

Estimation of Fuel Consumption in a Hypothesized Spoke-hub Airline Networks for the Transportation of Passengers

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Abstract

Transportation industry expands rapidly, fuel is a main cost element in air transportation industry. Worldwide airline networks strive to adopt a method, which enables them, to schedule their flights, with the minimum quantity of fuel to transport passengers through hub and spoke airlines networks. Therefore, fuel consumption should be investigated as economically as possible. To minimize the voyage fuel consumption for a set of aircraft routes, merging airfreight transportation routes through the hub and spoke networks plays a significant value in reducing the redundancy of long flights. A transshipment network of various numbers of international airports is developed; the developed transshipment network is then converted into a transportation network that is assumed to transfer passengers from their source airport to their intended destination airport directly. The methodology has been applied to three different real-life cases. Obtained solutions are tested and validated. Rationality of solutions are decided. A scientific methodology for the analysis and for the scheduling of passengers' transportation is presented. The methodology generated valid solutions that have been found workable which can improve transportation economics.

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Keywords: Spoke-hub modelling; Transportation; Transshipment; Airlines Fuel consumption; Passengers scheduling; Airline routing;

1. Introduction

Fuel costs are an airline's second largest expense. Only labor costs exceed fuel's cost. In 2000, airlines paid about \$5.4-billion in fuel costs, (Air Transport Association (ATA)). Any increase in fuel costs is usually passed onto passengers in the ticket price. Fuel price fluctuations and variations in the total operational cost of passenger transportation show that an increase (decrease) in fuel price leads to rise (fall) in total operation cost. Most of the passenger airlines in operation use a hub-and-spoke network to route their plane traffic. As shown by Figure 1, a hub is a central airport that flights are routed through, and spokes are the routes that planes take out of the hub airport. Most major airlines have multiple hubs. They claim that hubs allow them to offer more flights for passengers. Today, most airlines have at least one central airport that their flights must go through. From that hub, the spoke flights take passengers to select destinations. The purpose of the hub-and-spoke system is to save airlines money and to give passengers better routes to destinations. Airplanes are an airline's most valuable commodity, and every flight has certain set of costs. Each seat on the plane represents a portion of the total flight

cost. For each seat that is filled by a passenger, an airline lowers its break-even price, which is the seat price at which an airline stops losing money and begins to show a profit on the flight.



Figure1. Example for a hub-and-spoke network [1].

Growth and rapid transformation of the international air transport industry is increasing rapidly. Fuel is a main cost element in air transport industry. A typical airline spends 10% of its operating budget on fuel [2]. The global mobility and freight of passengers increase in international tourism, and the global growth of the economy at large. Various factors, such as the world's largest aviation market, the adoption of mid-sized long-haul aircraft,

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climate changes and depletion of stratospheric Ozone, and most importantly emissions of greenhouse gases will significantly create substantial fuel related emissions [3]. In 1990 airlines industry contributed to about 3.5% of global gas emissions [4]. This share depends on many factors, and energy efficiency is one of them.

Airlines fuel consumption is a very large cost element in transportation industry; it is highly correlated with emissions and contributes directly to transport externalities [3]. It has been reported how fuel efficiency of commercial aircraft has developed since 1930s [4]. Peeters comparing large piston-engine aircraft with both old and new jet engine, in their macro analysis they revealed an increase in fuel consumption per seat-kilometer as piston-engine aircraft were replaced by jet- one. As economy depends on fuel prices, fuel prices affect airlines operating cost, and affects the demand on travel and cargo. In 2003, fuel represented about 28% of total operating cost for a typical Airbus - A320, by 2006 fuel prices represented about 43% of all operating costs [5]. Many studies on fuel consumption and conservation have been conducted after Arab oil embargo after 1970. David A. Pilati evaluated and discussed various fuel saving strategies, [6]. In [7-9] various models for managing fuel consumption that resulted in fuel saving have been developed in, the operational parameters which effect fuel consumption have been studied in [10-12]. Alan J. Stolzer [2] examined extensively the literature related to fuel consumption efforts, and the related statistical methodologies, he stated that most fuel consumption studies are concerned with engineering rather than operational issues, he recommended that the implementation of a hypothesized Flight Operations Quality Assurance (FOQA) programs at airlines is helpful in this management effort. Recently, Megan et al. [13] developed a model for the estimation of fuel consumption based on two variables, distance and seats. It has been reported that airlines are seeking merger partners strategically to improved fuel efficiency, reduce cost, and boost revenue. That can eliminate network redundancy. The reduction of fuel consumption and its balance against the impact on passengers is also highlighted in [13], across two major US airline mergers, they find that the number of non-stop destinations and flight frequency per connection dropped significantly while the number of passengers increased considerably. It has been found in [13] that the fuel savings achieved by merged both airline networks. Modeling of fuel consumption cost has been considered by several researches. An operating cost model is developed in [14] by summing fuel consumption, labor costs, and other additional costs for specific aircraft types, with the goal of comparing aircraft costs parametrically over fuel price. In 1984 Oster, C. V., Jr., and A. McKey [15] compared the operating costs of different computer aircraft and they performed a parametric analysis of operating cost versus stage length.

A fuel burn in Kilos per seat per Nautical mile [Kg/seat NM] has been standardized in [3] for airplanes using aircraft inventory database. Comparative geographical heterogeneity of fuel burn rates among different distance and routes in the long-haul market is shown in [3], while controlling for seat configuration and stage distance. Zou and others [16] presented deterministic and stochastic

approaches to investigate the efficiency of fuel consumption, in their ratio-based analysis, 15 main airlines operators in the US. Are considered. A multi-objective optimization model has presented in [17] for a robust airline schedules, the considered the incremental changes of flight schedule and of the aircraft maintenance schedule. Their model is solved based simulation; the approach is tested by real world data from KLM Royal Dutch Airlines, significant improvements for the considered objectives was addressed. A network model for airline scheduling is proposed in [18], the scheduling model is solvable in a real-time environment, and it can be used in sophisticated operational and planning systems. The operational crew-scheduling problem is presented in [19] where both the crew pairing, and crew roistering problems are studied simultaneously. The importance of minimizing the total passenger waiting time is highlighted in [20, 21]. A heuristic that minimizes the number of canceled flights and the total passengers waiting time. A network models to determine aircraft swaps and flight cancellations are presented in [22, and 23]. Recent work on scheduling is found in [24-28]. This paper is expected to provide an assessment tool for reducing fuel consumption and consequently costs of transporting passengers and goods by airplanes through the hub and spoke networks. The main idea behind that is to minimize the voyage fuel consumption for a set of aircraft routes, as well as considering an adjustment on airports and the type of airplanes deployed on the destinations, merging airfreight transportation routes through the hub and spoke networks plays a significant value in reducing the redundancy of long flights. The proposed hub and spoke model is expected to enhance transportation efficiency through simplifying a network of routes. Airline companies may take advantage of the concept as it is expected to revolutionize the way airlines are run. A generic operation research model for the hub-and-spoke airlines network shown in figure 2 will be formulated subjected to traveling needs of passengers with the minimum fuel consumption airlines network.

The proposed model should capture some realistic shipping constraints, such as passengers transporting time, demand, destinations, and some other important factors. The main idea of our constructive insertion approach is to minimize fuel consumption for each main aircraft route, as well as considering an adjustment on airports called and the type of airplanes deployed on the main aircraft route. Satisfying the travelling needs of passengers through a hub and spoke networks will enable us to minimize the number of aircrafts travelling through the networks, yielding an expected savings in aircrafts fuel.

2. Materials and Methods

An attempt to formulate a generic model for a worldwide hub-and-spoke airlines network composed of N hubs, each hub composed of (mn) spokes as depicted in Figure 2, will be conducted. The proposed model should result in a route matrix of an adequate number of airplanes, satisfying travelling needs for long distance passengers, with the best fuel consumption, within the whole network paradigm. In this model, passengers are assumed to travel

from any airport source (spoke or hub) through any airport(s) (spoke or hub) to their destination airport.

2.1. Nomenclature

Nomenclature and notations shown in table 1 are used to formulate the proposed model.

2.2. Model Formulation

A suggested mathematical equation for the computation of the fuel consumption per passenger seat occupied for a travelling passenger between airport (i) and airport (j) is as follows:

$$c_{ij} = \frac{(d_{ij})(ac_{ij})(f_{ij})}{(os_{ij})^2(s_{ij})} \quad (1)$$

The objective is to minimize the total fuel consumption as follow:

$$\text{Min } z = \sum_{i=1}^L \sum_{j=1}^L c_{ij}(x_{ij}), c_{ij} = \infty \text{ for all } i = j \quad (2)$$

Subject to

$$L = N + \sum_{n=1}^N M_n \quad (3)$$

$$b_i = \sum_{j=1}^L x_{ij}, \quad i = \{1, 2, 3, \dots, L\} \quad (4)$$

$$a_j = \sum_{i=1}^L x_{ij}, \quad j = \{1, 2, 3, \dots, L\} \quad (5)$$

$$\sum_{i=1}^L b_i = \sum_{j=1}^L a_j \quad (6)$$

$$x_{ij} \geq 0, \forall (i), \text{ and } \forall (j) \quad (7)$$

The objective function in equation (2), and the constraints in equations (3), (4), (5), (6), and (7), constitute an operations research model, which can be used to estimate fuel consumption. The model depends on distance between airports (d_{ij}), aircraft performance in terms of rate of fuel consumption per hour of traveling from i to j , and the airplane speed of traveling from i to j in Kilometer per hour.

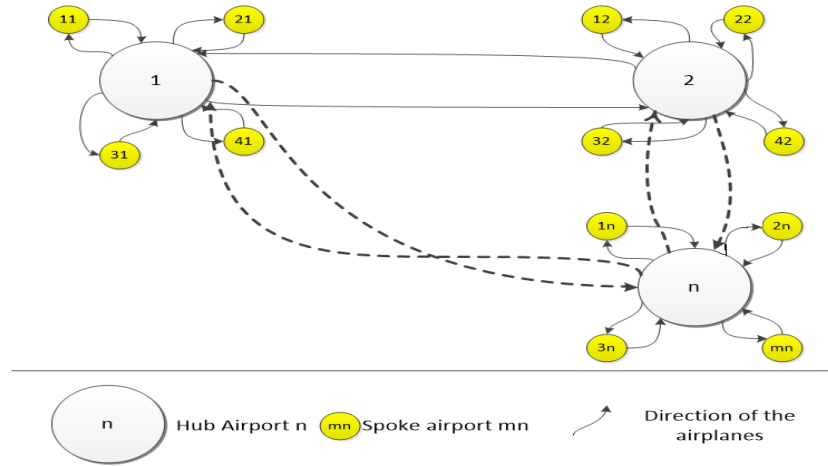


Figure 2. Schematic Illustration of (n) Hub (mn) Spoke Paradigm.

Table 1. Nomenclature and notations

N	Total Number of hubs in the paradigm
n	The n^{th} hub in the paradigm, $n = \{1, 2, \dots, N\}$
M_n	Total Number of spokes of the n^{th} hub paradigm
m_n	The m^{th} spoke of the n^{th} hub in the paradigm, $mn = \{1_n, 2_n, \dots, M_n\}$,
L	Total airports involved in the paradigm
i	Origin city airport index (Source node), $i = \{1, 2, \dots, L\}$,
j	Destination city airport index (Destination node) $j = \{1, 2, \dots, L\}$,
c_{ij}	Unit transportation cost from origin city airport (i) to destination city (j) in term of the fuel consumed per passenger per occupied seat.
x_{ij}	Unit to be shipped from origin city airport (i) to destination city airport (j)
b_i	Unit of supply on origin city airport (i), $b_i = \sum_{j=1}^L x_{ij}$, $i = \{1, 2, \dots, L\}$
a_j	Unit of demand on destination city airport (j) $a_j = \sum_{i=1}^L x_{ij}$, $j = \{1, 2, \dots, L\}$
OS_{ij}	Occupied Seats (OS) of an airplane traveling from origin city airport (i) to destination city airport (j). Seat/ airplane
ac_{ij}	Airplane Capacity travelling from origin city airport (i) to destination city airport (j).Seat/ airplane
d_{ij}	Travelling distance in Kilometer from origin city airport (i) to destination city airport (j).
f_{ij}	Rate of fuel consumption per hour of the airplane traveling from origin city airport (i) to destination city airport (j). Litters/(hour. airplane)
s_{ij}	Speed of the airplane traveling from origin city airport (i) to destination city airport (j) in Kilometer per hour.
k_{ij}	Number of airplanes required to be utilized between origin city airports (i) to destination city airport (j).

2.3. Solution Methodology

To obtain a valid solution, the following methodology, which is depicted in Figure 3, is followed:

1. Sketch the situation: A hub and spoke paradigm of (n) hub, (mn) spoke should be formulated as a transshipment network. A regular outline that describes the situation correctly and precisely with relevant data to the situation should be presented.
2. Identify N : Identifying the number of system hubs.
3. Identify the spokes mn for each hub (n), $n = 1, 2, \dots, N$: Identifying spokes and there related hubs.
4. Computing L : Total number of cities is computed using equation (3).
5. I- Vector construction: Identify indices of the airports, usually $i = 1, 2, \dots, L$.
6. J- Vector construction: Identify destination cities airports indices $j = 1, 2, 3 \dots L$.
7. D- Matrix construction: Identify the travel distance (d_{ij}) in Kilometer from origin city i to destination city j .
8. AC- Matrix construction: Identify airplane capacity (ac_{ij}) travelling from i to j .
9. OS- Matrix construction: Identify occupied seats (os_{ij}) of an airplane traveling from i to j .
10. F- Matrix construction: Identify rate of fuel consumed per hour (f_{ij}) by an airplane traveling from i to j .
11. S- Matrix construction: Identify the speed of an airplane (s_{ij}) traveling from i to j .
12. C- Matrix estimation: Estimate a passenger travelling fuel consumption (c_{ij}) from origin city i to destination city j based on equation (1), excel spreadsheets are recommended to be used.
13. A- Vector construction: Identify original demand at every destination city j (a_j), then compute total demand.
14. B- Vector construction: Identify original supply at every origin city i (b_i), then compute total supply.
15. Network balance check: To proceed correctly, total demand should equal total supply. If total demand does not equal total supply, a dummy source/ destination should be added to the network, with a quantity of supply/ demand equal to the difference between total supply and total demand. A zero-cost coefficient should be assigned to all dummy cells; accordingly, all the previous steps should be modified.
16. Buffer (Bu) computation: Buffer is computed using equation (6).
17. AA Vector construction: Compute total demand at every destination city j (aa_j), which is computed according to the following equation:

$$aa_j = \begin{cases} a_j & \text{Pure demand node} \\ a_j + B & \text{Pure transshipment node} \end{cases} \quad j = \{1, 2, 3, \dots, L\} \quad (8)$$
18. Computing BB vector, total supply at every origin city i , (bb_i), is computed according to equation (9):

$$bb_i = \begin{cases} b_i & \text{Pure demand node} \\ b_i + B & \text{Pure transshipment node} \end{cases} \quad i = \{1, 2, 3, \dots, L\} \quad (9)$$
19. Formulating the transportation model of the problem in tableau format.
20. Solving the problem: Tora software is used.
21. Solution explanation and validation.

3. Variant Real-Life Cases

The core of the problem of this research is to find passengers' routes, which will yield the minimum fuel consumption for the transferring of the passengers within the hub and spoke paradigm. The inputs to the estimation of the fuel consumed per passenger seat problem are just the result of another suboptimal planning problem – the location of hubs and spoke airports problem.

Even the fuel consumed per passenger seat problem described in this thesis is to some extent a simplification of the real problem. By employing a rule modeling tool and generic model, which do not make any assumption on the structure of the problem to be solved, the fuel consumption per passenger seat problem supports a sufficiently accurate modeling of the real-world estimation of fuel consumption in a hypothesized hub and spoke paradigm.

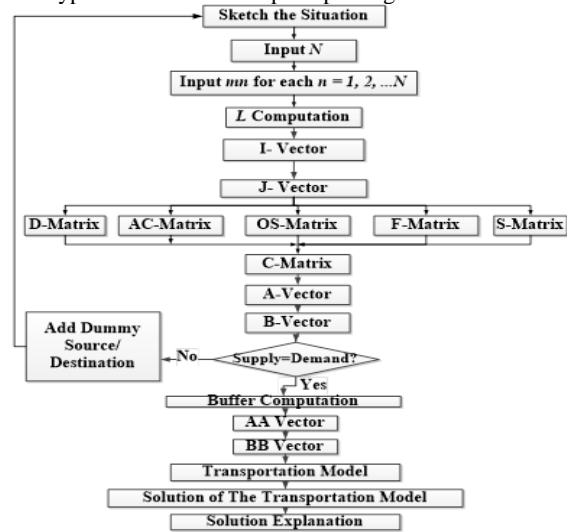


Figure 3. Two hub-spoke groups over other six airports with corresponding supply demand.

3.1. Case Study 1: Royal Jordanian Airplanes outing

The Royal Jordanian (RJ) airlines is the flag carrier airline of Jordan with its head office in Amman, Jordan. RJ offers international services from its main base at Queen Alia International Airport in Jordan. RJ is utilizing two hub-spoke groups connecting long haul passengers through their fleet over other six airports as indicated in figure 4. Cost parameters $\{s_{ij}, f_{ij}, oc_{ij}, ac_{ij}, d_{ij}\}$, demand (b_i) and supply (a_j) between cities airports, are reported in table 2. Using equation (1), the fuel consumed per passenger for a specific route (c_{ij}), (liters per occupied passenger seat) is calculated and recorded in table 3. The transshipment network should be balanced. In this model, the supply is ($= 3,746$) which equals to the total demand ($= 3,746$), the Buffer quantity (Bu) is determined; $\{Bu\} = \text{Total supply} = \text{Total Demand} = 3,746$. Airports are classified into: pure demand airport, transit airport, and pure supply airport.

Accordingly, supply and demand are adjusted using equation (8) and equation (9). The resultant transportation model is then solved using TORA Windows ® version 2.00, 2006. TORA is not a solver. It is a demo for students. More standard solvers should be used, but TORA can do the job for this research.

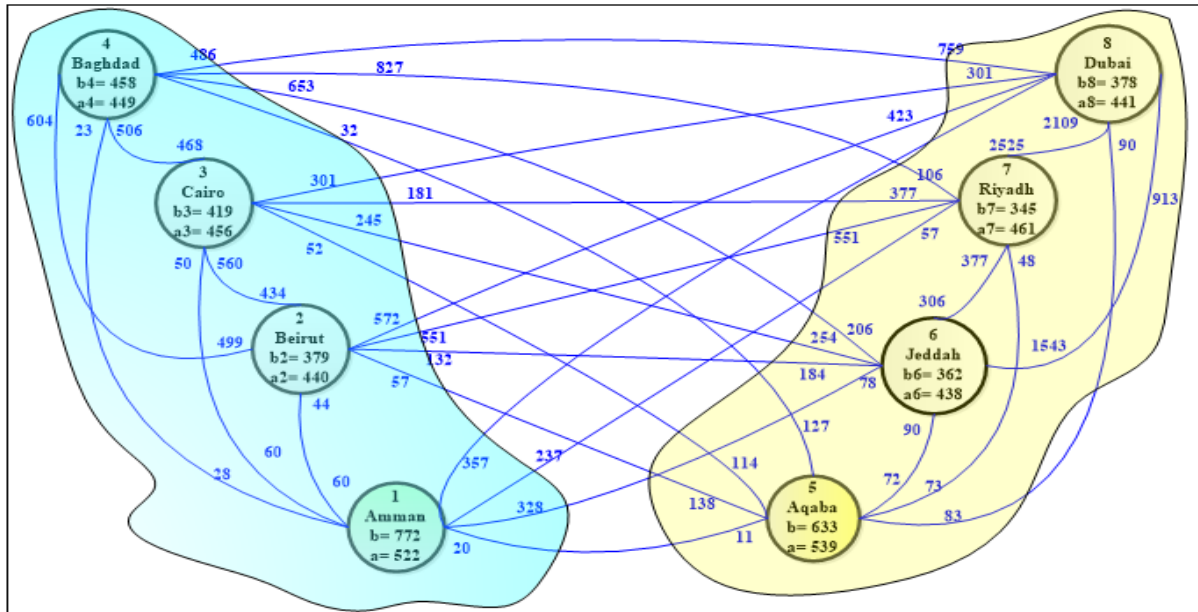


Figure 4. Two hub-spoke groups over other six airports with corresponding supply, demand, and unit transportation costs

Table 2. Cost function parameters between cities airports [14]

			Destination airport (<i>j</i>)								Supply (<i>b_i</i>)	Adjusted Supply (<i>bb_i</i>)
			1	2	3	4	5	6	7	8		
Origin airport (<i>i</i>)	1	<i>d_{ij}</i>	∞	238	473	631	253	1162	1291	2023	772	4518
		<i>ac_{ij}</i>	-	100	136	110	136	136	136	136		
		<i>oc_{ij}</i>	-	93	90	95	135	119	120	120		
		<i>f_{ij}</i>	-	2000	2000	2000	1600	3900	2800	2850		
		<i>s_{ij}</i>	-	124	317	680	276	559	596	676		
	2	<i>d_{ij}</i>	238	∞	563	814	447	473	1466	2142	379	4125
		<i>ac_{ij}</i>	100	-	136	136	136	136	136	136		
		<i>oc_{ij}</i>	80	-	30	50	64	50	55	50		
		<i>f_{ij}</i>	2000	-	4500	4500	2900	4500	3500	3000		
		<i>s_{ij}</i>	124	-	684	330	312	628	419	826		
	3	<i>d_{ij}</i>	473	563	∞	1263	381	1218	1614	2417	419	4165
		<i>ac_{ij}</i>	136	136	-	136	136	136	283	283		
		<i>oc_{ij}</i>	82	35	-	50	67	49	61	75		
		<i>f_{ij}</i>	2000	4500	-	4000	2950	2000	1750	1500		
		<i>s_{ij}</i>	317	648	-	543	300	563	570	606		
	4	<i>d_{ij}</i>	631	814	1263	∞	619	1427	614	1400	458	4204
		<i>ac_{ij}</i>	110	136	136	-	136	136	136	283		
		<i>oc_{ij}</i>	86	55	52	-	55	80	70	60		
		<i>f_{ij}</i>	2000	4500	4000	-	3000	3500	4000	4000		
		<i>s_{ij}</i>	680	330	543	-	656	514	646	580		
	5	<i>d_{ij}</i>	253	447	381	619	∞	986	1226	2054	633	4379
		<i>ac_{ij}</i>	136	136	136	136	-	136	136	136		
		<i>oc_{ij}</i>	100	100	99	109	-	60	90	75		
		<i>f_{ij}</i>	1600	2900	2950	3000	-	1300	1300	1300		
		<i>s_{ij}</i>	276	312	300	656	-	538	559	715		
	6	<i>d_{ij}</i>	1162	473	1218	1427	986	∞	853	1701	362	4108
		<i>ac_{ij}</i>	136	136	136	136	136	-	136	136		
		<i>oc_{ij}</i>	58	59	49	45	67	-	45	39		
		<i>f_{ij}</i>	3900	4500	2000	3500	1300	-	4500	3500		
		<i>s_{ij}</i>	559	628	563	514	538	-	683	583		
	7	<i>d_{ij}</i>	1291	1466	1614	614	1226	853	∞	874	345	4091
		<i>ac_{ij}</i>	136	136	136	136	136	136	-	136		
		<i>oc_{ij}</i>	59	55	61	25	73	50	-	22		
		<i>f_{ij}</i>	2800	3500	1750	4000	1300	4500	-	4500		
		<i>s_{ij}</i>	596	419	570	646	559	683	-	524		
	8	<i>d_{ij}</i>	2023	2142	2417	1400	2054	1701	874	∞	378	4124
		<i>ac_{ij}</i>	136	136	283	283	136	136	136	-		
		<i>oc_{ij}</i>	57	43	75	75	78	30	20	-		
		<i>f_{ij}</i>	2850	3000	1500	4000	1300	3500	4500	-		
		<i>s_{ij}</i>	676	826	606	580	715	583	524	-		
Demand (<i>a_j</i>)			522	440	456	449	539	438	461	441	3746	-
Adjusted demand (<i>aa_j</i>)			4268	4186	4202	4195	4285	4184	4207	4187	-	

Based on North-west corner method solution is obtained, solution is presented in figure 5, it represents the number of passengers who must be transferred between each route, with minimum fuel consumption of \$3,552,307 for the whole paradigm.

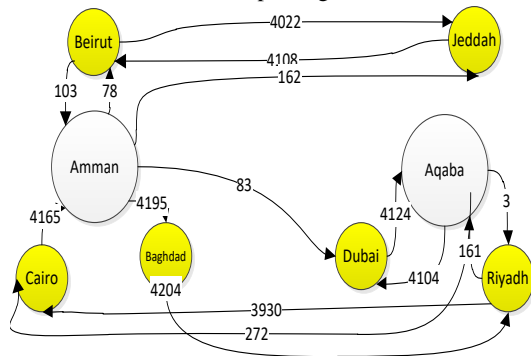


Figure 5. Optimal number of passengers routing through the considered case

Table 3. Fuel consumption per passenger (c_{ij}) computed by equation (1)

i	j	Destination airport							
		1	2	3	4	5	6	7	8
Origin airport	1	∞	44	50	23	11	78	57	81
	2	60	∞	560	604	138	184	551	423
	3	60	434	∞	506	114	245	377	301
	4	28	499	468	∞	127	206	106	759
	5	20	57	52	32	∞	90	48	90
	6	328	132	245	653	72	∞	377	913
	7	237	551	181	827	73	306	∞	2109
	8	357	572	301	486	83	1543	2552	∞

4. Results and discussion

This paper demonstrates a proposed methodology that focuses on creating a route of airplanes between airports of hub and spoke paradigm with the minimum fuel consumption. It develops an operations research model with single linear objective function. The intended model resulted in a routing solution that satisfies some travelling needs of passengers belonging to the hub and spoke network with the minimum fuel consumption of airlines fleets, which is expected to decrease the total costs incurred by airlines. The effectiveness of the proposed model and the efficiency of the proposed methodology have been assessed, using the shown numerical example based that mimic real-life situations.

The model has been created based on the relaxation of the considered hub and spoke paradigm resulting with a capacitated hub and spoke network model, then formulated as transshipment model. The generated transshipment model is then converted to a transportation problem, where hub and spoke airports are considered as a source nodes and a destination nodes. Each source airport has a fixed supply of passengers; the entire supply must be distributed to the destination airports. Similarly, each destination airport has a fixed demand for

passengers, where this entire demand must be received from the sources. A transportation problem will have a feasible solution if and only if, entire supply equals the entire demand, if this not the case the transportation model should be fit by introducing a dummy destination airport or a dummy source airport. The developed model is composed of:

1. A linear single objective function is derived. That depends on:

- Travelling distance $\{d_{ij}$ Kilometer from origin airport (i) to destination airport (j),
- Airplane capacity (ac_{ij}) travelling from i to j ,
- (c) standard rate of fuel consumed per hour by an airplane traveling from i to j ,
- Occupied seats (os_{ij}) of an airplane traveling from i to j , and
- The standard speed of an airplane (s_{ij}) traveling from i to j .

2. A total number of constraints equals to

$$\left(2L = 2N + 2 \sum_{n=1}^N M_n \right), \text{ these constraints assure}$$

that the model satisfying all the transshipment demand as well as the original external demand. When there is a need to introduce a dummy source/destination number of constraints equals to

$$\left(2L + 1 = 2N + 2 \sum_{n=1}^N M_n + 1 \right). \text{ Accordingly, the}$$

considered case has 16 constraints.

3. A total number of decision variables equals to

$$\left(L^2 = \left(N^2 + 2N \sum_{n=1}^N M_n + \left(\sum_{n=1}^N M_n \right)^2 \right) \right).$$

When there is a need to introduce a dummy source/destination number of decision variables equals to

$$\left(L^2 + L = \left(N^2 + 2N \sum_{n=1}^N M_n + \left(\sum_{n=1}^N M_n \right)^2 \right) + L \right)$$

Accordingly, the considered case has 64 decision variables.

4. The considered case has 16 constraints and 15 optimal basic variables.

5. Non-negativity constraints.

A linear formulation of the transshipment network of the considered case is developed as an alternative solution method based on the following; (a) Supply at each pure supply node (i), Table 4. (b) The transshipment quantity at each transshipment node (k) as shown in table 5 and table 6. Transshipment quantity is equal to the buffer quantity if the transshipment node is incapacitated and demand on the transshipment node if any, otherwise is equal to the assigned capacity. And (c) demand at each pure demand node (j) as shown in table 6.

Considering the transshipment network depicted in figure 6, if (x_{ik}) is the transshipped quantity from (i) to (k) with a unit transportation cost of (c_{ik}), and (y_{kj}) is the transshipped quantity from (k) to (j) with a unit transportation cost of (c_{kj}). Then the transshipment network can be modeled mathematically by the following set equations.

Table 4. Supply at each pure supply node

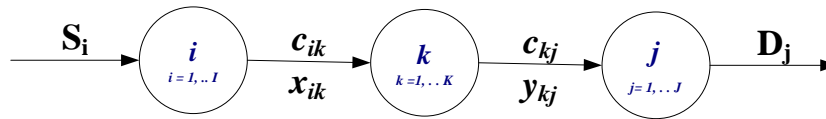
Node	Amman	Beirut	Cairo	Baghdad	Aqaba	Jeddah	Riyadh	Daubi	Total
i	1	2	3	4	5	6	7	8	3746
S_i	772	379	419	458	633	362	345	378	

Table 5. Transshipment quantity at each transshipment node (k)

Node	Amman	Beirut	Cairo	Baghdad	Aqaba	Jeddah	Riyadh	Daubi	Total
k	1	2	3	4	5	6	7	8	22968
T_k	3746	3746	3746	3746	3746	3746	3746	3746	

Table 6. Demand at each pure demand node (j)

Node	Amman	Beirut	Cairo	Baghdad	Aqaba	Jeddah	Riyadh	Daubi	Total
j	1	2	3	4	5	6	7	8	3746
D_i	522	440	456	449	539	438	461	441	

**Figure 6.** WIP flow and input-output relationship between resources and activity center

$$\text{Min } Z = \sum_{i=1}^I \sum_{k=1}^K c_{ik} x_{ik} + \sum_{k=1}^K \sum_{j=1}^J c_{kj} y_{kj}$$

$$\sum_{k=1}^K x_{ik} = s_i \quad \forall i = 1, 2, 3, \dots, I \quad (\text{Supply Constraint at Origin})$$

$$\sum_{i=1}^I x_{ik} = T_k \quad \forall k = 1, 2, 3, \dots, K \quad (\text{Transshipment Constraint})$$

$$\sum_{k=1}^K y_{kj} = D_j \quad \forall j = 1, 2, 3, \dots, J \quad (\text{Demand Constraint at destination})$$

$$\sum_{i=1}^I x_{ik} - \sum_{j=1}^J y_{kj} = 0 \quad \forall k = 1, 2, 3, \dots, K \quad (\text{Transshipment Balancing})$$

$$x_{ik}, y_{kj} \geq 0 \quad \forall i, k, j$$

The problem is solved as a linear operations research model using the roots of Simplex algorithm, Tora for windows version 2 is applied, then solutions are finalized, then the solution is converted into a schedule of airplanes. Validity of solutions has been upraised when the obtained solutions discussed with Royal Jordanian airline scheduling manager, and when model output compared with historical data under similar input conditions. The interpretation of equation (2), (4), (5), and (7) of the proposed model, is also a good indicator for solution validation, it has been found that the optimal fuel consumption cost;

$$z = \left(\sum_{i=1}^L \sum_{j=1}^L c_{ij} (x_{ij}) \right) = 3,552,30700$$

Subject to supply of each origin airport in the travelling paradigm is satisfied, accordingly;

$$\text{For Amman airport}(i=1) = \sum_{j=1}^L x_{Amman,j} = 4,518$$

$$= (x_{12}) + (x_{14}) + (x_{16}) + (x_{18}) \\ = 78 + 4195 + 162 + 83 \\ = 4,518$$

$$\text{For Beirut airport}(i=2) = \sum_{j=1}^L x_{Beirut,j} = 4,125 \\ = (x_{21}) + (x_{26}) = 103 + 4,022 \\ = 4,125$$

Similarly, supply of all other origin airport is checked and found to be satisfied. In addition, demand of each destination airport in the travelling paradigm is checked and it has been satisfied for all destination airport, for example;

$$\text{For Riyadh airport}(j=7) = \sum_{i=1}^L x_{i,Riyadh} \\ = (x_{47}) + (x_{57}) = 4204 + 3 \\ = 4,207$$

$$\text{For Daubi airport}(j=8) = \sum_{i=1}^L x_{i,Daubi} \\ = (x_{18}) + (x_{58}) = 83 + 4,104 \\ = 4,187$$

5. Conclusions

This research can act as a managerial guide for airlines scheduling as well as a reference for future research in this field. One major finding of this paper is an optimal airline routes that distribute the entire passengers from source locations to their intended destinations in a hub and spoke paradigm, with the minimum fuel consumption, accordingly an airline schedule can be produced. One advantage of the presented model is the ability to convert the solution into schedule of airplanes in term of the number of airplanes to be assigned between any two airports in the paradigm network.

Validation of the results indicates that such routes and schedules can be implemented in hub and spoke airline paradigm where an acceptable fuel consumption cost and a proper behavior of the system is expected. Obtained solutions reveal that the proposed model generates solutions that are adequately reasonable. Model output found to be acceptable it does not include surprises, and it does make sense airline scheduling manager. Therefore, solutions can be implemented.

Furthermore, simulation modeling, queuing theory, dynamic programming may be configured for this type of work as a future extension to obtain a valid model output.

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