.

Applying the Maxwell boundary conditions on the plate –fluid interface and at the same time applying the transverse magnetic field the obtained velocity profile for different values of magnetic fields and different values of Kn number is presented in figures 4 through 6. As shown in these figures, the effect of increasing Kn number from 0.001 up to 0.1on the velocity profile is to increase the velocity slip on the wall. Furthermore, the effect of increasing N of the velocity profile is to increase in the retardation of the velocity profile. This due creation of a retarding body force that acts in the opposite direction of the fluid.

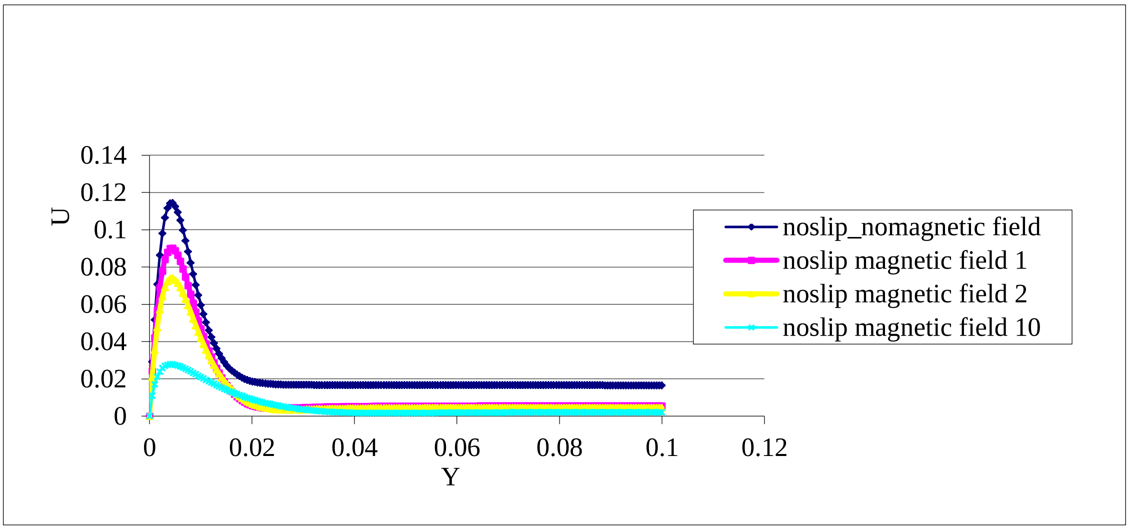

Figure 3: velocity profiles for Kn =0 and different values of N

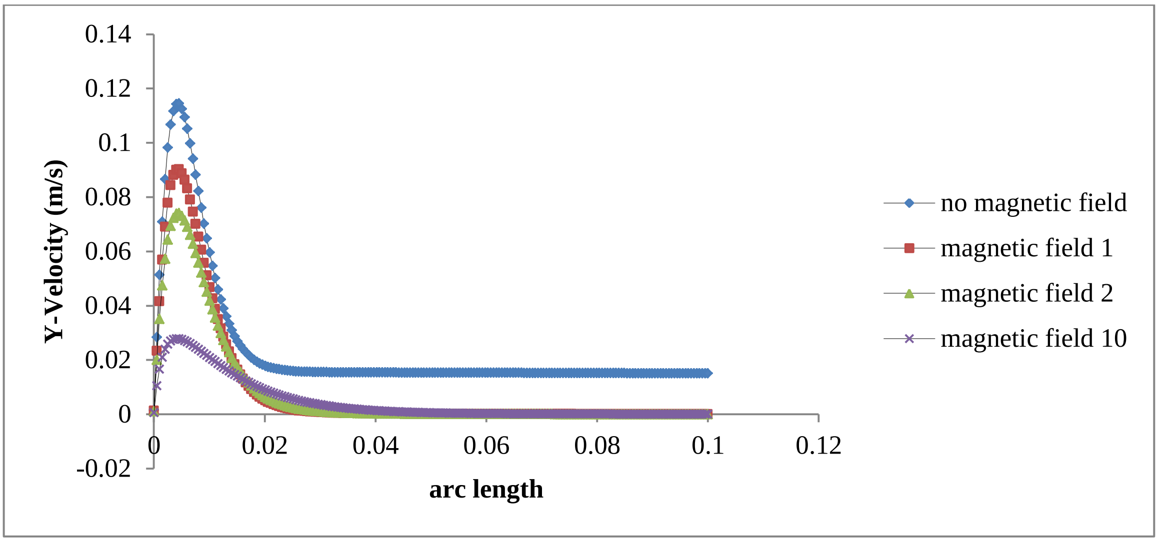

Figure 4: velocity profiles vertical plate Kn0.001 different N values.

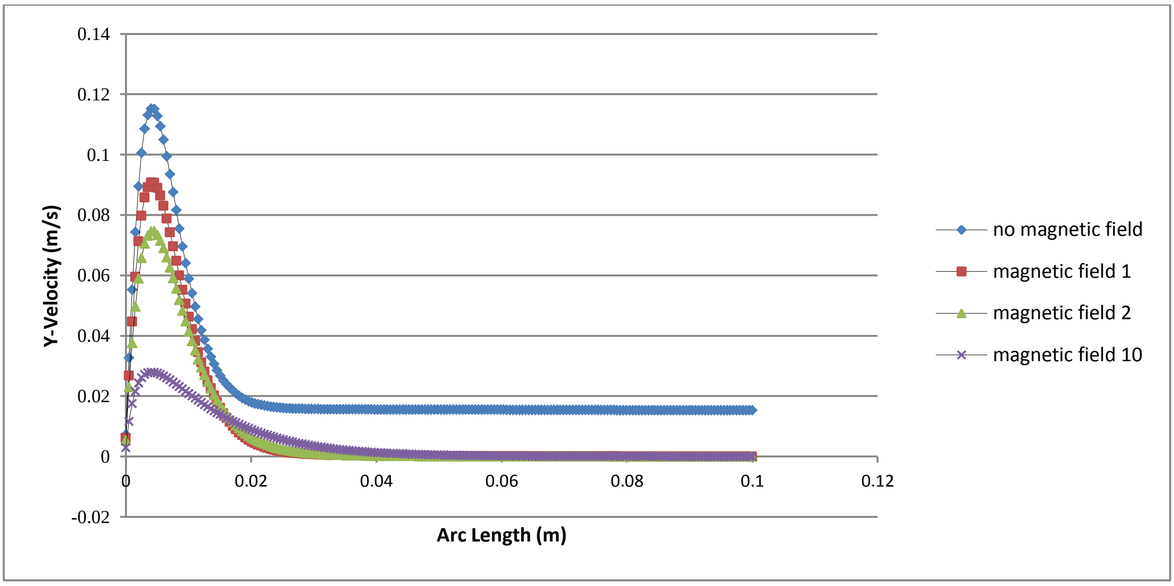

Figure 5: velocity profile for vertical plate for different N values at Kn=0.01

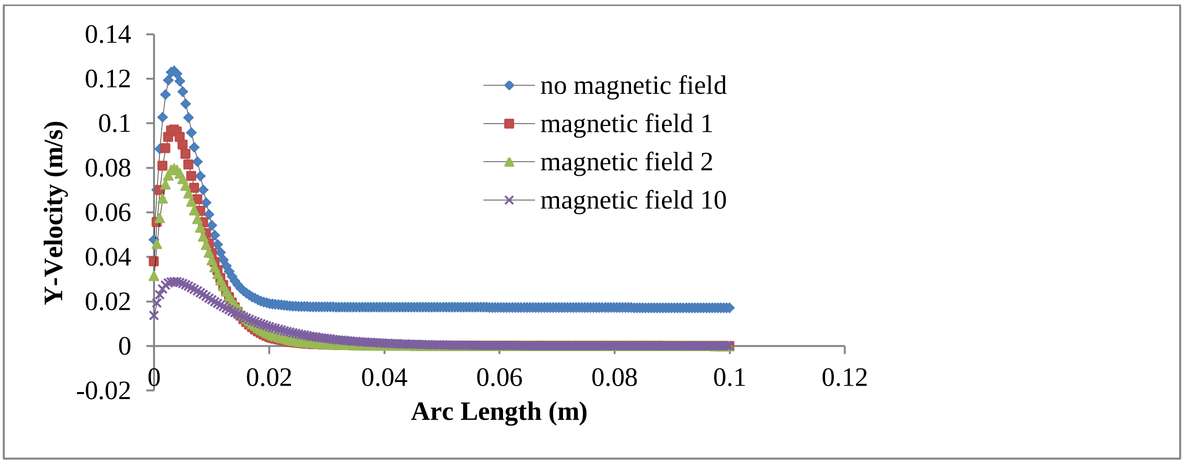

Figure 6: velocity profile for vertical plate for different N values at Kn=0.1

Figure 7 shows the temperature profiles under free convection conditions over vertical flat plate with different values of transverse magnetic field that is applied to the flow for the case of no jump (Kn=0). As indicated and in this figure, the temperature of the fluid increases with the magnetic field.

Figures 8 and 9 shows the temperature profiles under the conditions when the Smoluchowski boundary condition (Temperature jump) is applied on the fluid -plate interface and for two values of Kn (0.001and 0.1) along with different magnetic field values. It may be noted that the profile decreases rapidly in a direction away from the plate (in Y direction) as Y increases from zero value, and then, the rate decrease slows down significantly beyond, which the profile becomes independent of Y as it reaches zero value. It may be also noted that the temperature profile increases with N for any value of distance from the plate in Y direction.

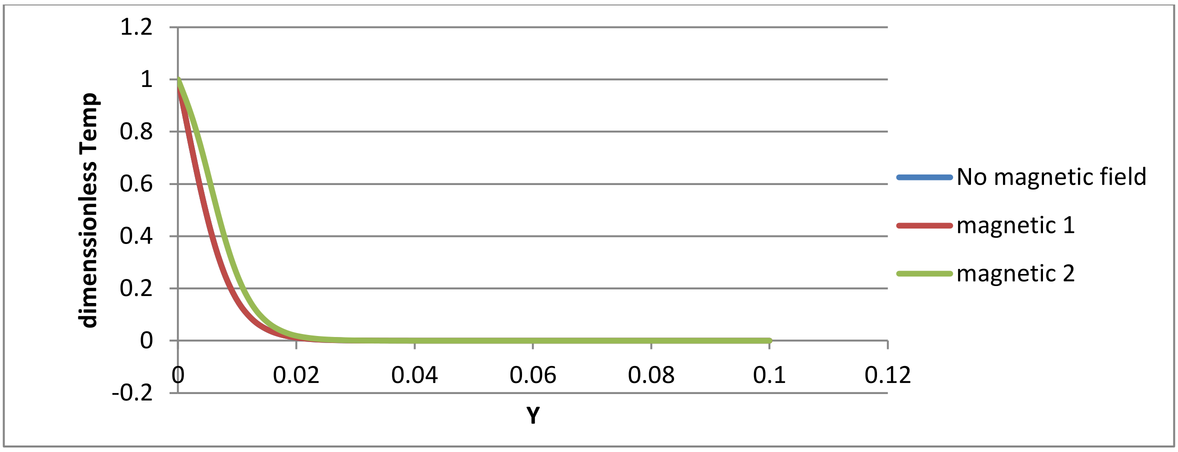

Figure 7: temperature profile over vertical plate with Kn=0 and different magnetic field

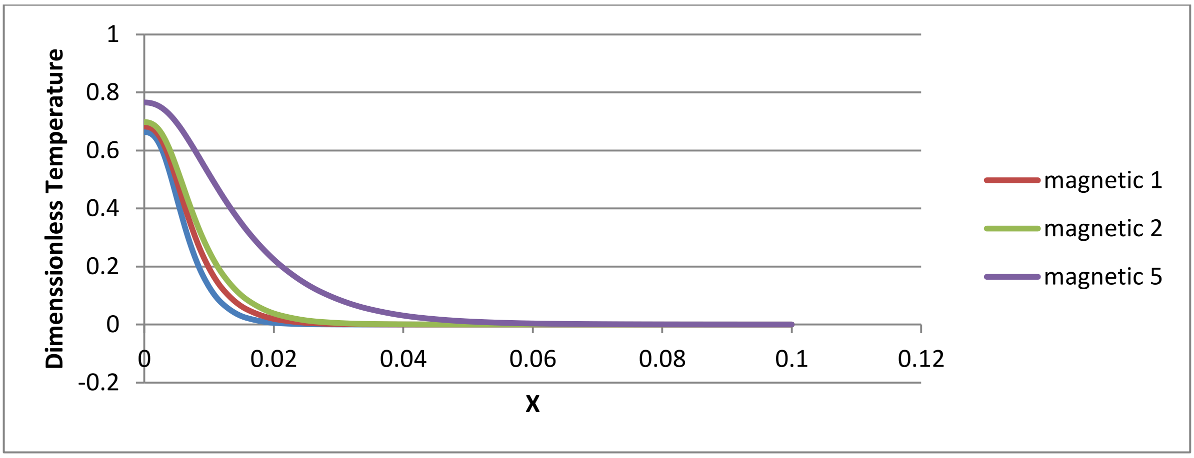

Figure 8: Temperature profiles for vertical plate with Kn=0.001 and different magnetic fields

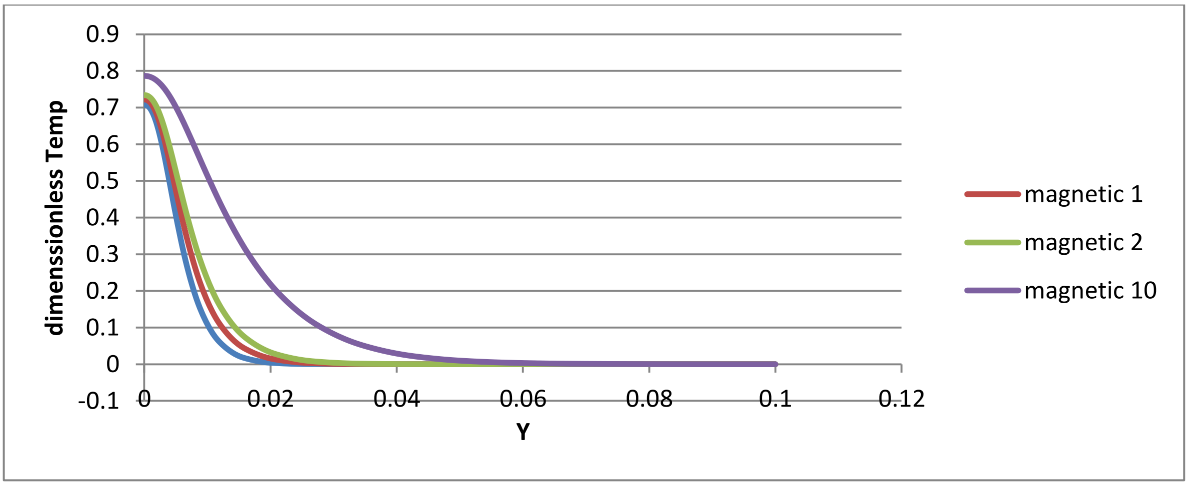

Figure 9: Temperature profiles for vertical plate with Kn=0. 1 and different magnetic fields.

The variation in skin friction with the magnetic influence number is presented in Fig. 10 as indicated the skin friction decreases in the direction of flow along the plate (in x direction) at high rate initially; then, this decrease in rate starts to flatten as x approaches a constant value.

3.2. Free convection between two vertical plates

In this case the behavior of the steady free convection between two vertical plates was investigated under the effect of the transverse magnetic field and the Maxwell boundary conditions on the plates –fluid interface taking into consideration that the walls are non -conducting.

Starting with the conventional conditions of case of no slip condition. Figure 11 shows the effect of the transverse magnetic field applied on the fluid flow. As indicated and as expected under the no slip conditions, the magnetic field as applied to the flow will tend to retard the flow.

The effect of the magnetic influence number on the velocity profile under different values of N and for the case when Kn = 00.01 is presented in figure 12. As it may be seen from this figure, the flow velocity retardation increases with the magnitude of the magnetic field. However, and under such conditions the slip velocity decreases as the magnetic field increases.

The effect of the transverse magnetic field on the temperature profile of a fluid between two vertical plates with Kn =0.001 is shown in figure 13 below. It is clear that at any specified location between the plates, the temperature of the flow increases with the applied magnetic field.

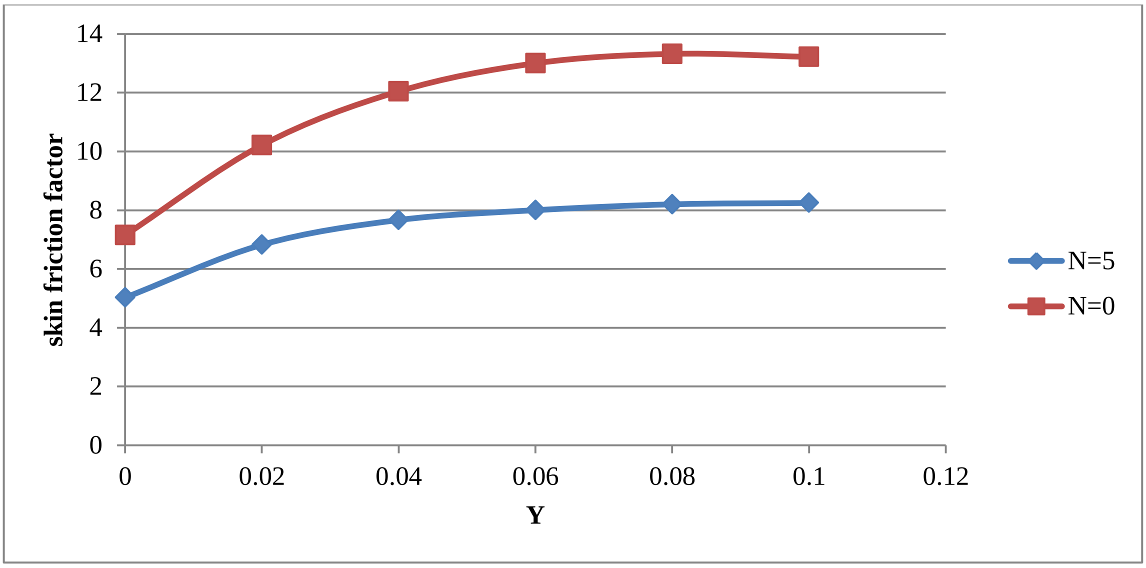

Figure 10: the effect of magnetic field on the skin friction value in a vertical

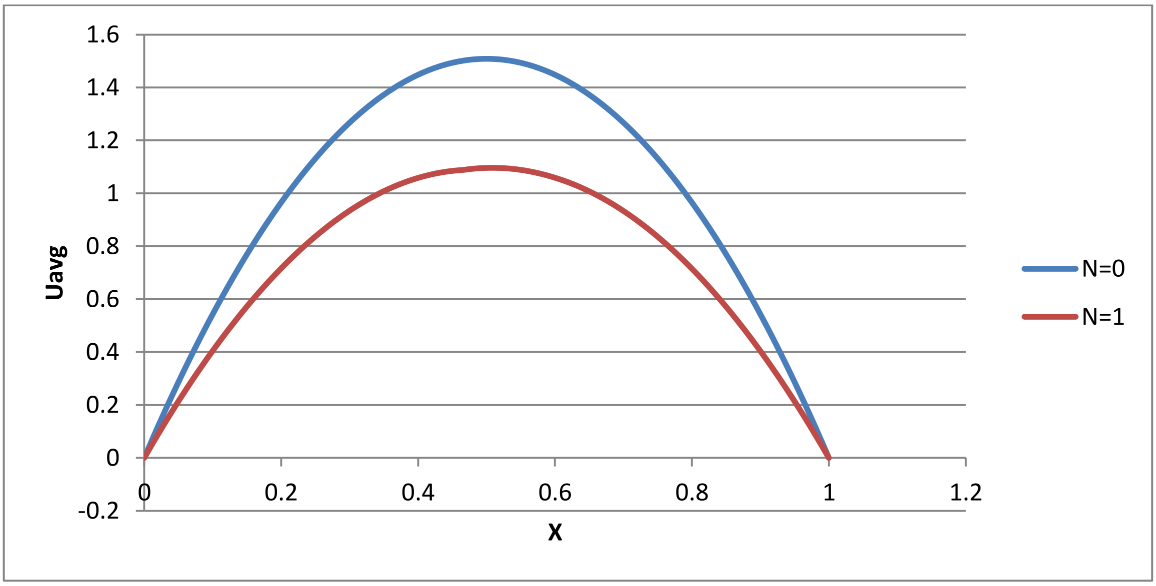

Figure 11: The effect of the transverse magnetic field on the flow velocity in the free convection between two parallel plates. Kn=0, N=0 and N=1

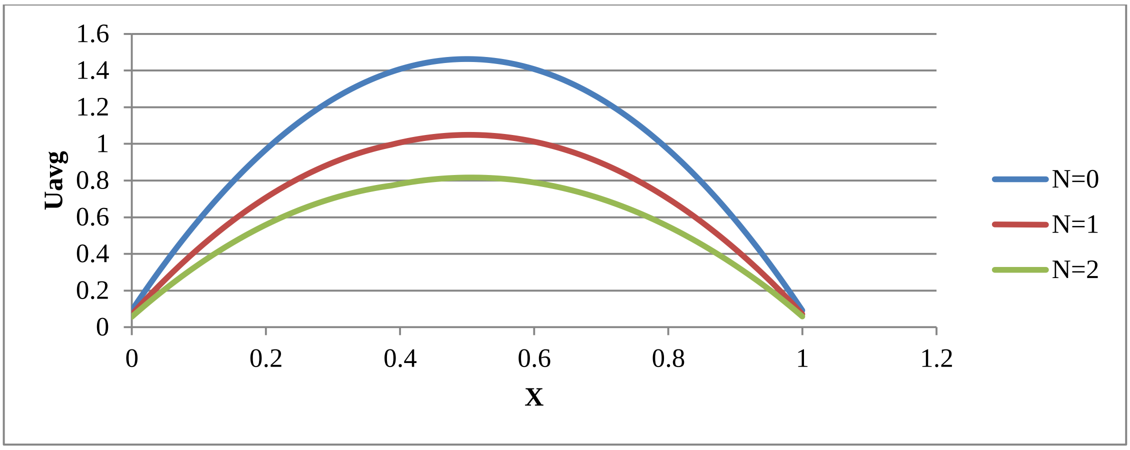

Figure 12: Velocity profile for free convection between two vertical parallel plates with Kn=0.001, N=0, N=1 and N=2

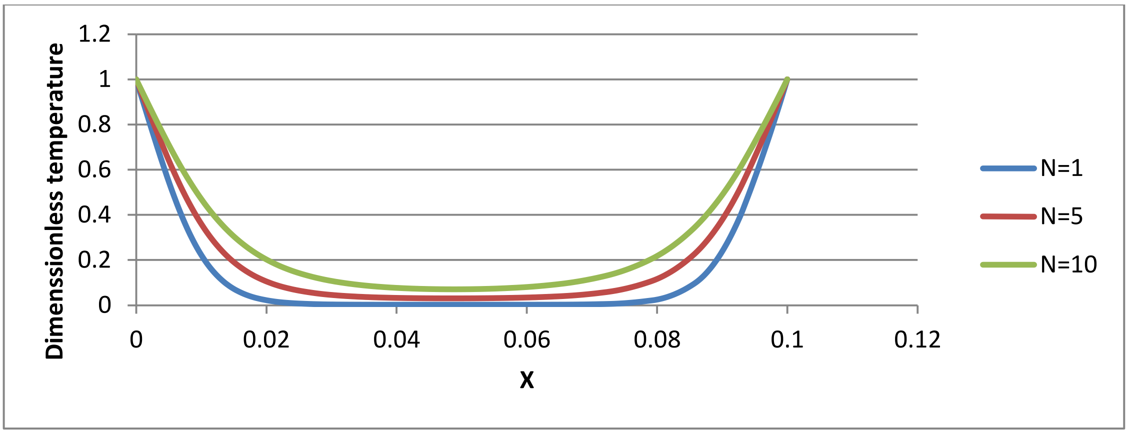

Figure 13: the effect of increasing the transverse magnetic field on the free convection velocity profile between two vertical plates with Kn=0.001

4. Conclusions

In this study the effect of induced magnetic field on hydrodynamic and thermal behaviors of MHD flows in the case of free convection have been studied. Four mathematical models that represent those cases have been developed; those are free convection over vertical flat plate, and free convection between two parallel vertical plates.

From this work, the following results may be concluded:

1. The applied transverse magnetic which produces Lorentz force that acts as an external body force tends to retard the flow velocity; this retardation was found to be directly proportional with both the magnetic field number (N) and the Knudsen Number (Kn).
2. The increase in the magnetic field applied and the increase of Kn number both will decrease the skin friction factor, the Nusselt number, and also decreases the thickness of the velocity boundary layer.

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Nomenclature

NOTATION	DESCRIPTION	UNITS
a_0	Speed of sound	m/s
B	Magnetic field	Tesla
c	specific heat of the heat transfer fluid	J/kg °C
c	Constant of proportionality	Dimensionless
C_f	Friction Coefficient factor	Dimenssionless
E	Electric field	V/m
F_e	Electric force	J/c
F_m	Magnetic force	N/A.m
g	Gravity accelaration	m/s^2
H	the magnetic field intensity	$F\ m^{-1}$
h	Convective heat transfer coeffiecient	W/m^2K
J	Current density	A/m^2
J_c	Conduction current density	A/m^2
J_{ind}	Induced current	A/m^2
J_{trans}	Current due to macroscopic velocity	A/m^2
k	Thermal conductivity	W/(m.K)
k	Boltzman constant	J/K
Kn	Knudsen number	Dimensionless
M	mass	Kg
Ma	Mach number	Dimensionless
$M_{diffuse}$	Diffused momentum	Kg.m/s
M_{out}	Out momentum	Kg.m/s
$M_{specular}$	Specular momentum	Kg.m/s
M_{wall}	Momentum at wall	Kg.m/s
M_{in}	In momentum	Kg.m/s
N	Magnetic influence number	Dimensionless
n	Number density	#of molecules/m^3
Nu	Nusselt number	Dimenssionless
R	Gas constant	Dimensionless
Re	Reynolds number	Dimensionless
Q_i	Energy of incoming molecules	J
Q_w	Reflected energy	J
T	temperature	K
T_∞	Tempereture at inlet	K
T_s	Fluid temperature	K
T_w	Wall temperature	K
u	X velocity component	m/s
U	Internal energy,dimenssionless velocity	J,dimenssionless
u_m	Mean velocity	m/s
u_s	Slip velocity	m/s
u_∞	Velocity at inlet	m/s
v_0	Characterstic velocity	m/s
V	Velocity vector in the conductor	m/s
V	Dimenssionless Y- velocity	Dimenssionless
v	y-component of velocity	m/s
W_{em}	Electromagnetic work	N/A.s
X	Dimenssionless width	Dimenssionless
Y	Dimenssionless height	Dimenssionless

GREEK NOTATIONS

β	Volumetric thermal coefficient	1/K
γ	Specific heat ratio	Dimensionless
λ_{mfp}	Mean free path	m
σ	Molecular diameter, electric conductivity	m
υ	Kinematic viscossity	m^2/s
σ_u	Momentum accomodation factor	Dimensionless
μ	Dynamic viscosity	$N.s/m^2$
ρ	density	Kg/m^3
σ_T	Temperature accomodation factor	Dimensionless
μ_0	magnetic field permeability	$H\ m^{-1}$
θ	Dimenssionless temperature	Dimenssionless