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Supplier Evaluation Using Fuzzy Analytical Network Process and Fussy TOPSIS

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Abstract

Supplier selection, which is the first step of the activities in the product realization process starting from the purchasing of material till to the end of delivering the products, is evaluated as a critical factor for the companies desiring to be successful in nowadays competition conditions. With the scope of this paper, supplier selection was considered as a multi criteria decision problem and its complexity is further aggravated if the highly important interdependence among the selection criteria is taken into consideration. The objective of this paper is to suggest a comprehensive decision method for identifying top suppliers by considering the effects of interdependence among the selection criteria. Proposed in this study is a hybrid model, which incorporates the technique of Analytic Network Process (ANP) in which fuzzy triangular priority weights using logarithmic least square method, Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is adopted to rank competing suppliers in terms of their overall performances. An example is solved to illustrate the effectiveness and feasibility of the suggested model also identified the most potential supplier.

Keywords: Supplier selection; Multiple criteria decision-making (MCDM); Fuzzy Analytic Network Process (FANP); Fuzzy Technique for Order Performance by Similarity to Ideal Solution (FTOPSIS); Logarithmic Least Square Method (LLSM)

1. Introduction

Due to the ever-mounting global competition, supplier management has come to play an increasingly crucial role as a key to business success. To secure competitive advantages, organizations have to integrate their internal core competencies and capabilities with those of their suppliers. How to choose capable suppliers is thus an imperative issue in the management of modern business organizations. Existing researches in the field of supplier selection can be divided into two major categories: those focusing on isolating different supply source selection criteria and assessing the degree of their importance from the purchasing firm’s point of view [1]; and those aiming to identify different alternative suppliers by developing and applying specific methods, such as cluster analysis [2], case based reasoning systems [3], statistical models [1], decision support systems [1, 3], data envelopment analysis [2, 4, 5], analytic hierarchy process [2, 6], total cost of ownership models [2, 7], activity based costing [8], artificial intelligence [2, 3], and mathematical programming [9, 5, 10].

Some of the above methods tend to treat each of the selection criteria and alternative suppliers as an independent entity. Price and quality, for example, are treated as two separate criteria without affecting each other. This is, however, seldom the case in the real world business context in which selection criteria and alternative suppliers are in fact characterized by interdependence. Analytic network process (ANP) can therefore be adopted to accommodate the concern of interdependence among selection criteria or alternatives.

The traditional ANP requires crisp judgments. However due to the complexity and uncertainty involved in real world decision problems, a decision maker(DM) may sometimes feel more confident to provide fuzzy judgments than crisp comparisons. This makes fuzzy logic a more natural approach to this kind of problems.

A number of methods have been developed to handle fuzzy comparison matrices. Van Laarhoven and Pedrycz [11] suggested a fuzzy logarithmic least squares method (LLSM) to obtain triangular fuzzy weights from a triangular fuzzy comparison matrix. Wang et al. [12] presented a modified fuzzy LLSM. Buckley [13] utilized the geometric mean method to calculate fuzzy weights. Chang [14] proposed an extent analysis method, which derives crisp weights for fuzzy comparison matrices. Xu[15] brought forward a fuzzy least squares priority method(LSM). Mikhailov[16] developed a fuzzy preference programming method (PPM), which also derives crisp weights from fuzzy comparison matrices. Csutora and Buckley [17] came up with a Lambda-Max method, which is the direct fuzzification of the well-known kmax method.

Among the above approaches, the extent analysis method has been employed in quite a number of applications [18-35] due to its computational simplicity. Shin-ichi Ohnishi [36] proposed fuzzy representation of

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criteria weights in order to reduce inconsistency in pairwise comparison matrix. Y-M. Wang [37] showed by examples that the priority vectors determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and that the misapplication of the extent analysis method to fuzzy AHP problems may lead to a wrong decision to be made and some useful decision information such as decision criteria and fuzzy comparison matrices not to be considered. In this work, modified fuzzy LLSM [12] is used to estimate the fuzzy priority weights in ANP.

The technique for order performance by similarity to ideal solution (TOPSIS) [38] is a widely accepted multi attribute decision-making technique due to its sound logic, simultaneous consideration of the ideal and the anti-ideal solutions, and easily programmable computation procedure [39]. This technique is based on the concept that the ideal alternative has the best level for all attributes, whereas the negative ideal is the one with all the worst attribute values. In fuzzy TOPSIS, attribute values are represented by fuzzy numbers. Using this method, the DM's fuzzy assignments with different rating viewpoints and the trade-offs among different criteria are considered in the aggregation procedure to ensure more accurate decision-making.

The objective of this paper is to suggest a comprehensive decision method for identifying top suppliers by considering the effects of interdependence among the selection criteria. The proposed method accordingly incorporates two stages: (i) Prioritizing criteria using FANP, where fuzzy triangular priority weights are obtained using modified logarithmic least square method (ii) Applying FTOPSIS for ranking of suppliers based on priority weights derived and to find the best supplier.

2. The ANP

The ANP is the most comprehensive framework for the analysis of corporate decisions. It allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay in human society, especially when risk and uncertainty are involved. The elements in a cluster may influence other elements in the same cluster and those in other clusters with respect to each of several properties. The main object is to determine the overall influence of all the elements. In that case, first of all properties or criteria must be organized and they must be prioritized in the framework of a control hierarchy. Then the comparisons must be performed and synthesized to obtain the priorities of these properties. Additionally, the influence of elements in the feedback system with respect to each of these properties must be derived. Finally, the resulting influences must be weighted by the importance of the properties and added to obtain the overall influence of each element [40, 41].

The modeling process can be divided into three steps for the ease of understanding which are described as follows:

2.1. Step I: the pairwise comparisons and relative weight estimation:

The pairwise comparisons and relative weight estimation before performing the pairwise comparisons, all criteria and clusters compared are linked to each other. There are three types of connections, namely one-way, two-way and loop. If there is only one-way connection between two clusters, only one-way dependencies exist and such a situation is represented with directed rows. If there is a two-way dependence between two clusters, bidirected arrows are used. Loop connections indicate the comparisons in a cluster and inner dependence. The pairwise comparisons are made depending on the 1–9 scale recommended by Thomas L. Saaty, where 1, 3, 5, 7 and 9 indicate equal importance, moderate importance, strong importance, very strong importance and extreme importance, respectively, and 2, 4, 6 and 8 are used for compromise between the above values. The score of a_{ij} in the pairwise comparison matrix represents the relative importance of the component on row (i) over the component on column (j), i.e., $a_{ij} = w_i/w_j$. The reciprocal value of the expression ($1/a_{ij}$) is used when the component j is more important than the component i. If there are n components to be compared, the matrix A is defined as

$$A = \begin{bmatrix} w_1/w_1 & w_1/w_2 \dots\dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 \dots\dots & w_2/w_n \\ \vdots & \vdots \cdots \dots\dots & \vdots \\ w_n/w_1 & w_n/w_2 \dots\dots & w_n/w_n \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \dots & 1 \end{bmatrix} \quad (1)$$

Once the pairwise comparisons are completed, like the AHP, a local priority vector (eigenvector) w is computed as an estimate of the relative importance accompanied by the elements being compared by solving the following equation:

$$Aw = \lambda_{max} w \quad (2)$$

where λ_{max} is the largest eigenvalue of matrix A.

Table 1: Linguistic variables describing weights of the criteria and values of ratings.

Linguistic scale	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equally important	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly important	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strong more important	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

2.2. Step II: formation of the initial supermatrix:

All obtained priority vectors are then normalized to represent the local priority vector. To obtain global priorities, the local priority vectors are entered in the appropriate columns of a matrix of influence among the elements, known as a supermatrix [41]. The supermatrix representation of a hierarchy with three levels is given as follows (Fig. 1a):

$$w = \begin{matrix} Goal(G) \\ Criteria(C) \\ Alternatives(A) \end{matrix} \begin{pmatrix} 0 & 0 & 0 \\ w_{21} & 0 & 0 \\ 0 & w_{32} & I \end{pmatrix} \quad (3)$$

where w_21 is a vector that represents the impact of the goal on the criteria, w_32 is a vector that represents the impact of the criteria on each of the alternatives, and I is the identity matrix. W is referred to as a supermatrix because its entries are matrices. For example, if the criteria are dependent among themselves, then the (2, 2) entry of W given by w_22 would be nonzero.

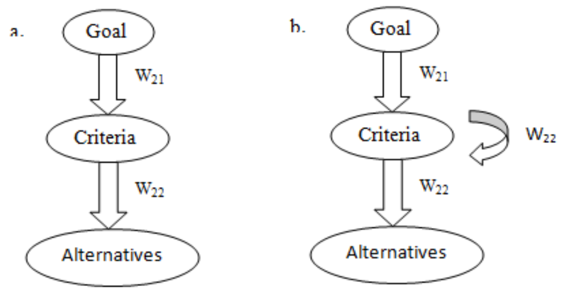

Figure 1: Hierarchy and Network (a). Hierarchy (b). Network [42].

The interdependence is exhibited by the presence of the matrix element $w_{22}$of the supermatrix W (Fig. 1b).

$$w = \begin{pmatrix} 0 & 0 & 0 \\ w_{21} & w_{22} & 0 \\ 0 & w_{32} & I \end{pmatrix} \quad (4)$$

The influence of a set of elements belonging to a cluster, on any element from another component, can be represented as a priority vector by applying pairwise comparisons [43]. Note that any zero value in the supermatrix can be replaced by a matrix if there is an interrelationship of the elements within a cluster or between two clusters. Fig. 1a and b shows hierarchy and network.

2.3. Step III: formation of the weighted supermatrix:

An eigenvector is obtained from the pairwise comparison matrix of the row clusters with respect to the column cluster, which in turn yields an eigenvector for each column cluster. The first entry of the respective eigenvector for each column cluster, is multiplied by all the elements in the first cluster of that column, the second by all the elements in the second cluster of that column and so on. In this way, the cluster in each column of the supermatrix is weighted, and the result, known as the weighted supermatrix, is stochastic. Raising a matrix to exponential powers gives the long term relative influences of the elements on each other [41].

3. Fuzzy Control

The fuzzy set theory introduced by Zadeh [44] and Zadeh [45] is suitable for dealing with the uncertainty and imprecision associated with information concerning various parameters. Human judgment is generally characterized by vague language, like ‘equally’, ‘moderately’, ‘strongly’, ‘very strongly’, and ‘extremely’. Using such language, DMs quantify uncertain events and objects. Fuzzy theory enables DMs to tackle the ambiguities involved in the process of the linguistic assessment of the data. The theory also allows mathematical operators and programming to apply to the fuzzy domain. It provides numerous methods to represent the qualitative judgment of the DM as quantitative data. Triangular fuzzy numbers are used in this paper to assess the preferences of DMs. Subsequently, a multi-criteria decision method can be applied to linguistic assessments to determine the best alternative [48].

Generally, the fuzzy sets are defined by the membership functions. The fuzzy sets represent the grade of any element x of X that have the partial membership to A. The degree to which an element belongs to a set is defined by the value between 0 and 1. If an element x really belongs to A, $\mu_A(x) = 1$ and clearly not, $\mu_A(x) = 0$ Higher is the membership value, $\mu_A(x)$ greater is the belongingness of an element x to a set A.

A triangular fuzzy number is defined as(l, m, u), where$l \leq m \leq u$. The parameters l, m and u respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event, (l, m, u) has the following triangular type membership function.

$$\mu_A(x) = \begin{cases} (x-l)/(m-l) & l \leq x \leq m \\ (u-x)/(u-m) & m \leq x \leq u \\ 0\ , & \text{otherwise} \end{cases} \quad (5)$$

By the extension principle [45], the fuzzy addition, the fuzzy multiplication, fuzzy division and the fuzzy subtraction of triangular fuzzy numbers are also triangular fuzzy numbers.

4. Fuzzy ANP

In the proposed methodology, the fuzzy Analytical Network Process has been used to solve the problem of supplier evaluation. It is very useful in situations where there is a high degree of interdependence between various attributes of the alternatives. In this approach, pair-wise comparison matrices are formed between various attributes of each level with the help of triangular fuzzy numbers. The FANP can easily accommodate the interrelationships existing among the functional activities [46]. The concept of supermatrices is employed to obtain the composite weights that overcome the existing interrelationships. The values of parameters such are transformed into triangular fuzzy numbers and are used to calculate fuzzy values.

In the pairwise comparison of attributes, DM can use triangular fuzzy numbers to state their preferences. Kahraman’s scale mentioned in Section 2 is precise and explicit.

To evaluate the DM preferences, pairwise comparison matrices are structured by using triangular fuzzy numbers(l,m,u)in fig2. The mxn triangular fuzzy matrix can be given as follows (Ramik, 2006).

$$\tilde{A} = \begin{pmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) & \cdots & (a_{1n}^l, a_{1n}^m, a_{1n}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) & \cdots & (a_{2n}^l, a_{2n}^m, a_{2n}^u) \\ \vdots & \vdots & \vdots & \vdots \\ (a_{m1}^l, a_{m1}^m, a_{m1}^u) & (a_{m2}^l, a_{m2}^m, a_{m2}^u) & \cdots & (a_{mn}^l, a_{mn}^m, a_{mn}^u) \end{pmatrix} \quad (6)$$

The element a_{mn} represents the comparison of component m (row element) with component n (column element). If $\tilde{A}$ is a pairwise comparison matrix, it is assumed that it is reciprocal, and the reciprocal value, i.e.,$1/a_{mn}$, is assigned to the element$\tilde{a}_{mn}$.

$$\tilde{A} = \begin{pmatrix} (1,1,1) & (a_{11}^l, a_{11}^m, a_{11}^u) & \cdots & (a_{1n}^l, a_{1n}^m, a_{1n}^u) \\ (1/a_{11}^u, 1/a_{11}^m, 1/a_{11}^l) & (1,1,1) & \cdots & (a_{2n}^l, a_{2n}^m, a_{2n}^u) \\ \vdots & \vdots & \vdots & \vdots \\ (1/a_{1n}^u, 1/a_{1n}^m, 1/a_{1n}^l) & (1/a_{2n}^u, 1/a_{2n}^m, 1/a_{2n}^l) & \cdots & (1,1,1) \end{pmatrix} \quad (7)$$

$\tilde{A}$ is also a triangular fuzzy pairwise comparison matrix. There are several methods for getting estimates for fuzzy priorities, $\tilde{w}_i$where $\tilde{w}_i = (w_i^l, w_i^m, w_i^u)$, i = 1, 2, . . ., n, from the judgment matrix $\tilde{A}$ which approximate the fuzzy ratios $\tilde{a}_{ij}$ so that $\tilde{a}_{ij} \approx \tilde{w}_i/\tilde{w}_j$One of these methods, logarithmic least squares method [12], is reasonable and effective, and it is used in this study. Hence the triangular fuzzy weights for the relative importance of the criteria, the feedback of the criteria and the alternatives according to the individual criteria can be calculated [47]. In our proposed model, only the triangular fuzzy weights for the relative importance of the criteria and the interdependence priorities of the criteria (Eq. (8)) will be used to support the fuzzy TOPSIS for selecting the best alternative.

$$w = \begin{pmatrix} 0 & 0 \\ w_{21} & w_{22} \end{pmatrix} \quad (8)$$

5. The Logarithmic Least Squares Method

Y.M. Wang et al. [12] presented a modified fuzzy LLSM for calculating triangular fuzzy weights as follows:

$$\text{Min } J = \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} \begin{pmatrix} (\ln w_i^L - \ln w_j^U - \ln a_{ij}^l)^2 + \\ (\ln w_i^M - \ln w_j^M - \ln a_{ij}^m)^2 + (\ln w_i^U - \ln w_j^L - \ln a_{ij}^u)^2 \end{pmatrix} \quad (9)$$

Subject to

$$\begin{cases} w_i^L + \sum_{j=1,j\neq i}^{n} w_j^U \geq 1, \\ W_i^U + \sum_{j=1,j\neq i}^{n} w_j^L \leq 1 \\ \sum_{i=1}^{n} w_i^M = 1, \qquad for\ i = 1,2,3,\ldots\ldots,n \\ \sum_{i=1}^{7}(w_i^L + w_i^U) = 2, \\ w_i^U \geq w_i^M \geq w_i^L > 1 \end{cases}$$

6. Evaluation of closeness coefficient for each alternative using Fuzzy TOPSIS

In the following subsection, some basic important definitions of fuzzy sets from Zimmermann [48], Buckley [13], Zadeh [45], Kaufmann and Gupta [49], Yang and Hung [50] and Chen et al. [51] are reviewed and summarized. It is often difficult for a DM to assign a precise performance rating to an alternative for the criteria under consideration. The merit of using a fuzzy approach is to assign the relative importance of criteria using fuzzy numbers instead of precise numbers. This subsection extends TOPSIS to the fuzzy environment.

Definition 1: Let $\tilde{a} = (l_1, m_1, u_1)$ and $\tilde{b} = (l_2, m_2, u_2)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them, as:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2\right]} \quad (10)$$

The problem can be described by following sets:

- A set of m possible candidates called K = {K_1, K_2 , . . . K_m}
- A set of n criteria, C = {C_1 ,C_2 , . . . C_i}.
- A set of performance ratings of K_k(k=1, 2, 3. . . m) with respect to criteria C_i(i =1, 2, 3 ..., n) called $\tilde{X}$= { $\tilde{x}_{ik}$ i = 1, 2, 3. . . n, k = 1, 2, 3, . . . m }.
- A set of importance weights of each criterion w_i(i = 1, 2, 3. . . n)

As stated above, decision matrix format can be expressed as follows:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

Definition 2: Considering the different importance values of each criterion, the weighted normalized fuzzy-decision matrix is constructed as:

$\tilde{V} = [\tilde{v}_{ik}]_{n\times k}$ for i=1, 2...n, k=1, 2… m, where $\tilde{v}_{ik} = \tilde{x}_{ik}$ (.) w_i (11)

According to the briefly summarized fuzzy theory above, fuzzy TOPSIS steps can be out lined as follows:

Step 1: Choose the linguistic ratings ($\tilde{x}_{ik}$ i = 1, 2, 3 . . . , n, k = 1, 2, 3 . . . , m) for alternatives with respect to criteria. The fuzzy linguistic rating ($\tilde{x}_{ik}$) preserves the property that the ranges of normalized triangular fuzzy numbers belong to [0, 1]; thus, there is no need for normalization.

Let $\tilde{x}_{ik} = (a_{ik}, b_{ik}, c_{ik})$, $\tilde{x}_k^- = (a_k^-, b_k^-, c_k^-)$ and $\tilde{x}_j^* = (a_k^*, b_k^*, c_k^*)$.

We have

$$\tilde{r}_{ik} = \begin{cases} \tilde{x}_{ik}(\div)\tilde{x}_k^* = \left(\frac{a_{ik}}{a_k^*}, \frac{b_{ik}}{b_k^*}, \frac{c_{ik}}{c_k^*}\right) \\ \tilde{x}_j^-(\div)\tilde{x}_{ij} = \left(\frac{a_k^-}{a_{ik}}, \frac{b_k^-}{b_{ik}}, \frac{c_k^-}{c_{ik}}\right) \end{cases} \quad (12)$$

Step 2: Calculate the weighted normalized fuzzy decision matrix. The weighted normalized value $\tilde{v}_{ij}$ calculated by Eq. (11)

Step 3: Identify positive ideal (K^*) and negative ideal (K^-) solutions. The fuzzy positive ideal solution (FPIS, K^*) and the fuzzy negative ideal solution (FNIS, K^-) are shown in Eqs. 13 and 14.

$K^{*}= \{\tilde{v}_1^{*}.........,\tilde{v}_i^{*}\} = \{(\max_k v_{ik}| i\in I')(\min_k v_{ik}|i\in I'')\}$
i =1, 2…n, k =1, 2 ….m (13)

$K^{-}= \{\tilde{v}_1^{-}.........,\tilde{v}_i^{-}\} = \{(\min_k v_{ik}|i\in I')(\max_k v_{ik}|i\in I'')\}$
i =1, 2…n, k =1, 2 ….m (14)

Where I' is associated with benefit criteria and I'' is associated with cost criteria.

Step 4: Calculate the distance of each alternative from K^* and K^- using Eqs. 15 and 16.

$$D_k^{*} = \sum_{i=1}^{n} d(\tilde{v}_{ik}, \tilde{v}_i^{*})\ k = 1, 2...m \quad (15)$$

$$D_j^{-} = \sum_{i=1}^{n} d(\tilde{v}_{ik}, \tilde{v}_i^{-})\ ,k = 1, 2...m \quad (16)$$

Step 5: Calculate similarities to ideal solution.

$$CC_k^{\ *} = \frac{D_k^{-}}{D_k^{*}+D_k^{-}}\ , kj = 1, 2...m \quad (17)$$

Step 6: Rank the alternatives based on closeness coefficient. Rank alternatives according to CC_k^* in descending order.

7. Application of Proposed Methodology for Supplier Evaluation

7.1. Step 1: Identifying criteria for supplier evaluation:

In the supplier evaluation process, an objective, unbiased decision is very hard to reach given the numerous criteria that need to be carefully considered and examined. One formal group management technique for determining a set of evaluation criteria is Nominal group technique (NGT) [52]. This well-known process forces everyone to participate and no dominant person is allowed to come out and control the proceedings. In NGT, all ideas have equal stature and will be judged impartially by the group. In this work, four potential evaluation criteria are determined as follows:

- Cost (C1): The total money, time and resources associated with a purchase or activity.
- Quality (C2): Quality is meeting the customer's needs in a way that exceeds the customer's expectations.
- Supply (C3): It is the ability to supply a good or service.
- Time to delivery (C4): It refers to the time required to a deliver a good or service according to the product specifications.

7.2. Step2: Structuring the ANP model hierarchically (goal, factors and alternatives):

The ANP model formed by the factors determined in the first step is shown in Fig. 2. ANP model is composed of three stages. In the first stage, there is the goal of determining factor weights. There are factors related to them in second. The factors of second stage are connected to the goal with a single directional arrow. The arrows in the second stage represent the inner-dependence among the factors. The third stage represents various alternate suppliers which are to be ranked.

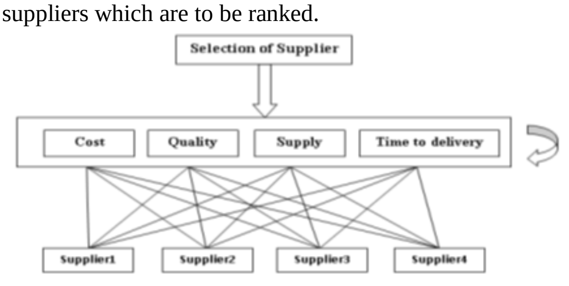

Figure 2: ANP model for supplier evaluation.

7.3. Step3: Recognition of the interdependence between criteria:

To simplify the process and avoid any misunderstandings, the interaction between any two of these criteria is not considered in the first instance. Next, in order to reflect the interdependence property between the criteria, we need to identify the exact relationship in a network structure of ANP. Another NGT process is taken to construct the relationship of interdependency.

7.4. Step4: Determination of local weights of the criteria:

In this step, local weights of the factors which take part in the second level of ANP model are calculated. Pairwise comparison matrices are formed by the decision committee by using the scale given in Table 1. For example the question ‘‘How important is Quality when it is compared with Cost?” and the answer ‘‘strongly more important”, to this linguistic scale is placed in the relevant cell against the triangular fuzzy numbers (2/5, 1/2, 2/3). All the fuzzy evaluation matrices are produced in the same manner. Pairwise comparison matrices are analyzed by Y.M. Wang et al. [12] modified logarithmic least square method to obtain the fuzzy priority weights. The local weights for the factors are calculated in a similar fashion to the fuzzy evaluation matrices, as shown under Table 2. Pair wise comparison matrices are given in Tables 3–6 together with the priority weights.

Table 2: Pairwise comparison matrix.

CRITERIA	Cost	Quality	Supply	Time to deliver	Local Priority weights
Cost	(1, 1, 1)	(3/2, 2, 5/2)	(2/7, 1/3, 2/5)	(5/2, 3, 7/2)	(0.0012, .0016,0.0064)
Quality	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/7, 1/3, 2/5)	(7/2, 4, 9/2)	(0.0013,0.0016,0.0063)
Supply	(5/2, 3, 7/2)	(5/2, 3, 7/2)	(1, 1, 1)	(5/2, 3, 7/2)	(0.7087,0.7476,0.7721)
Time to deliver	(2/7, 1/3, 2/5)	(2/9, 1/4, 2/7)	(2/7, 1/3, 2/5)	(1, 1, 1)	(0.2206,0.2492,0.2835)

Table 3: Interdependency matrix for "Cost".

CRITERIA	Quality	Time to deliver	Supply	Priority weights
Quality	(1, 1, 1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(0.2987, 0.3238, 0.4359)
Time to deliver	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/3, 1, 3/2)	(0.1846, 0.2253, 0.2478)
Supply	(2/5, 1/2, 2/3)	(2/3, 1, 3/2)	(1, 1, 1)	(0.3717, 0.4508, 0.4613)

Table 4: Interdependency matrix for "Quality".

CRITERIA	Cost	Time to deliver	Supply	Priority weights
Cost	(1, 1, 1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(0.0010, 0.0511, 0.2003)
Time to deliver	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/3, 1, 3/2)	(0.3996, 0.4745, 0.4798)
Supply	(2/5, 1/2, 2/3)	(2/3, 1, 3/2)	(1, 1, 1)	(0.3199, 0.4745, 0.5994)

Table 5: Interdependency matrix for "Supply".

CRITERIA	Cost	Quality	Time to deliver	Priority weights
Cost	(1, 1, 1)	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(0.0144, 0.0287,0.0376)
Quality	(2/7, 1/3, 2/5)	(1, 1, 1)	(2/7, 1/3, 2/5)	(0.2185, 0.2428,0.2757)
Time to deliver	(2/5, 1/2, 2/3)	(5/2, 3, 7/2)	(1, 1, 1)	(0.6892, 0.7285,0.7647)

Table 6 Interdependency matrix for "Time to Deliver".

CRITERIA	Cost	Quality	Supply	Priority weights
Cost	(1, 1, 1)	(2/9, 1/4 2/7)	(2/7, 1/3, 2/5)	(0.0144, 0.0287, 0.0376)
Quality	(7/2, 4, 9/2)	(1, 1, 1)	(2/7, 1/3, 2/5)	(0.2185, 0.2428, 0.2757)
Supply	(5/2, 3, 7/2)	(5/2, 3, 7/2)	(1, 1, 1)	(0.6892, 0.7285,0.7647)

7.5. Step 5: Determination of overall weights of the criteria:

In this step, interdependent weights of the factors are calculated and the dependencies among the factors are considered. Dependence among the factors is determined by analyzing the impact of each factor on every other factor using pair wise comparisons. Based on the dependencies, pair wise comparison matrices are formed for the factors (Tables 3–6). The following question, 'What is the relative importance of 'Quality' when compared with 'Time to deliver' on controlling 'Cost'?" may arise in pair wise comparisons and lead to a value of (3/2, 2, 5/2) as denoted in Table 3. The resulting relative importance weights are presented in the last column of Tables 3–6. Using the computed relative importance weights, the dependence matrix of the factors is formed. Interdependent weights of the factors are computed by multiplying the dependence matrix of the factors we obtained with the local weights of factors provided in Table 2. The interdependent weights of the factors are in last column of the Table 7.

Table 7: overall weights of factors.

CRITERIA	Local weights	Overall weights
Cost	(0.0012, .0016,0.0064)	(0.0012,0.0016,0.0064)
Quality	(0.0013,0.0016,0.0063)	(0.0013,0.0016,0.0063)
Supply	(0.7087,0.7476,0.7721)	(0.7087,0.7476,0.7721)
Time to deliver	(0.2206,0.2492,0.2835)	(0.2206,0.2492,0.2835)

7.6. Step6: Preparation of Decision matrix:

In this step, the decision makers are asked to establish the decision matrix by comparing candidates under each criterion separately. Table8 represents the decision matrix, in this some criteria Quality and Supply are assumed to be benefit criteria and Cost and Time to deliver are cost criteria. After the decision matrices are determined, we normalize these matrices via Eq. (12). Results are shown in Table 9. Then weighted normalized decision matrix is determined using Eq. (11). The results are shown in Table 10.

Table 8: Decision matrix.

SUPPLIER	Cost	Quality	Supply	Time to deliver
Supplier1	(3/2,2,5/2)	(1/2,1,3/2)	(3/2,2,5/2)	(2,5/2,3)
Supplier2	(1/2,1,3/2)	(1,3/2,2)	(5/2,3,7/2)	(2,5/2,3)
Supplier3	(1,3/2,2)	(3/2,2,5/2)	(1,3/2,2)	(1/2,1,3/2)
Supplier4	(1/2,1,3/2)	(1,3/2,2)	(1/2,1,3/2)	(1,3/2,2)
Supplier5	(5/2,3,7/2)	(1/2,1,3/2)	(1,3/2,2)	(3/2,2,5/2)

Table 9: Normalized decision matrix.

SUPPLIER	Cost	Quality	Supply	Time to deliver
Supplier1	(1/3,1/2,3/5)	(1/3,1/2,3/5)	(3/5,2/3,5/7)	(1/4,2/5,1/2)
Supplier2	(1,1,1)	(2/3,3/4,4/5)	(1,1,1)	(1/4,2/5,1/2)
Supplier3	(1/2,2/3,3/4	(1,1,1)	(2/5,1/2,4/7)	(1,1,1)
Supplier4	(1,1,1)	(2/3,3/4,4/5)	(1/5,1/3,3/7)	(1/2,2/3,3/4)
Supplier5	(1/5,1/3,3/7)	(1/3,1/2,3/5)	(2/5,1/2,4/7)	(1/3,1/2,3/5)

Table 10: Weighted Normalized Decision matrix.

SUPPLIER	Cost	Quality	Supply	Time to deliver
Supplier1	(0.0045,0.0143,0.0246)	(0.0678,0.1213,0.1763)	(0.0916,0.1218,0.1587)	(0.0205,0.0449,0.0654)
Supplier2	(0.0134,0.0287,0.041)	(0.1356,0.1819,0.2351)	(0.1527,0.1827,0.2222)	(0.0205,0.0449,0.0654)
Supplier3	(0.0067,0.0191,0.0307)	(0.2034,0.2425,0.2938)	(0.0611,0.0913,0.1269)	(0.082,0.1123,0.1308)
Supplier4	(0.0134,0.0287,0.041)	(0.1356,0.1819,0.2351)	(0.0305,0.0609,0.0952)	(0.041,0.0749,0.0981)
Supplier5	(0.0027,0.0096,0.0176)	(0.0678,0.1213,0.1763)	(0.0611,0.0913.0.1269)	(0.0273,0.0562,0.0785)

7.7. Step 7: Ranking of supplier based on closeness coefficient:

The positive ideal solution (K^*) and negative ideal solution (K^-) are determined by using the weighted normalized values. Equations 13–14 are used to determine the positive ideal solution and negative ideal solution. The positive triangular fuzzy numbers are in the range [0, 1]. Hence the fuzzy positive ideal reference point (FPIS, K^*) is (1, 1, 1) and fuzzy negative ideal reference point (FNIS, K^-) is (0, 0, 0). In the last step, the relative closeness to he ideal solution D_k^* and D_k^- are calculated. The relative closeness to the ideal solution is defined on Eqs. 15-16. Equation 10 is used to calculate distances to ideal solutions. Table 11 summarizes the results. The higher the closeness means the better the rank, so the relative closeness to the ideal solution of the alternatives can be substituted as follows:

Supplier3>Supplier2>Supplier4>Supplier1>Supplier5. Supplier2 is defined as the most potential supplier.

Table 11: The Results.

SUPPLIER	D_k^*	D_k^-	CC^*	Rank
Supplier1	3.6979	0.1063	0.0279	4
Supplier2	3.5604	0.2222	0.0587	2
Supplier3	3.5347	0.2525	0.0667	1
Supplier4	3.6563	0.14	0.0369	3
Supplier5	3.7229	0.089	0.0233	5

8. Conclusion

Supplier selection is a complex multi-criteria decision-making problem, and its complexity is further aggravated if the highly important interdependence among the selection criteria is taken into consideration. ANP, providing a systematic approach to set priorities among alternative suppliers, can effectively capture the interdependencies among various criteria. However, ANP handles only crisp comparison ratios. To tackle uncertain decision making judgments and to accommodate the criteria with interdependence Fuzzy Analytical Network Process is used to find the priority weights. To overcome the problem of inconsistency of pairwise comparison matrix fuzzy priority weights are derived using logarithmic least square method. FTOPSIS is used for supplier ranking based on criteria weights and supplier selection. As a result of the empirical study, we find that the proposed method is practical for ranking competing suppliers in terms of their overall performance with respect to multiple interdependence criteria.

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