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Nonlinear Dynamic Modeling of Double Helical Gear System

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Abstract

Double helical gear transmission is a very important transmission component. A lot of dynamic models of spur and helical gears for studying dynamic characteristics have been put forward, but the studies on double helical gears are scarce. To study the nonlinear dynamic characteristics of the double helical gear system, the nonlinear dynamic model of double helical gear system was established. Then the dimensionless dynamic model was summarized. Through numerical computation, the nonlinear dynamic characteristic was revealed in some aspect. The vibration amplitude of gear is smaller than the vibration amplitude of pinion at the same direction and the vibration amplitude of the same component at direction is larger than the amplitude of gear at direction. Through the numerical method, we find that there is a plenty of nonlinear dynamic characteristics in the double helical gear system, and thus more research is needed.

Keywords: *: Time-Varying Meshing Stiffness; Meshing Damping; Double Helical Gear; Nonlinear Characteristics.*

1. Introduction

Gear transmission is the most common transmission system and is widely used in the production of national industries. It is also a particularly important key component. It has three basic forms: spur, helical and double helical. Because of alternate meshing of single and double spur gear teeth in the transmission, the meshing stiffness changed with large amplitude when the coincidence degree is not an integer, motivating large vibration and noise [1-3]. There is not alternate meshing of single and double gear teeth in the helical gear system and the stiffness mutation, so it’s meshing smoother. But there are some shortcomings in the helical gear system, for example: the dynamic axial load, time-varying mesh wire length, time-varying frictional force and time-varying frictional torque [4-9]. The dynamic axial load of double helical gear was eliminated though the time-varying frictional force and time-varying frictional torque exist. Because of its high carrying capacity, smooth transmission, and no axial load, the double helical gear has been widely used. As we all known, many nonlinear factors such as time-varying mesh stiffness, time-varying mesh damping, meshing impact, backlash, time-varying frictional force and friction torque in double helical gear system will generate complex nonlinear vibration. However, studies on system dynamics of double helical gear is seldom, the research on double helical gear system dynamics is very necessary.

Wesley Blankenship G. and Singh R. [10] studied the dynamic meshing force, stiffness and transfer matrix in helical gear system, and researched backlash nonlinear characteristics under different modal parameters. IMAMURA.Y and SATO.S [11] studied the distribution of dynamic stress in gear systems. Walha L. [12] and his partners studied the nonlinear dynamic response of helical gear system with mass eccentricity. Wei [13-14] studied the friction nonlinear characteristics in-depth in the spur gear system. Ma Hui [15] analyzed the influence of eccentric load in helical gear system in detail and studied the dynamic characteristic in eccentric helical gear system by the method of modal analysis and coupling analysis. Wang Qing [16] established the coupling dynamic model and system differential equations of cylindrical helical gear transmission considering time-varying mesh stiffness, backlash and transmission error. N. Leiba [17] studied the vibro-impact phenomenon by experimental and numerical method. Song Xiaoguang [18] studied the nonlinear characteristics of the flexible shaft helical gear system considering backlash, bearing radial clearance and gear unbalanced force. Tang Jinyuan and Chen Siyu [19-20] proposed an improved nonlinear dynamics model of the spur gear. On the basis of the model, the gap nonlinear characteristics were analyzed. Wang Feng and Wang Cheng [21-23] established a dynamic model of herringbone gears, then the herringbone gears’ dynamic characteristics were analyzed, but the calculation and analysis of time-varying parameters and nonlinear factors were not concerned. Li Wenliang [24] analyzed the friction force in helical gear and the bending effect. Wei Jing [25] discussed the backlash nonlinear characteristic in helical gear system. Guo Jiashun [26] analyzed dynamic profile modification with herringbone gear tooth based on

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analytical solutions. Bu Zhonghong [27-28] studied the inherent characteristic and free vibration characteristics of herringbone planetary gear set. Feng Zhigang [29] studied the meshing static-dynamic vibration characteristics of herringbone gear. Yang Xiaofang [30] analyzed the distribution of power flow of the three branches of herringbone gear drive system by simulation. Ma Shangjun [31] analyzed the steady-state response of plate convergence herringbone gears. Wu Xinyue [32, 33] established a simplified kinetic model of herringbone gear and got preliminary solution. Xie Zuiwei [34] analyzed the stress on herringbone gear tooth surface by using the contact stress finite element method. Ahmad Al-Shhyab and Ahmet Kahraman [35] developed a non-linear time-varying dynamic model to study torsional dynamic behavior for a multi-mesh gear system.

Referring to the modeling process of helical gear system and combining its own characteristics, bending-torsion-axis coupled nonlinear vibration model with 16 degrees of freedom was established. Nonlinear factors considered in the process of modeling include time-varying mesh stiffness, meshing impact and backlash. Dimensionless operation was adopted to process the system dynamics equations. To solve the dimensionless differential equations, the 4-5 Runge-Kutta method was used. Then the nonlinear characteristic of time history diagram, phase diagrams, Lyapunov exponent, Lyapunov dimension, Poincare map and global bifurcation graphs in the double helical gear system was studied.

2. Nonlinear Dynamic Model of the Double Helical Gear System

2.1. The Nonlinear Dynamic Model

The double helical gear is wildly used in industry, such as truck, train and other overload machines. A pair of double helical gear is shown in the Figure.1. From the Figure1, we find that the double helical gear could regard as two helical gear with opposite helical angle. Then the 12 degrees bend-twist-axis coupling vibration model of double helical gear pair is established just as Figure2 shown.

Figure 1. A pair of double helical gear

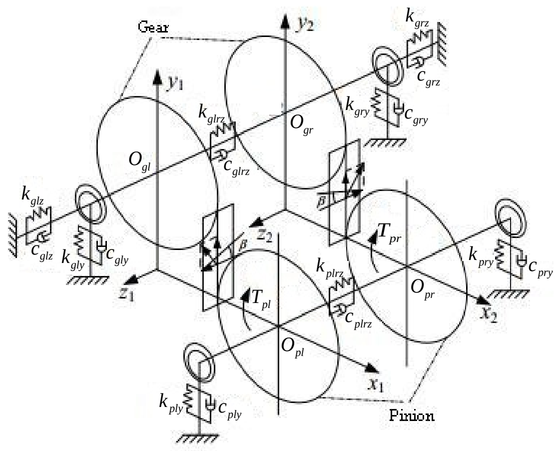


Figure 2. 12 degrees bend-twist-axis coupling vibration model of double helical gear pair

As shown in Figure 2, to establish the nonlinear dynamic model of double-helical gear system of 12 degrees, the tooth surface friction and the friction torque are not considered and the generalized displacement matrix of 12 degrees can be expressed as:

$$\delta = (y_{pl}, z_{pl}, \theta_{pl}; y_{pr}, z_{pr}, \theta_{pr}; y_{gl}, z_{gl}, \theta_{gl}; y_{gr}, z_{gr}, \theta_{gr})^T \quad (1)$$

Where, $y_{ij}, z_{ij}, \theta_{ij} (i = p, g; j = l, r)$ is the translational and rotational vibration displacement of center of pinion and gear $O_{ij} (i = p, g; j = l, r)$ at y, z direction, respectively.

Vibration displacement of meshing point P_l, P_r and G_l, G_r can be expressed as:

$$\bar{y}_{pi} = y_{pi} + \theta_{pi} r \quad (2a)$$

$$\bar{z}_{pi} = z_{pi} - \bar{y}_{pi} \tan\beta = z_{pi} - (y_{pi} + \theta_{pi} r)\tan\beta \quad (2b)$$

$$\bar{y}_{gi} = y_{gi} - \theta_{gi} R \quad (2c)$$

$$\bar{z}_{gi} = z_{gi} - \bar{y}_{gi} \tan\beta = z_{gi} - (y_{gi} - \theta_{gi} R)\tan\beta \quad (2d)$$

Where, $i = l, r$.

Taking the double helical gear, seen as two helical gears with opposite helical gears direction, and assuming the two helical gears have the same parameters except the helix angle, normal meshing stiffness of unilateral can be expressed as:

$$K_{mn}(t) = K_{mn} + \sum_{\xi=1}^{\infty}\{a_{n1}\cos[\xi\omega_h(t)t] + b_{n1}\sin[\xi\omega_h(t)t]\} = K_{mn} + K_{mna}\cos(\omega_h t) \quad (3a)$$

Where: K_m is the normal average mesh stiffness of double helical gear pair; $\omega_h(t)$ is the meshing frequency of double helical gear pair; a_n and b_n are the Fourier coefficients in Eq. (3a).

Mesh stiffness at the tangential and axial directions can be expressed as:

$$K_{mry}(t) = K_{mly}(t) = K_{mn}(t)\cos\beta \quad (3b)$$

$$K_{mrz}(t) = K_{mlz}(t) = K_{mn}(t)\sin\beta \quad (3c)$$

Where, $i = l, r$.

The normal damping is expressed as:

$$C_{mn}(t) = 2\zeta_v \sqrt{K_{mn}(t) m_e} \quad (4a)$$

Where: m_e is the equivalent mass of double helical gear pair, $m_e = \dfrac{J_p J_g}{J_g R_b^2 + J_p r_b^2}$; ζ_v is the relative damping of double helical gear pair, and ζ_v =0.070。

Mesh damping at the tangential and axial can be expressed as:

$$C_{mry}(t) = C_{mly}(t) = C_{mn}(t)\cos\beta \quad (4b)$$

$$C_{mrz}(t) = C_{mlz}(t) = C_{mn}(t)\sin\beta \quad (4c)$$

Where, $i = l, r$.

The tangential and axial dynamic meshing force at left and right ends of double helical gear pair can be expressed as:

$$F_{yl}(t) = K_{mly}(t) f(\bar{y}_{pl} - \bar{y}_{gl} - e_{ly}) + C_{mly}(t)(\dot{\bar{y}}_{pl} - \dot{\bar{y}}_{gl} - \dot{e}_{ly}) \quad (5a)$$

$$F_{zl}(t) = K_{mlz}(t) f(\bar{z}_{pl} - \bar{z}_{gl} - e_{lz}) + C_{mlz}(t)(\dot{\bar{z}}_{pl} - \dot{\bar{z}}_{gl} - \dot{e}_{lz}) \quad (5b)$$

$$F_{yr}(t) = K_{mry}(t) f(\bar{y}_{pr} - \bar{y}_{gr} - e_{ry}) + C_{mry}(t)(\dot{\bar{y}}_{pr} - \dot{\bar{y}}_{gr} - \dot{e}_{ry}) \quad (5c)$$

$$F_{zr}(t) = K_{mrz}(t) f(\bar{z}_{pr} - \bar{z}_{gr} - e_{ry}) + C_{mrz}(t)(\dot{\bar{z}}_{pr} - \dot{\bar{z}}_{gr} - \dot{e}_{ry}) \quad (5d)$$

Where: $e_{ij}(t)\ (i = l, r;\ j = y, z)$ are the tangential and axial meshing error at left and right ends of double helical gear pair, respectively; they can be then expressed in sine function forms as: $e_{ij}(t) = \tilde{e}_{ij} + e_{aij}\sin[\omega_h(t)t + \phi_{eij}]\ (i = l, r;\ j = y, z)$.

From Eq. (5a) to Eq. (5d), $f(j_i)(i = l, r;\ j = y, z)$ is the backlash nonlinear function, and also the tangential and axial relative displacement at left and right ends of double helical gear pair, respectively. Assume that the backlash at left and right ends of double helical gear pair is equal, and the normal backlash is $2b_n$, then the tangential and axial backlash is $2b_t = 2b_n\cos\beta$ and $2b_a = 2b_n\sin\beta$, respectively, where β is the helical angle of the double helical gear system.

$$f(y_i) = \begin{cases} y_i - b_t & (y_i > b_t) \\ 0 & (|y_i| \le b_t) \\ y_i + b_t & (y_i < -b_t) \end{cases} \quad (6a)$$

$$f(z_i) = \begin{cases} z_i - b_a & (z_i > b_a) \\ 0 & (|z_i| \le b_a) \\ z_i + b_a & (z_i < -b_a) \end{cases} \quad (6b)$$

Where, $i = l, r$.

Through the Newton's second law and considering the dynamic meshing force and backlash in double helical gear pair shown in Figures 1 and 2, the kinetic equation of gear system shown in Figure 2 can be established as:

$$\begin{cases} m_{pl}\ddot{y}_{pl} + c_{ply}\dot{y}_{pl} + k_{ply}y_{pl} = -F_{yl}(t) + m_{pl}g \\ m_{pl}\ddot{z}_{pl} + c_{plrz}(\dot{z}_{pl} - \dot{z}_{pr}) + k_{plrz}(z_{pl} - z_{pr}) = -F_{zl}(t) \\ J_{pl}\ddot{\theta}_{pl} = -F_{yl}(t)r + T_{pl} \end{cases} \quad (7a)$$

$$\begin{cases} m_{pr}\ddot{y}_{pr} + c_{pry}\dot{y}_{pl} + k_{pry}y_{pl} = -F_{yr}(t) + m_{pr}g \\ m_{pr}\ddot{z}_{pr} - c_{plrz}(\dot{z}_{pl} - \dot{z}_{pr}) - k_{plrz}(z_{pl} - z_{pr}) = -F_{zr}(t) \\ J_{pr}\ddot{\theta}_{pr} = -F_{yr}(t)r + T_{pr} \end{cases} \quad (7b)$$

$$\begin{cases} m_{gl}\ddot{y}_{gl} + c_{gly}\dot{y}_{gl} + k_{gly}y_{gl} = F_{yl}(t) + m_{gl}g \\ m_{gl}\ddot{z}_{gl} + c_{glz}\dot{z}_{gl} + k_{glz}z_{gl} + c_{glrz}(\dot{z}_{gl} - \dot{z}_{gr}) \\ \quad + k_{glrz}(z_{gl} - z_{gr}) = F_{zl}(t) \\ J_{gl}\ddot{\theta}_{gl} = F_{yl}(t)R - T_{gl} \end{cases} \quad (7c)$$

$$\begin{cases} m_{gr}\ddot{y}_{gr} + c_{gry}\dot{y}_{gr} + k_{gry}y_{gr} = F_{yr}(t) + m_{gr}g \\ m_{gr}\ddot{z}_{gr} + c_{grz}\dot{z}_{gr} + k_{grz}z_{gr} - c_{glrz}(\dot{z}_{gl} - \dot{z}_{gr}) \\ \quad - k_{glrz}(z_{gl} - z_{gr}) = F_{zr}(t) \\ J_{gr}\ddot{\theta}_{gr} = F_{yr}(t)R - T_{gr} \end{cases} \quad (7d)$$

Where, $m_{ij}(i = p, g;\ j = l, r)$ are the mass at left and right ends of pinion and gear, respectively; $J_{ij}(i = p, g;\ j = l, r)$ are the moment of inertia at left and right ends of pinion and gear respectively; r, R are reference radius of pinion and gear, respectively.

2.2. Dimensionless of Kinetic Equations

In order to obtain these kinetic equations dimensionless, define the system dimensionless time and dimensionless excitation frequency, respectively as:

$$\tau = t \cdot \omega_n \quad (8a)$$

$$\omega = \omega_h / \omega_n \quad (8b)$$

Where: ω_n is the natural frequency of the system, and $\omega_n = \sqrt{K_{mn} / m_e}$; m_e is the equivalent mass of gear pair; K_{mn} is the normal average mesh stiffness of gear pair.

Take b_n as the nominal dimension to take the Eq. (7) dimensionless. The dimensionless displacement of double helical gear system can be expressed as：

$$\begin{aligned}&p_1 = y_{pl}/b_n,\ p_2 = z_{pl}/b_n,\ p_3 = r\theta_{pl}/b_n\\&p_4 = y_{pr}/b_n,\ p_5 = z_{pr}/b_n,\ p_6 = r\theta_{pr}/b_n\\&p_7 = y_{gl}/b_n,\ p_8 = z_{gl}/b_n,\ p_9 = R\theta_{gl}/b_n\\&p_{10} = y_{gr}/b_n,\ p_{11} = z_{gr}/b_n,\ p_{12} = R\theta_{gr}/b_n\\&p_{1,1} = y_l/b_n,\ p_{1,2} = z_l/b_n\\&p_{2,1} = y_r/b_n,\ p_{2,2} = z_r/b_n\end{aligned} \tag{9}$$

Then the dimensionless kinetic equation of gear system can be expressed as:

$$\begin{cases}\ddot{p}_1+2\xi_{ply}\dot{p}_1+\eta_{ply}p_1+\eta_{mlyp}(\tau)f(p_{1,1})\\ \quad +2\xi_{mlyp}(\tau)\dot{p}_{1,1}=\tilde{F}_1\\ \ddot{p}_2+2\xi_{plrz}(\dot{p}_2-\dot{p}_5)+\eta_{plrz}(p_2-p_5)\\ \quad +\eta_{mlzp}(\tau)f(p_{1,2})+2\xi_{mlzp}(\tau)\dot{p}_{1,2}=\tilde{F}_2\\ \ddot{p}_3+2[\eta_{mlyp}(\tau)f(p_{1,1})+2\xi_{mlyp}(\tau)\dot{p}_{1,1}]=\tilde{F}_3\end{cases} \tag{10a}$$

$$\begin{cases}\ddot{p}_4+2\xi_{pry}\dot{p}_4+\eta_{pry}p_4+\eta_{mryp}(\tau)f(p_{2,1})\\ \quad +2\xi_{mryp}(\tau)\dot{p}_{2,1}=\tilde{F}_4\\ \ddot{p}_5-2\xi_{plrz}(\dot{p}_2-\dot{p}_5)-\eta_{plrz}(p_2-p_5)\\ \quad +\eta_{mrzp}(\tau)f(p_{2,2})+2\xi_{mrzp}(\tau)\dot{p}_{2,2}=\tilde{F}_5\\ \ddot{p}_6+2[\eta_{mryp}(\tau)f(p_{2,1})+2\xi_{mryp}(\tau)\dot{p}_{2,1}]=\tilde{F}_6\end{cases} \tag{10b}$$

$$\begin{cases}\ddot{p}_7+2\xi_{gly}\dot{p}_7+\eta_{gly}p_7-\eta_{mlyg}(\tau)f(p_{1,1})\\ \quad -2\xi_{mlyg}(\tau)\dot{p}_{1,1}=\tilde{F}_7\\ \ddot{p}_8+2\xi_{glz}\dot{p}_8+\eta_{glz}p_8+2\xi_{glrz}(\dot{p}_8-\dot{p}_{11})+\eta_{glrz}(p_8\\ \quad -p_{11})-\eta_{mlzg}(\tau)f(p_{1,2})-2\xi_{mlzg}(\tau)\dot{p}_{1,2}=\tilde{F}_8\\ \ddot{p}_9-2[\eta_{mlyg}(\tau)f(p_{1,1})+2\xi_{mlyg}(\tau)\dot{p}_{1,1}]=\tilde{F}_9\end{cases} \tag{10c}$$

$$\begin{cases}\ddot{p}_{10}+2\xi_{gry}\dot{p}_{10}+\eta_{gry}p_{10}-\eta_{mryg}(\tau)f(p_{2,1})\\ \quad -2\xi_{mryg}(\tau)\dot{p}_{2,1}=\tilde{F}_{10}\\ \ddot{p}_{11}+2\xi_{grz}\dot{p}_{11}+\eta_{grz}p_{11}-2\xi_{glrz}(\dot{p}_8-\dot{p}_{11})-\eta_{glrz}(p_8\\ \quad -p_{11})-\eta_{mrzg}(\tau)f(p_{2,2})-2\xi_{mrzg}(\tau)\dot{p}_{2,2}=\tilde{F}_{11}\\ \ddot{p}_{12}-2[\eta_{mryg}(\tau)f(p_{2,1})+2\xi_{mryg}(\tau)\dot{p}_{2,1}]=\tilde{F}_{12}\end{cases} \tag{10d}$$

Where:

$\xi_{ijk}(i=p,g;\ j=l,r;k=y,z)$ is the dimensionless support damping at bearing at left and right ends of pinion and gear at y,z direction, respectively;

$\eta_{ijk}(i=p,g;\ j=l,r;k=y,z)$ is the dimensionless support stiffness at bearing at left and right ends of pinion and gear at y,z direction, respectively;

$\xi_{mij}(\tau)\ (i=l,r;\ j=y,z)$ is the dimensionless meshing damping at left and right ends of gear at y,z direction, respectively;

$\eta_{mij}(\tau)\ (i=l,r;\ j=y,z)$ is the dimensionless meshing stiffness at left and right ends of gear at y,z direction, respectively;

ξ_{ilrj} and $\eta_{ilrj}\ (i=p,g;\ j=y,z)$ are dimensionless internal damping and internal stiffness at left and right ends of pinion and gear, respectively;

$\tilde{F}_i(i=1,12)$ is the dimensionless external excitation.

The expressions of dimensionless parameter in Eq. (10a) to Eq. (10d) are:

$\xi_{ply}=\dfrac{c_{ply}}{2m_{pl}\omega_n}$, $\eta_{ply}=\dfrac{k_{ply}}{m_{pl}\omega_n^2}$, $\eta_{mlyp}(\tau)=\dfrac{K_{mly}(t)}{m_{pl}\omega_n^2}$

$\xi_{mlyp}(\tau)=\dfrac{C_{mly}(t)}{2m_{pl}\omega_n}$, $\xi_{plrz}=\dfrac{c_{plrz}}{2m_{pl}\omega_n}$, $\eta_{plrz}=\dfrac{k_{plrz}}{m_{pl}\omega_n^2}$

$\eta_{mlzp}(\tau)=\dfrac{K_{mlz}(t)}{m_{pl}\omega_n^2}$, $\xi_{mlzp}(\tau)=\dfrac{C_{mlz}(t)}{2m_{pl}\omega_n}$, $\xi_{pry}=\dfrac{c_{pry}}{2m_{pr}\omega_n}$

$\eta_{pry}=\dfrac{k_{pry}}{m_{pr}\omega_n^2}$, $\eta_{mryp}(\tau)=\dfrac{K_{mry}(t)}{m_{pr}\omega_n^2}$, $\xi_{mryp}(\tau)=\dfrac{C_{mry}(t)}{2m_{pr}\omega_n}$

$\eta_{mrzp}(\tau)=\dfrac{K_{mrz}(t)}{m_{pr}\omega_n^2}$, $\xi_{mrzp}(\tau)=\dfrac{C_{mrz}(t)}{2m_{pr}\omega_n}$, $\xi_{gly}=\dfrac{c_{gly}}{2m_{gl}\omega_n}$

$\eta_{gly}=\dfrac{k_{gly}}{m_{gl}\omega_n^2}$ $\xi_{glz}=\dfrac{c_{glz}}{2m_{gl}\omega_n}$, $\eta_{glz}=\dfrac{k_{glz}}{m_{gl}\omega_n^2}$

$\xi_{mlzg}(\tau)=\dfrac{C_{mlz}(t)}{2m_{gl}\omega_n}$, $\eta_{mryg}(\tau)=\dfrac{K_{mry}(t)}{m_{gr}\omega_n^2}$, $\xi_{mryg}(\tau)=\dfrac{C_{mry}(t)}{2m_{gr}\omega_n}$

$\eta_{mrzg}(\tau)=\dfrac{K_{mrz}(t)}{m_{gr}\omega_n^2}$, $\xi_{mrzg}(\tau)=\dfrac{C_{mrz}(t)}{2m_{gr}\omega_n}$, $\tilde{F}_3=\dfrac{2T_{pl}}{b_n m_{pl}\omega_n^2}$

$\tilde{F}_1=\tilde{F}_4=\tilde{F}_7=\tilde{F}_{10}=\dfrac{g}{b_n\omega_n^2}$, $\tilde{F}_2=\tilde{F}_5=\tilde{F}_8=\tilde{F}_{11}=0$,

$\tilde{F}_6=\dfrac{2T_{pr}}{b_n m_{pr}\omega_n^2}$, $\tilde{F}_9=-\dfrac{2T_{gl}}{b_n m_{gl}\omega_n^2}$, $\tilde{F}_{12}=-\dfrac{2T_{gr}}{b_n m_{gr}\omega_n^2}$ 。

The dimensionless backlash nonlinear function can be expressed as:

$$f(p_{i,j})=\begin{cases}p_{i,j}-\dfrac{b_k}{b_n} & (p_{i,j}>\dfrac{b_k}{b_n})\\ 0 & (|p_{i,j}|\le\dfrac{b_k}{b_n})\\ p_{i,j}+\dfrac{b_k}{b_n} & (p_{i,j}<-\dfrac{b_k}{b_n})\end{cases} \tag{11}$$

Where, $i=1,2;\ j=1,2,k=t,a$.

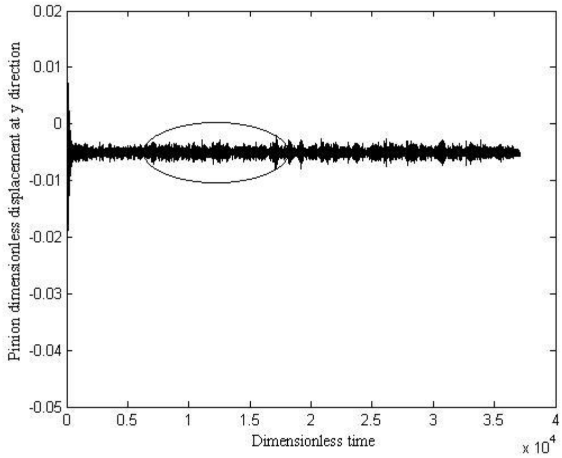
The dimensionless transmission error can be expressed as:

$$e_{ij}(\tau)=\frac{\tilde{e}_{ij}}{b_n}+\frac{e_{aij}}{b_n}\sin(\omega\tau+\phi_{eij})\ (i=l,r;\ j=y,z) \tag{12}$$

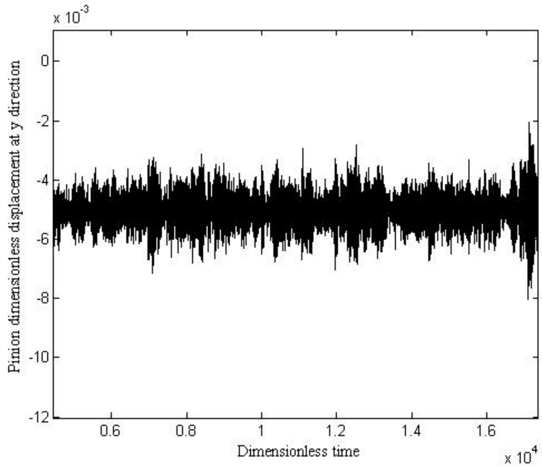
3. Numerical Results and Discussion

Take the basic parameters as:
$z_p = 30$, $z_g = 90$, $m = 3$, $\alpha = 20°$, $\beta = 10°$, $g = 9.8N / kg$, $b_n = 0.1mm$, $B = 60mm$, T_p=300$N \cdot m$, $T_g = 900N \cdot m$, $f = 5000r / \min$, $m_p = 5kg$, $m_g = 45kg$, $K_{mna} = 2\times10^8 N / m$, $K_{mn} = 6\times10^8 N / m$, ξ_{plrz}=ξ_{glrz}=0.1 , k_{ply}=k_{pry}=k_{gly}=k_{gry} $= 6.15\times10^9 N / m$., ξ_{glz}=ξ_{grz} $= 0.008$, k_{glz}=k_{grz} $= 3.03\times10^6 N / m$, ξ_{ply}=ξ_{pry}=ξ_{gly}=ξ_{gry} $= 0.008$, k_{plrz}=k_{glrz}=$6.02\times10^{10} N / m$, $e_{ij}(\tau)$=0 .

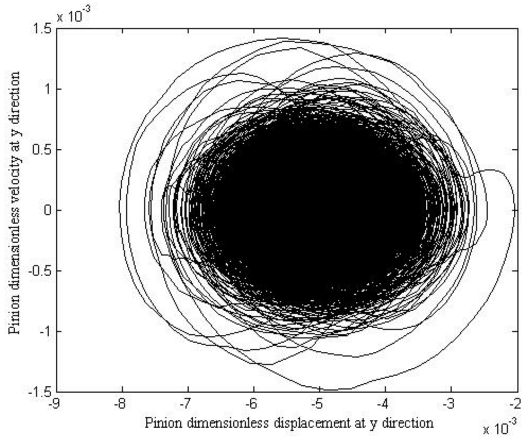
Then Figure 3 to Figure 5 can be obtained by 4-5 Runge-Kutta method, in which the nonlinear characteristics of double helical gear system can be revealed.



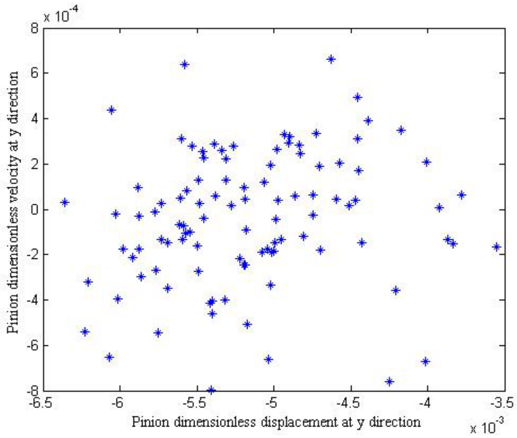
(a)Time history



(b) Time history partial enlarged view

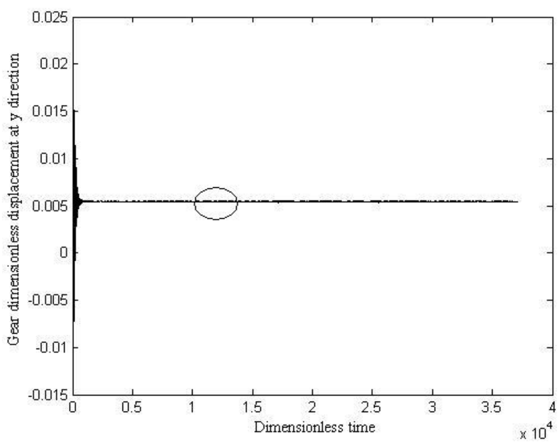


(c)Speed-displacement phase diagram

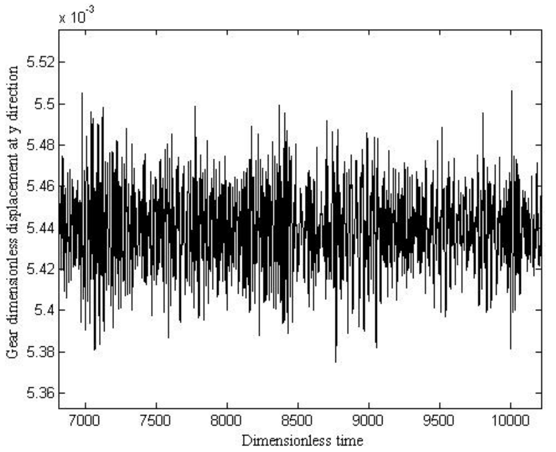


(d) Poincare section

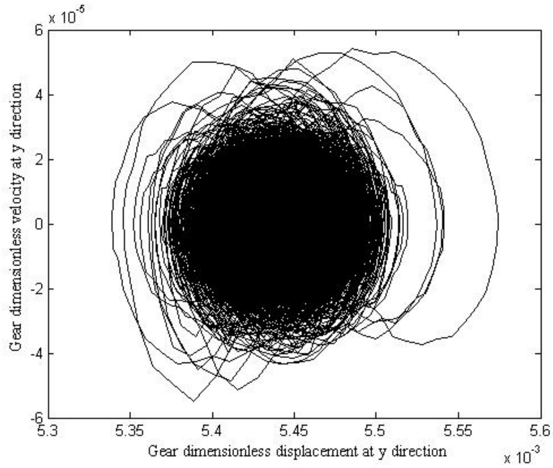
Figure 3. The numerical result of pinion at y direction



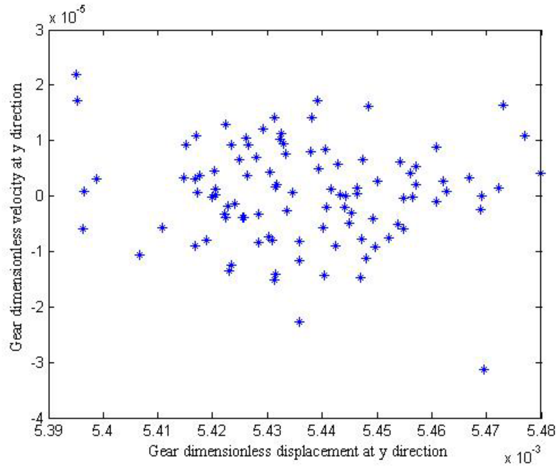
(a) Time history



(b) Time history partial enlarged view



(c) Speed-displacement phase diagram

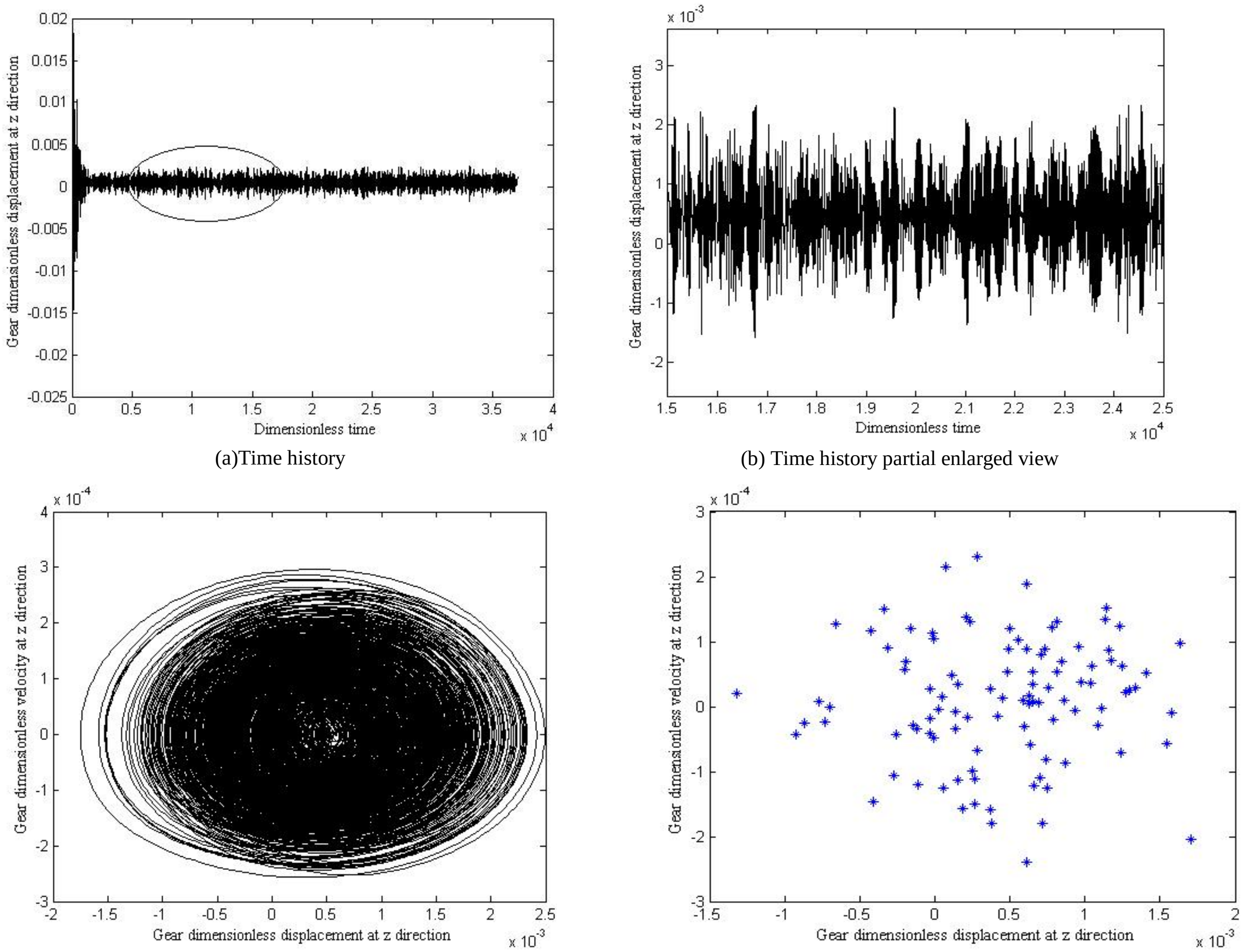


(d) Poincare section

Figure 4. The numerical result of gear at y direction

In Figures 3 to 5, some nonlinear characteristics are revealed. From Figure (3a), Figure (4a) and Figure (5a), we get that the vibration is in the double helical gear system. From Figure (3a), Figure (4a), Figure (3b) and Figure (4b), we find that the amplitude of gear at y direction is smaller than the amplitude of pinion at direction. From Figure (4a), Figure (5a), Figure (4b) and Figure (5b), we find that the amplitude of gear at z direction is larger than the amplitude of gear at y direction. To a certain extent, we could obtain that the vibration amplitude of gear is smaller than the vibration amplitude of pinion at the same direction and the vibration amplitude of the same component at z direction is larger than the amplitude of gear at y direction.

From Figures 3 to 5, we find that there are abundant nonlinear characteristics existing in the double helical gear system. According to their nonlinear characteristics, we should take the dynamic performance into consideration when designing the double helical gear system. At the same time, we could use the vibration signal to monitor the operation of double helical gear system.



(a)Time history
(b) Time history partial enlarged view
(c) Speed-displacement phase diagram
(d) Poincare section

Figure 5. The numerical result of gear at z direction

4. Conclusions

In this paper, we obtained the nonlinear dynamic vibration model of the double helical gear system by taking the time-varying nonlinear factors into consideration. The time-varying nonlinear factors in this model include time-varying mesh stiffness, time-varying meshing damping, backlash, time-varying transmission error and time-varying meshing force.

Based on the nonlinear dynamic vibration model, the dimensionless nonlinear dynamic vibration model had been formed. Then take the Runge-Kutta method to solve the differential equations.

Through the numerical result, we find that there are a lot of nonlinear vibration characteristics in the double helical gear system. The vibration amplitude of gear is smaller than the vibration amplitude of pinion at the same direction and the vibration amplitude of the same component at z direction is larger than the amplitude of gear at y direction. From the study of nonlinear characteristics and performance of double helical gear system, the double helical gear can be optimistic designed. What is more, there are many unknown nonlinear vibration characteristics that need to be researched further.

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