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The Effect of Temperature on the Stresses Analysis of Composites Laminate Plate

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Abstract

In this paper, the derivation of analytic formulation of stresses analysis has been done using the theory of classical laminate plate. The method of Navier solution is used in the calculation. The composite laminate plate is exposed to out-off plane temperature. The temperature gradient of thermal shock is varied between (60 C^o and -15 C^o). The analytic results of normal stresses are checked and verified using ANSYS software. The normal stress and strain values are decreased sinusoidal and exponentially with the increasing of aspect ratio and fiber volume fraction respectively under the effect of temperature (60 C^o) along x and y directions. For (-15 C^o), the value of normal strain is decreased with the increasing of fiber volume fraction along x and y directions.

Keywords: Classical Plate Theory, Composite Laminate Plate, Temperature Effect, Stresses Analysis;

Nomenclature

σ_{xx}, σ_{yy}: Lamina Stresses Vector, N/m^2.

$\varepsilon_{xx}^{(0)}, \varepsilon_{yy}^{(0)}$: Extensional strain vector.

$\varepsilon_{xx}^{(1)}, \varepsilon_{yy}^{(1)}$: Bending strain vector.

a,b: Length of large and small span of the rectangular plate respectively, m.

N: Total number of layers.

Q_{ij}: Reduced stiffness elements, N/m^2.

B_{ij}: Extension coupling stiffness elements, N.

D_{ij}: Bending stiffness elements, N.m.

M_{xx}, M_{yy}, M_{xy}: Bending and twisting moments, N.m.

ΔT: Gradient uniform temperature, C^o.

A_1, A_2: Bending moment due to temperature, N.m/C^o.

m,n: Double trigonometric of Furrier series.

u_o, v_o: Mid plane in-plane displacement along x and y directions, m.

w_o : Mid plane deflection along z-direction.

x,y,z: Cartesian coordinate system, m.

z_k, z_{k+1}: Upper and lower lamina surfaces coordinates along z direction, m.

α_1, α_2: Thermal expansion coefficient in longitudinal and lateral directions, 1/C^o.

1. Introduction

The effect of temperature on the thermal stress distribution of composites laminate plate are one of the primary life limiting factors of machinery components. The main contribution of this work is the mismatch of the resin and fiber due to thermal loading through a plate thickness. The mismatch of the composite laminate plate occurs since the matrix shear stress around the fiber could exceed the allowable matrix shear stress or the bond between the fiber and the matrix might be broken, [1]. The application of this work is in reusable launch vehicle, satellite, and rocket engine. Thangaratnam R.K. et al. presented the thermal stresses in cross-ply and angle-ply of laminated plates and shells under the effect of thermal gradients a cross the thickness, [2]. They used semiloof finite element formulation to extend the thermal stress analysis for different boundary conditions taking into consideration the temperature dependence of the material properties. An exact closed-form solutions are presented by Carrera E. et al. using the applications of Ritz method and Finite element method, [3]. Sen F. et al. used ANSYS finite element software to evaluate the thermal stress analysis of symmetric angle-ply laminated thermoplastic composite plates with different square hole dimensions. They selected a uniform temperature such as (50 C^o, 60C^o, 70 C^o, 80 C^o, 90 C^o, and 100 C^o) and examined the effect of increasing uniform temperature on the value of thermal stresses, [4]. Sit M. et al. analyzed laminated composite plates in which is subjected to a constant temperature using third order shear deformation theory. The results of deflection and stresses are validated using ANSYS Ver. 14 based on the first order shear deformation theory. Finite element modeling has been generated using an eight nodes isoperimetric element with seven degrees of freedom per node, [5,6]. Rohwer K. et al. used first order shear

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deformation theory to determine the transverse shear and normal stresses for any plate under a sinusoidal temperature distribution, [7]. Vnuec Z. performed a numerical analysis of stress and strain values of angle-ply with four symmetric layered of laminated plate, [8]. Moreover EL Moufari and Meryem proposed a several numerical simulation to describe the interaction between thermal and mechanical stresses, [9,10]. In this paper, the effect of temperature on the stresses analysis of composites laminate plate is studied.

2. Equations of Motion In Terms of Displacement

In this paper, classical laminate plate theory represents the three dimensional of experimental stress analysis in two dimensional form using the state of plane stress. For especially orthotropic (that the material axes coincide with respect to laminates coordinates), the stress-strain relation is as below, [11]:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}^k = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix} * \left(\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} - \begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix} * \Delta T\right) \quad (1a)$$

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z * \begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} \quad (1b)$$

Where:

$$\{\varepsilon^{(0)}\} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u_o}{\partial x}\\ \frac{\partial v_o}{\partial y}\\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\end{Bmatrix}$$

And;

$$\{\varepsilon^{(1)}\} = \begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} = \begin{Bmatrix}-\frac{\partial^2 w_o}{\partial x^2}\\ -\frac{\partial^2 w_o}{\partial y^2}\\ -2 * \frac{\partial^2 w_o}{\partial x\,\partial y}\end{Bmatrix}$$

The differential laplacian equation of motion for an isotropic thin rectangular plate is as follows:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2 * \frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} = 0 \quad (2)$$

The resultant moment including the temperature effects are:

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \begin{Bmatrix}M_{xx}^{Mech.}\\ M_{yy}^{Mech.}\\ M_{xy}^{Mech.}\end{Bmatrix} - \begin{Bmatrix}M_{xx}^{Ther.}\\ M_{yy}^{Ther.}\\ M_{xy}^{Ther.}\end{Bmatrix} \quad (3)$$

Where:

$$\begin{Bmatrix}M_{xx}^{Mech.}\\ M_{yy}^{Mech.}\end{Bmatrix} = \begin{bmatrix}B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix} * \begin{Bmatrix}\frac{\partial u_o}{\partial x}\\ \frac{\partial v_o}{\partial y}\\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\end{Bmatrix} - \begin{bmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix} * \begin{Bmatrix}\frac{\partial^2 w_o}{\partial x^2}\\ \frac{\partial^2 w_o}{\partial y^2}\\ 2 * \frac{\partial^2 w_o}{\partial x\,\partial y}\end{Bmatrix}$$

And;

$$\begin{Bmatrix}M_{xx}^{Ther.}\\ M_{yy}^{Ther.}\end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12} & Q_{22}\end{bmatrix}^{(k)} * \begin{Bmatrix}\alpha_1\\ \alpha_2\end{Bmatrix}^{(k)} * \int_{z_k}^{z_{k+1}} \Delta T * z\ dz$$

It can be assumed that all layers have the same orientation (ϴ =0), and the same thickness in which (B_{ij} = 0). Substitute Eqn.(3) into Eqn. (2) it can be obtained the equation of motion in terms of stiffness bending:

$$D_{11} * \left(\frac{\partial^4 w_o}{\partial x^4}\right) + 2 * (D_{12} + 2 * D_{66}) * \left(\frac{\partial^4 w_o}{\partial x^2\,\partial y^2}\right) + D_{22} * \left(\frac{\partial^4 w_o}{\partial y^4}\right) = -\left(\frac{\partial^2 M_{xx}^{Ther.}}{\partial x^2} + \frac{\partial^2 M_{yy}^{Ther.}}{\partial y^2}\right) \quad (4)$$

3. Calculation of Thermal Stress by Using Navier Solution

The thermal stress is derived based on the solution of classical laminate plate theory using Navier equation taking into consideration the simply supported boundary condition from all edges. It can be assumed that the temperature is varied linearly through the plate thickness, is as below:

$$\Delta T(x, y, z) = z * T_1(x, y) \quad (5)$$

Where:

T_1 is the out-off plane uniform temperature when the heater is in the out-off plane direction.

$$\Delta T(x, y, z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_{mn}(z) * \sin(\alpha_m * x) * \sin(\beta_n * y) \quad (6)$$

Equation (6) can be written in another form,is as below:

$$T_{mn}(z) = \frac{4}{a*b} * \int_0^a \int_0^b \Delta T(x, y, z) * \sin(\alpha_m * x) * \sin(\beta_n * y)\, dxdy \quad (7)$$

Where:

$$\alpha_m = \frac{m * \pi}{a}, m = 1,3,5, \dots, \infty$$

$$\beta_n = \frac{n * \pi}{b}, n = 1,3,5, \dots, \infty$$

Substitute Eqn.(5) into Eqn.(7):

$$T_{mn}(z) = \frac{16*T_1*z}{m*n*\pi^2} \quad (8)$$

The thermal loading is defined as follows:

$$\begin{Bmatrix} M_{xx}^{Ther.} \\ M_{yy}^{Ther.} \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} M_{mn}^{(1)} \\ M_{mn}^{(2)} \end{Bmatrix} * \sin(\alpha_m * x) * \sin(\beta_n * y) \quad (9)$$

Where:

$$M_{mn}^{(1)} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} (Q_{11}*\alpha_1 + Q_{12}*\alpha_2) * T_{mn}(z) * z\,dz$$

$$M_{mn}^{(2)} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} (Q_{12}*\alpha_1 + Q_{22}*\alpha_2) * T_{mn}(z) * z\,dz$$

After simplify Eqn.(9), the thermal loading is as below:

$$\text{thermal loading} = \frac{\partial^2 M_{xx}^{Ther.}}{\partial x^2} + \frac{\partial^2 M_{yy}^{Ther.}}{\partial y^2} = \frac{-16*T_1}{3*\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(A_1*\alpha_m^2 + A_2*\beta_n^2)}{m*n} * \sin(\alpha_m * x) * \sin(\beta_n * y)$$

The solution of normal deflection is as below:

$$w_o = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} * \sin(\alpha_m * x) * \sin(\beta_n * y) \quad (10)$$

Substitute Eqn.(9) and Eqn.(10) into Eqn.(4):

$$w_{mn} = \frac{\frac{16*T_1}{3*\pi^2*m*n}*(A_1*\alpha_m^2 + A_2*\beta_n^2)}{D_{11}*\alpha_m^4 + 2*(D_{12}+2*D_{66})*\alpha_m^2*\beta_n^2 + D_{22}*\beta_n^4} \quad (11a)$$

Then:

$$w_o(x,y) = \frac{16*T_1}{3*\pi^2} * \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(A_1*\alpha_m^2 + A_2*\beta_n^2)*\sin(\alpha_m*x)*\sin(\beta_n*y)}{m*n*[D_{11}*\alpha_m^4 + 2*(D_{12}+2*D_{66})*\alpha_m^2*\beta_n^2 + D_{22}*\beta_n^4]} \quad (11b)$$

Where:

$$A_1 = \sum_{k=1}^{N}(Q_{11}*\alpha_1 + Q_{12}*\alpha_2) * (z_{k+1}^3 - z_k^3)$$

$$A_2 = \sum_{k=1}^{N}(Q_{12}*\alpha_1 + Q_{22}*\alpha_2) * (z_{k+1}^3 - z_k^3)$$

Derive eqn.(11b) once and twice to obtain the thermal stresses analysis:

$$\sigma_{xx}^{(k)}(x,y,z) = -z * [-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(Q_{11}*\alpha_m^2 + Q_{12}*\beta_n^2) * w_{mn} * \sin(\alpha_m * x) * \sin(\beta_n * y)] \quad (12a)$$

$$\sigma_{yy}^{(k)}(x,y,z) = -z * [-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(Q_{12}*\alpha_m^2 + Q_{22}*\beta_n^2) * w_{mn} * \sin(\alpha_m * x) * \sin(\beta_n * y)] \quad (12b)$$

4. Numerical Simulation

In this paper, finite element discretization is carried out using ANSYS Ver. 19.2. (SHELL 132) element is used to mesh the composite laminate plate, [12]. SHELL 132 is defined by eight nodes having six degrees of freedom at each node to calculate the normal stress-strain analysis. The maximum value of stress and strain of the middle point has been selected in which it has maximum normal bending deflection. In numerical simulation, the effect of temperature is considered. Fiber volume fraction (25.076%) has been selected from the experimental results. The simply supported boundary condition from all edges is used to compare the value of thermal stress of Navier solution. The out-off plane temperature is constant and distributed along the length and the width of the composite laminate plate. The loading condition are shown in Figure 1.

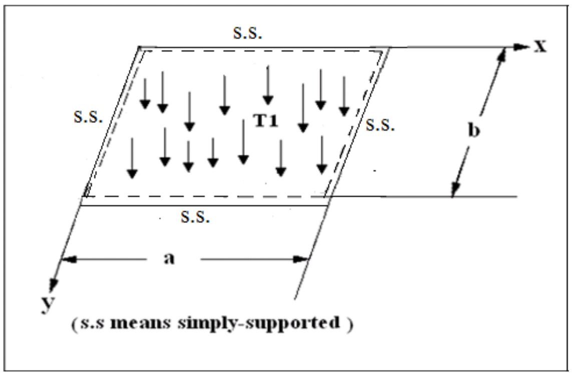

Figure 1. Loading condition on composite laminate plate.

5. Results and Discussions

Figure 2 and 3 show the verification of normal stress versus the aspect ratio along x and y directions. Navier method and ANSYS Ver. 19.2 are verified and checked for thermal stress analysis under the effect of temperature which is varied between (60 C^o) and (-15 C^o). The normal stress value is decreased sinusoidal with the increasing of aspect ratio for (60 C^o) along x and y directions, as illustrated in Figures 2 and 3, while it is decreasing for (-15 C^o) along x direction, as shown in Figure 2. The value of stress is increased with the increasing of aspect ratio for (-15 C^o) in which the normal stress settlesat aspect ratio between (1.8-2.3) along y direction, as depicted in Figure 3. In general, the value of normal stress at (60 C^o) along x and y directions is bigger than the value of normal stress at (-15 C^o).

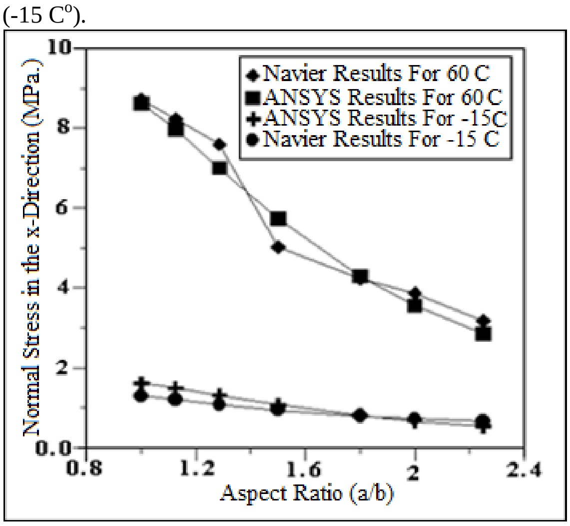

Figure 2. Verification of normal stress in the x-direction for (60 C^o) and (-15 C^o).

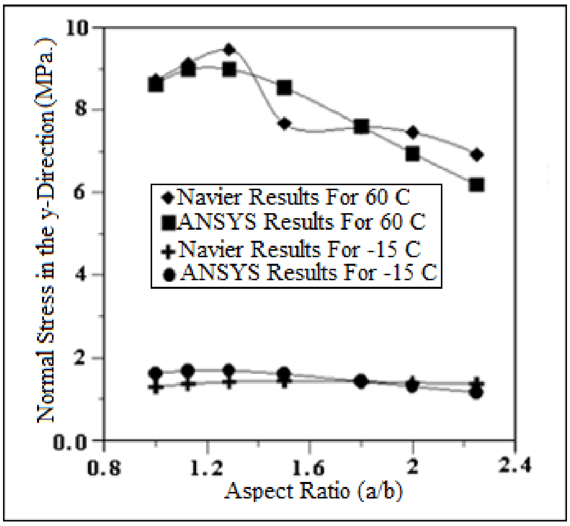

Figure 3. Verification of normal stress in the y-direction for (60 C^o) and (-15 C^o).

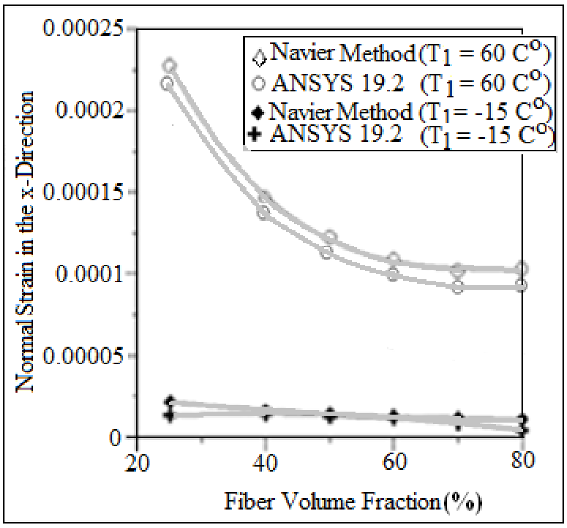

Figure 4. Verification of normal strain in the x-direction for (60 C^o) and (-15 C^o).

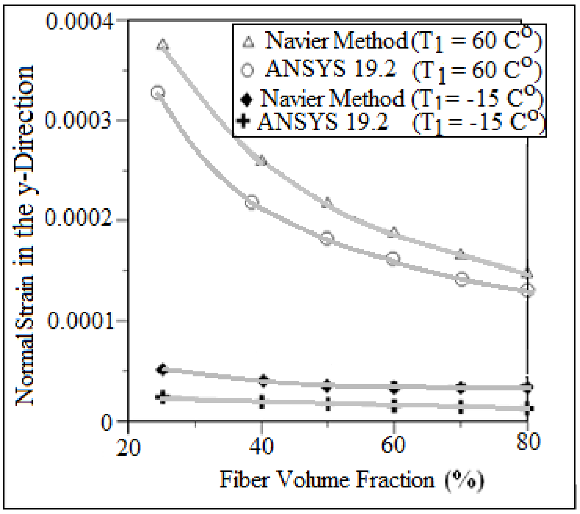

Figure 5. Verification of normal strain in the y-direction for (60 C^o) and (-15 C^o).

Figures 4 and 5 show the verification of normal strain against the percent of fiber volume fraction along x and y directions. Navier method and ANSYS Ver. 19.2 are verified and checked for normal strain under the effect of temperature which is varied between (60 C^o) and (-15 C^o). The normal strain is decreased exponentially with the increasing of fiber volume fraction for (60 C^o), as illustrated in Figures 4 and 5, while it is decreasing for (-15 C^o) along x and y directions, as shown in Figures 4 and 5 respectively. The value of normal strain settlesat the volume ratio between (50-70 %) for (-15 C^o). In general, the value of normal strain at (60 C^o) along x and y directions is bigger than the value of normal strain at (-15 C^o).

Figures 6 and 7 show the verification of normal bending deflection using Navier solution taking into consideration ANSYS 19.2 results in the comparison. The normal bending deflection decreased with the increasing of plate aspect ratio because of the increasing in plate bending stiffness at (60 C^o) and (-15 C^o) for fiber volume fraction (25.075%). The bending deflection value when (60 C^o) is higher than the value of bending deflection when (-15 C^o).

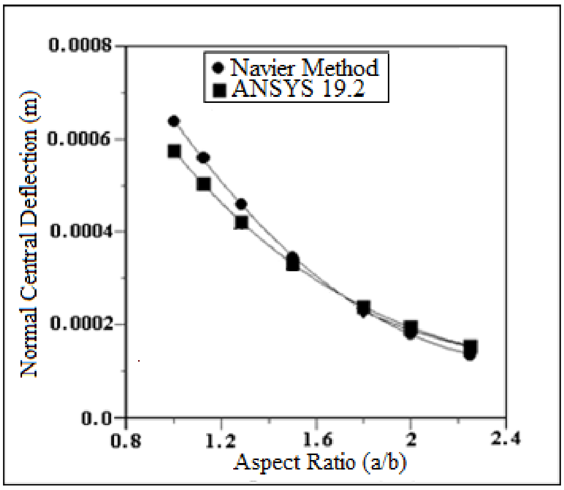

Figure 6. Normal bending deflection versus with laminate plate aspect ratio at (60 C^o).

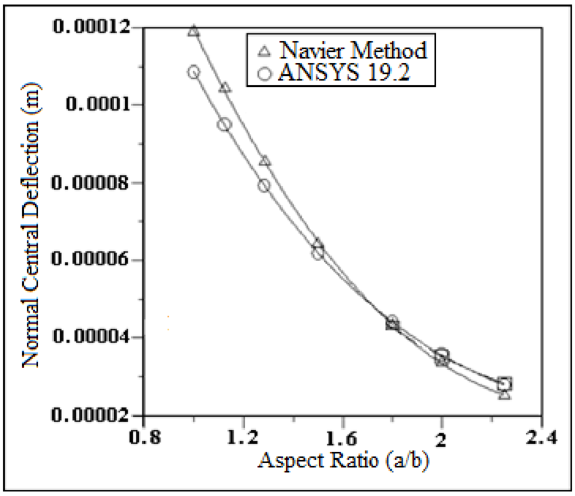

Figure 7. Normal bending deflection versus with laminate plate aspect ratio at (-15 C^o).

6. Conclusions

In this paper, the derivation of analytic formulation of thermal stress analysis has been done using the theory of classical laminate plate. The method of Navier solution is used in the calculation. ANSYS software is used in the verification. The normal stress-strain values are decreased sinusoidal and exponentially with the increasing of aspect ratio and fiber volume fraction respectively under the effect of temperature (60 C^o) along x and y directions. For (-15 C^o), the value of normal strain is decreased with the increasing of fiber volume fraction along x and y directions.

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