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The Effect of the Couple Stress Fluid Flow on MHD Peristaltic Motion with Uniform Porous Medium in the Presence of Slip Effect

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Abstract

The present paper investigates the effects of the couple stress fluid flow on the magnetohydrodynamic peristaltic motion with a uniform porous medium in the presence of slip effect. The analysis is carried out under the assumption of long wavelength approximations. Expressions of the axial velocity, transverse velocity, pressure gradient, volume flow rate, average volume flow rate, pressure rise and shear stress are all obtained. The effects of various emerging axial velocity, transverse velocity, pressure gradient shear stress are discussed through graphs. It was observed that the velocity distribution (u) decreased by increasing the couple stress parameter (S) with $\beta \geq 0.2$; we also noticed that the transverse velocity increased by increasing the couple stress parameter (S) $\beta \geq 0.2$. We noticed that the pressure gradient decreased by increasing the slip parameter (β).

Keywords: *Peristaltic Fluid Flow, Porous Medium, Magnetic Field, Slip Condition, Couple Stress*.

1. Introduction

Peristaltic motion is now well-known to the physiologists as one of the major mechanisms for fluid transport in many biological systems and it is also an important research topic due to its great amount of applications in engineering. A fluid transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube containing fluid is called peristaltic transport. It is an automatic and vital process that moves food through the digestive tract, urine from the kidneys through the ureters into the bladder, and bile from the gallbladder into the duodenum and transport of blood through the artery with mild stenosis. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems. This mechanism also has many applications in roller and finger pumps, some bio-mechanical instruments (e.g., heart-lung machine, blood pump machine and dialysis machine). Thus, peristaltic transport has been the core interest of many recent studies of researchers/scientists owing to the above-mentioned applications in bio-mechanical engineering and bio-medical technology. Latham [1] first initiated the concept of peristaltic mechanism. Later, this mechanism became an important topic of research owing to the above-mentioned applications in biomechanical engineering and biomedical technology.

Researchers have considered different geometries and fluids to understand the mechanism of peristalsis under different assumptions. Some recent investigations on this topic have been reported in Refs. [2 - 13]. Magnetohydro-dynamic (MHD) is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field; the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid [14]. As early as 1937, this was due to the fact that such studies were useful, particularly for getting a proper understanding of the functioning of different machines used by clinicians for pumping blood (Misra *et al.* [15]). Misra *et al.* [16] surveyed the theoretical studies with the aim of exploring the effect of the magnetic field on the flow of blood in atherosclerotic vessels; they also found an application in a blood pump used by cardiac surgeons during the surgical procedure. The MHD principles may be used to reaccelerate the flow of blood in a human artery system and thereby they found it useful in the treatment of certain cardiovascular disorders [17] and in the diseases with an accelerated blood circulation like hemorrhages and hypertension, etc.

MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation

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was studied by ziya uddin [18]. Static and dynamic analysis of hydrodynamic four-lobe journal bearing with couple stress lubricants was studied by boualem chetti [19].

The couple stress fluid is a special case of the non-newtonian fluids where these fluids consist of rigid randomly oriented particles suspended in a viscous medium and their sizes are taken into account. This model can be used to describe the human and animal blood, infected urine from a diseased kidney and liquid crystals. There have been only few attempts for studying the peristaltic flow of couple stress fluids, first discussed by Stokes [20]. From the recent attempts dealing with the couple stress model, we referred to Mekheimer [21], as he investigated the problem of the peristaltic transport of couple stress fluids in a uniform and non-uniform channel. Also, Nadeem and Akram [22] investigated the peristaltic flow of couple stress fluids under the effect of induced magnetic field in an asymmetric channel. Similarly, Sobh [23] studied the effect of slip velocity on peristaltic flow of couple stress fluids in uniform and non-uniform channels. The peristaltic fluid flow, through channels with flexible walls, was studied by Ravi Kumar *et al*. [24-31].

Flow through porous media has been of a considerable interest in the recent years due to its potential application in all fields of Engineering, Geo-fluid dynamics and Biomechanics. The study of flow through porous media is immensely vital for understanding the transport process in lungs, kidneys, gallbladder with stones, the movement of small blood vessels and tissues, cartilage and bones, etc. Most of the tissues in the body (e.g., bone, cartilage, muscle) are deformable porous media. The proper functioning of such materials depends crucially on the flow of blood, nutrients and so forth through them. Porous-medium models are used to understand various medical conditions (such as tumor growth) and treatments (such as injections). One class contains the rarefied gases (Kwang and Fang [32]), while the other fluids have much more an elastic character. In such fluids, some slippage occurs under a large tangential traction. It was claimed that the slippage can occur in non-Newtonian fluids, concentrated polymer solution, and molten polymer. Furthermore, in the flow of dilute suspensions of particles, a clear layer is sometimes observed next to the wall. Poiseuille, in a work that won a prize in experimental physiology, observed such a layer with a microscope in the flow of blood through capillary vessels [33]. The effects of the induced magnetic field and the slip condition on peristaltic transport with heat and mass transfer in a non-uniform channel were studied by Najma Saleem *et al*. [34]. Peristaltic Transport of Visco-Elasto-Plastic Fluids in a Planar Channel was investigated by Zaheer Asghar *et al.* [35]. Slip Effects on Peristaltic Transport of a Particle-Fluid Suspension in a Planar Channel was studied by Mohammed H. Kamel *et al.* [36].

To the best of our knowledge, no attempt has yet been reported to discuss the effects of couple stress fluid flow on MHD peristaltic motion with uniform porous medium in the presence of slip effect. The aim of the present study is to investigate the effects of couple stress fluid flow on magnetohydrodynamic peristaltic motion with uniform porous while taking into consideration of slip effect during this work. This investigation may have an application in many clinical applications such as endoscopes problem. Since the present study was carried out for a situation when the human body is subjected to an external magnetic field, it bears the promise of a significant application in magnetic or electromagnetic therapy, which has gained a considerable popularity. The present study is also useful for evaluating the role of porosity and slip condition when the body is subjected to magnetic resonance imaging (MRI).

2. Mathematical Formulation and Solution

We considered the unsteady hydromagnetic flow of a viscous, incompressible and electrically conducting couple-stress fluid through a two-dimensional channel of non- uniform thickness with a sinusoidal wave travelling down its wall. The plates of the channel were assumed to be electrically insulated. We chose a rectangular coordinate system for the channel with x along centerline in the direction of wave propagation and y transverse to it.

The geometry of the wall surface is defined as:

$$h\,(x,t) = a(x) + b\sin\frac{2\pi}{\lambda}(x - ct) \qquad (1)$$

with $a(x) = a_0 + kx$

where $a(x)$ is the half-width of the channel at any axial distance x from inlet, a_0 is the half-width at inlet, $k(<<1)$ is a constant whose magnitude depends on the length of the channel and exit and inlet dimensions, b is the wave amplitude, λ is the wave length, c is the propagation velocity and t is the time.

The constitutive equations and equations of motion for a couple stress fluids are (Stocks, 1966):

$$T_{j\,i,\,j} + \rho\, f_i = \rho\frac{\partial v_i}{\partial t}$$

$$e_{i\,j\,k} + T^{A}_{j\,k} + M_{j\,i,\,j} + \rho\, C_i = 0$$

$$\tau_{i\,j} = -p'\delta_{i\,j} + 2\,\mu\, d_{i\,j}$$

$$\mu_{i\,j} = 4\,\eta\,\omega_{j,\,i} + 4\eta'\,\omega_{j,\,i}$$

Where f_i is the body force vector per unit mass, C_i is the body moment per unit mass, v_i is the velocity vector, τ_{ij} and $T^{A}_{j\,k}$ are the symmetric and anti-symmetric parts of the stress tensor $T_{j\,k}$ respectively, $M_{i\,j}$ is the couple stress tensor, $\mu_{i\,j}$ is the deviatory part of $M_{i\,j}$, ω_i is the vorticity vector , $d_{i\,j}$ is the symmetric part of the velocity gradient , η and η' are constants associated with the couple stress , p' is the pressure, and the other terms have their usual meaning from tensor analysis.

We introduced a wave frame of reference (x, y) moving with velocity c in which the motion became independent of time when the channel length was an integral multiple of the wavelength and the pressure difference at the ends of the channel was a constant [38]. The transformation from the fixed frame of reference (X,Y) to the wave frame of reference (x, y) is given by:

$x = X - c\,t$, $y = Y$, $u = U - c$, $v = V$ and $p(x) = P(X\,,\,t)$, where (*u, v*) and (*U,V*) are the velocity components, *p* and *P* are pressures in the wave and fixed frames of reference, respectively.

Neglecting the body force and the body couples, the continuity equations and equations of motion are [37]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \left[\sigma B_0^2\right]u - \left[\frac{\mu}{k_1}\right]u - \eta\nabla^4 u \quad (3)$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \left[\sigma B_0^2\right]v - \left[\frac{\mu}{k_1}\right]v \quad (4)$$

u and v are the velocity components in the corresponding coordinates p is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of the viscosity, η is the coefficient of couple stress, k_1 is the permeability of the porous medium and *k* is the thermal conductivity.

Since it is presumed that the couple stress is caused by the presence of the suspending particles, obviously the clear fluid cannot support the couple stress at the boundary, hence we tactically assumed that the components of the couple stress tensor at the wall vanishes.

Using the following the non-dimensional variables:

$$x^* = \frac{x}{\lambda} \quad y^* = \frac{y}{a_0} \quad u^* = \frac{u}{c} \quad v^* = \frac{\lambda v}{a_0 c}$$

$$p^* = \frac{a_0^2 p}{\lambda \mu c} \quad t^* = \frac{c t}{\lambda} \quad \mathrm{Re} = \frac{\rho c a_0}{\mu}$$

$$M = \sqrt{\frac{\sigma}{\mu}} B_0 a_0 \qquad S = \frac{a_0^2 \eta}{\mu a_0^4}$$

$$\delta = \frac{a_0}{\lambda} \quad h = 1 + \frac{\lambda k x}{a_0} + \phi \sin\{2\pi(x - t)\}$$

here ϕ (amplitude ratio) = $\frac{b}{a_0} < 1$

Equations of motion and boundary conditions in the dimensionless form become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

$$\mathrm{Re}\,\delta\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - s\delta^4\frac{\partial^4 u}{\partial x^4} - s\frac{\partial^4 u}{\partial y^4} - 2s\delta^2\frac{\partial^4 u}{\partial x^2 \partial y^2} - \frac{1}{D}u - M^2 u \quad (6)$$

$$\mathrm{Re}\,\delta\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \delta^4\frac{\partial^2 v}{\partial y^2} + \delta^4\frac{\partial^2 v}{\partial y^2} + \delta^2\frac{\partial^2 v}{\partial y^2} - \delta^2\frac{1}{D}v - \delta^2 M^2 v \quad (7)$$

Using long wavelength (i.e., $\delta << 1$) and negligible inertia (i.e., $\mathrm{Re} \to 0$) approximations, we have:

$$S\frac{\partial^4 u}{\partial y^4} - \frac{\partial^2 u}{\partial y^2} + Au = -\frac{\partial p}{\partial x} \quad (8)$$

where $A = \left[M^2 + \frac{1}{D}\right]$ (9)

$$\frac{\partial p}{\partial y} = 0$$

The associated boundary conditions are:

Slip condition: $u = -\beta\frac{\partial u}{\partial y}$ at y = h (10)

where β is the slip parameter.

Regularity condition: $\frac{\partial u}{\partial y} = 0$ at y = 0 (11)

Vanishing of couple stresses, $\frac{\partial^2 u}{\partial y^2} = 0$ at y = h

$\frac{\partial^3 u}{\partial y^3} = 0$ at y = 0 (12)

Solving equation (8) using the boundary conditions (10 and 12), we get

$$u = -B N_1 Cosh[a y] + B N_2 Cosh[b y] - B \quad (13)$$

where $a = \sqrt{\dfrac{1+\sqrt{1-4S(M^2+\frac{1}{D})}}{2S}}$ $\quad b = \sqrt{\dfrac{1-\sqrt{1-4S(M^2+\frac{1}{D})}}{2S}}$ $\quad B = \dfrac{\frac{\partial p}{\partial x}}{(M^2+\frac{1}{D})}$

$$N_1 = \left[\frac{-B b^2 Cosh[bh]}{a^2 Cosh[ah]\left[Cosh[bh] + \beta b\, Sinh[bh]\right] - b^2 Cosh[bh]\left[Cosh[ah] + \beta a\, Sinh[ah]\right]} \right]$$

$$N_2 = \left[\frac{B a^2 Cosh[ah]}{a^2 Cosh[ah]\left[Cosh[bh] + \beta b\, Sinh[bh]\right] - b^2 Cosh[bh]\left[Cosh[ah] + \beta a\, Sinh[ah]\right]} \right]$$

From equation (5)

$$v = C\, Sinh[a y] - D\, Sinh[b y] \tag{14}$$

where $C = \dfrac{B N_3}{a}$ $\; D = \dfrac{B N_4}{b}$ $\; N_3 = \dfrac{a_1\left(b^3 Sinh[bh]\right) - \left(b^2 Cosh[bh]\right)\left(a_2 - a_3\right)}{a_1^2}$

$$N_4 = \frac{a_1\left(a^3 Sinh[ah]\right) - \left(a^2 Cosh[ah]\right)\left(a_2 - a_3\right)}{a_1^2}$$

$$a_1 = a^2 Cosh[ah]\left[Cosh[bh] + \beta b\, Sinh[bh]\right] - b^2 Cosh[bh]\left[Cosh[ah] + \beta a\, Sinh[ah]\right]$$

$$a_2 = a^2 Cosh[ah]\left[b\, Sinh[bh] + \beta b^2 Cosh[bh]\right] + \left[Cosh[bh] + \beta b\, Sinh[bh]\right]\left[a^3 Sinh[ah]\right]$$

$$a_3 = b^2 Cosh[bh]\left[a\, Sinh[ah] + \beta a^2 Cosh[ah]\right] + \left[Cosh[ah] + \beta a\, Sinh[ah]\right]\left[b^3 Sinh[bh]\right]$$

3. Shear Stress, Pressure Gradient and Pressure Rise

The shear stress at the upper wall y = h (x), in the dimensional form is given by:

$$T = \frac{\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\left[1 - \left(\frac{dh}{dx}\right)^2\right] + \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right]\left(\frac{dh}{dx}\right)}{\left[1 + \left(\frac{dh}{dx}\right)^2\right]}$$

and it is solution is given by:

$$\tau = \frac{\left(\frac{E+F}{2}\right)\left(1 - h'^2\right) + \left(G - H\right)h'}{\left(1 + h'^2\right)} \tag{15}$$

Where

$$E = a_4\, Sinh[a y] + a_5\, Sinh[b y]$$

$$F = a_6\left[\left(\frac{\partial}{\partial x} N_3\right) Sinh[a y]\right] - a_7\left[\left(\frac{\partial}{\partial x} N_4\right) Sinh[b y]\right]$$

$$G = a_8 Cosh[a y] - a_9\, Cosh[b y]$$

$$H = -B\left[\left(\frac{\partial}{\partial x} N_1\right) Cosh[a y]\right] + B\left[\left(\frac{\partial}{\partial x} N_2\right) Cosh[b y]\right]$$

$a_4 = -B N_1 a$ $\quad a_5 = B N_2 b$ $\quad a_6 = \dfrac{B}{a}$

$a_7 = \dfrac{B}{b}$ $\quad a_8 = B N_3$ $\quad a_9 = B N_4$

The rate of volume flow 'q' through each section is a constant (independent of both x and t). It is given by:

$$q = \int_0^h u\, dy = a_{10}\, Sinh[bh] - a_{11}\, Sinh[ah] - Bh \tag{16}$$

where $a_{10} = \dfrac{B N_2}{b}$ $\quad a_{11} = \dfrac{B N_1}{a}$

Hence the flux at any axial station in the fixed frame is found to be given by:

$$Q(x,t) = \int_0^h (u+1)dy = q + h \quad (17)$$

while the expression for the time-averaged volumetric flow rate over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is obtained as:

$$\bar{Q} = \frac{1}{T}\int_0^T Q dt \;=\; q+1 \quad (18)$$

The pressure gradient obtained from equation (18) can be expressed as:

$$\frac{dp}{dx} = \frac{B(\bar{Q}-1)}{\left[\frac{N_2}{b}Sinh[bh] - \frac{N_1}{a}Sinh[ah] - h\right]} \quad (19)$$

The pressure rise Δ_{PL} (at the wall) in the channel of length L, non-dimensional form is given by:

$$\Delta_p = \int_0^1 \frac{dp}{dx}dx$$

$$\Delta_p = \int_0^1 \left[\frac{B(\bar{Q}-1)}{\left[\frac{N_2}{b}Sinh[bh] - \frac{N_1}{a}Sinh[ah] - h\right]}\right]dx$$

4. Numerical Results and Discussion

The analytical expressions for the axial velocity, transverse velocity, shear stress, pressure gradient and pressure rise are derived in the last section. The numerical and computational results are discussed through the graphical illustration. Mathematica software is used to find out numerical results. The axial and transverse velocities are shown in the Figures 1 - 4 for various governing parameters, like couple stress parameter (S), porous parameter (D), magnetic field (M) and slip parameter (β). Figures 1 - 2 reveal the axial velocity distribution (u) decreases by increasing the couple stress parameter (S) with β ≥ 0.2 for fixed D = 10, M = 0.2, *dp/dx* = 0.5, Ø = 0.7, x = t = *π/4*, λ = 10, k = 0.0005, a_0= 0.01.

The transverse velocity distribution (v) with couple stress parameter (S) as depicted in figures (3) to (4) with β ≥ 0.2. We notice that the transverse velocity increases by increasing the couple stress parameter (S) for fixed D = 10, M = 0.2, *dp/dx* = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01. Figures 5-6 illustrate the variations of *dp/dx* with β. It is interesting to note that the pressure gradient decreases by increasing the slip parameter (β). We observed that through the region $x\epsilon\,(0.3, 0.7)$, i.e., narrowing part of the channel, the flow cannot pass easily. Therefore, it required a large pressure gradient to maintain the same flux to pass it in the narrow part of the channel. Figures 7 and 8 illustrate the influence of time t on pressure gradient dp/dx. From these Figures, it can be seen that the axial pressure gradient decreases with the increase in time t. It is interesting to note that the pressure gradient is maximum at x = 0.3. Figures 9 - 10 corresponds to the behavior of the shear stress in a cycle of oscillations at different points of wave length for various governing parameters S, D, M and β. We notice that no separation occurs in the flow field for S = 0.2, M = 0.2, D = 10, β ≥ 0.2, Ø = 0.7, λ = 10, k = 0.0005, a_0= 0.01, y =1.0043.

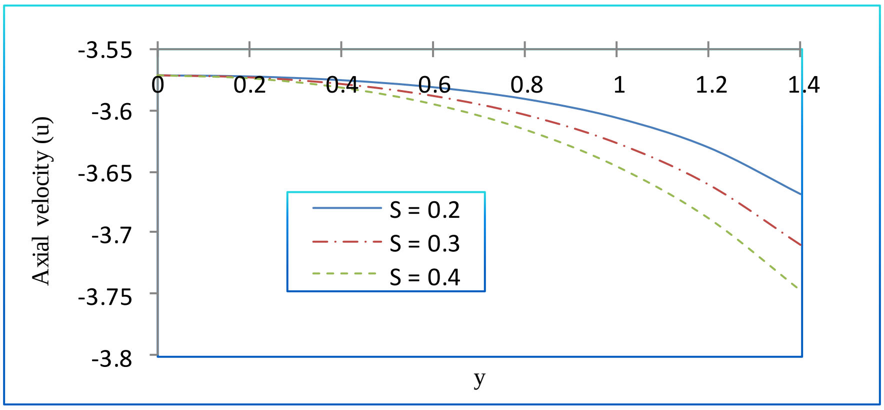

Figure 1. Distribution of axial velocity for different values of S with fixed D = 10, M = 0.2, β = 0.2, *dp/dx* = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01

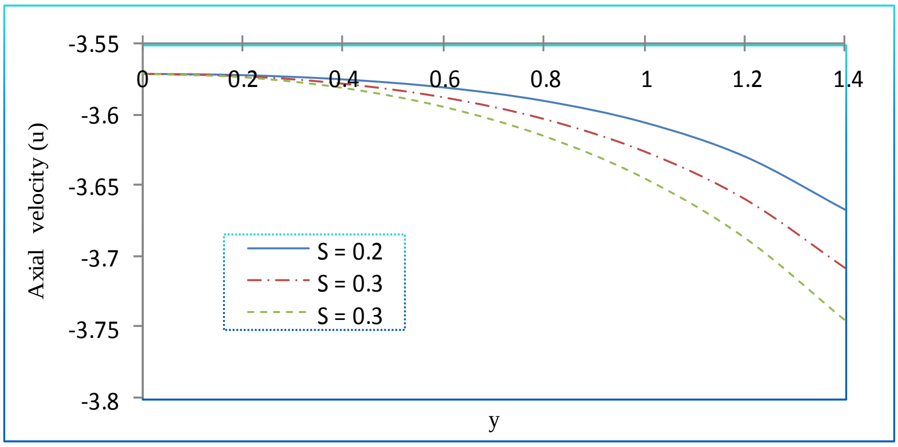

Figure 2. Distribution of axial velocity for different values of S with fixed D = 10, M = 0.2, β = 0.3, *dp/dx* = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01

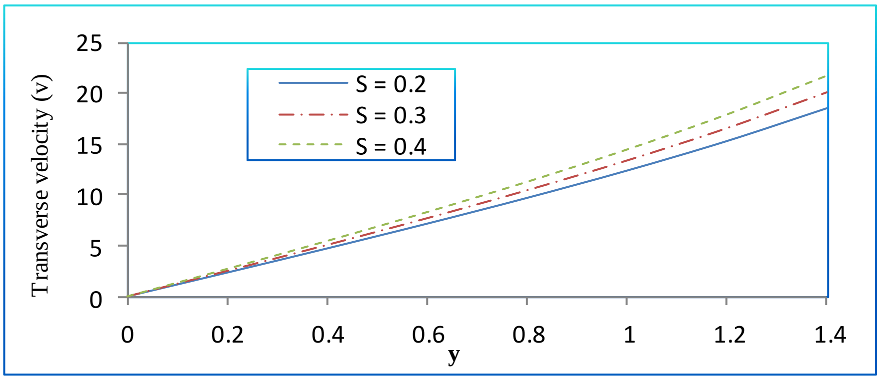

Figure 3. Distribution of transverse velocity for different values of S with fixed D = 10, M = 0.2, β = 0.2, *dp/dx* = 0.5, Ø = 0.7 x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01

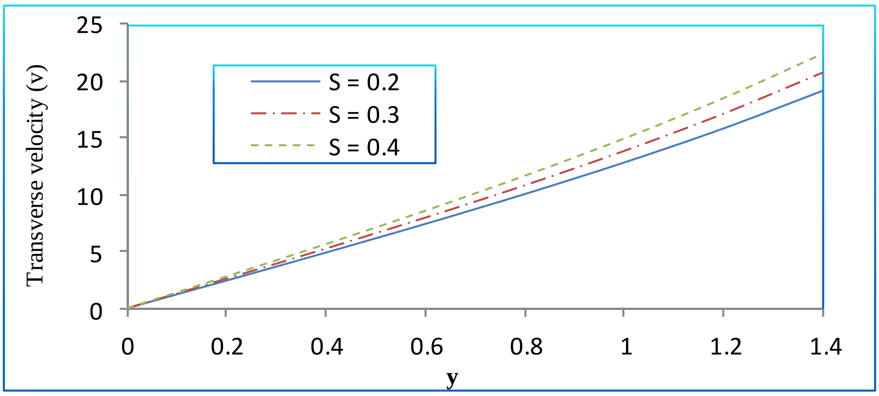

Figure 4. Distribution of transverse velocity for different values of S with fixed D = 10, M = 0.2, β = 0.3, *dp/dx* = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01

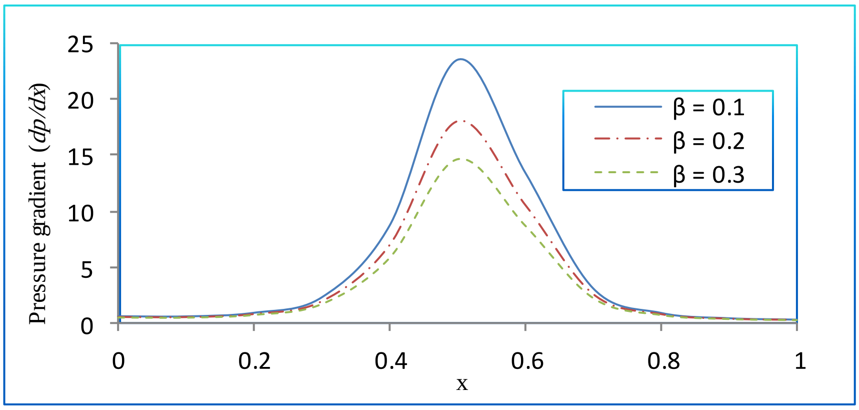

Figure 5. Distribution of Pressure gradient versus x with β for fixed D = 10, M = 0.1, S = 0.2, $\bar{Q}$ = 0.2 , Ø = 0.7, t = π/4, λ = 10, k = 0.0005, a_0= 0.01

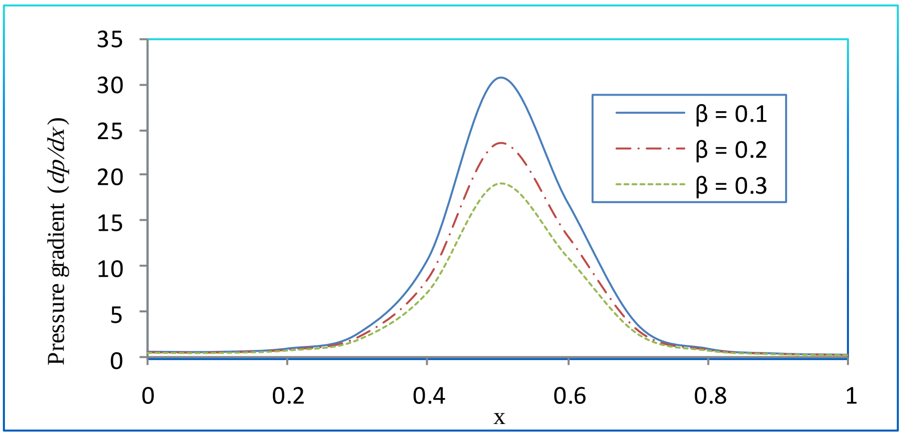

Figure 6. Distribution of Pressure gradient versus x with β for fixed D = 10, M = 0.1, S = 0.3, $\bar{Q}$ = 0.2 , Ø = 0.7, t = π/4 , λ = 10, k = 0.0005, a_0= 0.01

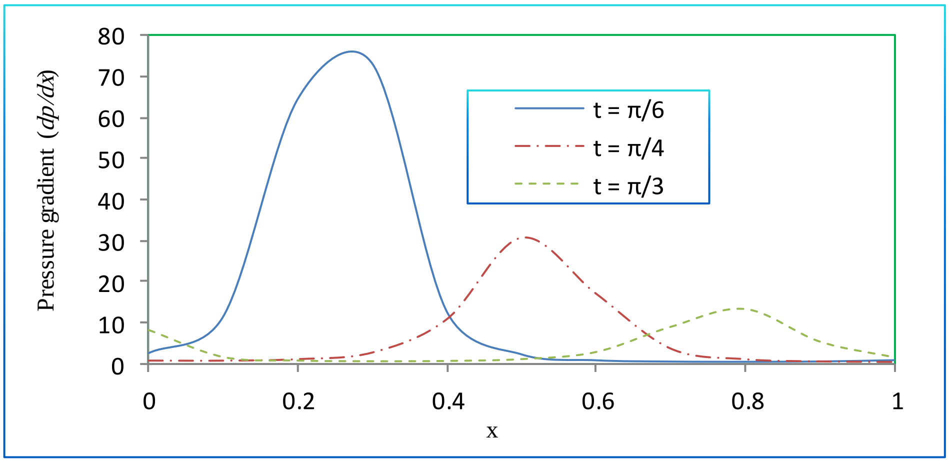

Figure 7. Distribution of Pressure gradient versus x with t for fixed D = 10, M = 0.1, β = 0.1 S = 0.3, $\bar{Q}$ = 0.2 , Ø = 0.7, λ = 10, k = 0.0005, a_0= 0.01

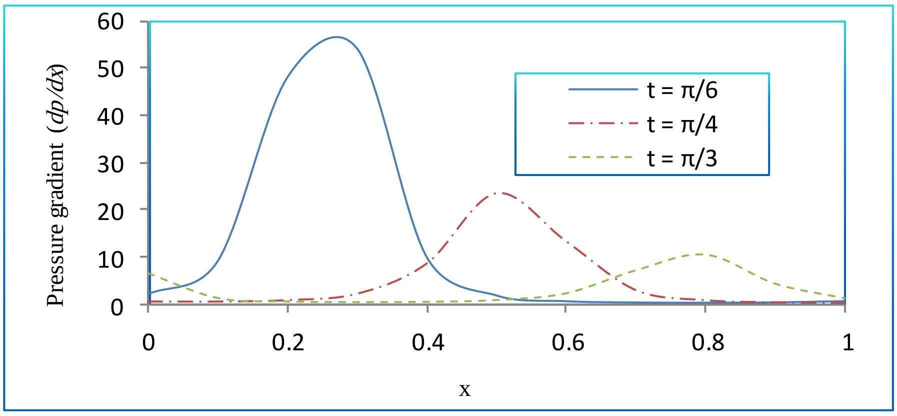

Figure 8. Distribution of Pressure gradient versus x with t for fixed D = 10, M = 0.1, β = 0.2 S = 0.3, $\bar{Q}$ = 0.2 , Ø = 0.7, λ = 10, k = 0.0005, a_0= 0.01

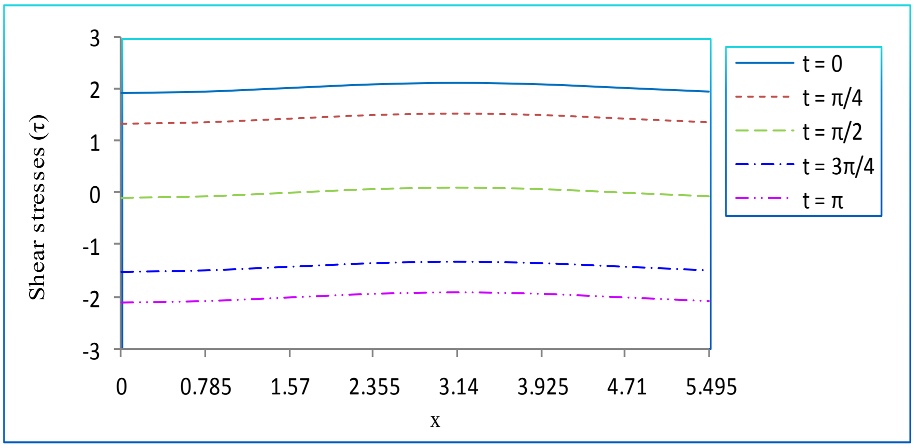

Figure 9. Distribution of Shear stresses τ for S = 0.2, M = 0.2, D = 10, β = 0.2, Ø = 0.7, λ = 10, k = 0.0005, a_0= 0.01, y =1.0043

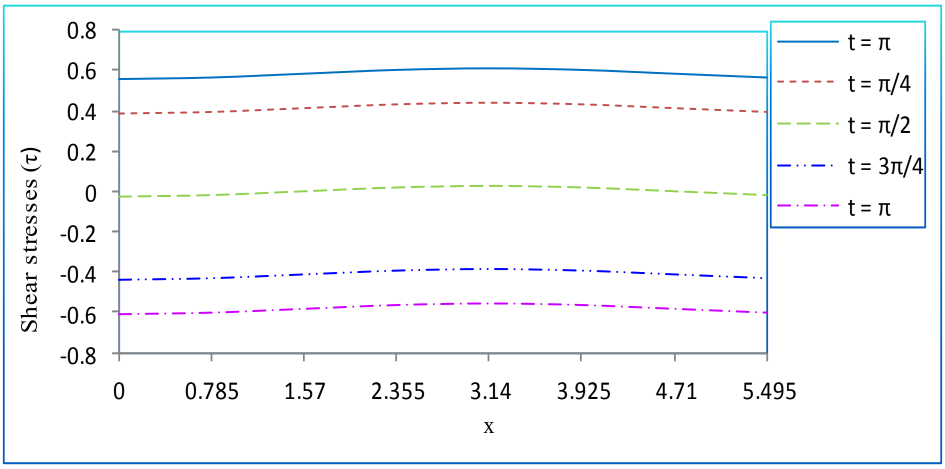

Figure 10. Distribution of Shear stresses τ for S = 0.2, M = 0.2, D = 10, β = 0.3, Ø = 0.7, λ = 10, k = 0.0005, a_0= 0.01, y =1.0043

4. Conclusions

The effect of the couple stress fluid flow on magnetohydrodynamic peristaltic flow of blood with a uniform porous medium in the presence of slip effect is investigated under the assumption of long wavelength approximation. Moreover, the effect of the various values of parameters on axial velocity, transverse velocity, pressure gradient and shear stress were computed numerically and explained graphically. We conclude the following observations:

- The axial velocity (u) decreases with the increase in the couple stress parameter (S) with β ≥ 0.2 for fixed D = 10, M = 0.2, *dp/dx* = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01.
- The transverse velocity (v) increases with the increase in the couple stress parameter (S) for fixed D = 10, M =

0.2, dp/dx = 0.5, Ø = 0.7, x = t = π/4, λ = 10, k = 0.0005, a_0= 0.01.

- The axial pressure gradient (dp/dx) decreases with the increase in the slip parameter β.
- We notice that the pressure gradient (dp/dx) cannot pass easily through the region $x\epsilon\ (0.3,0.7)$, i.e., the narrowing part of the channel (Figs. 5 and 6).
- The behavior of the shear stress in a cycle of oscillations at different points of wave length for various governing parameters S, D, M and β. We notice that no separation occurs in the flow field.

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