JJMIE

Volume 9 Number 4, August.2015
ISSN 1995-6665
Pages 253 - 261

**Jordan Journal of Mechanical and Industrial Engineering**

# Numerical Simulation and Nonlinear Stability Analysis of Francis Hydraulic Turbine-Seal System

Qianqian Wu[a], Leike Zhang[b *] and Zhenyue Ma[a]

[a]*Faculty of Infrastructure Engineering, School of Hydraulic Engineering, Dalian University of Technology, Dalian, China.*

[b]*College of Water Resources Science and Engineering, Taiyuan University of Technology, Taiyuan, China.*

*Received 28 Dec 2014* *Accepted 22 Sep 2015*

**Abstract**

Hydraulic and electromagnetic forces are main causes of vibrations in a hydraulic generating set. The Unbalanced Magnetic Pull (UMP) and nonlinear sealing force can produce large oscillations which will be dangerous to the shaft system. In the present paper, the kinetic model of a simplified unit shaft system, which mainly considers 6 degrees of freedom of rotor and turbine, under UMP, and Muszynska nonlinear sealing force is established. On this basis, the nonlinear stability of Francis turbine-seal system is emphasized, and the effect of the seal parameters on system stability is also discussed, according to the Lyapunov first approximation theory. The results show that there is very little likelihood of rub-impact phenomenon occurrence between the turbine and seal structure with nonlinear sealing force, if no other external interferences are taken into account. In addition, some suggestions about reducing self-excited vibration by parameters sensitivity analysis are given, which can be beneficial to the safety operation and vibration fault diagnosis for the units.



**Keywords:** *Hydroelectric Generating Set; Sealing Force; Turbine-Seal System; Critical Instable Speed*.

## 1. Introduction

The hydraulic generating set is a typical hydro-mechanical-electrical coupling system; meanwhile, as the key equipment of hydropower plant, it is also the important guarantee of economic and social benefits produced from hydro-junction project eventually. With the rapid development of manufacturing and processing technology, the hydraulic generating set is moving in the direction to large-scale, high-speed and high-power. At the same time, the unit vibration phenomenon has been increasingly prominent that the corresponding stability problem, which is one of the hottest research area in the field of hydropower industry, is concerned extensively by engineering and academic circles.

The hydraulic generating set is mainly affected by hydraulic, mechanical and electromagnetic vibration sources. The mechanical vibration sources, such as mass unbalance, misalignment and bearing's looseness, are widely present in rotary machinery, and related studies have been relatively mature [1]. In contrast, it is difficult to grasp the effect of the hydraulic and electromagnetic vibration sources on the dynamic characteristics of unit, due to their complexity and uncertainty. In particular, the interactive relationship between the hydraulic vibration source and the turbine; in spite of numerous works done, its influence in the shaft system, cannot be accurately mastered.

The unequal clearance of turbine seal belongs to one of the hydraulic vibration sources. The sealing structure is used to control fluid leakage in turbine with the features including complex device, small clearance as well as the large pressure between front and back device. When the periodic eccentric movement of unit occurs due to some reasons, the pressure pulsations will appear generating the sealing force which has an influence on the stability of the unit. The 8-paramerters model [2] was employed for analyzing the rotor-seal structure in the past; however, it is difficult to reflect the nonlinear characteristics of sealing force. The nonlinear model proposed by Muszynska and Bently [3-5] has the milestone meaning. Although the form is simple, it can well embody the effect of rotor disturbance on the movement stiffness, damping and inertia. Meanwhile, the

* Corresponding author. e-mail: lkzhang@hotmail.com.

nonlinear characteristics of sealing force can be reflected with the confirmation by experiment. This model has currently become the most widely used sealing model.

UMP is the manifestation of electromagnetic vibration source. When the generator runs normally, the rotor rotates in a uniform magnetic field, and the magnetic pull applied on rotor radial points is also uniform. However, the UMP will act on the rotor if the gap between the stator and rotor is not equal, the rotor has eccentricity or the initial deflection of shaft appears [6]. Currently, there are two ways for the complete expression derivation of UMP, the first one is using air-gap potential and permeability to obtain the flux density $B(a,t)$ , then getting the unit area radial magnetic force of rotor surface or stator inner surface according to the formula $\sigma = \frac{B^2}{2\mu_0}$ ; here, $\sigma$ is stress, $\mu_0$ is air permeance, finally the expression can be achieved through integration [7-10]. The second one is taking direct derivative of $x$ and $y$ displacement according to air-gap magnetic field energy [11-13].

Although there are many studies on sealing force [14-19] and UMP [20-24] on the dynamics of unit; however, most discussions were contributed to the effect of a single vibration source on the system responses, and the stability analyses were insufficient.

In the present paper, the nonlinear dynamical model of shaft system for hydroelectric generating set under both UMP and sealing force is established. On this basis, the dynamic characteristics and stability with different sealing parameters for turbine-seal system are numerically analyzed and compared with other studies. Some different conclusions are also obtained according to the numerical calculation results.

## 2. Dynamic Model of System

### 2.1. Shaft System

The hydroelectric generating set system model consists of upper, lower, water guide bearings, rotor as well as turbine components, as seen in Figure 1. Due to the complex vibration characteristics of system under UMP and sealing force, only the lateral vibration of the unit is considered, while the effect of thrust bearing and shaft on the system is ignored.

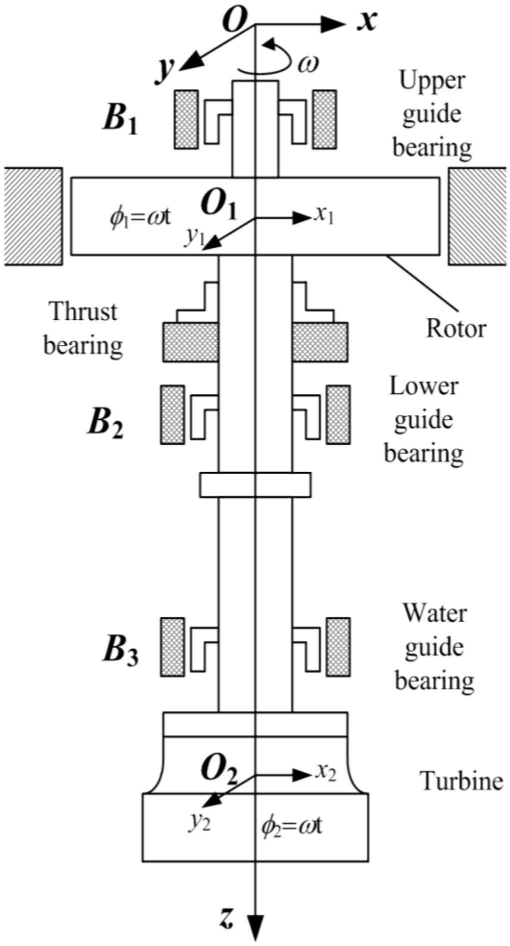


**Figure 1** Shaft system of hydroelectric generating unit

The total kinetic energy of the system includes that from generator and that from turbine, namely

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + e_1^2 \dot{\phi}_1^2 + 2e_1 \dot{\phi}_1 \dot{y}_1 \cos\phi_1 - 2e_1 \dot{\phi}_1 \dot{x}_1 \sin\phi_1) + \frac{1}{2}(J_1 + m_1 e_1^2)\dot{\phi}_1^2 + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + e_2^2 \dot{\phi}_2^2 + 2e_2 \dot{\phi}_2 \dot{y}_2 \cos\phi_2 - 2e_2 \dot{\phi}_2 \dot{x}_2 \sin\phi_2) + \frac{1}{2}(J_2 + m_2 e_2^2)\dot{\phi}_2^2 \quad (1)$$

where $m_1$, $e_1$, $\phi_1=\omega t$, $J_1$ are the mass, eccentricity, rotating angle and rotary inertia of rotor, respectively; $m_2$, $e_2$, $\phi_2=\omega t$ and $J_2$ are the mass, eccentricity, rotating angel and rotary inertia of turbine, respectively; $\omega$ is rotating speed, ($x_1$, $y_1$), ($x_2$, $y_2$) are the centre coordinates of rotor and turbine, respectively.

In Figure 1, $B_1$, $O_1$, $B_2$, $B_3$ and $O_2$ are the geometric centres of upper guide bearing, rotor, lower guide bearing, turbine and water guide bearing, respectively. Denote $|B_1O_1|$=a, $|O_1B_2|$=b, $|B_2B_3|$=c, and $|B_3O_2|$=d, $r_1$ and $r_2$ are the radial displacement of rotor and turbine, respectively; then $r_1 = \sqrt{x_1^2 + y_1^2}$ , $r_2 = \sqrt{x_2^2 + y_2^2}$ . According to the geometrical relationship, expressions of $r_3$, $r_4$ and $r_5$ can be obtained as:

$$\begin{aligned} r_3 &= \frac{[(a+b)(b+c+d) - a(c+d)]\cdot r_1 - ab\cdot r_2}{b(b+c+d)} = \frac{A_1 \cdot r_1 - A_2 \cdot r_2}{B} \\ r_4 &= \frac{(c+d)\cdot r_1 + b\cdot r_2}{b+c+d} \\ r_5 &= \frac{d\cdot r_1 + (b+c)\cdot r_2}{b+c+d} \end{aligned} \quad (2)$$

where $A_1 = (a+b)(b+c+d)$, $A_2 = ab$, $B = b(b+c+d)$.

Without a consideration of the change of gravitational potential energy, assuming that each guide bearing's stiffness including the upper one $k_1$, lower one $k_2$ and water one $k_3$, is isotropic, then the elastic potential energy of system can be expressed as follows:

Without a consideration of the change of gravitational potential energy, assuming that each guide bearing's stiffness including the upper one $k_1$, lower one $k_2$ and water one $k_3$, is isotropic, then the elastic potential energy of system can be expressed as follows:

$$U=\frac{1}{2}(k_1r_3^2+k_2r_4^2+k_3r_5^2)$$
$$=\left[\frac{A_1^2}{B^2}k_1+\frac{(c+d)^2}{(b+c+d)^2}k_2+\frac{d^2}{(b+c+d)^2}k_3\right]\frac{x_1^2+y_1^2}{2}+\left[\frac{A_2^2}{B^2}k_1+\frac{b^2}{(b+c+d)^2}k_2+\frac{(b+c)^2}{(b+c+d)^2}k_3\right]\frac{x_2^2+y_2^2}{2}+ \tag{3}$$
$$\left[-\frac{A_1A_2}{B^2}k_1+\frac{b(c+d)}{(b+c+d)^2}k_2+\frac{d(b+c)}{(b+c+d)^2}k_3\right]\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$$

It is proposed that the various damping of system is linear, existing in the direction of $x_1$, $y_1$, $x_2$ and $y_2$, considering the UMP acted on rotor and sealing force acted on turbine, the generalized force can be written as follows:

$$\begin{cases} Q_{x_1}=-c_1\dot{x}_1+F_{x\_ump} \\ Q_{y_1}=-c_1\dot{y}_1+F_{y\_ump} \\ Q_{x_2}=-c_2\dot{x}_2+F_{x\_seal} \\ Q_{y_2}=-c_2\dot{y}_2+F_{y\_seal} \end{cases} \tag{4}$$

where $c_1$ and $c_2$ are the damping coefficient of rotor and turbine, respectively; $F_{\text{ump}}$ and $F_{\text{seal}}$ are the UMP and nonlinear sealing force, respectively, whose x and y direction components are as follows:

$$\begin{bmatrix} F_{x\_ump} \\ F_{y\_ump} \end{bmatrix}=\frac{R_rL_r\pi k_j^2I_j^2}{4\mu_0}(2\Lambda_0\Lambda_1+\Lambda_1\Lambda_2+\Lambda_2\Lambda_3)\begin{bmatrix}\cos\gamma \\ \sin\gamma\end{bmatrix} \tag{5}$$

$$\begin{bmatrix} F_{x\_seal} \\ F_{y\_seal} \end{bmatrix}=-\begin{bmatrix} K-m_f\tau_f^2\omega^2 & \tau_f\omega D \\ -\tau_f\omega D & K-m_f\tau_f^2\omega^2 \end{bmatrix}\begin{Bmatrix} x \\ y\end{Bmatrix}- \begin{bmatrix} D & 2\tau_f m_f\omega \\ -2\tau_f m_f\omega & D\end{bmatrix}\begin{Bmatrix}\dot{x} \\ \dot{y}\end{Bmatrix}-\begin{bmatrix} m_f & \\ & m_f\end{bmatrix}\begin{Bmatrix}\ddot{x} \\ \ddot{y}\end{Bmatrix} \tag{6}$$

In equation (5), Ij is the excitation current, kj is the coefficient of fundamental magnetomotive force, Rr is the radius of rotor, Lr is the length of rotor, Λn (n=0, 1, 2, ...) are the Fourier coefficients, □ is the rotary angle of rotor. The formula derivation can be found in reference [9].

In equation (6), $K$, $D$, $m_f$, respectively, stands for equivalent stiffness, damping and mass of sealing force. $K$, $D$, $\tau_f$ are nonlinear function of perturbation motion displacement $X$ and $Y$, where:

$$K=K_0\left(1-e^2\right)^{-n},\quad D=D_0\left(1-e^2\right)^{-n},\quad \tau_f=\tau_0\left(1-e\right)^b,\ n=\frac{1}{2}\sim 3,\ 0<b<1 \tag{7}$$

More details about the above expressions can be seen in reference [4].

According to the lagrange function, the differential equations of the shaft system can be obtained:

$$\begin{cases} m_1\ddot{x}_1+c_1\dot{x}_1+\left[\frac{A_1^2}{B^2}k_1+\frac{(c+d)^2}{(b+c+d)^2}k_2+\frac{d^2}{(b+c+d)^2}k_3\right]x_1+\left[-\frac{A_1A_2}{B^2}k_1+\frac{b(c+d)}{(b+c+d)^2}k_2+\frac{d(b+c)}{(b+c+d)^2}k_3\right]\frac{\sqrt{x_2^2+y_2^2}}{\sqrt{x_1^2+y_1^2}}x_1= \\ m_1e_1\omega^2\cos\phi+F_{x\_ump} \\ m_1\ddot{y}_1+c_1\dot{y}_1+\left[\frac{A_1^2}{B^2}k_1+\frac{(c+d)^2}{(b+c+d)^2}k_2+\frac{d^2}{(b+c+d)^2}k_3\right]y_1+\left[-\frac{A_1A_2}{B^2}k_1+\frac{b(c+d)}{(b+c+d)^2}k_2+\frac{d(b+c)}{(b+c+d)^2}k_3\right]\frac{\sqrt{x_2^2+y_2^2}}{\sqrt{x_1^2+y_1^2}}y_1= \\ m_1e_1\omega^2\sin\phi+F_{y\_ump} \\ m_2\ddot{x}_2+c_2\dot{x}_2+\left[\frac{A_2^2}{B^2}k_1+\frac{b^2}{(b+c+d)^2}k_2+\frac{(b+c)^2}{(b+c+d)^2}k_3\right]x_2+\left[-\frac{A_1A_2}{B^2}k_1+\frac{b(c+d)}{(b+c+d)^2}k_2+\frac{d(b+c)}{(b+c+d)^2}k_3\right]\frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}x_2= \\ m_2e_2\omega^2\cos\phi+F_{x\_seal} \\ m_2\ddot{y}_2+c_2\dot{y}_2+\left[\frac{A_2^2}{B^2}k_1+\frac{b^2}{(b+c+d)^2}k_2+\frac{(b+c)^2}{(b+c+d)^2}k_3\right]y_2+\left[-\frac{A_1A_2}{B^2}k_1+\frac{b(c+d)}{(b+c+d)^2}k_2+\frac{d(b+c)}{(b+c+d)^2}k_3\right]\frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}y_2= \\ m_2e_2\omega^2\sin\phi+F_{y\_seal} \end{cases} \tag{8}$$

For the convenience of calculation, we assume that $|B_1O_1|=|O_1B_2|=|B_3O_2|=|B_2B_3|/2$, and equation (8) can be rewritten as:

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + \dfrac{1}{16}\left[(25k_1+9k_2+k_3)+(-5k_1+3k_2+3k_3)\dfrac{\sqrt{x_2^2+y_2^2}}{\sqrt{x_1^2+y_1^2}}\right]x_1 = m_1e_1\omega^2\cos\phi + F_{x\_ump} \\ m_1\ddot{y}_1 + c_1\dot{y}_1 + \dfrac{1}{16}\left[(25k_1+9k_2+k_3)+(-5k_1+3k_2+3k_3)\dfrac{\sqrt{x_2^2+y_2^2}}{\sqrt{x_1^2+y_1^2}}\right]y_1 = m_1e_1\omega^2\sin\phi + F_{y\_ump} \\ m_2\ddot{x}_2 + c_2\dot{x}_2 + \dfrac{1}{16}\left[(k_1+k_2+9k_3)+(-5k_1+3k_2+3k_3)\dfrac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}\right]x_2 = m_2e_2\omega^2\cos\phi + F_{x\_seal} \\ m_2\ddot{y}_2 + c_2\dot{y}_2 + \dfrac{1}{16}\left[(k_1+k_2+9k_3)+(-5k_1+3k_2+3k_3)\dfrac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}\right]y_2 = m_2e_2\omega^2\sin\phi + F_{y\_seal} \end{cases} \quad (9)$$

Define $T=\omega t$, $X_1=\dfrac{x_1}{\delta_0}$, $Y_1=\dfrac{y_1}{\delta_0}$, $X_2=\dfrac{x_2}{c_0}$, $Y_2=\dfrac{y_2}{c_0}$

where $\delta_0$ is the mean air-gap length when the rotor is centered, then

$$\frac{d}{dt}=\omega\frac{d}{dT},\ \frac{d^2}{dt^2}=\omega^2\frac{d^2}{dT^2},\ X'=\frac{dX}{dT},\ Y'=\frac{dY}{dT},$$

$$X''=\frac{d^2X}{dT^2},\ Y''=\frac{d^2Y}{dT^2}$$

After the dimensionless transformations, the state equation of (9) can be obtained:

$$\begin{Bmatrix} X_1'' \\ Y_1'' \\ X_1' \\ Y_1' \\ X_2'' \\ Y_2'' \\ X_2' \\ Y_2' \end{Bmatrix} = \begin{bmatrix} -\dfrac{c_1}{m_1\omega} & 0 & -\dfrac{K_3}{m_1\omega^2} & 0 & & & & \\ 0 & -\dfrac{c_1}{m_1\omega} & 0 & -\dfrac{K_3}{m_1\omega^2} & & & & \\ 1 & 0 & & & & & & \\ 0 & 1 & & & & & & \\ & & & & -D_1 & -D_2 & -K_1 & -K_2 \\ & & & & D_2 & -D_1 & K_2 & -K_1 \\ & & & & 1 & 0 & & \\ & & & & 0 & 1 & & \end{bmatrix} \begin{Bmatrix} X_1' \\ Y_1' \\ X_1 \\ Y_1 \\ X_2' \\ Y_2' \\ X_2 \\ Y_2 \end{Bmatrix} + \begin{Bmatrix} \dfrac{e_1\cos T}{\delta_0}+\dfrac{F_{x\_ump}}{m_1\omega^2\delta_0} \\ \dfrac{e_1\sin T}{\delta_0}+\dfrac{F_{y\_ump}}{m_1\omega^2\delta_0} \\ 0 \\ 0 \\ \dfrac{m_2e_2\cos T}{c_0M} \\ \dfrac{m_2e_2\sin T}{c_0M} \\ 0 \\ 0 \end{Bmatrix} \quad (10)$$

where

$$K_1=\frac{K_4+K-\tau_f{}^2\omega^2m_f}{M\omega^2},\ K_2=\frac{\tau_f D}{M\omega},\quad D_1=\frac{c_2+D}{M\omega},$$

$$D_2=\frac{2\tau_f m_f}{M},\ M=m_2+m_f,$$

$$K_3=\frac{1}{16}\left[(25k_1+9k_2+k_3)+\frac{c_0}{\delta_0}(-5k_1+3k_2+3k_3)\frac{\sqrt{X_2^2+Y_2^2}}{\sqrt{X_1^2+Y_1^2}}\right]$$

$$K_4=\frac{1}{16}\left[(k_1+k_2+9k_3)+\frac{\delta_0}{c_0}(-5k_1+3k_2+3k_3)\frac{\sqrt{X_1^2+Y_1^2}}{\sqrt{X_2^2+Y_2^2}}\right]$$

and $c_0$ is seal clearance.

Order $e_1$=$e_2$=0, when the system (10) is balanced, the acceleration and rotating speed equal zero, namely $X_1''=Y_1''=X_2''=Y_2''=X_1'=Y_1'=X_2'=Y_2'=0$.

Substituting

$X_1''=Y_1''=X_2''=Y_2''=X_1'=Y_1'=X_2'=Y_2'=0$

into equation (10) yields:

$$\begin{cases} K_3-\dfrac{F_{\_coeffi}}{\delta_0}\left(\dfrac{1}{2}+\dfrac{5}{8}(X_1{}^2+Y_1{}^2)\right)=0 \\ K_3-\dfrac{F_{\_coeffi}}{\delta_0}\left(\dfrac{1}{2}+\dfrac{5}{8}(X_1{}^2+Y_1{}^2)\right)=0 \\ K_1X_2+K_2Y_2=0 \\ K_2X_2-K_1Y_2=0 \end{cases} \quad (11)$$

where $F_{\_coeffi}=\dfrac{R_rL_r\pi\mu_0k_j^2I_j^2}{\delta_0{}^2}$ is the constant item of UMP.

Making use of numerical algorithms, such as iterative method, can solve the complicated equations above. After getting the numerical solutions (the balance points ($X_{10}$, $Y_{10}$, $X_{20}$, $Y_{20}$)), substitute them into Jacobian matrix, then determine the stability of system according to the eigenvalues of Jacobian matrix.

However, it is observed that the first two equations in (11) are the same, which means that it is impossible to obtain the numerical results in the absence of other known conditions. Therefore, the determination whether the system is instable cannot be conducted.

Song and Ma [24] discussed the stability of rotor system for hydroelectric generating set considering the UMP. It was pointed out that the main reason behind the system instability can be attributed to the cross items of electromagnetic stiffness and nonlinear oil film force. Only the main diagonal items of UMP model, used in the present paper, are included, while the effect of cross ones is ignored. Hence, from the perspective of rotor model, the instability phenomenon will not happen. In addition, compared to the rotor structure, the turbine-seal system is affected by hydraulic loads, the performance is more complex and the instability condition is more obvious after the occurrence of the self-excited vibration. In light of this, the turbine-seal system is selected to analyze the instability characteristics under sealing force, instead of the whole shaft system in this paper.

*2.2. Turbine-Seal Model*

For the convenience of analysis and comparison, it is supposed that the turbine is single disc system with two ends simply-supported, and the quality is focused on the middle. The nonlinear sealing force, mainly producing in the gap between crown and head-cover, as well as the shaft seal, is equivalently acted on the disk, as seen in Figure 2, and then the system equation is as follows:

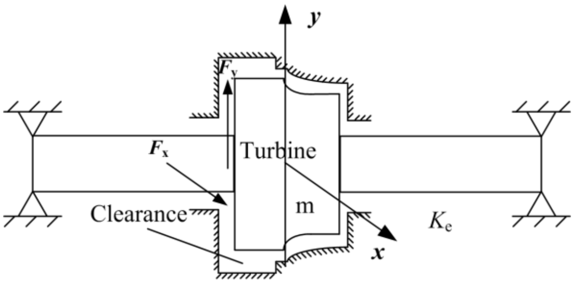


**Figure 2** Model of turbine-seal system

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{X} \\ \ddot{Y} \end{Bmatrix}+\begin{bmatrix} D_e & 0 \\ 0 & D_e \end{bmatrix}\begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix}+\begin{bmatrix} K_e & 0 \\ 0 & K_e \end{bmatrix}\begin{Bmatrix} X \\ Y \end{Bmatrix}=\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix}+\begin{Bmatrix} 0 \\ -mg \end{Bmatrix} \quad (12)$$

where $m$ is the mass of turbine, $K_e$ is axis stiffness, $D_e$ is turbine damping. $F_X$ and $F_Y$ are the nonlinear sealing force. Substitute equation (6) into equation (12) and define:

$$x=\frac{X}{c_0},\ y=\frac{Y}{c_0},\ \tau=\omega t,\ \dot{x}=\frac{dx}{dT},$$

$$x'=\frac{dx}{d\tau}=\frac{1}{\omega}\frac{dx}{dt},\ M=m+m_f$$

Then

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} x'' \\ y'' \end{Bmatrix}+\begin{bmatrix} D_1 & D_2 \\ -D_2 & D_1 \end{bmatrix}\begin{Bmatrix} x' \\ y' \end{Bmatrix}+\begin{bmatrix} K_1 & K_2 \\ -K_2 & K_1 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}=\begin{Bmatrix} 0 \\ G \end{Bmatrix} \quad (13)$$

where

$$K_1=\frac{K_e+K-\tau_f{}^2\omega^2 m_f}{M\omega^2},\ K_2=\frac{\tau_f D}{M\omega},\ D_1=\frac{D_e+D}{M\omega},$$

$$D_2=\frac{2\tau_f m_f}{M},\ G=-\frac{mg}{Mc\omega^2}$$

*2.3. Stability Analysis*

Equation (13) can be rewritten as the state equation:

$$\begin{Bmatrix} x'' \\ y'' \\ x' \\ y' \end{Bmatrix}=\begin{bmatrix} -D_1 & -D_2 & -K_1 & -K_2 \\ D_2 & -D_1 & -K_2 & -K_1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{Bmatrix} x' \\ y' \\ x \\ x \end{Bmatrix}+\begin{Bmatrix} 0 \\ G \\ 0 \\ 0 \end{Bmatrix}=\begin{Bmatrix} f_1(x',y',x,y,\omega) \\ f_2(x',y',x,y,\omega) \\ f_3(x',y',x,y,\omega) \\ f_4(x',y',x,y,\omega) \end{Bmatrix} \quad (14)$$

When the system is balanced, substituting $x''=y''=x'=y'=0$ into equation (14) yields:

$$\begin{bmatrix} K_1 & K_2 \\ -K_2 & K_1 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}=\begin{Bmatrix} 0 \\ \mathrm{G} \end{Bmatrix} \quad (15)$$

The balance value ($x_0$, $y_0$), which is related to $\omega$, can be solved through iteration method. Expand equation (14) as the first approximation at the balance point ($x_0$, $y_0$), then yield Jacobian matrix. According to the Lyapunov first approximation theory, nonlinear system stability is determined with the eigenvalues nature of Jacobian matrix. If all of the eigenvalues have negative real part, the system is asymptotically stable. If a pair of eigenvalues with zero real part exist while others whose real parts are nonzero, the system is in a critical state. And the rotating speed calculated for this condition is called critical rotating speed $\omega_c$, at this time the system behaves as a pair of conjugate eigenvalues crossing the imaginary axis, losing stability at the balance point with the occurrence of Hopf bifurcation. The amplitude of turbine centre orbit increases as the rotating speed increases, and the impact phenomenon may happen between turbine and seal.

## 3. Numerical Simulation and Discussion

*3.1. System Stability Analysis under Sealing Force*

The parameters for linear and nonlinear runner-sealing are listed in Table 1 and Table 2, respectively. The first critical rotating speed of the system is $\omega_n=\sqrt{\frac{K_e}{m}}=\sqrt{\frac{0.5\times10^{10}}{3\times10^5}}=129rad/s$. In the present paper, the trial and error method is adopted to obtain the critical instable rotating speed $\omega_c$ = 240 rad/s under nonlinear sealing force, with accuracy 1 rad/s. When $\omega=\omega_c$ = 240 rad/s, the balance position is (0.1137, -0.2467), the eigenvalues = (0.0000±0.5407i, -0.2754±0.5219i). When $\omega$ = 200 rad/s, the eigenvalues = (-0.0259±0.6395i, -0.3056±0.6157i), all of them have negative real part, which signifies the stability of system, as depicted in Figure 4(a). When $\omega$ = 300 rad/s, the eigenvalues = (0.0243±0.4378i, -0.2405±0.4202i), the

 

**Table 1**. Parameters of runner linear sealing system

| $a_0$ | $\rho$ (kg/m$^3$) | $\lambda$ | $\xi$ | $m$ (t) | $R$ (m) | $l$ (m) | $v$ (m/s) | $c$ (mm) | $K_e$ (N/m) | $D_e$ (N.s/m) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1000 | 0.1 | 1.5 | 300 | 3.0 | 0.50 | 3.0 | 2.0 | $0.5\times10^{10}$ | $0.5\times10^{5}$ |

**Table 2**. Parameters of runner nonlinear sealing system

| $b$ | $n$ | $n_0$ | $m_0$ | $\tau_0$ | $\xi$ | $m$ (t) | $R$ (m) | $l$ (m) | $v$ (m/s) | $c$ (mm) | $\Delta P$ (pa) | $K_e$ (N/m) | $D_e$ (N.s/m) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0.5 | 0.079 | -0.25 | 0.5 | 1.5 | 300 | 3.0 | 0.50 | 3.0 | 2.0 | $0.5\times10^{6}$ | $0.5\times10^{10}$ | $0.5\times10^{5}$ |

emergence of positive real part indicates that the system is instable at the balance point, as seen in Figure 4(b). With the further increase of $\omega$ the orbit amplitude arises consecutively, being close to the seal clearance size, as shown in Figure 4 (c)-(d).

The critical instable speed $\omega_{cl}$ under linear sealing force is 261 rad/s. It can be seen from Figure 5 that the system is stable and the orbit is approximated by a point when $\omega$ = 200 rad/s < $\omega_{cl}$. When $\omega$ = 270 rad/s > $\omega_{cl}$, the instability occurs as the orbit amplitude increases rapidly, which is obviously larger than the seal clearance. Hence, the rub-impact phenomenon between turbine and seal happens, causing a damage to the system.

In references [14 and 17], the mechanical models of turbine-seal system are established, with almost the same numerical parameters as adopted in the present paper. According to several turbine orbit trends with increasing rotating speed, instead of giving a further calculation verification, it was pointed out that when $\omega$ increases to a certain extent, the rub-impact phenomenon will appear between the seal device and turbine for the system under nonlinear sealing force, due to the turbine amplitude exceeding seal clearance. However, the calculation results described in Figures 4(e)-(f) demonstrates that even if the rotating speed reaches such a high level like $\omega$= 800 rad/s (in fact, it is out of question for hydraulic generating set to run at extra high speed), the amplitude is only close to the seal clearance rather than exceeding it.

Gustavsson and Aidanpää [25] took a practical unit in Sweden as an example, analyzing the nonlinear rub-impact dynamic behavior between the turbine and seal structure. It was indicated that the combined effects of turbine eccentricity and hydrodynamic pulsation was the main reason resulting in the rub-impact phenomenon, while the influence of the nonlinear sealing force was not mentioned.

The parameters in linear sealing force are constant, which is different from that in nonlinear sealing force. After the former is acted on the turbine system, the common differential equation of rotor system under external force can be formed. If the values of parameters satisfy the requirement, the similar rub-impact condition between rotary and stationary parts, as described in reference [26], will appear.

In view of this, it is considered in the present paper that the rub-impact between the turbine and seal structure for turbine-seal system under nonlinear sealing force may not happen, due to the increase of the rotating speed, in the absence of other external interferences.

### *3.2. Influence of Seal Parameters in the Dynamic Characteristics of System*

Keep other parameters unchanged, choose seal radius $R$, seal length $L$, axial velocity $\upsilon$, seal pressure dropping, seal coefficients $\tau_0$ and seal clearance $c_0$ to discuss their effect on the critical instable speed, as shown in Figure 6.

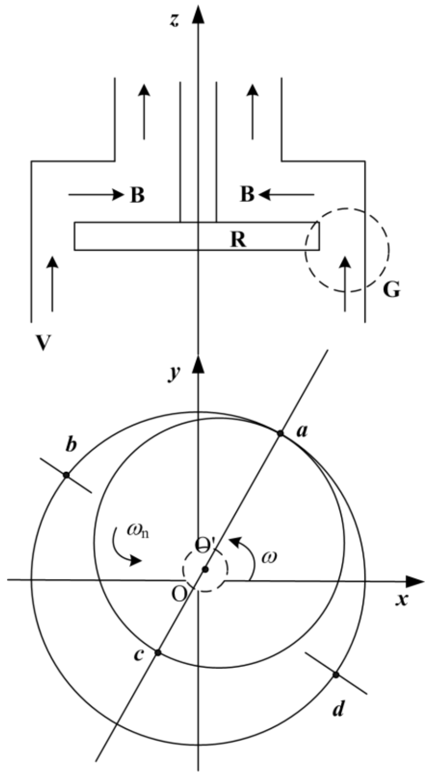


**Figure 3.** Hydraulic turbine simplified model

With the increase of seal radius, seal length and seal pressure dropping, the system critical speed decreases first and then increases, and the effect behaves obvious nonlinear characteristics, as seen in Figures 6 (a, b, d). While the increase of axial velocity and seal parameter $\tau_0$ reduces $\omega_c$ continuously, as depicted in Figures 6(c) and 6(e). However, it is found, through comparison, that the effect of axial velocity increased to a certain value on $\omega_c$ is relatively limited, while that of $\tau_0$ remains the notably linear trend. The observation of equation (6) signifies that the tangential cycling force formed by cross stiffness $\tau_f\omega D$ is the main source resulting in instability. The larger the fluid damping ratio is, the more serious the system instability becomes. The stability can be improved by reducing $\tau$. It is known from equation (7) that $\tau$ is proportional to $\tau_0$ when other parameters remain unchanged, therefore, changing $\tau_0$, by means of anti-swirling, setting circumferential retaining piece, is an effective way to affect the critical instable speed. Increasing the seal clearance will raise $\omega_c$ nonlinearly, as seen in Figure 6(f).

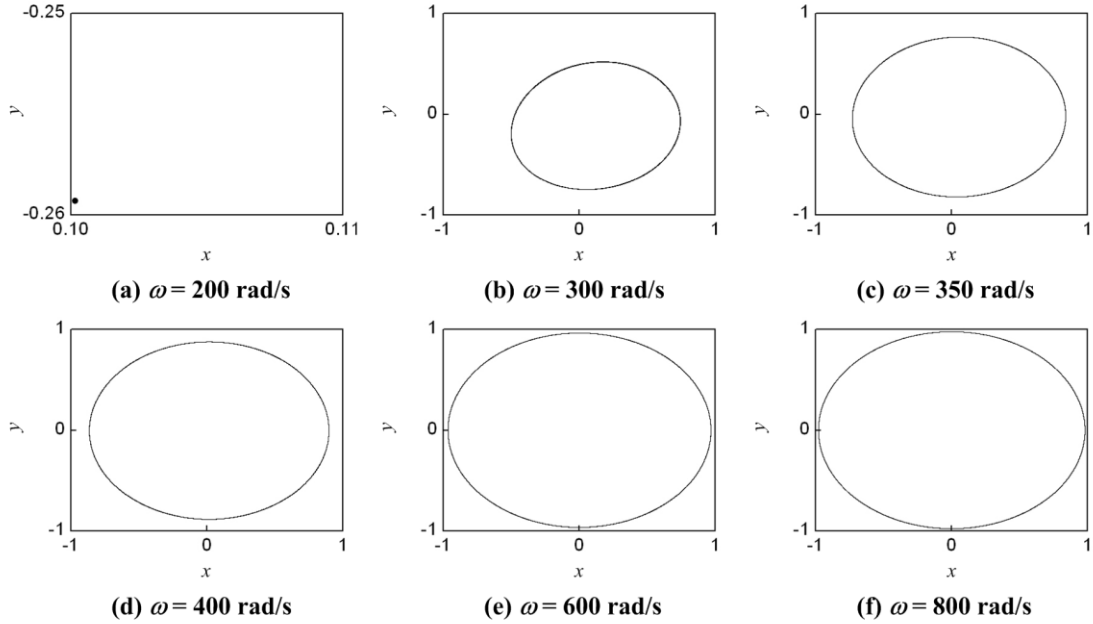

**Figure 4** Trajectories of turbine under the nonlinear seal force with different rational speed

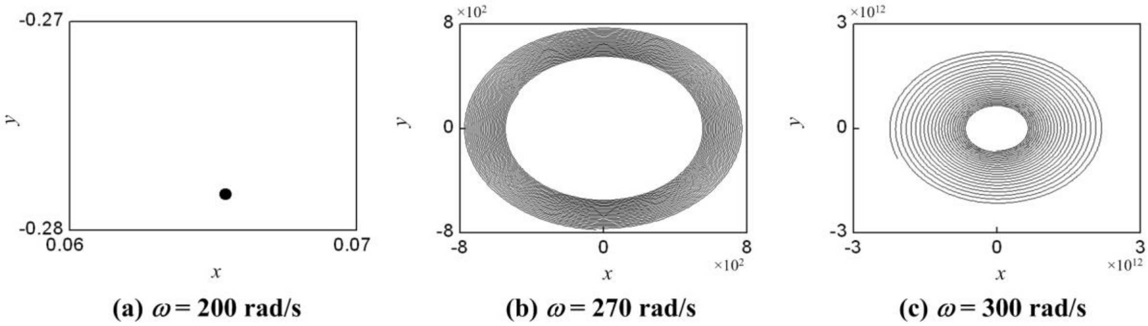

**Figure 5** Trajectories of turbine under the linear seal force with different rational speed

The above analysis shows that the increase of the axial velocity, seal pressure dropping and $\tau_0$ is disadvantageous to the stability of the system. The seal length, seal radius and seal pressure dropping have a different impact on $\omega_c$ according to their different values. The increase of the seal clearance can effectively reduce the influence of excited force in system. Nevertheless, it is noted that the increase of the seal clearance will also weaken the ability of seal to prevent fluid leakage, causing adverse effect to unit operation. Every corn has two sides; therefore, adjusting and optimizing the seal structure within a reasonable range of design parameters is the key to protect the stable operation for turbine-seal system.

The poor concentricity between turbine and chamber, and curved shaft axis are likely to induce self-excited vibration. The improvement of manufacturing, installation in order to reduce eccentricity and ensure the quality of barring shaft, is an important prerequisite to avoid the occurrence of self-excited vibration. In the event of instability, supplementary compressed air to cavity back changing axial velocity, and site polish expanding seal clearance are the better ways.

## 4. Conclusions

Based on the model establishment and analysis of the whole shaft system, the stability and critical instable speed of turbine-seal system under linear and nonlinear sealing force are emphasized, according to nonlinear vibration theory. Depending on the characteristics of the effect of different seal parameters on critical instable speed, some suggestions about reducing self-excited vibration are given to provide support with dynamic design and stable operation for turbine-seal system of unit. The main conclusions are as follows:

- Either linear or nonlinear seal system, when the rotating speed exceeds a critical instable speed, will lose stability at the balance point. With the increase of $\omega$, the turbine orbit amplitude increases drastically and consecutively. Compared to the nonlinear seal structure, the amplitude under linear sealing force is more obvious.
- The critical instable speed, calculated with Muszynska model, is lower than that with 8-parameters model, namely $\omega_c$ is advanced and closer to the working speed. It is demonstrated that using nonlinear sealing force to analyze the dynamic characteristics of turbine-seal system is more advantageous to the stable operation of unit.
- The rub-impact between the turbine and seal structure for turbine-seal system under nonlinear sealing force may not happen with the increasing rotating speed after losing instability, in the absence of other external interferences.

 

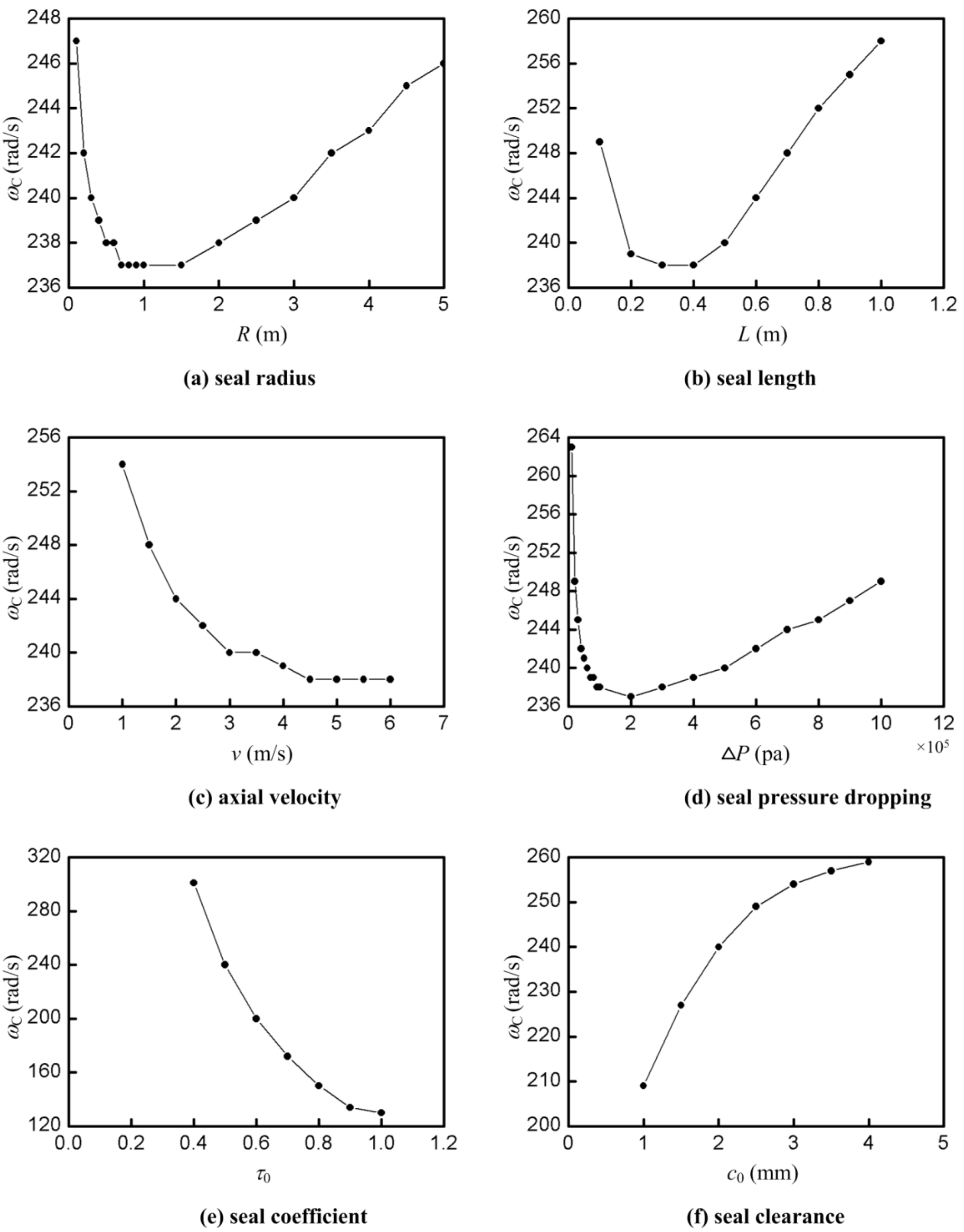


**Figure 6**. The effect of different sealing parameters on the system critical speed

- The effect of seal radius, seal length and seal pressure dropping on critical instable speed behaves obvious nonlinear characteristics as these parameters are increased, $\omega_c$ decreases first then increases, and is more sensitive to the change of lower parameters values. The increase of axial velocity and $\tau_0$ reduces $\omega_c$, aggravating the impact of the sealing force on the system vibration. For the seal clearance $c_0$, on the one hand, increasing $c_0$ can improve $\omega_c$, which is good for reducing the system vibration. On the other hand, it weakens the ability of the seal to prevent fluid leakage, which is disadvantageous to the system stability. Therefore, an appropriate adjustment of the seal parameters, based on the actual operation of the unit in order to improve the situation of the dynamic stability of the system, is necessary.

The UMP model, which is applicable to hydraulic generating set, does not include the electromagnetic cross items in the present paper; therefore, the stability analysis for the whole shaft system is not conducted. In fact, the influence of the electromagnetic stiffness in the system response is obvious and cannot be ignored, when UMP changes. The dynamic characteristics of unit shaft system, under UMP, with consideration of cross electromagnetic items and sealing force, will be more deeply explored in a future work.

**Acknowledgement**

This research paper is supported by the National Natural Science Foundation of China (No.51379030).

**References**

[1] Zhang Y. Mechanical Vibration. Beijing: Tsinghua University Press; 2007.

[2] Ma Z. Dynamic analysis of hydraulic turbine generator set and penstock. Dalian: Dalian University of Technology, 1988.

[3] D. E. Bently, A. Muszynska. "Role of circumferential flow in the stability of fluid-handling machine rotors". The 5th Workshop on Rotordynamic Instability Problems in High Performance Turbomachinery, A&M University, Taxas, United states, 1988.

[4] A. Muszynska, D. E. Bently, "Frequency-swept rotating input perturbation techniques and identification of the fluid force models in rotor/bearing/seal systems and fluid handling machines". Journal of Sound and Vibration, Vol. 143 (1990) No. 1, 103-124.

[5] A. Muszynska, "Model testing of rotor /bearing systems". The International Journal of Analytical and Experimental Modal Analysis, Vol. 1 (1996) No. 3, 15-34.

[6] Ma Z, Dong Y. Dynamics of Water Turbine Generator Set. Dalian: Dalian University of Technology Press; 2003.

[7] H. Jordan, R. D. Schroeder, H. O. Seinsch, "Zur Berechnung einseitigmagnetischer Zugkräfte in Drehfeldmaschienen". Electrical Engineering (Archiv fur Elektrotechnik), Vol. 63 (1981) No. 2, 117-124.

[8] H. Ohishi, S. Sakabe, K. Tsumagari, et al, "Radial magnetic pull in salient pole machines with eccentric rotors". IEEE Transactions on Energy Conversion, Vol. EC-2 (1987) No. 3, 439-443.

[9] D. Guo, F. Chu, D. Chen, "The unbalanced magnetic pull and its effects on vibration in a three-phase generator with eccentric rotor". Journal of Sound and Vibration, Vol. 254 (2002) No. 2, 297-312.

[10] P. Pennacchi, L. Frosini, "Dynamical behaviour of a three-phase generator due to unbalanced magnetic pull". IEE Proceedings: Electric Power Applications, Vol. 152 (2005) No. 6, 1389-1400.

[11] Y. F. Wang, L. H. Huang, Y. Li, "Nonlinear vibration and stability of a jeffcott rotor under unbalanced magnetic excitation". International Journal of Nonlinear Sciences and Numerical Simulation, Vol. 8 (2007) No. 3, 375-384.

[12] Qiu J. Electromechanical Analytical Dynamics. Beijing: Science Press; 1992.

[13] Y. Wang, L. Huang, J. Hu, "The effect of unbalanced magnetic pull on the motor rotor dynamic characteristics". Mechanics in Engineering, Vol. 18 (1996) No. 4, 32-34.

[14] Huang Z. Vibration mechanism for hydroelectric generating unit based on nonlinear rotor dynamics. Wuhan: Huazhong University of Science and Technology, 2011.

[15] W. Ma, W. Liang, H. Nan, et al, "The influence of sealing clearance value of francis runner on the unit stability". Journal of Hydroelectric Engineering, Vol. 29 (2010) No. 4, 219-223.

[16] Z. Ma, Y. Dong, "The dynamic characteristics on seal of turbine runner and it's effect on the behaviour of free vibration of unit". Large Electric Machine and Hydraulic Turbine, (1987) No. 6, 43-50.

[17] Yang X. Research on generating set vibration and coupled that and hydropower house vibration in hydropower station. Dalian: Dalian University of Technology, 2006.

[18] L. Zhang, Z. Ma, B. Song, "Characteristics analysis of hydroelectric generating set under unbalanced magnetic pull and sealing force of runner". Water Resources and Power, Vol. 28 (2010) No. 9, 117-120.

[19] D. Kong, Z. Li, Y. Jiao, et al, "Models of the rotor-seal system for hydraulic turbine and its nonlinear dynamics analysis". Journal of Hydraulic Engineering, Vol. 44 (2013) No. 4, 462-469.

[20] Y. Calleecharan, J. Aidanpää, "Dynamics of a hydropower generator subjected to unbalanced magnetic pull ". Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 225 (2011) No. 9, 2076-2088.

[21] L. Lundström, R. Gustavsson, J. O. Aidanpää, et al, "Influence on the stability of generator rotors due to radial and tangential magnetic pull force". IET Electric Power Applications, Vol. 1 (2007) No. 1, 1-8.

[22] Y. Xu, Z. Li, "Computational model for investigating the influence of unbalanced magnetic pull on the radial vibration of large hydro-turbine generators". Journal of Vibration and Acoustics, Vol. 134 (2012) No. 5, 51013-51019.

[23] Y. Y. Zeng, Y. Zeng, Z. Wu, et al, "Impact of the unbalanced magnetic pull on the shafting vibration of hydro turbine generating sets". Applied Mechanics and Materials, Vol. 644-650 (2014), 406-411.

[24] Z. Song, Z. Ma, "Nonlinear vibration analysis of an eccentric rotor with unbalance magnetic pull". Journal of Vibration and Shock, Vol. 29 (2010) No. 8, 169-173.

[25] R. K. Gustavsson, J. O. Aidanpää, "Evaluation of impact dynamics and contact forces in a hydropower rotor due to variations in damping and lateral fluid forces". International Journal of Mechanical Sciences, Vol. 51 (2009) No. 9-10, 653-661.

[26] F. Chu, Z. Zhang, "Periodic, quasi-periodic and chaotic vibrations of a rub-impact rotor system supported on oil film bearings". International Journal of Engineering Science, Vol. 35 (1997) No. 10-11, 963-973.