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Numerical Simulation and Performance Evaluation of Stirling Engine Cycle

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Abstract

A thermodynamic based model was developed. The influences of thermo-physical parameters on the Stirling engine performance were investigated. The analysis is mainly performed inside the compression and expansion cylinders. The instantaneous temperatures of the working fluid inside the engine cylinders, and the interconnected heat exchangers were investigated. The system of governing equations (mass and energy) was developed and employed to investigate the variations of all the process parameters. The step-by-step crank angle intervals were applied to investigate the pressure, temperature, volume, mass flow, and convective heat transfer during the compression and expansion processes. Losses of mass due to the leakage, and energy losses were investigated under the adopted working conditions and phase difference. Variation of work generation due to the leakage of gas through the clearance between the piston and cylinder was evaluated. The numerical solution was adequately performed with a large number of iterations and for each angular interval. A noticeable stable evolutional behavior of the simulated parameters was observed. The stable evolutional behavior of the simulated parameters confirms the model suitability, the capability of pointing out the engine performance, and the capability of converting the thermal energy into useful mechanical work. Additionally, the use of other related correlations ensures the model accuracy, while the resulted small differences are within the expected ranges. In comparison with similar studies, the calculation procedure can be used to investigate the engine performance under different operating conditions.

Keywords: Bio-Diesel; Stirling Engine; Hot Expansion; Cold Compression; Gas Leakage; Polytropic.

Nomenclature

A: Area [m^2]
B.D.C: Bottom Dead Center
c: compression
C.A: Crank angle
c.c: Crank case
c_P, cv: Specific heats [J/Kg.k]
D: Diameter [m]
d: expansion
E, e: Total energy [KJ]
f_c: Cyclic frequency [H_Z]
H, h: Enthalpy [KJ]
$\dot{m}$: Mass flow rate [kg/s]
m : Mass of gas [kg]
n: Polytropic exponent
P: Power [KW]
p: pressure [N/m^2]
Q: Heat transfer [KJ]
R: Gas particular constant [J/Kg.k]
S: Stroke [m]
S_p: Mean piston speed [m/s]
T: Temperature [K]
T.D.C: Top Dead Center
T_H: High Temperature [K]
T_L: Lower temperature [K]
W: Work [KJ]
V_S: Swept volume [dm^3]
t: Time [s]
δ : Clearance between piston and cylinder [m]
ρ : Gas density [kg/m^3]
η_{cycle} : Cycle efficiency [%]
λ : Thermal conductivity [W/m K]
ν : Dynamic viscosity [Ns/m^2]
ω : Angular velocity [rpm]
θ : Crank angle (C.A)

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1. Introduction

Robert Stirling invented Stirling engine in 1816. Engines based on his invention were built in many forms and sizes. The new materials, developments, and models are the keys of the success for this engine. The increased interest in Stirling engines is due to several important advantages over the classical engines, Stirling engines are simple in design, easily operated and externally heated. They operate on a closed regenerative thermodynamic cycle using compressed gases. In comparison with the internal combustion engines, Stirling engines are more attractive, because they offer several important advantages over the traditional internal combustion engine. If the fuel combustion is considered as the fueling heat for this engine, the Stirling engine appears to be more clean and efficient engine, because the combustion is performed externally and the possibility of controlling the amount of oxygen will result in getting a complete combustion, and less fuel consumption with lower pollution. Unlike the internal combustion engine with greater difficulty of controlling the amount of oxygen needed for a complete combustion [1,2]. The demands for clean environment and energy saving can be satisfied [3]. The ability to use a wide variety of cheapest and available thermal sources of energy such as: sources of renewable energy, biomass, biogas, waste heat recovery, and fossil fuel which meet the effective use and saving of energy [4]. Unfortunately, the real Stirling engine suffers from significant losses and disadvantages; flow loses, gas leakage, heat losses, the negative influence of dead volume, and the poor reputation of the heat exchangers under a continuous heat transfer at higher temperature and pressure.

Once these disadvantages are minimized or eliminated, the Stirling engine solution becomes more attractive due to its adaptability to the new demands and operational standards. The reported literature review with valuable base line information, important developments, and performance optimization about Stirling engines have been reviewed and evaluated [5,6]. Usually, the design point of the engine will be between the limits of maximum efficiency and maximum power output. Several authors have studied the thermodynamic performance, and evaluated the effects of different losses on the engine performance. The first classical analysis of Stirling-cycle engine was performed Schmidt [7]; he obtained a theory that provides sinusoidal volume variations in the compression and expansion spaces, and uniform pressure through the system. Considerable efforts have been made to develop and improve the Schmidt analysis. Urieli et al. [9] improved the analysis to cover the effects of leakage, heat losses, and flow losses, indicated work for all the configurations of Stirling engine. Fenkelsein [10] provide the nodal thermodynamic analysis. In this analysis the engine was divided into 13 sub-volumes, the temperature variation at sub-volumes is calculated with differential time intervals by the first law of thermodynamics. Several investigators applied the nodal analysis; they studied the effect of different losses on basis of the nodal analysis. Intensive research works, developments, analysis, and simulation methods were applied to improve the engine cycle performance [11-16]. Petrescu [17] presented a method based on the first law of thermodynamics for calculating the efficiency and power output; they showed that the significant reduction in the engine performance is due to the non-adiabatic and incomplete heat regeneration. The effects of pressure drop due to the mechanical and fluid friction in heat exchangers, the energy losses due to internal and external conduction between the hot and cold parts, and the shuttle effect were studied [3,4] A thermodynamic analysis to investigate the effect of dead volumes on the engine network, and the heat transfer efficiency [18]. The significant amount of lost power is directly related to the lost mass and gas leakage through the clearance between the piston and cylinder [4, 15].

In this paper, the greatest interest was focused on developing a precise and practical numerical simulation, to adequately investigate the operational behavior of the working fluid (hot air) inside the compression and expansion engine cylinders. The proposed thermodynamic model, with the appropriate data input related to the design and operational characteristics, is primarily based on solving the system of governing (mass and energy balance equations). The step-wise basis of the angular increments or intervals was applied over whole the engine operating cycle. The evolutional behavior of the simulated parameters during the hot expansion, and the cold compression of the gas inside the engine cylinders are presented. The same results could be obtained when applying the time or spatial variation methods. Finally, the model was developed; the obtained results of the simulated properties were compared with similar studies [12,14,15,16]. The results are graphically presented on a proper performance curves. The remarkable degree of stability of the gas evolutional behavior was observed during the compression and expansion processes. The resulting indicated performance parameters demonstrate the engine capability of converting the thermal energy into a useful mechanical work.

2. Thermodynamics of Stirling cycle engine

2.1. The ideal cycle engine

Stirling engines are classified into three main configurations (alpha, beta, and gamma). They have the same thermodynamic cycle; but each configuration has its particular mechanical design [5]. Alpha-type is the simplest configuration; it consists of two pistons placed in separate cylinders and mounted on the same crankshaft. The phase difference between both cylinders is 90º crank angles, resulting to have a pure sinusoidal reciprocating motion. The ideal cycle of Stirling engine consists of four processes, namely isothermal compression and expansion, and isentropic heat addition and rejection processes. The engine cycle invented by Robert Stirling presented on PV and TS diagrams as shown in Figure 1 [5]. The expansion and compression cylinders are interconnected via a series of heat exchangers (heater, regenerator, and the cooler).

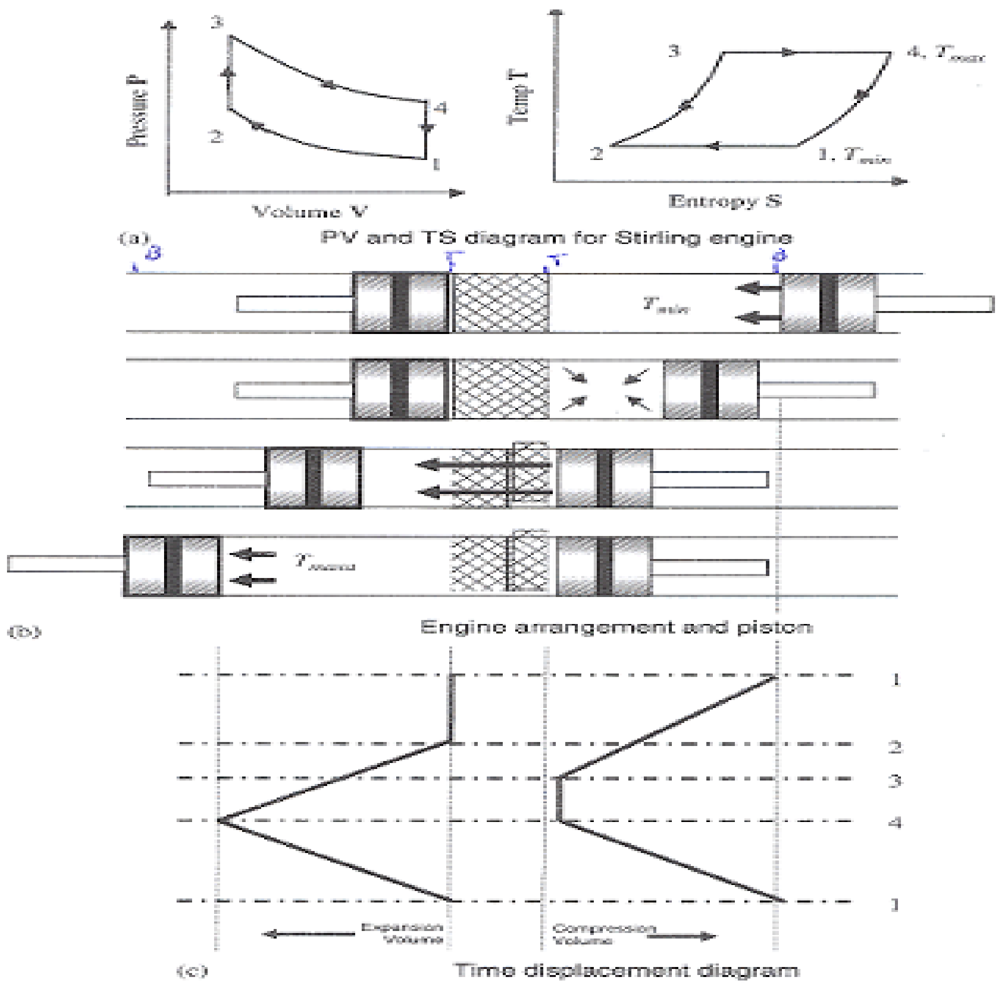

Fig.1. Ideal cycle of Stirling engine

Expansion volume is maintained at the same high temperature of the thermal source (T_H), while the compression volume is kept at low temperature, which is the same temperature of the cold sink (T_C). The regenerator is like a thermal storage; it absorbs and rejects the heat from and to the working fluid. The working gas transports energy from the high temperature heat source to the low temperature heat sink. Hence, work is obtained during one complete cycle. The initial state of the cycle is identified by the position of the crank angle for both pistons. The power piston in the compression volume is maintained at its BDC, and the compression volume is at its maximum, while the pressure and temperature of the working fluid are at their minimum values. The displacer is contained in the expansion volume at its TDC and close to the regenerator. The phase difference between the power piston and displacer is 90º crank angles. In Figure1, both P-V and T-S ideal diagrams, the engine arrangements, and time displacement diagram are clearly shown. The main four ideal processes are:

1-2: Isothermal Compression Process: D*uring this process, the power piston moves from the BDC toward reaching its TDC, helped by the flywheel momentum, and the created partial vacuum in the cold space (cooler) due to the cooling of the working fluid. The total mass of fluid* *is still maintained in the cold space, which follows to be compressed by the power piston. In this process the pressure and temperature of the fluid are increased to their values in state 2. The displacer is still maintained stationary at its TDC.*

2-3: Constant volume regenerative heating: *In this process, the power piston is kept at its TDC close to the regenerator. The displacer starts moving from its TDC away from the regenerator. The volume between both pistons remains constant. The working fluid is forced to pass from the cooler through the regenerator to the heater. The stored heat in the regenerator from the previous cycle is added to the flowing fluid. The temperature is increased and causes the pressure increase to achieve their maximum values (state 3).*

3-4: Isothermal Expansion: *After which the displacer has introduced the working fluid into the hot space, it will be held stationary at its BDC, and the volume, pressure, temperature achieved their maximum values in the expansion space. The power piston being pushed to move back toward its BDC under the increased pressure, it forces the flywheel thus creating a mechanical work.*

4-1: Constant volume regenerative cooling: *Heat is transferred from the fluid and absorbed into the regenerator as a result of flowing through the regenerator from the heater to the cooler and then to the compression space. After which the power piston has reached its BDC, the pressure and temperature start dropping until achieving their initial values (state1).*

2.2. Key factors influencing the cycle engine performance.

Several practical factors affecting the engine performance were reported by various authors. Schmidt [7] performed the first analysis of Stirling engine. However, the working spaces of the real engines have the tendency to be adiabatic rather than isothermal. Fenkelstein [19] performed the first analysis with non-isothermal working spaces, and introduced the concept of conditional temperatures, which depends on the flow direction of the fluid; he assumed that the heat transfer within the working spaces occurs by convection. Using the state equation as well as the laws of energy and mass conservations the analysis was performed. Real engines have many losses. The influence of several practical factors, which causes the real engine to deviate from the ideal one, should be highlighted. Neither heating nor cooling takes place exactly at constant volume or at constant temperature. The working fluid inside the engine cylinders tends to be influenced in an adiabatic rather than isothermal manner. Therefore, the temperature in the compression and expansion spaces will vary over the cycle according to the adiabatic nature. The working fluid should provide high thermal conductivity; high specific heat; low viscosity, and low density. The heat transfer coefficient varies linearly with respect to the local temperatures of the hot components. The heat transfer coefficient must then be determined using the Reynolds analogy, which relates heat transfer to the fluid frictional resistance [20]. Factors governing the performance of a well-designed engine are proportional to pressure, volume, speed and temperature [5,6,17]. A significant effect of friction factor becomes as a function of Reynolds number, the friction factor decreases gradually with the increased value of Reynolds number [21] In real engines, the dead volumes accounts up of 50% of the total internal volume [15]. The increased dead volume within the engine results in reducing the power output [18]. The dead volumes of hot and cold spaces will reduce the engine power and efficiency, and increases the external heat input and output [13]. The leakage of the working fluid is inevitable, this is due to the higher pressure that forces the sealing elements to leak the gas out of the system. Therefore, both pistons need to be perfectly sealed to prevent the leakage tendency [17]. Other important disadvantages related to the flow and energy losses were reported [9,15]

3. Operational principles Design characteristics

The Stirling engine components and operating cycle are shown in Figure 2 [6]. The engine operates on a closed regenerative thermodynamic cycle with a cyclic hot expansion and cold compression. The expansion piston acts as an expander while the compression piston as a compressor. The regenerator is placed between the cooler and a heater with the scope of using the removed heat from the gas during the energy transfer to the cold cylinder. The stored energy in the regenerator is added to preheat the fluid when it is transferred back to the hot cylinder. The gas (air) is initially compressed in the compression space (cold compression), and forced to flow at a uniform rate through the regenerator to recover the stored heat. The maximum working temperature is reached inside the heater, due to the transferred heat from the higher temperature source (T_H) to the fluid. The working gas transports energy from the high temperature heat source to the low temperature heat sink. Hence work is obtained during one complete cycle. Work is exerted to compress the cold working gas, and a useful work is extracted by expanding the gas after it has been heated in order to increase its pressure. Heating and cooling of the gas is achieved by moving the gas back and fourth through the connected heat exchangers (heater, regenerator, and cooler). The deviation between the ideal and real cycles is clearly shown in Figure 3 [6], where the effects of irreversible factors are presented on the pressure-volume diagram (P-V). Alpha-type Stirling engine consists of two separate cylinders with symmetrical geometry. The expansion cylinder is advanced with a 90º crank angles than the compression cylinder. Some valuable conclusions regarding to the effects of the operating parameters were appreciated. An efficient heat transfer to the working fluid can be satisfied by high mass flow rate of the working fluid [12]. The power output of Stirling engine is assumed to be proportional to the increased mean cycle pressure [3]. The high pressure is necessary to reduce the need for high mass flow rate [3,4,22]. The friction factor and flow discharge coefficient are considered as a function of Reynolds number, the fluid friction in the heat exchangers decreases gradually with the increased value of Reynolds number [21]. In this work, the geometrical and operational parameters are tabulated in Table 1.

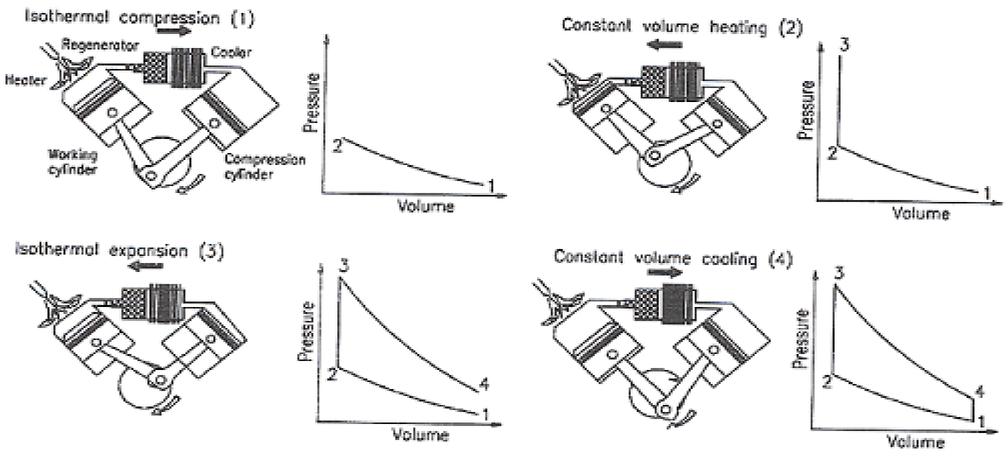

Figure 2. Stirling engine operation principles

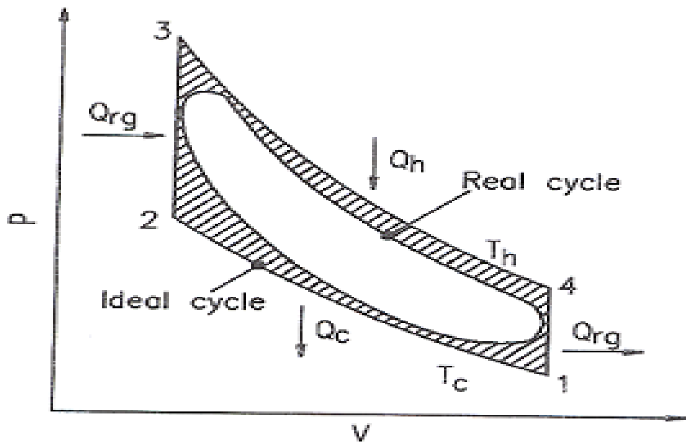

Figure 3. Deviation between the ideal and real cycles of Stirling engine

Table 1. Stirling engine characteristics

Swept volume, V_S = 1dm^3	Working fluid is air
Cylinder diameter = 108 mm	Engine speed= 1500 rpm
Stroke length = 108 mm	Initial pressure = 50 bar
Phase angle advance = 90º	Hot source temperature = 1000K
Number of nodal volumes = 72	Low sink temperature = 300 K

The cheap and available air is taken as the working agent, and a pressure of 50 bars is used. The passageway between both cylinders is formed by a number of small tubes; it has relatively a total cross-sectional area of 42.6% of the total frontal cross-sectional area of the piston. The constant discharge coefficient of 70% was taken as a function of Reynolds number. The engine operating cycle is divided into 72 angular intervals or increments; each interval is represented by 0.2º of one full crankshaft rotation. . The difference between two successive intervals is 5º of crank angles. The first step of calculation will be started by setting the initial state and aligning the expansion piston to its (TDC), which corresponds to a crank angle ($\theta_{exp.}$ = 0), respectively the swept volume at the initial crank angle ($V_{exp.}$=1dm^3). At the same time, the swept volume in the compression cylinder is identified by the position of the compression piston with the crank angle of ($\theta_{comp.} = 270^o$).

4. The Stirling engine model

4.1. Governing equations and assumptions

Modeling activities have major contributions to predict the engine performance over a wide range of design and operating variables, and at different conditions. Modeling is considered as a powerful tool to identify the key controlling variables without having to conduct the costly experiments and tests, or due to the difficulty of performing direct measurements for all the flow parameters and characteristics.

The thermodynamic-based model is formed by the combination between the system of governing equations (mass and energy balance equations), with the time or crank angle increments as the independent variables. In order to accurately evaluate the variation of the simulated parameters, the system of equations was solved on a step-wise spatial variation in each angular interval. Each cylinder is divided into an active swept volume (V_S) and inactive dead volume. The total internal volume occupied by working fluid is divided into a number of small nodal volumes. The first nodal volume is taken in expansion cylinder, and the last nodal volume in the compression cylinder. The other nodal volumes are located in the flow passage between the cold and hot spaces. All the nodal volumes are assumed to be open systems. From the thermodynamic point of view, every cell in an open system is controlled by the periodic condition at the inlet and outlet. Generally, the main thermodynamic assumptions for an open system were adopted to formulate the mathematical model:

- The air is taken as working agent; it behaves as an ideal gas.
- The state of the mass at each does not vary with time
- Total mass in the system does not change with time
- The volume in hot and cold cylinders vary according to the crank angle, while in all the heat exchangers the volume is kept constant
- The gas pressure and temperature does not vary with time at the boundaries of any control volume.
- Uniform working fluid with uniform states of mass at any point throughout the entire control volume.

The engine consists of two cylinders with symmetrical geometry; but they differ in their working conditions. The phase difference between the power piston and displacer is 90º crank angles. The expansion piston is set to be in advance with 90º-phase angle than the compression piston. In this work, the calculation procedure was performed in the expansion cylinder, a cylinder in which heat energy is converted into a useful mechanical work. Therefore, the variations and evolutional behavior of the simulated parameters inside the cylinder are described by solving the system of governing equations on a stepwise basis of crank angle intervals:

The mass balance equation for open control volume:

When the gas leakage from the engine is

$$m_i = m_e \tag{1}$$

considered, it can be calculated by:

$$\frac{\partial m}{\partial t} = \dot{m}_i - \dot{m}_e - \dot{m}_{lost} \tag{2}$$

The mass flow rate of gas leakage through the clearance between the piston and cylinder as due to the bad sealing conditions can be calculated by:

$$m_{leakage} = \frac{\pi D \delta^3}{12 \nu L}(p_{cylinder} - p_{c.c}) \tag{3}$$

The first law of thermodynamics (energy balance) is rearranged into the following form: (equation 4)

$$dQ_{wall} - dW_{\text{intern}} = mde + e_{out}dm_{out} - e_{in}dm_{in} + e_{lost}dm_{lost}$$

Generally, for an open control volume, only the change in enthalpy is considered. The kinetic and potential energies are assumed of negligible effects, because they are small enough. The pressure difference between nodal volumes due to flow friction can be calculated by

$$p = \frac{m_t R}{\sum_{i=1}^{n} V_i / T_I} \tag{5}$$

The temperature variation between two steps of time within a nodal volume is calculated by using the first law of thermodynamics for open system as:

$$\Delta T_i = [h_i A_i (T_{wi} - T_i)\Delta t - \Delta m_i C_v T + E_i - P\Delta V_i)/(m_i C_V) \tag{6}$$

Where E_i, represents the enthalpy flow in or out the nodal volume, and calculated by:

$$E_i = -C_P \frac{T_i + T_{i+1}}{2}\sum_{j=i+1}^{n} \Delta m_j - C_P \frac{T_{i-1} + T_i}{2}\sum_{j=1}^{i-1} \Delta m_j \tag{7}$$

Boundary temperature of flow passage is assumed to be constant with time and varies linearly in flow direction from T_C to T_H. Specific heats are assumed constant. The flow across boundaries is taken as positive in the direction of flow. The influence of flow friction on the mass distribution in the engine, is directly included in the mass balance equation, and it could be using the pressure losses correlations. If $\sum_{j=i+1}^{n} \Delta m_j$ has a negative value, enthalpy flow occurs from the nodal volume i+1 to the nodal volume i. else the enthalpy flow will be from the nodal volume i to the nodal volume i+1. Similarly, if $\sum_{j=1}^{i-1} \Delta m_j$ has a negative value, then enthalpy flow will be from the nodal volume i-1 to the nodal volume i; else the flow of enthalpy will be from the nodal volume i to the nodal volume i-1. Instantaneous values of the mass m_j, in the nodal volumes are calculated using the general state equation of perfect gases. The included terms in equation (6) are the flow of energy (enthalpy) between neighboring control volumes, the convective heat transfer between the gas and metal, and the work as due to the volume variation. The heat transfer between the gas and the internal walls of the cylinders, is assumed take place by convective heat transfer only:

$$dQ_{walls} = h_c A(T_{wall} - T_{gas})d\tau \quad (8)$$

Different correlations can be used to determine the heat transfer coefficient such as Woschni correlation number 9 or equation number 10 can be applied to calculate the heat transfer coefficient [17,20].

$$h_c = 4.14(1 + 1.24\bar{S}_p)(p^2T)^{1/3} \quad (9)$$

$$h_c = 2.1\frac{\lambda}{D}\left(\rho \bar{S}_p \frac{D}{\eta}\right)^3 \quad (10)$$

both correlations give nearly the same value of the heat transfer coefficient.

The indicated work term is expressed as:

$$dW_{indicated} = pdV - mvdv - dW_{friction} \quad (11)$$

In equation (11), only work due to the volume variation is considered, while the kinetic energy, and the frictional work are assumed with negligible effects, because of their insignificant effects.

The indicated cyclic work can be expressed as:

$$dW_{indicated} = \frac{p_j + p_{j+1}}{2}(V_{j+1} - V_j) \quad (12)$$

By considering the polytropic exponent, the indicated work is calculated in each angular interval using the following equation:

$$dW_{indicated} = \frac{n}{n-1}mRT\left[\left(\frac{p+dp}{p}\right)^{\frac{n-1}{n}} - 1\right] \quad (13)$$

where, n is the polytropic exponent, and it can be determined using with the following equation:

$$n = \frac{kdW_{indicated}}{(k-1)dQ_{wall} + dW_{intern}} \quad (14)$$

The instantaneous values of the indicated power and heat flux are expressed by:

Power: $P_{inst.,j+1} = \frac{p_{mean,j+1}(V_{j+1} - V_j)}{\Delta\tau}$. (15)

Heat flux: $\dot{Q}_{wall,inst.,j+1} = \frac{dQ_{wall,j+1}}{\Delta\tau}$. (16)

Finally, the numerical program provides the following indicated performance parameters at the end of each operating cycle:

Indicated work per cycle: $W_{indicated} = \oint pdV$ (17)

The work given by the cycle is: $W_{net} = W_{expansion} + W_{compression}$ (18)

where, the work done during the expansion and compression are calculated by considering the volume variation in each cylinder.

$$\frac{dW}{dt} = P_c\frac{dV_c}{dt} + P_d\frac{dV_d}{dt}$$

Indicated power: $P_{indicated} = W_{cycle,indicat.} f_c$. (19)

Where, f_c is the cyclic frequency expressed by: $f_c = \frac{\omega}{2\pi}$ (20)

Heat added from the higher temperature source: $Q_{added} = \oint dQ_{added}$ (22)

Heat rejected to the lower temperature sink: $Q_{rejected} = \oint dQ_{rejected}$ (23)

Indicated cycle efficiency: $\eta_{indicated} = \dfrac{P_{indicat.}}{Q_{added} f_c} = \dfrac{Q_{added} - Q_{rejected}}{Q_{added}}$ (24)

Indicated specific heat: $C_{indicat.} = \dfrac{Q_{added}}{P_{indicated} / f_c}$ (21)

4.2. Particular Aspects of The Model.

Defining the initial state of the fluid inside the cylinder starts the first step of calculation. The thermodynamic based model allows the possibility of calculation all the instantaneous and average values of the key parameters. It is capable to simulate all the processes in proper sequence until making up the engine cycle. It is often required to model the region of interest (engine cylinders) as an open thermodynamic system; such a model is appropriate when the fluid inside the open system is assumed to be uniform at each point in time. Convergence with the cycle simulations occurs within a few iterations. The developed model can be used to calculate all of the instantaneous properties, and the indicated parameters at any working condition. The particular aspects of the model are presented as following:

1. A simplified iterative numerical solution is developed to solve the system of governing equations. The analysis is started at the beginning of the expansion stroke (initial state), and the values of the simulated properties are determined at the starting point of the cycle with the expansion piston at its position of TDC. While the compression piston is still maintained at its BDC. The instantaneous properties are calculated on a step-wise basis of crank angle intervals. The operating cycle should ends with the initial state of the working fluid that was started out. The variables representing mass and energy must be the same at the end and starting points of the cycle. The model should results in a periodic response with less error. In case that the values of the fluid properties at the end state are different than those of the initial state, the iterative solution will continue until the discrepancy is sufficiently small and the converged solution is achieved.
2. Nodal volumes technique provides a precise evaluation of the real effects of non-adiabatic processes. In this work, the working space is divided into 72 of finite volumes, which are sufficiently enough to obtain accurate results. The simulated properties are calculated on a step-wise basis of angular intervals, and over each operating cycle.
3. The initial state of the working fluid at the starting point of the cycle was defined. The key parameters that describe the evolutional behavior of the working fluid inside the engine cylinders were calculated. These parameters include: the cylinder dimensions, dead volumes, sizes and cross sectional areas of the heat exchangers, initial pressure and temperature of the working fluid, temperatures of the hot and cold sources, temperature of the cylinder walls, and the engine rpm.
4. The calculated parameters at the end of the first angular interval are obtained as a function of their values at the beginning of the same angular interval. The first property that will be determined at the starting and end points is the displacement volume. The mass of gas in the cylinder was initially considered as known value. The calculation procedure will continue to point out the other properties such as; the pressure and temperature of the fluid at the start and end of each angular interval, the gas leakage through the clearance between the piston and cylinder. Using the mass balance equation, the available mass that still exists in the cylinder is calculated at the end of each interval. The same procedure will be applied from (1to n angular intervals). The simulated properties at the start of each angular interval are function of (θ_j): V_j, p_j, T_j, ΔW_j, ΔQ_j, m_j, and n_j. While their properties at the end of each angular interval are function of (θ_{j+1}), and their values are: V_{j+1}, p_{j+1}, T_{j+1}, ΔW_{j+1}, ΔQ_{j+1},m_{j+1} and n_{j+1}).
5. The stable evolutional behavior of the main parameters is a good indicator for determining the indicated work and power, and the cycle efficiency. The main parameters are: the gas pressure and temperature in each cylinder and for each moment of calculation, the gas flow rates entering and leaving both cylinders, the transferred heat between the gas and internal walls, and the work transfer from the gas to the drive mechanism. Actually, the calculation procedure was performed with sufficient number of iterations. The resulted stable evolutional behavior of the working fluid ensures the model correctness. The same values of the thermo-physical properties at the end and starting points of the cycle were satisfied.

4.3. *Model Verification*

A great effort has been made to verify the correctness of the model. The great challenge is related the model capability of computing the amount of gas leakage, and energy losses from the engine. Testing for mass conservation has been made by the sum of gas in all control volumes. Using other correlations ensures correctness of the model as well as the implemented correlation, and checking for possible differences. The differences have been found within the expected ranges. The small differences are due to the effects of four design parameters: temperature ratio, gas leakage, swept volume ratio, and dead volume ratio. The Stirling engine power output is a good indicator for measuring and evaluating the engine performance, and the model accuracy. The Stirling engine power output can be calculated by using different formulas. The simpler approach is to use the Beale formula; this formula requires few design parameters and can be used for various configurations and sizes of Stirling engine. Beale formula considers that the power output of many Stirling engines is roughly proportional to the pressure, swept volume, and speed [5,8]. The relationship between all parameters in this formula is expressed by an empirical formula as called Beale number:

$$P/(p_m f V_P) = \text{Constant.} \qquad (22)$$

Walker [23] proposed an approximated Beale formula for all types and sizes of Stirling engines in order to calculate the power output as:

$$P = 0.015 p_m f V_P \qquad (23)$$

Where P is the engine power in W, p_m is the mean cycle pressure in bar, V_P is the displacement volume in cm^3, f is the cycle frequency in H_Z.

Walker [23] introduced the temperature ratio and modified Beale formula in order to suit all the real engines, the new correlation is:

$$P = F p_m f V_P \frac{T_H - T_C}{T_H - T_C} \qquad (24)$$

The last developed correlation (eq.24) is considered as a powerful tool during the first stages of design, it reflects the effects of the real parameters by providing an experience factor F with a practical value ranged between (0.25-0.35). The indicated power can also be calculated by using other correlation:

$$P_{indicated} = W_{net} f \qquad (25)$$

Comparing the obtained results with the ideal isothermal or the adiabatic available models is the best method of verification. Several thermodynamic models were reviewed and evaluated. The comparison with similar thermodynamic models [4,12,15,16] and a good agreement between the results was observed. Nearly, the same values of simulated parameters at the starting and end points of the cycle is an important aspect of the model validity. The converged values of simulated parameters are due to the iterative procedure, and the increased number of computational points, where the total volume was divided into 72 of finite volumes. The resulted stable behavior of the simulated parameters demonstrates the engine capability of converting the heat energy into a useful mechanical work. The model can be used to predict different key parameters under different working conditions, and encourage us to start other investigations in the next future. .

5. Results and Discussion

The evolutional behavior of the simulated parameters was investigated in each cylinder as a function of crank angle variation. These parameters are: pressure, temperature, flow rates in and out of the cylinders, mass of gas in and out, polytropic exponent, convective heat transfer coefficient, transferred heat, work done on and by the system, heat rejection and addition, and the power output. The model validation is investigated by studying the effect different design and operational parameters on the engine performance. The adiabatic expansion and compression of the gas inside the engine cylinders was investigated, The influence of the high temperature (T_H) and the lower temperature (T_L) with respect to their values is evaluated, the effect of hydraulic resistances on the flowing fluid, the significant effect of dimensional parameters such as: the length and diameter of the passageway between both cylinders. Based on the accurate data input, the system of equations was solved iteratively on a step-wise basis of angular increments. The variations of operating parameters are presented on a proper performance curves. A detailed analysis and illustration for each gas variable was performed separately.

5.1. Gas pressure and temperature variations.

The pressure and temperature variations in both cylinders are represented over each operating cycle. In Fig.4 (p-θ) diagram, illustrates the pressure variation in the engine cylinders as a function of crank angle variation. The remarkable observation that could be observed is focused the small differences between the calculated pressures in both cylinders. These differences are due to the friction and other resistances against the flowing fluid in the passageway between the cylinders. The increased number of the interconnected small tubes, and their cross sectional areas are important factors that control the pressure difference. The reduced cross sectional areas of these tubes will increase the pressure difference between the cylinders. The pressure difference between the expansion and compression cylinders having to indicate the pressure losses in both cylinders.

In Fig.5, the temperature variation in the hot and cold cylinders versus the crank angle variation is illustrated. Temperature variation within the nodal volumes is obtained by using the first law of thermodynamics for open systems. The influence of the source temperature (1000K) at which heat is added, and the sink temperature (300K) to which hear is rejected is clearly observed. The maximum temperature and pressure are achieved between 60 and 80º crank angle, this is due the heat added during this period. The temperature and pressure start to decrease at around 90 to 100º of crank angles, this decrease is due to the energy conversion into mechanical work during the expansion process.

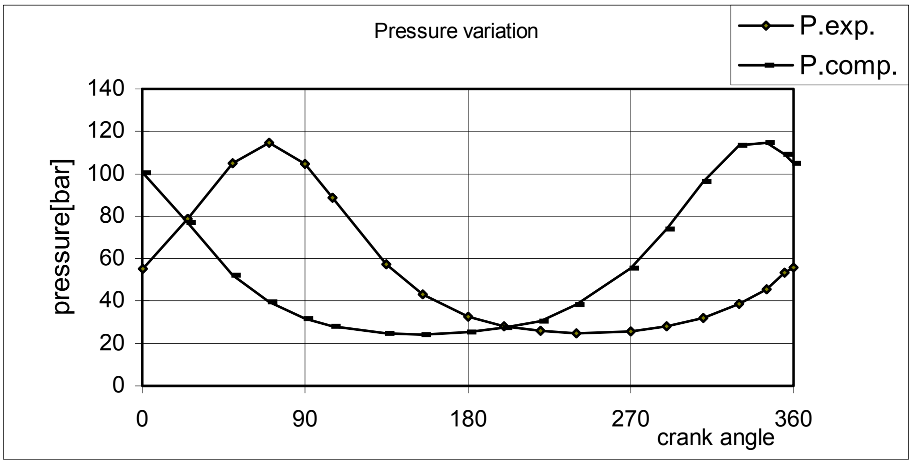

Figure 4. Pressure variation

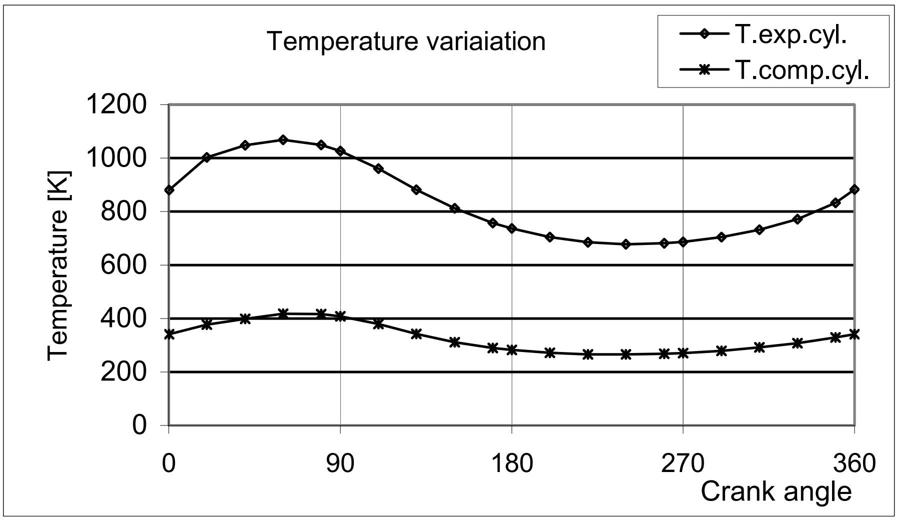

Figure 5. Temperature variation of the working gas

5.2. Mass flow rates and accumulated mass inside the cylinders

The mass transfer between both cylinders depends on various factors. The mass transfer is governed by different factors such as: the cross sectional areas of the pistons (A_P), the pistons speed (S_P) as a function of the engine rpm and stroke length (S), the cross sectional areas of the small pipes in the interconnected passageway, and the density variation between the engine components. The gas density is highly dependent on the operating temperature and pressure. In fact, the mass losses are evaluated as due to the pressure increase. The state equation of ideal gases is employed to calculate the gas density in each crank interval: $p = \rho RT$. Based on the calculated densities, the mass of gas was calculated in each interval. The transferred mass between the cylinders occurs under the effect of pressure difference. The discharge coefficient with a value of 0.7 is considered for all the passages with similar shape. In Fig.6, the flow velocity is illustrated through the variation of the gas entering the expansion cylinder, and the gas leaving the expansion cylinder in Fig. 7. As shown in Figure 6, the gas leakage and mass losses is observed between 15 and 25º Crank Angle, this is mainly due to the high pressure and friction that forces the gas to leak through clearance between the pistons and cylinder. The maximum flow rate was achieved at 70º crank angles. The smoother mass flow rate leaving the expansion cylinder than that entering the cylinder is clearly shown; this is due the reduced pressure. In Fig. 8, the variation of the total mass of the gas in both cylinders versus the crank angle variation is observed with great dependence on the phase difference between the cylinders. it was observed that by lowering the mass of working gas, the temperature of the fluid approaches rapidly the level of hot source temperature after heating. Similarly, the gas approaches the cold temperature sink after cooling. Using the higher pressure or the lower viscosity, or the combination between them could reduce the need for high mass flow rate. As shown in Figure 8, the gas inside the cold cylinder is denser than that in the hot cylinder, except at the angular intervals between (75-145º), as due to the closed position of the piston to the Top Dead Center of the cold cylinder. Actually the total mass in this period is smaller than that in the hot cylinder.

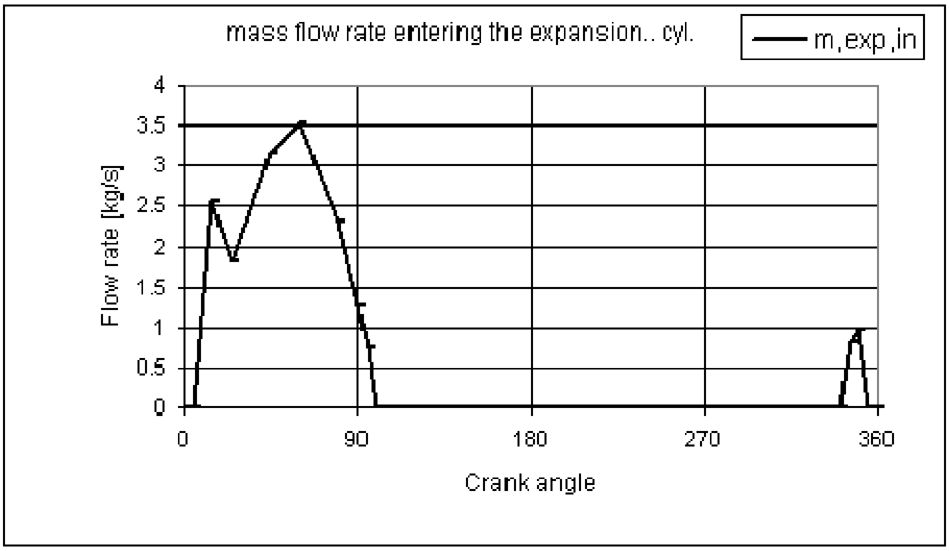

Figure 6. Variation of mass flow rate when entering the expansion cylinder

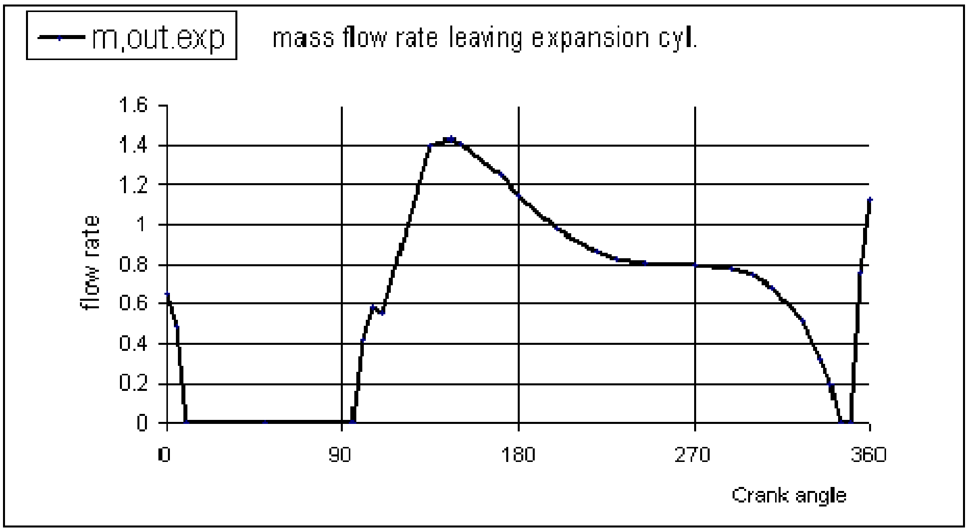

Figure 7. Mass flow rate variations when leaving the expansion cylinder

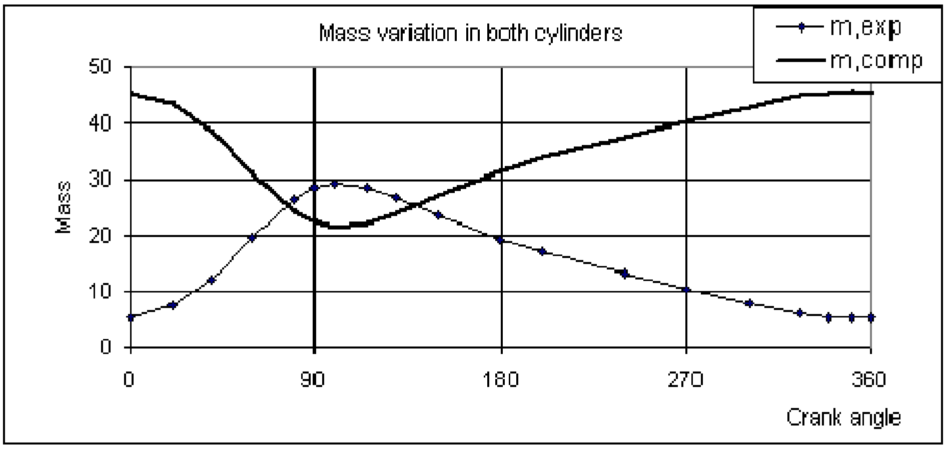

Figure 8. Total mass variation in both cylinders during the adiabatic expansion and compression

5.3. Polytropic Exponent Variation

Practically, in real cycles it is hard to achieve an isothermal behavior. The compression and expansion processes of working gas inside the engine cylinders are associated with the heat and mass transfer, and they are performed with polytropic nature. The work done on or by the system during these processes is expressed by the general expression: (PV^n=constant). The polytropic exponent (n) is assumed to be between the isothermal and adiabatic exponents: 1(isothermal) ≤ n (polytropic) ≤ k (adiabatic). The polytropic exponents were considered and calculated in the hot and cold cylinders under the adiabatic conditions. The polytropic exponent variations versus the crank angle variation are calculated during the hot expansion of the working fluid as shown in Fig.9, and the variation during the cold compression is shown in Fig.10.

$$n_{\exp.(adiab)} = f(\theta_{\exp.})$$

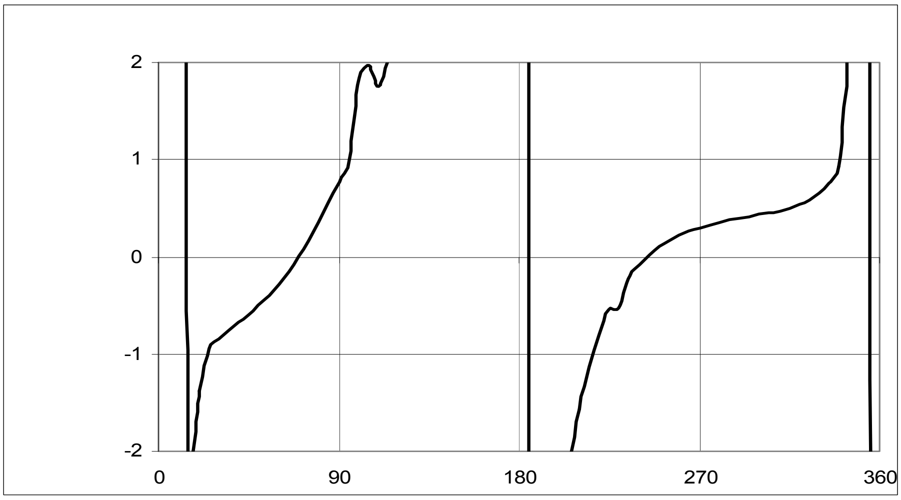

Figure 9.variation of polytropic exponent for the adiabatic expansion of gas inside the hot cylinder.

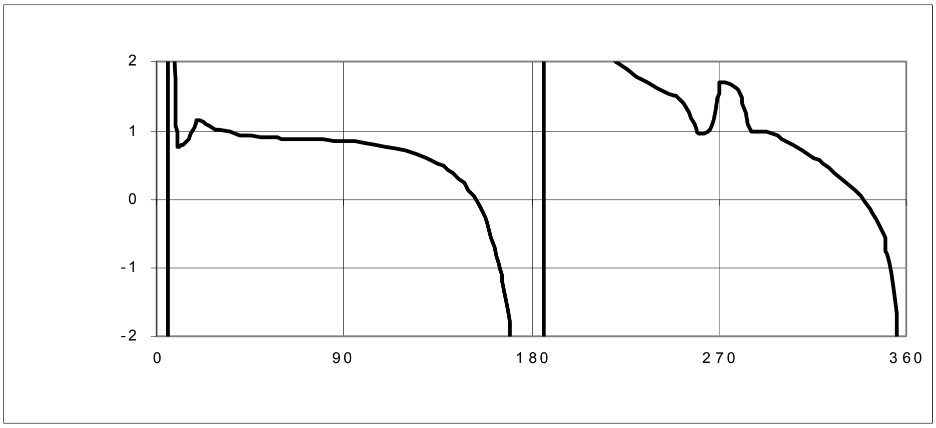

Figure 10. Polytropic exponent variation for the adiabatic compression of the gas inside the cold cylinder

The pronounced variations of this exponent are clearly observed over the total cyclic duration. Practically, the greater variations of the polytropic exponent are observed to occur near the dead centers of both cylinders. The variations of the exponents are primarily provoked by the effect of mass exchange with a slight significance of the heat transfer between the cylinder gas and walls.

5.4. Indicated work per cycle

Several authors demonstrated that the maximum power output of well-developed Stirling engine, is proportionally governed by to the pressure, volume, speed, and temperature. In real engines, the Cooler and heater volumes contribute to large portions of dead volumes; therefore the hot and cold volumes should be as small as possible [18]. The dead volumes accounts up to 50% of the total engine internal volume, they negatively produce an exponential drop in the power output [15,16,18]. In this work, the effect of dead volumes was observed to have a significant reduction on the power output. The optimum dead volume should be accounted to accommodate the necessary heat transfer. The effect gas leakage in real Stirling engines is inevitable; this is due to the higher pressure that forces the sealing elements to leak the gas out of the system. Under the operating pressure a significant power loss was observed to be as a function of gas leakage. Additionally, it was observed that the flow friction consumes significant amount of power from the net engine power. The heat energy is developed and converted into mechanical energy in each operating cycle. The produced work is transferred to the drive mechanism during each operating cycle. Variations of the compression work, expansion work, and network versus the crank angles are presented in Fig.11. The remarkable aspect that can be seen is the similar variation between the compression and expansion works. Another observation that could be detected is the point of origin from which the compression and expansion works are started.

5.5. Evolutional behavior of transferred heat.

The transferred heat from the higher temperature source (1000K) to the gas, and the rejected heat from the gas to the lower temperature sink (300K) are presented against the crank angle variation in Fig.12. Thermal communication between the gas and both reservoirs can be expressed by the instantaneous fluxes of heat within the different cells. The mass flow rate, the specific heat of the working agent and temperature difference between both reservoirs are observed with significant influence on the instantaneous fluxes of heat. The reduced performance of the real Stirling engine was demonstrated as due some deficiencies such as: the non-ideal regeneration, non-isothermal heat transfer, and the non-adiabatic expansion and compression processes. The variations of transferred heat As shown in the figure, the greater amount of heat is added to the flowing mass that enters into the hot cylinder is achieved during the initial period of the gas expansion, during the first quarter of the cycle (90º crank angles).

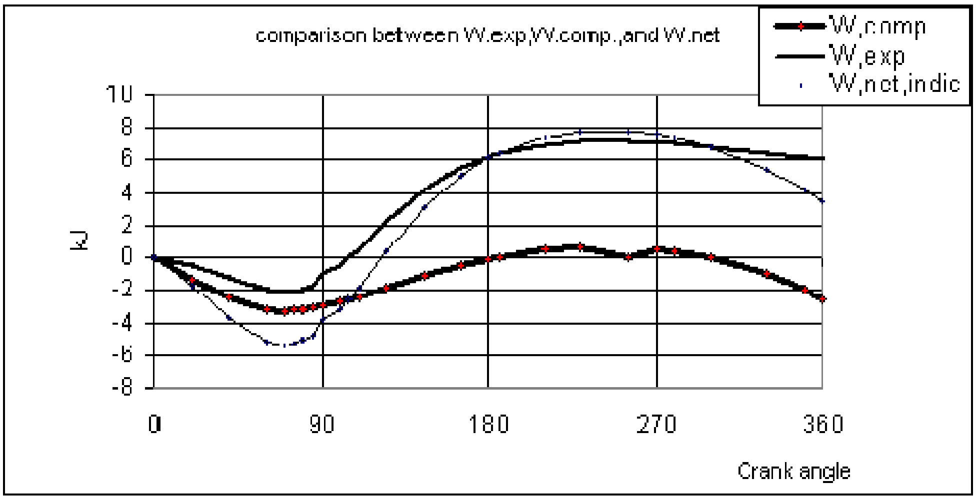

Figure 11. Variations of the work energy versus the crank angles (expansion, compression, and network per cycle.

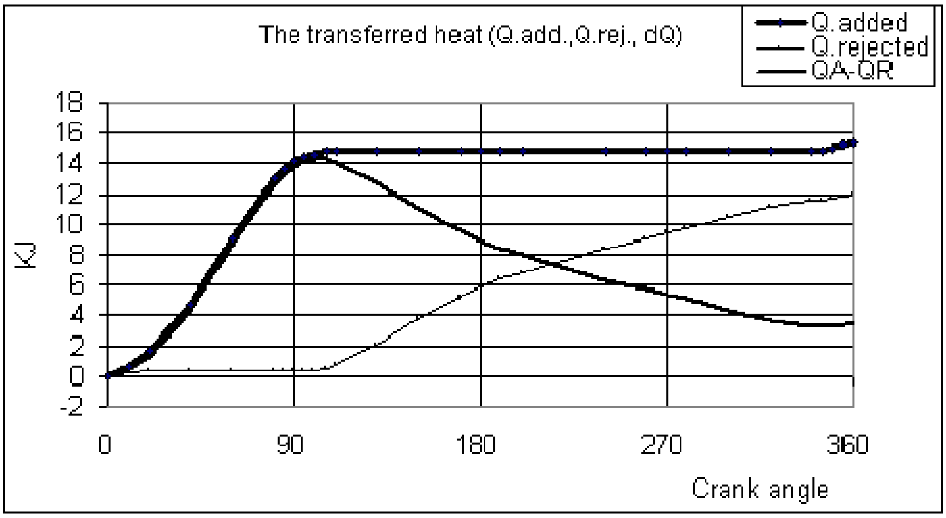

Figure 12. Variations of the transferred heat to and from the gas (Q_{added}, $Q_{\cdot rejected}$, and the heat difference

This is due to the position of heater that is closed to the expansion cylinder. The heat rejection from the working gas is observed with an opposite behavior, the different behavior can be interpreted as due to the effect of gas leakage. Practically, while the cold piston continues moving toward the end point of the compression stroke, the gas which is already heated leaves the compression cylinder back to the hot cylinder without to reject the associated heat.

The effects of mass and heat losses on the engine performance were considered. The obtained results demonstrated that lowering the mass of gas, and minimizing the rejected heat to the surroundings through the cylinder walls could achieve the greater amount of heat. Also, it should be mentioned that the regeneration conditions are much far away than those of the ideal regenerator.

6. Conclusion

In this work, a thermodynamic analysis of Stirling engine was performed. The analysis provides the necessary data for evaluating several operating aspects, which contributes to the improvement of the Stirling engine. The employed model is capable to stabilize all the thermo-physical parameters of the flowing gas between the engine cylinders. The model provides necessary information regarding the irreversible factors, which influence the conversion of thermal energy into a useful mechanical work per cycle. With respect to the gas leakage and phase difference, the variations of simulated parameters were calculated and represented on proper performance curves. The proposed numerical simulation permits including the gas leakage through the clearance between the piston and cylinder. The system of governing equations (mass and energy) equations was iteratively solved in each control volume on a step-wise basis of crank angles. The obtained results show that the lowered performance of the real Stirling engine is related to some deficiencies such as: the non-ideal regeneration, the non-isothermal heat transfer, the gas leakage, and the significant friction associated with the flowing agent through the heat exchangers. The energy losses due to the significant amount of friction associated with the flowing agent through the heat exchangers, and the shuttle effect are additional factors.

It was demonstrated that lowering the charging mass and minimizing the gas leakage could reach the higher thermal efficiency. The working fluid should be with low viscosity to reduce the pumping losses. The higher pressure, or the lower viscosity, or the combination between them could reduce the need for high mass flow rate.

The effect of dead volume in real Stirling engines is unavoidable; it reduces the engine network and thermal efficiency. In real engines, the leakage tendency of the fluid is inevitable and it negatively affects the engine power output, the gas leakage is due to the higher pressure that forces the sealing elements to leak the gas out of the system.

Finally, the obtained stable evolutional behavior is good indicators that encourage the future research and development of finding proper measures to overcome the engine deficiencies. In the next future, a comparison between the adiabatic and non-adiabatic performance will be made using the same model. This comparison will be focused on evaluating the effect of higher temperature source on the work output and engine efficiency under the adiabatic and non-adiabatic conditions, in order to minimize the deviation between the real and ideal cycles. Based on the expected results, the model will be validated against experimental data.

References

[1] J.B.Heywood, Internal Combustion Engines Fundamentals, McGraw-Hill Book Company, 1988.

[2] A. D. Gosman, Flow Processes in Cylinders, in J.H. Horlock, and D.E. Winterbone, The Thermodynamics and Gas Dynamics of Reciprocating Engines, (1986), Vol. II, Oxford University Press

[3] Youssef Timoumi et al. Design and performance optimization of GPU-3 Stirling engines. Energy (2008), 33: 1100-1114.

[4] Youssef Timoumi et al. Performance optimization of Stirling engines. Renewable Energy (2008), 33: 2134-2144.

[5] D.G.Thombare, S.K.Verma, Technological development in the Stirling cycle engines. Renewable and Sustainable Energy (2008), 12: 1-38.

[6] Bancha Kongtragool, Somchai Wongwises, A review of solar-powered Stirling engine and low temperature differential Stirling engine. Renewable and Sustainable Energy (2003), 7: 31-154.

[7] Schmidt G. Classical analysis of Stirling engine operation. A report published in German Engineering Union (Original German), vol. XV; 1871, p.1-12.

[8] Bancha Kongtragool, Somchai Wongwises, Investigation on power output of the gamma-configuration low temperature differential Stirling engines, (2005), 30: 465-476.

[9] Urieli U, Berchowitz DM. Stirling cycle machine analysis. Bristol Hilger Ltd: 1984.

[10] Finkelstein T, Thermodynamic analysis of Stirling engines, Spacecraft Rockets (1967), 4(9): 1184-1189.

[11] Stig Kilgard et al., Preliminary results from simulation of temperature oscillations in Stirling engine regenerator matrices, Energy, (2006), 31: 1371- 1383.

[12] Ihsan Batmaz, Suleiman Ustun, Design and manufacturing of a V-type Stirling engine with double heaters, Applied Energy (2008), 85: 1041-1049.

[13] Bancha Kongtragool, Somchai Wongwises, Thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space, and regenerator, Renewable Energy (2006), 31: 345-359.

[14] H. Karabulut et al., Nodal analysis of a Stirling engine with concentric piston and displacer, Renewable Energy (2006), 31: (2188-2197.

[15] Nezaket Parlak et al., Thermodynamic analysis of a gamma type Stirling engine in non-ideal adiabatic conditions, Reneable Energy (2009), 34: 266-273.

[16] H. Karbulut et al., Thermodynamic analysis of a βtype Stirling engine with a displacer driving mechanism by means of a lever, Renewable Energy (2009), 34: 202-208.

[17] Petrescu S, Costea M. Application of the direct method to irreversible Stirling cycle with finite speed. Int. J Energy Conver, Conserv. Environ. (2002), 26: 586-609

[18] Kongtragool B, Wongwises S. Thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space, and regenerator. Renewable Energy (2006), 31: 345-359.

[19] Finkelstein T. Insights into the thermodynamic analysis of Stirling cycle machines, AIAA-94-3951-CP (1994): 1829-1834.

[20] Costea M, Fedit M. The effect of the overall heat transfers coefficient variation on the optimal distribution of the heat transfer surface conductance of area in Stirling engine. J Energy Conversion Manag. (1998), 39 (16-18): 1753-1761.

[21] Isshiki S et al., Studies of flow resistance and heat transfer of regenerator wire meshes of Stirling engines in oscillatory flow. JSME Int. J Fluid Therm. Eng, Series B (1997), 40 (2): 281-289.

[22] Berrin Erbay L, Yavuz H. Analysis of the Stirling heat engine at maximum power conditions. Energy (1997), 22 (7): 645s-650

[23] Walker G. Stirling engines, Oxford Clarendon press; 1980