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Reliability Modelling of a Computer System with Priority to H/W Repair over Replacement of H/W and Up-gradation of S/W Subject to MOT and MRT

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Abstract

In the present paper, some reliability measures of a computer system of two cold standby identical units are obtained using a semi-Markov process and a regenerative point technique. For this purpose, a reliability model is proposed by considering independent h/w and s/w failure. Initially, one unit is operative and another is kept as spare in cold standby. A single server is provided to conduct preventive maintenance, repair, replacement and up-gradation of the components upon their failure. Then, a maximum operation time system undergoes preventive maintenance, directly from normal mode. If the repair of the h/w is not possible by the server up to a pre-specific time, it is replaced by a new one with some replacement time. However, only up-gradation of the software by a new one is made whenever s/w fails to execute the desired function properly. Priority to h/w repair is given over h/w replacement and s/w up-gradation. The failure time distribution of the h/w and s/w follows a negative exponential while the distributions of preventive maintenance, repair, up-gradation and replacement time are taken as arbitrary with different probability density functions. To depict the importance of the study, graphs are drawn for a particular case with fixed values of other parameters and costs.

Keywords: *Computer System, H/W and S/W Failure, Maximum Operation and Repair Times, Priority and Preventive Maintenance.*

Nomenclature

E	The set of regenerative states
NO	The unit is operative and in normal mode
Cs	The unit is cold standby
a/b	Probability that the system has hardware / software failure
λ_1/λ_2	Constant hardware / software failure rate
α_0	Maximum Operation Time
β_0	Maximum Repair Time.
Pm/PM	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state
WPm/WPM	The unit is waiting for preventive Maintenance/ waiting for preventive maintenance from previous state
HFur/HFUR	The unit is failed due to hardware and is under repair / under repair continuously from previous state
HFurp/HFURP	The unit is failed due to hardware and is under replacement / under replacement continuously from previous state
HFwr / HFWR	The unit is failed due to hardware and is waiting for repair/ waiting for repair continuously from previous state
SFurp/SFURP	The unit is failed due to the software and is under up-gradation/ under up-gradation continuously from previous state
SFwrp/SFWRP	The unit is failed due to the software and is waiting for up-gradation / waiting for up-gradation continuously from previous state
h(t) / H(t)	pdf / cdf of up-gradation time of unit due to software
g(t) / G(t)	pdf / cdf of repair time of the hardware
m(t)/ M(t)	pdf / cdf of replacement time of the hardware
f(t) / F(t)	pdf / cdf of the time for PM of the unit
pdf / cdf	Probability density function/ Cumulative density function

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qij (t)/ Qij(t)	pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]
qij.kr (t)/Qij.kr(t)	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]
μi(t)	Probability that the system up initially in state Si ∈ E is up at time t without visiting to any regenerative state
Wi(t)	Probability that the server is busy in the state Si upto time 't'without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
mij	Contribution to mean sojourn time (μi) in state Si when system transit directly to state Sj so that $\mu_i = \sum_j m_{ij}$ and mij = $\int t dQ_{ij}(t) = -q_{ij}^{*}{}'(0)$
Ⓢ/©	Symbol for Laplace-Stieltjes convolution/Laplace convolution
~ / *	Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)

1. Introduction

In our modern society, there is an increased reliance on computer systems due to the rise of technology in everyday life. These systems control everything from banking to telecommunications, to nuclear power plants, to air traffic. Their failure can be catastrophic. Therefore, ensuring the reliability and performance of such systems has become a growing need for engineers and academicians. Currently, redundancy is one of the best methods of improving the reliability of any operating system. During the past few years, some reliability models of computer systems with independent h/w and s/w failures have been suggested by some researchers, including Malik and Anand [1] and Malik and Kumar [3].

Also, it is a well known fact that preventive maintenance can slow down the deterioration process of a computer system and restore the system as new. Thus, the method of preventive maintenance can be used to improve the reliability and the profit of the system. The concept of preventive maintenance was used by Malik and Nandal [2] while discussing a cold standby system with a maximum operation time. Also, it sometimes becomes necessary to give priority to repairing one unit over the repair activities of another unit, not only to reduce the down time but also to minimize the operating cost. Malik *et al.* [4,5] analyzed stochastically a computer system with priority to h/w repair over s/w replacement. Kumar *et al.* [6] developed a stochastic model for a computer system using the concept of s/w up-gradation over h/w repair activities. Furthermore, the reliability and availability of a system can be increased by making a replacement of the failed component by a new one in case the repair time is too long. Recently, Kumar *et al.* [4,5] proposed a reliability model for a computer system with preventive maintenance and repair subject to maximum operation and repair times.

In light of the above facts and practical utility, here is a reliability model proposed taking into account the independent h/w and s/w failure. Initially, one unit is operative and the other is kept as spare in cold standby. A single server is provided to conduct preventive maintenance, repair, up-gradation and a replacement of the components upon their failure. After that, a maximum operation time system undergoes preventive maintenance directly from the normal mode. If the repair of the h/w is not possible by the server up to a pre-specific time, it is replaced by a new one with some replacement time. However, only up-gradation of the software by new one is made whenever s/w fails to execute the desired function properly. Priority to h/w repair is given over h/w replacement and s/w up-gradation. The failure time h/w and s/w follows exponential distribution while the distributions of preventive maintenance, repair, up-gradation and replacement time are taken as arbitrary with different probability density functions. Various system effectiveness measures, such as mean time to system failure, availability, busy period of the server due to preventive maintenance, busy period of the server due to repair, busy period of the server due to hardware replacement , busy period of the server due to software up-gradation, expected number of software up-gradations, expected number of hardware replacement and expected number of visits of the server are derived by using semi-Markov process and regenerative point technique. All random variables are statistically independent and uncorrelated. Switch devices are perfect. The graphical study of the results for a particular case has also been made to highlight the importance of the results.

2. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions by taking A $= a\lambda_1 + b\lambda_2 + \alpha_0$, B= $a\lambda_1+b\lambda_2+\alpha_0+\beta_0$ for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \tag{1}$$

$$p_{16} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}[1-f^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{12.6}, \qquad p_{02} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0},$$

$$p_{18} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}[1-f^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{13.8}, \qquad p_{01} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0},$$

$$p_{1.13} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}[1-f^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{11.13}, \qquad p_{03} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0},$$

$$p_{24} = \frac{\beta_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)], \qquad p_{10} = f^*(a\lambda_1+b\lambda_2+\alpha_0),$$

$p_{25} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]$,

$p_{20}=g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)$,

$p_{2.11} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]$, $p_{30} = h^*(a\lambda_1+b\lambda_2+\alpha_0)$,

$p_{2.12} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]$, $p_{40} = m^*(a\lambda_1+b\lambda_2+\alpha_0)$,

$p_{37} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)]$, $p_{51} = g^*(\beta_0)$, $p_{5,16} = 1-g^*(\beta_0)$,

$p_{39} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)]= p_{3,1.9}$, $p_{62} = f^*(0)$, $p_{73} = g^*(0)$,

$p_{3,10} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)]= p_{33.10}$, $p_{83} = f^*(0)$, $p_{93} = f^*(0)$,

$p_{4.17} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)]= p_{4,1.17}$, $p_{10.3} = h^*(0)$, $p_{11.3} = g^*(\beta_0)$,

$p_{4,18} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{43.18}$, $p_{11.14} = 1-g^*(\beta_0)$, $p_{14.3} = m^*(0)$,

$p_{4.19} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)]$, $p_{15.2} = m^*(0)$, $p_{16.1} = m^*(0)$,

$p_{21.5} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]\, g^*(\beta_0)$, $p_{12.2} = g^*(\beta_0)$, $p_{13.1} = f^*(0)$,

$p_{21.16,5} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)]$, $p_{17.1} = m^*(0)$,

$p_{23.11} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][g^*(\beta_0)]$, $p_{18.3} = m^*(0)$,

$p_{23.11,14} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)]$, $p_{19.4} = g^*(0)$,

$p_{22.12} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]\, g^*(\beta_0)$, $p_{12.15} = 1-g^*(\beta_0)$,

$$p_{22.12,15} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)] \quad (2)$$

It can be easily verified that

$$p_{01}+p_{02}+p_{03} = p_{10}+p_{16}+p_{18}+p_{1.13} = 1$$

$$p_{20}+p_{24}+p_{25}+p_{2,11}+p_{2.12}= p_{30}+p_{37}+p_{39}+p_{3.10} = 1$$

$$p_{40}+p_{4.17}+p_{4.18}+p_{4.19} = p_{5.1}+p_{5.16}= p_{62}= p_{73} = p_{83} = p_{91} = 1$$

$$p_{10.3} = p_{11.3} + p_{11.14} = p_{12.2} + p_{12.15} = p_{13.1} = p_{14.1} = p_{15.2} = 1 \quad (3)$$

$$p_{16.1} = p_{17.1} = p_{18.3}= p_{19.4} = p_{10} +p_{12.6}+ p_{11.13} +p_{13.8} =1$$

$p_{20} + p_{24} + p_{21.5} + p_{21, 16.5} + p_{23, 11} + p_{23.11, 14} + p_{22, 12} + p_{22.12, 15} = 1$

$p_{30} + p_{31.9} + p_{32.7} + p_{33.10} = p_{40} + p_{41.17} + p_{42.19} + p_{43.18} = 1$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{(a\lambda_1 + b\lambda_2 + \alpha_0)}, \qquad \mu_1 = \frac{1}{(a\lambda_1 + b\lambda_2 + \alpha_0 + \alpha)},$$

$$\mu_2 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \theta + \beta_0}, \qquad \mu_3 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta},$$

$$\mu_3' = \frac{\beta^2 + a\lambda_1\beta + (b\lambda_2 + \alpha_0)(a\lambda_1 + b\lambda_2 + \alpha_0)}{\beta(a\lambda_1 + b\lambda_2 + \alpha_0 + \theta + \beta_0)^2}, \qquad \mu_4 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \gamma},$$

(4)

$$\mu_4' = \frac{\gamma^2 + a\lambda_1\gamma + (b\lambda_2 + \alpha_0)(a\lambda_1 + b\lambda_2 + \alpha_0)}{\gamma(a\lambda_1 + b\lambda_2 + \alpha_0 + \theta + \beta_0)^2}, \qquad \mu_1' = \frac{1}{\alpha}, \qquad \mu_{19} = \frac{1}{\theta},$$

$$\mu_2' = \frac{(\beta_0 + \theta)}{(B + \theta)^2} + \frac{(A)\{-\theta^2\gamma(\theta + \beta_0)^2 + \gamma\theta(B) + \beta_0(\beta_0 + \theta)(\theta + B)(B) - \beta_0\theta\gamma(\theta + \beta_0)(\theta + B) + (B + \beta_0)\gamma(B)(\theta + \beta_0)}{\gamma(\theta + B)^2(\theta + \beta_0)^2(B)},$$

$$\mu_7 = \frac{1}{\theta}$$

3. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for ϕ_i (t):

$$\phi_i(t) = \sum_j Q_{i,j}(t) ® \phi_j(t) + \sum_k Q_{i,k}(t) \qquad (5)$$

Where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking LT of the above relation (5) and solving for $\tilde{\phi}_0(s)$.

We have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \qquad (6)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to o} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \qquad (7)$$

Where

$N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{24}p_{02}\mu_4$

And $D_1 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} - p_{02}p_{24}p_{40}$

4. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \qquad (8)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t},$$

$$M_1(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{F(t)},$$

$$M_2(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t}\overline{G(t)} \qquad (9)$$

$$M_3(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{H(t)},$$

$$M_4(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{M(t)}$$

Taking LT of the above relations (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \quad \text{Where} \tag{10}$$

$$\begin{aligned}
N_2 =\ & (-p_{24})\{\mu_0 \{p_{12.6}[p_{43.18}p_{31.9}+p_{41.17}(1-p_{33.10}-p_{37}p_{73})]\}-\mu_1 [p_{02}\{p_{43.18}p_{31.9}+(1-p_{33.10}-p_{37}p_{73})p_{41.17}\}]+\mu_3 [-p_{01}p_{43.18} \\
& p_{12.6}+p_{03}p_{41.17}p_{12.6}-p_{02}\{p_{43.18}(1-p_{11.13})+p_{41.17}p_{13.8}\}]+\mu_4 [p_{01}(1-p_{33.10}-p_{37}p_{73})p_{12.6}+p_{02}\{(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73})- \\
& p_{13.8}p_{31.9}\}+p_{03}p_{31.9}p_{12.6}]\}+(1-p_{4.19}p_{19.4})\{\mu_0 [(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73})(1-p_{22.12}-p_{22.12.15})-p_{12.6}\{(1-p_{33.10}-p_{37}p_{73}) \\
& (p_{21.5}+p_{21.5,16})+(p_{23.11}+p_{23,11.14})p_{31.9}\}-p_{13.8}(1-p_{22.12}-p_{22.12.15})p_{31.9}\}]+\mu_1[p_{01}[(1-p_{33.10}-p_{37}p_{73})(1-p_{22.12}-p_{22.12.15})+p_{02}\{ \\
& (1-p_{33.10}-p_{37}p_{73})(p_{21.5}+p_{21.5,16})+(p_{23.11}+p_{23,11.14})p_{31.9}\}+p_{03}(1-p_{22.12}-p_{22.12.15})p_{31.9}]+\mu_2 [p_{01}\{(1-p_{33.10}-p_{37}p_{73})p_{12.6}+ \\
& p_{02}\{(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73})-p_{13.8}p_{31.9}\}+p_{03}p_{31.9}p_{12.6}]+\mu_3 [p_{01}[p_{13.8}(1-p_{22.12}-p_{22.12.15})+(p_{23.11}+p_{23,11.14})p_{12.6}]+p_{02}\{ \\
& (p_{21.5}+p_{21.5,16})p_{13.8}+(1-p_{11.13})(p_{23.11}+p_{23,11.14})\}+p_{03}\{-p_{12.6}(p_{21.5}+p_{21.5,16})+(1-p_{22.12}-p_{22.12.15})(1-p_{11.13})\}]\}
\end{aligned}$$

And

$$\begin{aligned}
D_2 =\ & (-p_{24})\{\mu_0 \{p_{12.6}[p_{43.18}p_{31.9}+p_{41.17}(1-p_{33.10}-p_{37}p_{73})]\}-\mu_1' [p_{02}\{p_{43.18}p_{31.9}+(1-p_{33.10}-p_{37}p_{73})p_{41.17}\}]+(\mu_3'+p_{37}\mu_7)\ [- \\
& p_{01}p_{43.18}p_{12.6}+p_{03}p_{41.17}p_{12.6}-p_{02}\{p_{43.18}(1-p_{11.13})+p_{41.17}p_{13.8}\}]+(\mu_4'+p_{4.19}\mu_{19})[p_{01}(1-p_{33.10}-p_{37}p_{73})p_{12.6}+p_{02}\{(1- \\
& p_{11.13})(1-p_{33.10}-p_{37}p_{73})-p_{13.8}p_{31.9}\}+p_{03}p_{31.9}p_{12.6}]\}+(1-p_{4.19}p_{19.4})\{\mu_0 [(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73})(1-p_{22.12}-p_{22.12.15})- \\
& p_{12.6}\{(1-p_{33.10}-p_{37}p_{73})(p_{21.5}+p_{21.5,16})+(p_{23.11}+p_{23,11.14})p_{31.9}\}-p_{13.8}(1-p_{22.12}-p_{22.12.15})p_{31.9}\}]+\mu_1' [p_{01}[(1-p_{33.10}-p_{37}p_{73}) \\
& (1-p_{22.12}-p_{22.12.15})+p_{02}\{(1-p_{33.10}-p_{37}p_{73})(p_{21.5}+p_{21.5,16})+(p_{23.11}+p_{23,11.14})p_{31.9}\}+p_{03}(1-p_{22.12}-p_{22.12.15})p_{31.9}]+\mu_2' [p_{01} \\
& \{(1-p_{33.10}-p_{37}p_{73})p_{12.6}+p_{02}\{(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73})-p_{13.8}p_{31.9}\}+p_{03}p_{31.9}p_{12.6}]+(\mu_3'+p_{37}\mu_7)[p_{01}[p_{13.8}(1-p_{22.12}- \\
& p_{22.12.15})+(p_{23.11}+p_{23,11.14})p_{12.6}]+p_{02}\{(p_{21.5}+p_{21.5,16})p_{13.8}+(1-p_{11.13})(p_{23.11}+p_{23,11.14})\}+p_{03}\{-p_{12.6}(p_{21.5}+p_{21.5,16})+(1- \\
& p_{22.12}-p_{22.12.15})(1-p_{11.13})\}]\}
\end{aligned}$$

5. Busy Period Analysis for Server

5.1. Due to Preventive Maintenance (PM)

Let $B_i^P(t)$ be the probability that the server is busy in the preventive maintenance of the unit at an instant 't' given that the system entered state i at t = 0. The recursive relations $B_i^P(t)$ for are as follows:

$$B_i^p(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^p(t) \tag{12}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_1(t)$ is the probability that the server is busy in state S_i due to the preventive maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{F}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{F}(t) + (a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t})\overline{F}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{F}(t)$$

5.2. Due to Hardware Failure

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \tag{13}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_2(t)$ is the probability that the server is busy in state S_i due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{G}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{G}(t) + (a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t})\overline{G}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{G}(t)$$

5.3. Due to Up-Gradation of the Software

Let B_i^S (t) be the probability that the server is busy due to up-gradation of the software at an instant 't' given that the system entered the regenerative state i at t = 0. We have the following recursive relations for B_i^S (t):

$$B_i^S(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^S(t) \tag{14}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_3(t)$ is the probability that the server is busy in state S_i due to up-gradation of the software up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{H}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{H}(t) + (a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t})\overline{H}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} © 1)\overline{H}(t)$$

5.4. Due to Hardware Replacement

Let $B_i^{HRp}(t)$ be the probability that the server is busy in replacement of the unit due to hardware failure at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i^{HRp}(t)$ are as follows:

$$B_i^{HRp}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^{HRp}(t) \tag{15}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_4(t)$ is the probability that the server is busy in state S_i due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{M}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} ©1)\overline{M}(t) + (a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t})\overline{M}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} ©1)\overline{M}(t)$$

Taking the LT of the above relations (12) to (15), and solving for B_0^{*H} (s) and B_0^{*S} (s), the time for which server is busy due to repair and replacements, respectively, is given by

$$B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2} \qquad B_0^R = \lim_{s\to 0} sB_0^{*R}(S) = \frac{N_S^R}{D_2} \tag{16}$$

$$B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2} \qquad B_0^{HRp} = \lim_{s\to 0} sB_0^{*HRp}(S) = \frac{N_S^{HRp}}{D_2}$$

$$N_3^P = W_1^*(0)\{p_{24}p_{02}[p_{31.9}p_{43.18} + p_{41.17}(1 - p_{33.10} - p_{37}p_{73})]\} + (1 - p_{4.19}p_{19.4})[p_{01}(1 - p_{33.10} - p_{37}p_{73})(1 - p_{22.12} - p_{22.12,15}) + p_{02}\{(p_{23.11} + p_{23.11,14})p_{31.9} + (1 - p_{33.10} - p_{37}p_{73})(p_{21.5} + p_{21.5.15})\} + p_{03}\{(1 - p_{22.12} - p_{22.12,15})p_{31.9}\}]$$

$$N_3^R = p_{24}[W_7^*(0)p_{37}[p_{01}p_{12.6}p_{43.18} - p_{41.17}p_{03}p_{12.6} + p_{02}\{(1 - p_{11.13})p_{43.18} + p_{41.17}p_{13.8}\}] + W_{19}^*(0)p_{4.19}\{p_{01}(1 - p_{33.10} - p_{37}p_{73})p_{12.6} + p_{02}\{(1 - p_{11.13})(1 - p_{33.10} - p_{37}p_{73}) - p_{13.8}p_{31.9}\} + p_{03}p_{31.9}p_{12.6}\}] + (1 - p_{4.19}p_{19.4})[W_2^*(0)\{p_{01}(1 - p_{33.10} - p_{37}p_{73})p_{12.6} + p_{02}\{(1 - p_{11.13})(1 - p_{33.10} - p_{37}p_{73}) - p_{13.8}p_{31.9}\} + p_{03}p_{31.9}p_{12.6}\} + W_7^*(0)p_{37}[p_{01}[p_{13.8}(1 - p_{22.12} - p_{22.12,15}) + (p_{23.11} + p_{23,11.14}) p_{12.6}] + p_{02}\{(1 - p_{11.13})(p_{23.11} + p_{23.11,14}) + p_{13.8}(p_{21.5} + p_{21.5.15})\} + p_{03}\{(1 - p_{22.12} - p_{22.12,15})(1 - p_{11.13}) - p_{12.6}(p_{21.5} + p_{21.5.15})\}]$$

$$N_3^S = W_3^*(0)[p_{24}\{p_{01}p_{43.18}p_{12.6} - p_{03}p_{41.17}p_{12.6} + p_{02}\{(1 - p_{11.13})p_{43.18} + p_{41.17}p_{13.8}\}] + (1 - p_{4.19}p_{19.4})\{p_{01}[p_{13.8}(1 - p_{22.12} - p_{22.12,15}) + (p_{23.11} + p_{23,11.14}) p_{12.6}] + p_{02}\{(1 - p_{11.13})(p_{23.11} + p_{23.11,14}) + p_{13.8}(p_{21.5} + p_{21.5.15})\} + p_{03}\{(1 - p_{22.12} - p_{22.12,15})(1 - p_{11.13}) - p_{12.6}(p_{21.5} + p_{21.5.15})\}]$$

$$N_3^{HRp} = W_4^* p_{24}[p_{01}(1 - p_{33.10} - p_{37}p_{73})p_{12.6} + p_{02}\{(1 - p_{33.10} - p_{37}p_{73})(1 - p_{11.13}) - p_{13.8}p_{31.9})\} + p_{03}p_{12.6}p_{31.9}]$$ and D2 is already mentioned.

6. Expected Number of H/W Replacements and S/W Up-Gradations

6.1. Due to Hardware Failure

Let $R_i^H(t)$ be the expected number of replacements of the failed hardware components by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^H(t)$ are given as

$$R_i^H(t) = \sum_j q_{i,j}^{(n)}(t) ® \left[\delta_j + R_j^H(t)\right] \tag{17}$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does the job afresh, otherwise δj = 0.

6.2. Due to Software Failure

Let $R_i^S(t)$ be the expected number of up-gradations of the failed software by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^S(t)$ are given as

$$R_i^S(t) = \sum_j q_{i,j}^{(n)}(t) ® \left[\delta_j + R_j^S(t)\right] \tag{18}$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LT of the relations (17) and (18), and solving for $\tilde{R}_0^H(s)$ and $\tilde{R}_0^S(s)$. The expected numbers of replacements per unit time to the hardware and software failures are respectively given by

$$R_0^H(\infty) = \lim_{s\to 0} s\tilde{R}_0^H(s) = \frac{N_4^H}{D_2}$$

and (19)

$$R_0^S(\infty) = \lim_{s\to 0} s\tilde{R}_0^S(s) = \frac{N_4^S}{D_2}$$

Where D_2 is already mentioned.

$$N_4^H = (p_{40} + p_{41.17} + p_{43.18})(p_{24})\{p_{01}(1-p_{33.10}-p_{37}p_{73})p_{12.6} + p_{02}[(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73}) - p_{31.9}p_{13.8}] + p_{03}p_{31.9}p_{12.6}\} + (p_{22.12,15} + p_{21.5,16} + p_{23.11,14})(1-p_{19.4}p_{4.19})\{p_{01}(1-p_{33.10}-p_{37}p_{73})p_{12.6} + p_{02}[(1-p_{11.13})(1-p_{33.10}-p_{37}p_{73}) - p_{31.9}p_{13.8}] + p_{03}p_{31.9}p_{12.6}\}$$

$$N_4^S = (p_{30} + p_{33.10} + p_{31.9})[(p_{24})\{[p_{01}p_{43.18}p_{12.6} - p_{03}p_{41.17}p_{12.6} + p_{02}\{(1-p_{11.13})p_{43.18} - p_{41.17}p_{13.8}\}] + (1-p_{19.4}p_{4.19})\{p_{01}[p_{13.8}(1-p_{22.12}-p_{22.12,15}) + (p_{23.11} + p_{23,11.14})p_{12.6}] + p_{02}\{(1-p_{11.13})(p_{23.11}+p_{23.11,14}) + p_{13.8}(p_{21.5}+p_{21.5.16})\} + p_{03}\{(1-p_{22.12}-p_{22.12,15})(1-p_{11.13}) - p_{12.6}(p_{21.5}+p_{21.5.16})\}]]$$

7. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j(t)\right] \quad (20)$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj=1, if *j* is the regenerative state where the server does the job afresh, otherwise δj= 0. Taking LT of the relation (20) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}, \text{ where} \quad (21)$$

N_5 =(-p_{24}) $p_{12.6}$\{ $p_{43.18}p_{31.9}$+ (1- $p_{33.10}$)$p_{41.17}$\} +(1- $p_{4.19}p_{19.4}$) [(1- $p_{11.13}$) (1- $p_{33.10}$ –$p_{37}p_{73}$) (1- $p_{22.12}$- $p_{22.12.15}$)– $p_{12.6}$ \{ ($p_{21.5}$+ $p_{21.5.16}$)(1- $p_{33.10}$–$p_{37}p_{73}$)+ $p_{31.9}$ ($p_{23.11}$+ $p_{23.11,14}$) \}- $p_{13.8}p_{31.9}$ (1- $p_{22.12}$- $p_{22.12.15}$)]

8. Profit Analysis

The profit incurred to the system model in a steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3B_0^S - K_4B_0^{HRp} - K_5R_0^H - K_6R_0^S - K_7N_0 \quad (22)$$

K_0 =Revenue per unit up-time of the system

K_1 =Cost per unit time for which server is busy due preventive maintenance

K_2 =Cost per unit time for which server is busy due to hardware failure

K_3 =Cost per unit replacement of the failed software component

K_4 =Cost per unit replacement of the failed hardware component

K_5 =Cost per unit replacement of the failed hardware

K_6 =Cost per unit replacement of the failed software

K_7 =Cost per unit visit by the server

9. Conclusion

In the present study, the numerical results considering a particular case $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \alpha e^{-\alpha t}$ and $m(t) = \gamma e^{-\gamma t}$ are obtained for some of the reliability measures of a computer system of two identical units with h/w and s/w components. The graphs of the mean time of the system failure (MTSF), the availability and the profit are drawn with respect to preventive maintenance rate (α) for fixed values of other parameters including a=.7 and b=.3 as shown, respectively, in Figure 2, 3 and 4. These figures reveal that MTSF, availability and profit increase with the increase of the preventive maintenance rate (α), h/w repair rate (θ) and by interchanging the values of a and b, i.e., a=.3 and b=.7 . But the values of these measures decrease with the increase of the maximum operation time (α_0) and maximum repair time (β_0). Thus, on the basis on the results obtained for a particular case, it is suggested that the reliability and profit of a system in which the chances of h/w failure are high can be improved by

- By taking one more computer system in cold standby.
- By performing preventive maintenance after a maximum operation time.
- By making replacement of the outdated s/w by new one immediately.

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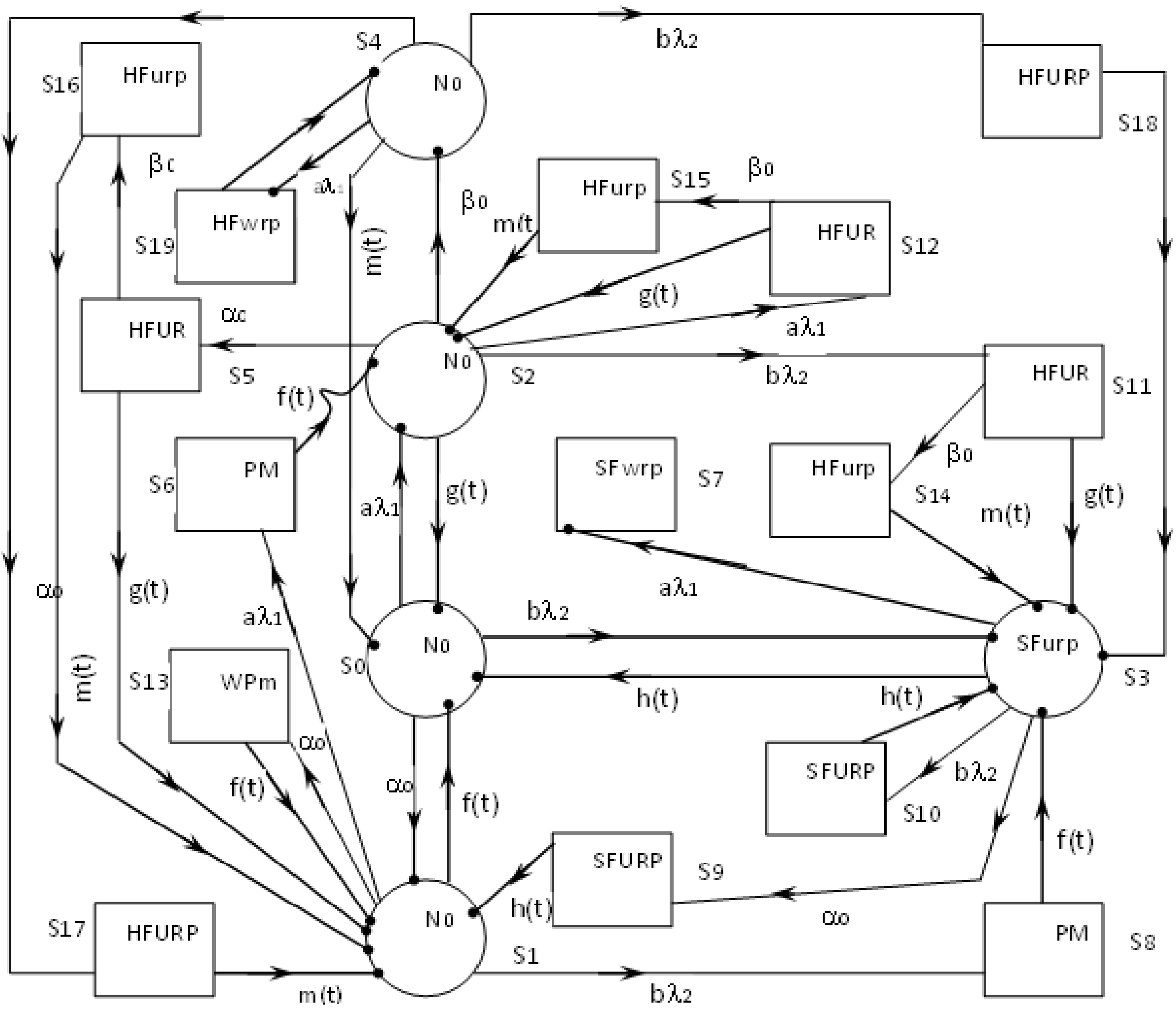

Figure 1: State transition diagram

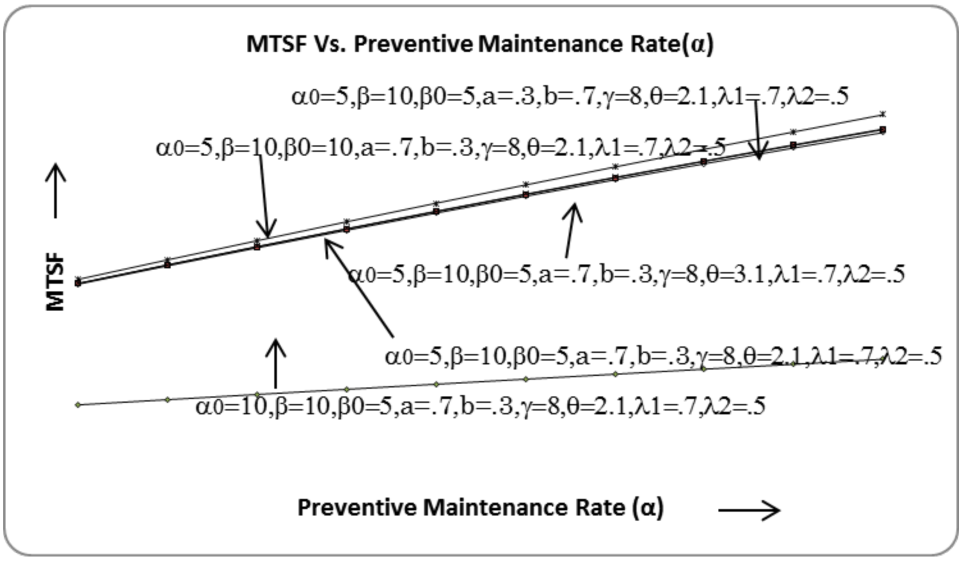

Figure 2. MTSF Vs. Preventive Maintenace Rate

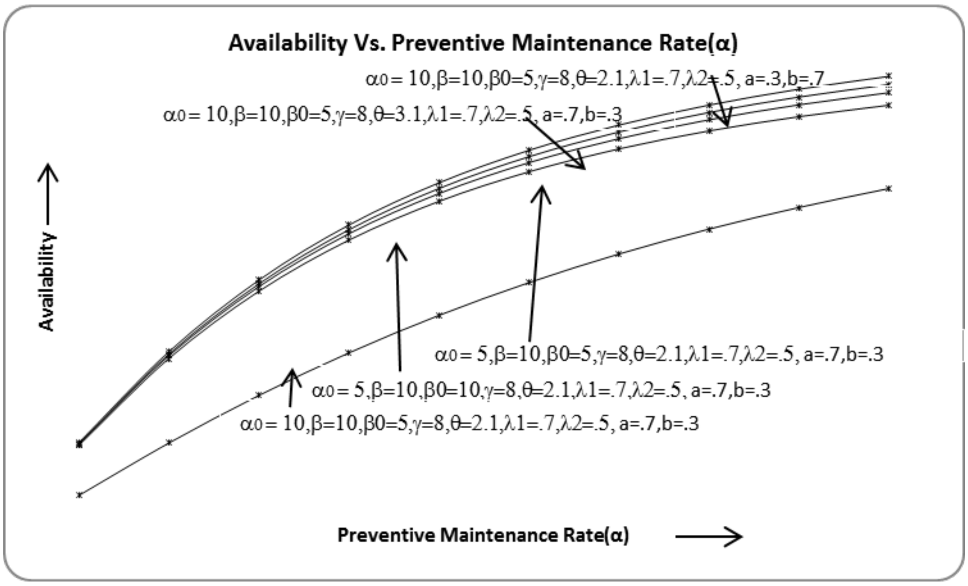

Figure 3. Availability Vs. Preventive Maintenace Rate

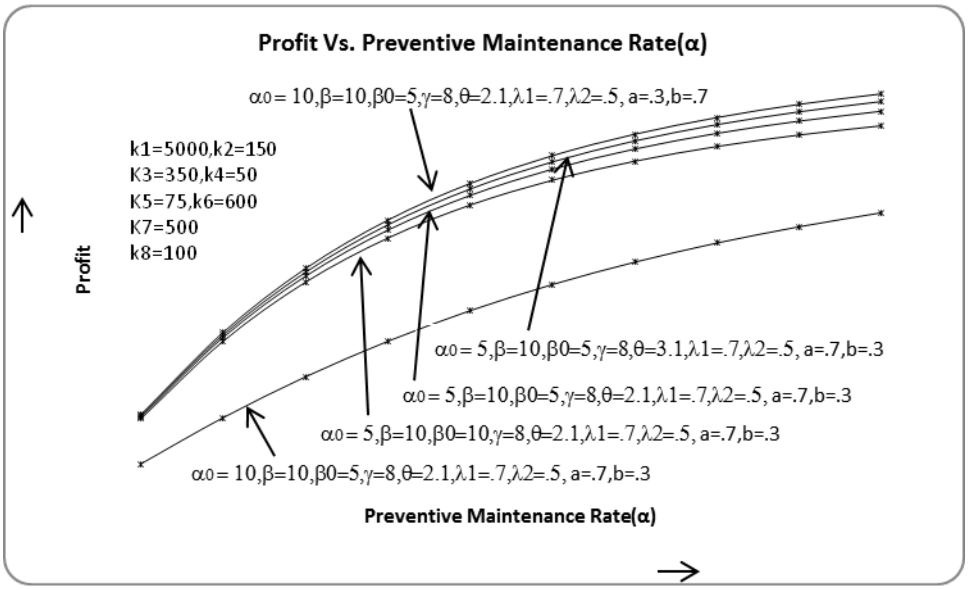

Figure 4. Profit Vs. Preventive Maintenance Rate

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