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A New Method for Calculating Notch Tip Stresses and Strains Based on Neuber Method and ESED Method under Multiaxial Loading

Liu Jianhui*, Wang Shengnan, Su Yi

School of aeronautics, Northwestern Polytechnical University 127 Youyixi Road, xi'an shaanxi, 710072, P.R.China

Abstract

A modified version of Neuber method and ESED method for calculating elasto-plastic notch tip stresses and strains in bodies subjected to proportional multiaxial loading, in which only the heat energy is considered as a dissipation and the stored energy is regarded as a contribution to local stress and strain ranges, has been developed in this paper. The method considers the material constant of yield stress in response to the difference between before and after the plastic phase. This approach makes the calculated results tend to be more precise and reveals its energy meaning, considers the elastic-plastic properties of material itself and avoids the blindness of selecting coefficient values. Finally, the calculated results using modified model are validated with the Finite Element Method. It is shown that, for the case of cyclic loading, the modified method further improves the accuracy of the original Neuber method and ESED method in predicting the nonlinear stress-strain behavior of notches. It is also shown that the modified model proposed in this paper can easily be used for a calculation of the local stress-strain relationship.

***Keywords**: Neuber Method; Equivalent Strain Energy Density Method; Notch Tip Stresses and Strains; Proportional Loading, Finite Element Method.*

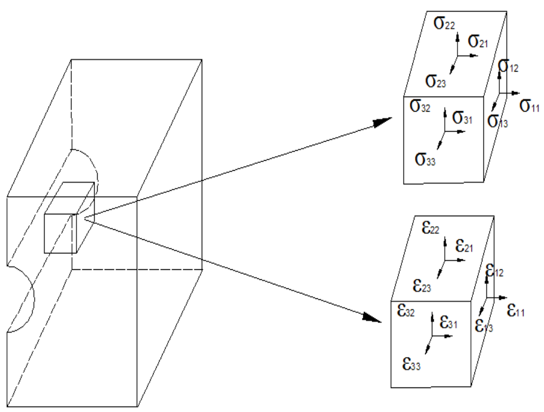
1. Introduction

There are various kinds of notches in engineering applications, which lead to stress concentration. The mechanical structures in practical application mostly work under cyclic loading. The loading mode may be uniaxial cyclic loading, or likely to be multiaxial cyclic loading [1]. Fatigue life prediction of notched components in complex service conditions requires the local stress-strain relationship to be known. Although accurate calculations are not intractable, they are difficult and lengthy especially for a long arbitrary cyclic loading. Therefore, approximate methods [2-3] are widely used in engineering practice. So far, a few approximate methods [4-6] for description of the nonlinear stress-strain behavior of notches have been developed. Ayhan Ince and Grzegorz Glinka [7] has developed a computational modeling method of the multiaxial stress-strain notch analysis to compute elastic-plastic notch-tip stress-strain responses using linear elastic finite element results of notched components. In this paper, a modified method is proposed considering the material constants of yield stress. The benefit of this approach is to consider the elastic-plastic properties of material itself and avoid the blindness of selecting coefficient values. The Neuber method and ESED method were used to calculate the stresses and strains at the notch tip of the LD5 shaft under the tensile-torsional proportional loading. A modified model was proposed to calculate the stresses and strains of the same specimen on the basis of the analysis of the above methods [8-9].

It was proved that the proposed modified model is available and practicable by the comparing the calculated results with the FEM results.

2. Stress-Strain Analysis and Constitutive Relations

When the stress range of notch tip is under the uniaxial state, the state of stress and strain can be expressed as:



Figuer1. Notch tip stress and strain state

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \qquad (1)$$

* Corresponding author. e-mail: liujianhui2010@163.com.

At the same time, in the case of multiaxial loading, the state of stress and strain can be represented as:

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & \varepsilon_{23} \\ 0 & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \quad (2)$$

The elastic-plastic stress-strain constitutive relation is usually derived from uniaxial stress strain curve based on elastic-plastic theory and it can be written as:

$$\varepsilon_{ij} = \frac{1+v}{E}\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij} + \frac{3}{2}\frac{\varepsilon_{eq}^{p}}{\sigma_{eq}}S_{ij} \quad (3)$$

where v is poisson's ratio, E is elastic modulus, ε_{ij} are the strain components, σ_{ij} are the stress components, δ_{ij} is Karen Necker coefficient, σ_{eq} is the equivalent stress, ε_{eq}^{p} is the equivalent plastic strain and S_{ij} are stress deviators.

$$\sigma_{eq} = \sqrt{\frac{3}{2}S_{ij}S_{ij}}\ ,\ \varepsilon_{eq}^{p} = \sqrt{\frac{2}{3}\varepsilon_{ij}^{p}\varepsilon_{ij}^{p}}\ ,$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\ ,$$

$$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}\ ,\ \varepsilon_{eq}^{p} = f\left(\sigma_{eq}\right).$$

Here, $f\left(\sigma_{eq}\right)$ is the function of stress and plastic strain in the case of uniaxial tension and compression.

3. Neuber Method

The Neuber rule was initially proposed for a notched body under pure shear stress state, but is most often used for notches under a tensile or bending load (Fig. 2). Neuber rule is written in the form of eq.(4) which relates the theoretical stress concentration K_t, the actual stress concentration K_σ and the actual strain concentration K_ε factors:

$$K_t^2 = K_\sigma K_\varepsilon \quad (4)$$

where：

$$K_t = \frac{\sigma_{22}^{e}}{\sigma_n}, K_\sigma = \frac{\sigma_{22}^{N}}{\sigma_n}, K_\varepsilon = \frac{\varepsilon_{22}^{N}}{\varepsilon_n},\ \varepsilon_n = \frac{\sigma_n}{E} \quad (5)$$

where, K_t is the theory elastic stress concentration factor, K_ε is the strain concentration factor, K_σ is the stress concentration factor, σ_n is the nominal stress, superscript e represents the corresponding item analyzed by linear elastic and superscript N represents the corresponding item calculated by Neuber rule.

In the case of notched bodies in plane stress resulting in an uniaxial stress state in the notch tip, Neuber's rule eq.(4) can also be written in the form which relates the elasto-plastic strain ε_{22}^{N} and stress σ_{22}^{N} components to the hypothetical linear elastic notch tip strain ε_{22}^{e} and stress σ_{22}^{e}.

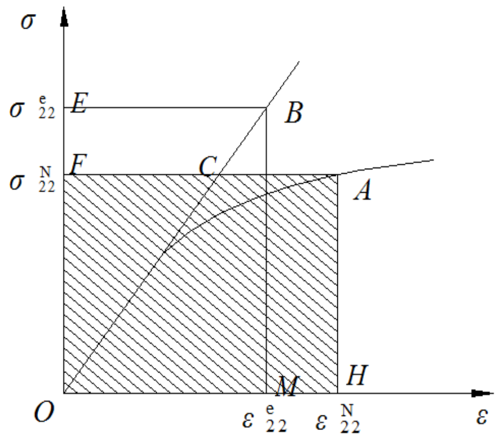
$$\sigma_{22}^{e}\varepsilon_{22}^{e} = \sigma_{22}^{N}\varepsilon_{22}^{N} \quad (6)$$

Thus, when the notch tip is subjected to a uniaxial stress state (as in plane stress), eq.(6) represents the equality of the total strain energy density at the notch tip as shown in Fig.3. The total strain energy density is defined as the sum of the strain energy density and the complementary energy density. A relationship similar to eq.(6) can also be written for notched bodies in strain.

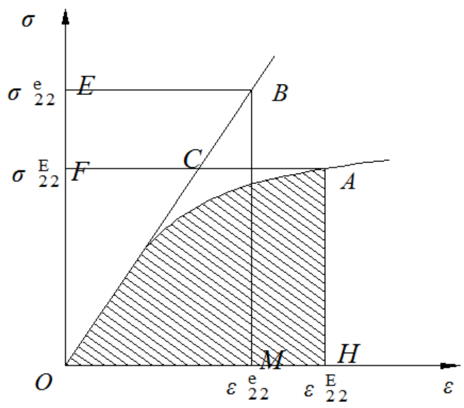
There are three stress components and four strain components when the structures are subjected to multiaxial loading. That is to say, there are a total of seven unknown parameters. However the constitutive equations can only provide four equations. Therefore three additional equations are required. Eq.(6) can be expanded in the case of multiaxial loading and its tensor form is given by

$$\sigma_{ij}^{e}\varepsilon_{ij}^{e} = \sigma_{ij}^{N}\varepsilon_{ij}^{N} \qquad \text{i,j=1,2,3} \quad (7)$$

It has been proved that this equation has energy meaning. Fig.2 shows that, in the case of uniaxial stress state (such as plane stress), the total strain energy density in plastic state equals to that in the linear elastic state, namely the rectangular AFOH and the rectangular BEOM have the same area.



Figuer2. Principle of Neuber Method



Figuer3. Principle of ESED Method

It has been reported that it is more convenient to describe Eq.(7) using the principal stress and strain in the case of proportional loading, so there are five unknown parameters and five equations are required. Then Eq.(7) can be rewritten as:

$$\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e=\sigma_2^N\varepsilon_2^N+\sigma_3^N\varepsilon_3^N \quad (8)$$

Meanwhile it has been experimentally proven that in the case of multiaxial loading the stress and strain distribution ratio can be assumed as:

$$\frac{\sigma_2^e\varepsilon_2^e}{\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e}=\frac{\sigma_2^E\varepsilon_2^E}{\sigma_2^E\varepsilon_2^E+\sigma_3^E\varepsilon_3^E} \quad (9)$$

The constitutive equations can provide the following three equations:

$$\varepsilon_1^N=-\frac{\nu}{E}\left(\sigma_2^N+\sigma_3^N\right)-\frac{f\left(\sigma_{eq}^N\right)}{2\sigma_{eq}^N}\left(\sigma_2^N+\sigma_3^N\right) \quad (10)$$

$$\varepsilon_2^N=\frac{1}{E}\left(\sigma_2^N-\nu\sigma_3^N\right)+\frac{f\left(\sigma_{eq}^N\right)}{2\sigma_{eq}^N}\left(2\sigma_2^N-\sigma_3^N\right) \quad (11)$$

$$\varepsilon_3^N=\frac{1}{E}\left(\sigma_3^N-\nu\sigma_2^N\right)+\frac{f\left(\sigma_{eq}^N\right)}{2\sigma_{eq}^N}\left(2\sigma_3^N-\sigma_2^N\right) \quad (12)$$

where:

$$\sigma_{eq}^N=\sqrt{\left(\sigma_2^N\right)^2-\sigma_2^N\sigma_3^N+\left(\sigma_3^N\right)^2}$$

So Eqs.(8-12) can be employed to calculate the stresses and strains by Neuber method.

4. ESED Method.

The equivalent strain energy density (ESED) relationship was initially proposed [15] for a notched body in plane stress, and is given as an equality of strain energy densities at the notch tip by Molski, Glinka et al.[16-17], which is based on the assumption that the strain energy density distribution in the plastic zone ahead of a notch tip is the same as that determined on the basis of the pure elastic stress-strain solution. In the case of monotonic loading and uniaxial stress condition at the notch tip, ESED method can be expressed in the form

$$\int_0^{\varepsilon_{22}^e}\sigma_{22}^e d\varepsilon_{22}^e=\int_0^{\varepsilon_{22}^E}\sigma_{22}^E d\varepsilon_{22}^E \quad (13)$$

where superscript E represents the corresponding item calculated by ESED method.

Fig.3 shows that the strain energy density equals to that in elastic state although the stress range of notch is in the plastic region, namely the shadow area and the triangle area OBM are equal. The equivalent strain energy density method needs the following expression to solve the problem under multiaxial loading

$$\frac{1}{2}\left(\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e\right)=\frac{1}{3E}(1+\nu)\left(\sigma_{eq}^E\right)^2+\frac{1-2\nu}{6E}\left(\sigma_2^E+\sigma_3^E\right)+\int_0^{\varepsilon_{eq}^{pE}}\sigma_{eq}^E d\varepsilon_{eq}^{pE} \quad (14)$$

where

$$\varepsilon_{eq}^{pE}=f\left(\sigma_{eq}^{pE}\right),\quad \sigma_{eq}^E=\sqrt{\left(\sigma_2^E\right)^2-\sigma_2^E\sigma_3^E+\left(\sigma_3^E\right)^2}$$

Similarly the stress and strain distribution ratio can be given as:

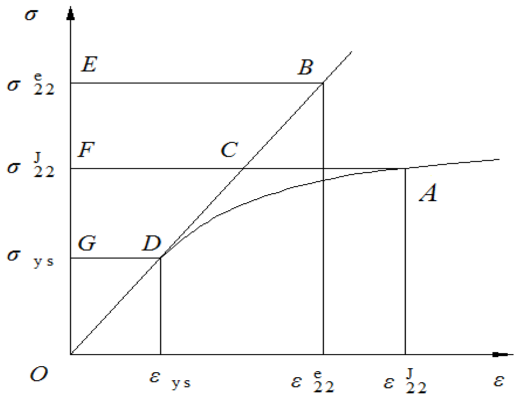
$$\frac{\sigma_2^e\varepsilon_2^e}{\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e}=\frac{\sigma_2^E\varepsilon_2^E}{\sigma_2^E\varepsilon_2^E+\sigma_3^E\varepsilon_3^E} \quad (15)$$

Therefore Eqs.(14-15) and constitutive equations are used to calculate the stresses and strains by ESED method.

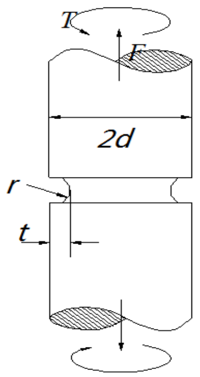
5. Modified Model

On the basis of the analysis of results of notch tip subjected to multiaxial loading, a quantitative relationship between Neuber method and the equivalent strain energy density (ESED) method is found. It is shown that, in the case of elastic range, both Neuber method and ESED method get the same estimation of the local stresses and strains, whereas in the case of elastic-plastic range, Neuber method normally overestimates the notch tip stresses and strains, while ESED method tends to underestimate the notch tip stresses and strains. In other words, the upper limit of the stresses and strains can be obtained by Neuber method and the lower limit is given by ESED method. Through the above analysis, the following points can be given: first, the yield strength limit should be reflected in response to the difference between before and after the plastic phase; secondly, the results calculated by modified model should locate between the upper limit and lower limit in order to avoid large errors of calculated results. So calculation model should be modified based on the above two reasons [18-19].

Providing the further quantitative analysis for Fig.2 and Fig.3 and considering the stress-strain relationship, it can be found that the shaded area calculated by Neuber method is larger than that calculated by the equivalent strain energy density method. Difference area roughly equals to trapezoidal area BCFE (Fig.4), which provides the idea to modify the stress and strain calculation model under multiaxial loading.



Figuer4. Principle of Modified Model



Figuer5. Shape and Dimensions of Specimen

In this paper, the essence of the modified model is based on Neuber method and ESED method. According to the above analysis, the results calculated by modified model should locate between the results calculated by Neuber method and ESED method. Then calculation model is processed by using the principal stress and strain:

$$\left(\frac{\sigma_{ys}}{\sigma_2^e}\right)^2\left(\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e\right)=\frac{1}{2}\left[\frac{2}{3E}(1+\nu)\left(\sigma_{eq}^E\right)^2+\frac{1-2\nu}{3E}\left(\sigma_2^E+\sigma_3^E\right)+2\int_0^{\varepsilon_{eq}^{pE}}\sigma_{eq}^E\, d\varepsilon_{eq}^{pE}+\sigma_2^N\varepsilon_2^N+\sigma_3^N\varepsilon_3^N\right] \quad (16)$$

In this paper the yield strength of material should be defined as:

$$\sigma_{ys}=\sigma_{ys}\left(\sigma_{ys}\le\sigma_2^e\right)$$
$$\sigma_{ys}=\sigma_2^e\left(\sigma_{ys}>\sigma_2^e\right) \quad (17)$$

The modified model makes the calculated results tend to be more precise and reveals its energy meaning. Similarly, the distribution ratio can be assumed as

$$\frac{\sigma_2^e\varepsilon_2^e}{\sigma_2^e\varepsilon_2^e+\sigma_3^e\varepsilon_3^e}=\frac{\sigma_2^J\varepsilon_2^J}{\sigma_2^J\varepsilon_2^J+\sigma_3^J\varepsilon_3^J} \quad (18)$$

Therefore Eq(16), Eq.(18) and constitutive equations enable the notch tip stress and strain components to be calculated.

6. Application Example

The shape and dimensions of specimen are simplified as shown in Fig.5 and the material constants of LD5 aluminum alloy are listed in table 1

Table 1. Material Constants of LD5 Aluminum Alloy

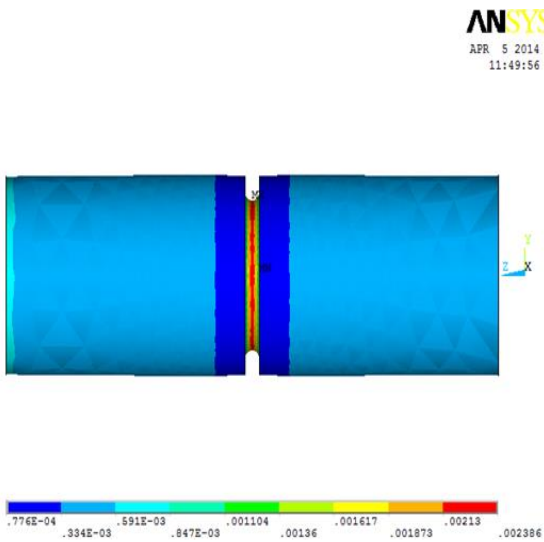
elastic modulus, E	68GPa
shear elastic modulus, G	26GPa
yield stress, σ_{ys}	120Mpa
yield strain, ε_y	0.004
poisson's ratio, υ	0.3

It is easily obtained that σ_{nF} and τ_n can be calculated by the formula as follows:

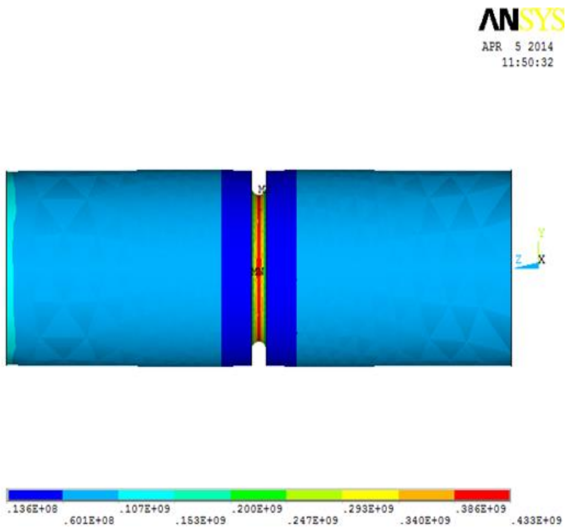
$$\sigma_{nF}=\frac{F}{\pi(d-t)^2},\quad \tau_n=\frac{2T}{\pi(d-t)^3} \quad (19)$$

where, $\tau_n=2.5\sigma_{nF}$, σ_{nF} is nominal stress under loading F, τ_n is nominal shear stress, F is axial loading, D is cylinder radius, t is the depth of the notch, R is radius of notch and T is torque.

The finite element method (FEM) has a widespread application in engineering because of its high precision and economy. Finite element model, boundary conditions and loading process, which directly affects the calculation of the degree of correctness and precision, should close to the actual situation of the project as much as possible, so that the results can be more accurate. Comparing with the eight-node hexahedral elements, the twenty- node hexahedral element, which is an advanced unit setting up on the basis of eight-node hexahedral elements using for the analysis of structure, is more suitable for high precision. In this paper, the ANSYS software is used and element unit is SOLID95. The stresses and strains can be read from ANSYS results(Fig.6 and Fig.7).

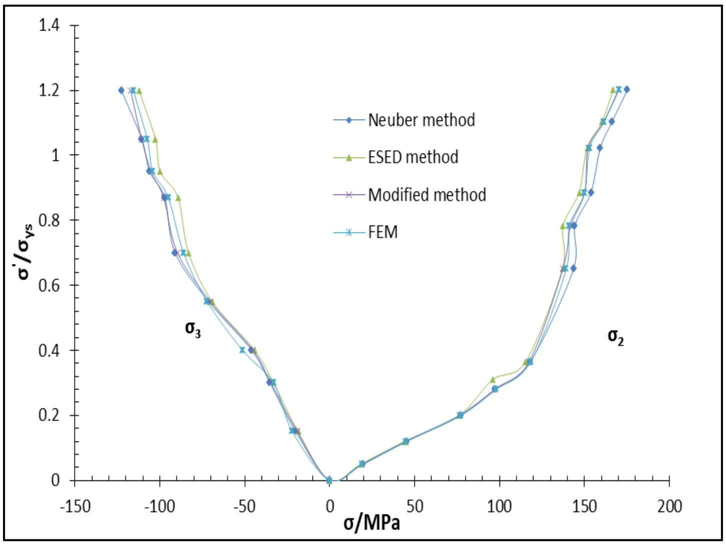


Figuer6. Equivalent stress under multiaxial loading

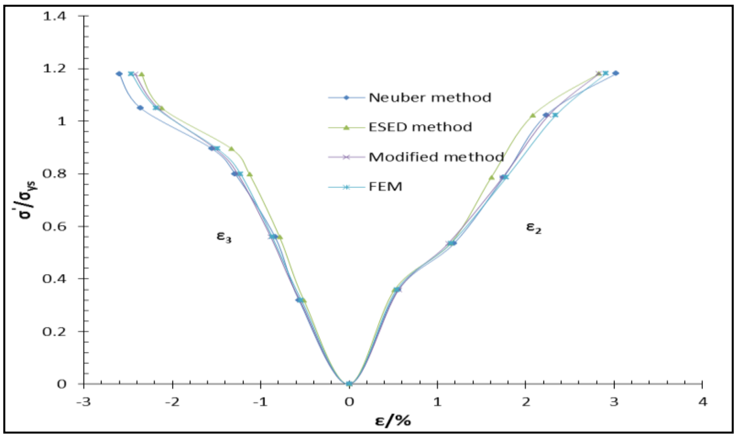


Figuer7. Equivalent strain under multiaxial loading

The stresses and strains can also be calculated by Neuber method, ESED method and modified model. The calculated results using above three methods are compared with the FEM results, and the comparison are shown in Fig.8 and Fig.9:



Figuer8. comparison of stress between different methods



Figuer9. comparison of strain between different methods

Fig. 8 and Fig. 9 show the plots of the calculated and measured local stress and strain amplitude against the ratio of the average nominal stress S' to yield stress σ_{ys} and $S' = \sqrt{\sigma_{nF}^2 + 3\tau_n^2}$. It is obvious from the figures that the results calculated by Neuber method are slightly larger than FEM results, namely Neuber method gives the upper limit of stresses and strains. However the results of ESED method are smaller than FEM results, that is, the lower limit can be got by ESED method. It can be found that the results of the two methods locate both sides of FEM results, and the results calculated by modified model are validated with the FEM results. As a consequent, the correction model can be applied to calculate the stresses and strains in the case of multiaxial loading.

7. Conclusion

It is easily obtained that the results calculated by the above three methods are the same in the case of elastic range; However both Neuber method and ESED method usually have large errors under proportional loading. In this paper, a new modified model is proposed on the basis of the analysis of difference between the two methods. The comparisons of the calculated results with FEM results show that the modified model can relatively accurately calculate the stresses and strains of notch tip and it is convenient for engineering application.

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