The Effect of Chemical Reaction and Double Stratification on MHD Free Convection in a Micropolar Fluid with Heat Generation and Ohmic Heating

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Abstract

The present paper deals with the study of flow, heat and mass transfer characteristics of the free convection on a vertical plate in porous media with variable wall temperature and concentration in a doubly stratified and viscous dissipating micropolar fluid in presence of chemical reaction, heat generation and Ohmic heating. A uniform magnetic field is applied normal to the plate. The governing non-linear partial differential equations are transformed into a system of non-linear ordinary differential equations using similarity transformations and, then, solved numerically using the Runge-Kutta-Fehlberg method with a shooting technique. The non-dimensional velocity, microrotation, temperature and concentration are presented graphically for various values of magnetic parameter, coupling number, thermal stratification parameter, solutal stratification parameter and chemical reaction parameter. The skin-friction coefficient, the wall couple stress coefficient, the Nusselt number and the Sherwood number are shown in a tabular form.

1. Introduction

The study of free convective flow, heat transfer in non-Newtonian fluid in porous media has been an active field of research as it plays an important role in diverse applications, for example, thermal insulation, extraction of crude oil and chemical catalytic reactors, the thermal designing of industrial equipment dealing with molten plastic, polymeric liquids, foodstuffs, etc. Many transport processes occurring both in nature and in industries involve fluid flows with the combined heat and mass transfer. Such flows are driven by the multiple buoyancy effects arising from the density variations caused by the variations in temperature as well as species concentrations. Convective flows in porous media were extensively examined during the last several decades due to many practical applications which can be modeled or approximated as transport phenomena in porous media. Eringen [1-3] introduced the micropolar fluids theory that is capable to describe those fluids by taking into account the effect arising from local structure and micromotions of the fluid element. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory and in the theory of porous media are presented by Lukaszewicz [4]. J. Wright and W. Shyy [5] investigated the time-dependent development of convective intrusions in a thermohaline stratification using a composite grid method with local refinement. Ching-Yang Cheng [6,7] studied the effects of thermal stratification and mass stratification on the coupled heat and mass transfer throw a vertical wavy plate with constant wall temperature and concentration in porous media saturated with non-Newtonian power law fluids. The thermal and solutal gradients and fluid flow velocities, increasing the thermal and concentration boundary layer thicknesses, and decreasing the dimensionless total heat and mass transfer rates between the fluid and the wall. Moreover, increasing the thermal stratification parameter or concentration stratification parameter leads to a smaller fluctuation of the local Nusselt number and the local Sherwood number with the stream wise coordinate. Srinivasacharya and Ram Reddy [8] analysed the natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified micropolar fluid saturated by non-Darcy porous medium and found that an increase in the both thermal and solutal stratification parameters, the velocity, skin friction parameter and non-dimensional heat and mass transfer

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coefficients were decreasing but the wall couple stress was increasing. Ibrahim and Makinde [9] studied the boundary layer analysis for free convection flow in a doubly stratified nanofluid over a vertical plate with uniform surface and mass flux conditions and they got an increase in solutal stratification parameter increases both the local Nusslet number and local Sherwood number. Murthy *et al.* [10] studied the effect of magnetic field on free convection in a thermally stratified non-Darcy porous medium saturated with nanofluid with convective boundary condition. Upendar and Srinivasacharya [11] investigated the flow and heat and mass transfer characteristics of the natural convection on a vertical plate with variable wall temperature and concentration in a doubly stratified MHD micropolar fluid and founded that an increase coupling number reduce velocity but enhance the temperature and concentration distributions. An increase in the magnetic parameter decreases the velocity, skin friction coefficient and heat and mass transfer rates. An increase in the thermal (solutal) stratification parameter reduces the velocity, temperature (concentration), skin friction, heat and mass transfer rates but enhances the concentration (temperature) and wall couple stress. Das [12] investigated the effect of the first order chemical reaction and the thermal radiation on hydro-magnetic free convection heat and mass transfer flow of a microplar fluid through a porous medium. Naraya *et al.* [13] investigated the effects of Hall current and radiation absorption on MHD free convection mass transfer flow of a micropolar fluid in a rotating frame of reference. Hayat and Qasim [14] investigated the magneto hydrodynamic two-dimensional flow with heat and mass transfer over a stretching sheet in the presence of Joule heating and thermophoresis. Mahmoud and Waheed [15] presented the effect of slip velocity on the flow and heat transfer for an electrically conducting micropolar fluid through a permeable stretching surface with variable heat flux in the presence of heat generation (absorption) and a transverse magnetic field. They found that the local Nusselt number decreased as the heat generation parameter is increased with an increase in the absolute value of the heat absorption parameter. Bataller [16] proposed the effects of viscous dissipation, work due to deformation, internal heat generation (absorption) and thermal radiation. It was shown that internal heat generation/absorption enhances or damps the heat transformation. Ravikumar *et al.* [17] studied unsteady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing, Rivlin–Ericksen flow fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal buoyancy effect. They observed that the heat absorption coefficient increase results in a decrease in the velocity and temperature. Chien-Hsin Chen [18] presented the problem of combined heat and mass transfer in buoyancy-induced MHD natural convection flow of an electrically conducting fluid along a vertical plate is investigated with Ohmic and viscous heating. Narayana and Sravanthi [19] studied the simultaneous effects of

soret and ohmic heating on MHD free convective heat and mass transfer flow for a micro polar fluid bounded by a vertical infinite surface. Kumar [20] analysed the problem of MHD mixed convective flow of a micropolar fluid with the effect of Ohmic heating, radiation and viscous dissipation over a chemically reacting porous plate with constant heat flux. Tamayol and Bahrami [21] reported that porous materials can be used to enhance the heat transfer rate from stretching surfaces to improve processes such as hot rolling and composite fabrication. Sharma and Gupta [22] studied the effect of medium permeability on thermal convection in micropolar fluids and found that the presence of coupling between thermal and micropolar effects may introduce oscillatory motions in the system.

By the same token, Raptis [23] investigated the boundary layer flow of a micropolar fluid through a porous medium. The unsteady MHD boundary layer flow of a micropolar fluid near the stagnation point of a two-dimensional plane surface through a porous medium was studied by Nadeem *et al.* [24]. Rashad [25] proposed the effect of thermal radiation with a regular three-parameter perturbation analysis in some free convection flows of Newtonian fluid in saturated porous medium. El-Aziz [26] studied the unsteady mixed convection flow of a viscous incompressible micropolar through a heated vertical surface in the presence of viscous dissipation and buoyancy force. E1-Hakiem [27] presented an analysis for the effect of thermal dispersion, viscous and Joule heating on the flow of an electrically conducting and viscous incompressible micropolar fluid past a semi-infinite plate whose temperature varies linearly with the distance from the leading edge in the presence of uniform transverse magnetic field. The skin friction factor and the rate of heat transfer decrease with the magnetic parameter and the micropolar parameter and they increase with the thermal dispersion parameter increase. Oahimire and Olajuwon [28] studied heat and mass transfer effects on an unsteady flow of a chemically reacting MHD micropolar fluid through an infinite vertical porous plate in porous medium with hall effect and thermal radiation. Hussain *et al.* [29] reported on the radiation effects on the unsteady boundary layer flow of a micropolar fluid over a stretching sheet. The sheet is considered to be permeable and the problem is non-dimensionalized by using similarity transformations. They found that temperature increases for radiation parameter, however it decreases for stagnation point.

The aim of the present work is to investigate the effects of transverse magnetic field, thermal and solutal stratification on the MHD free convection heat and mass transfer from a vertical plate embedded in micropolar fluid in porous medium with viscous dissipation, heat generation and Ohmic heating. The equations thus obtained were solved numerically using Runge–Kutta–Fehlberg fourth fifth order method with shooting technique. The effects of different parameters on velocity, microrotation, temperature and concentration are presented graphically. The skin friction coefficient, wall couple stress coefficient, Nusselt number and Sherwood number are tabulated.

2. Mathematical formulation

The graphical model of the problem is given along with the flow configuration and coordinate system, Figure 1. The system deals with a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semi-infinite vertical plate in porous media embedded in a doubly stratified, electrically conducting micropolar fluid. The heat generation, as well as viscous dissipation and Ohmic heating terms, was retained in the energy equation. Choosing the coordinate system, such that x axis, is along the vertical plate and y axis normal to the plate. The plate is maintained at temperature $T_w(x)$ and concentration $C_w(x)$. The temperature and the mass concentration of the ambient fluid are assumed to be linearly stratified in the form $T_\infty(x) = T_{\infty,0} + A_1 x$ and $C_\infty(x) = C_{\infty,0} + B_1 x$, respectively, where A_1 and B_1 are constants and varied to alter the intensity of stratification in the medium and $T_\infty 0$ and $C_\infty 0$ are the beginning ambient temperature and concentration at $x = 0$, respectively. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field; neglecting the Hall current effect, electrically non-conducting wall, local thermal equilibrium between fluid and solid.
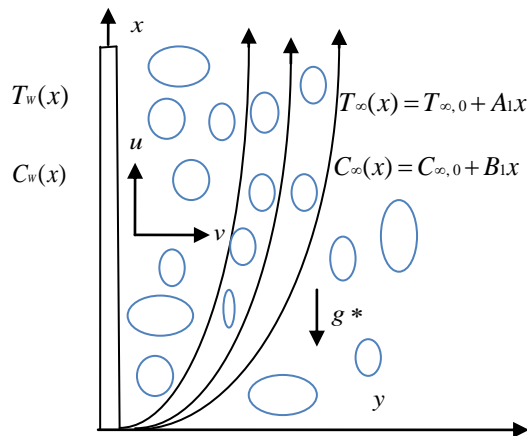

Figure 1. Physical model and coordinate

Using the Boussinesq boundary layer approximations, the governing equations for the micropolar fluid are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu + \kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial \omega}{\partial y} + g^*(B_T(T - T_\infty) + B_C(C - C_\infty)) - \frac{\sigma B_0^2}{\rho}u + \frac{\mu\varepsilon}{\rho k_1}u \tag{2}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{(\mu + \kappa)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho c_p}u^2 + \frac{\theta_0}{\rho c_p}(T - T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R_r(C - C_\infty) \tag{5}$$

where u and v are the component of velocity along x and y directions, respectively, ω is the component of microrotation whose direction of rotation lies in the xy - plane, g* is the gravitational acceleration, ρ is fluid density, T is the temperature, C is the concentration, B_T is the coefficient of thermal expansions, B_C is the coefficient of solutal expansions, B_o is the coefficient of the magnetic field, ε is porosity of porous media, k_1 is permeability of porous media, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, υ is the kinematic viscosity, α is the thermal diffusivity, k is the thermal conductivity of the fluid and D is the molecular diffusivity, C_p is the specific heat, θ_o is the internal heating, R_r is the chemical reaction rate constant.

The boundary conditions are:

$$at\ y = 0 \quad u = 0, \quad v = 0, \quad \omega = 0,$$
$$T = T_w(x),\ C = C_w(x),$$
$$as\ y \to \infty \quad u \to 0, \quad \omega \to 0, \tag{6}$$
$$T \to T_\infty(x),\ C \to C_\infty(x)$$

where the subscripts w and ∞ indicate the conditions at wall and at the outer edge of the boundary layer, respectively.

The continuity equation (1) is satisfied by introducing the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

Equations 2, 3, 4 and 5 can be transformed into a set of nonlinear ordinary differential equations by using the following similarity variables:

$$\psi = A\ x\ f(\eta),$$
$$\eta = B\ y,$$
$$\omega = E\ x\ g(\eta)$$
$$\theta(\eta) = \frac{T - T_{\infty,0}}{\Delta T} - \frac{A_1 x}{\Delta T}, \tag{8}$$
$$\Delta T = T_w(x) - T_{\infty,0} = M_1(x)$$
$$\emptyset(\eta) = \frac{C - C_{\infty,0}}{\Delta C} - \frac{B_1 x}{\Delta C},$$
$$\Delta C = C_w(x) - C_{\infty,0} = N_1(x)$$

where the constants A, B, E, M_1 and N_1 have, respectively, the dimension of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of temperature and length and the ratio of concentration and length.

The transformed ordinary differential equations are:

$$\left(\frac{1}{1-N}\right)f''' + ff'' + \left(\frac{N}{1-N}\right)g' - (f')^2 + \theta + L\emptyset - Mf' + \Lambda f' = 0 \tag{9}$$

$$\lambda g'' - \left(\frac{N}{1-N}\right)£(2g + f'') + fg' - f'g = 0 \tag{10}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta + H\theta + MEcf'^2 +$$

$$\left(\frac{1}{1-N}\right)Ecf''^2 - \varepsilon_1 f' = 0 \tag{11}$$

$$\frac{1}{Sc}\emptyset'' + f\emptyset' - f'\emptyset - R\emptyset - \varepsilon_2\emptyset = 0 \tag{12}$$

where primes denotes differentiation with respect to similarity variable η, $Pr = \upsilon/\alpha$ is Prandtl number, $Sc = \upsilon / D$ is the Schmidt number, $£ = 1/(j\,b^2)$ is the micro-inertia density, $N = \kappa/\mu + \kappa\,(0 \le N < 1)$ is the Coupling number, $\lambda = \gamma / j\,\rho\,\upsilon$ is the spin – gradient viscosity, $L = \frac{B_C}{B_T}\frac{\Delta C}{\Delta B}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $\Lambda = \frac{\varepsilon}{k_1 B^2}$ is the porous parameter, $H = \frac{\theta_0}{\mu C_p B^2}$ is the heat generation parameter, $Ec = \frac{(\upsilon x B^2)^2}{C_p \Delta T}$ is the Eckert number, $R = \frac{R_r}{\upsilon B^2}$ is the chemical reaction parameter, $\varepsilon_1 = \frac{x}{\Delta T}\frac{d}{dx}[T_\infty(x)]$ is the thermal stratification parameter and $\varepsilon_2 = \frac{x}{\Delta C}\frac{d}{dx}[C_\infty(x)]$ is the solutal stratification parameter.

The boundary conditions (6) in terms of f, g, θ and $\emptyset$ become:

at $\eta = 0$ $f(0) = 0, f'(0) = 0, g(0) = 0,$
$\qquad\theta(0) = 1 - \varepsilon_1, \emptyset(0) = 1 - \varepsilon_2$
as $\eta \to \infty$ $f'(\infty) \to 0, g(\infty) \to 0, \theta(\infty) \to 0,$
$\qquad\qquad\emptyset(\infty) \to 0$ $\tag{13}$

The physical parameters of interest are the skin friction coefficient C_f, the Couple stress coefficient M_w, the local Nusselt number Nu and the Sherwood number Sh which are defined as

$$C_f = \frac{2\tau_w}{\rho A^2}, \quad M_w = \frac{B}{\rho A^2}m_w, \, Nu_x = \frac{q_w}{B\,k\,(T_w - T_\infty)}$$

$$\text{and } Sh = \frac{q_m}{D\,B[C_w - C_\infty]} \tag{14}$$

Where τ_w, m_w, q_w and q_m are the wall shear stress, the wall couple stress, the wall heat flux and the wall mass flux, respectively, are given by:

$$\tau_w = \left[(\mu + \kappa)\left(\frac{\partial u}{\partial y}\right) + \kappa\omega\right]_{y=0}, m_w = \gamma\left(\frac{\partial \omega}{\partial y}\right)_{y=0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{and } q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{15}$$

Hence using (8), we get:

$$C_f = \left(\frac{2}{1-N}\right)f''(0)\bar{x}, \; M_w = \frac{\lambda}{£}g'(0)\bar{x},$$

$$Nu = -\theta'(0) \quad \text{and } Sh = -\emptyset'(0) \tag{16}$$

where $\bar{x} = B\,x$.

3. Method of Solution

The non-linear ordinary differential equations (9)-(12) subject to the boundary condition (13) were solved using the Runge-Kutta-Fehlbe fourth fifth order method along with shooting technique. This method is based on the discretization of the problem domain and the calculation of unknown boundary conditions. The domain of the problem is discredited and the boundary conditions for η_∞ are replaced by $f'(\eta_\infty) = 0$, $g(\eta_\infty) = 0$, $\theta(\eta_\infty) = 0$, and $\emptyset(\eta_\infty) = 0$, where η_∞ is sufficiently large value of η at which boundary conditions (13) for $f(\eta)$ are satisfied. We ran the computer code written in MATLAB for $\eta_\infty = 4$ and step size $\Delta\eta = 0.01$. To solve the problem the non-linear equations, (9) - (12) were converted into nine first orders linear ordinary differential equations. There are five conditions at boundary $\eta = 0$ and four conditions at boundary $\eta = \infty$. To find the solution of the problem, one will need four more conditions $f''(0)$, $g'(0)$, $\theta'(0)$ and $\emptyset'(0)$ at $\eta = 0$. These conditions were found by the shooting technique. Finally, the problem was solved by the Runge-Kutta-Fehlberg fourth fifth method along with calculated boundary conditions.

4. Results and discussion

In order to study the effects of the coupling number N, magnetic field parameter M, thermal stratification parameter ε_1, solutal stratification parameter ε_2 and chemical reaction parameter R on the physical quantities of the flow, the other parameters were fixed as $Ec = 0.01$, $H = 0.2$, $\Lambda = 0.3$, $L = 1$, $Pr = 1$, $Sc = 0.2$, $\lambda = 1$ and $£ = 0.1$. The values of micropolar parameter λ and $£$ were chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [1].

To validate the results obtained, the authors compared the results in the absence porous parameter Λ, heat generation parameter H, Eckert number Ec, chemical reaction parameter R with reported by Srinivasacharya and Upendar [11] and found that they were in good agreement, as shown in Table1.

The non-dimensional velocity, microrotation, temperature and concentration profiles for various values of magnetic parameter M are illustrated in Figures 2-5 with $N = 0.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.2$ and $R = 0.3$. It is depicted from Figure 2 that the velocity decreases as the magnetic parameter (M) increases. This is due to the transverse magnetic field, normal to the flow direction, has a tendency to generate the drag known as the Lorentz force which tends to resist the flow. Hence, the horizontal velocity decreases as the magnetic parameter M increases. Figure 3 depicts that the microrotation component increases near the plate and decreases far away from the plate for increasing values of M. It is clear from Figure 4 that the temperature increases with the increasing values of magnetic parameter. The transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and so increases its temperature and concentration.

Table 1. Comparison between $f''(0)$ and $-\theta'(0)$ calculated by the present method and Ref. [11] for $\Lambda = R = H = Ec = 0$, $Pr = 1$ and $Sc = 0.2$

N	M	ε_1	ε_2	Ref. [11]		Present	
				$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
0.0	1.0	1.0	2.0	0.97648	0.62178	0.97649	0.62178
0.3	1.0	1.0	2.0	0.78120	0.58833	0.78120	0.58833
0.5	2.0	1.0	2.0	0.55448	0.51043	0.55449	0.51042
0.5	1.0	2.0	2.0	0.59187	0.52654	0.59187	0.52653
0.5	1.0	1.0	4.0	0.55160	0.52112	0.55161	0.52111
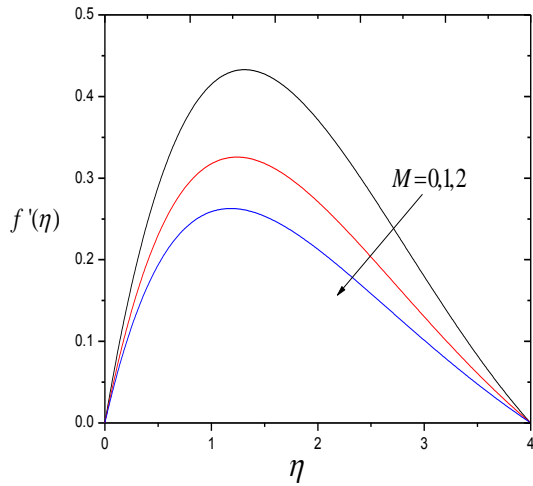
Figure 2. Velocity profile for various values of magnetic parameter M.
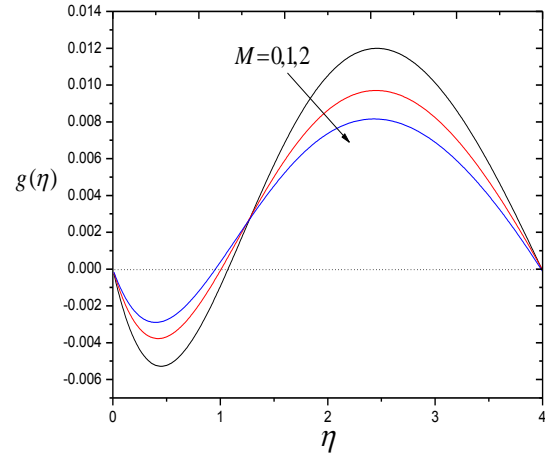
Figure 3. Microrotation profile for various values of magnetic parameter M.
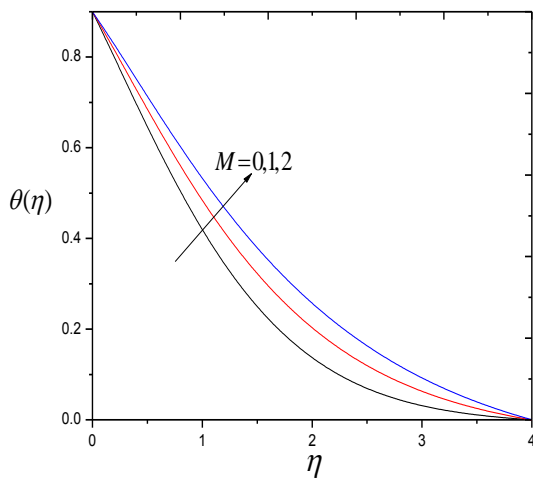
Figure 4. Temperature profile for various values of magnetic parameter M.
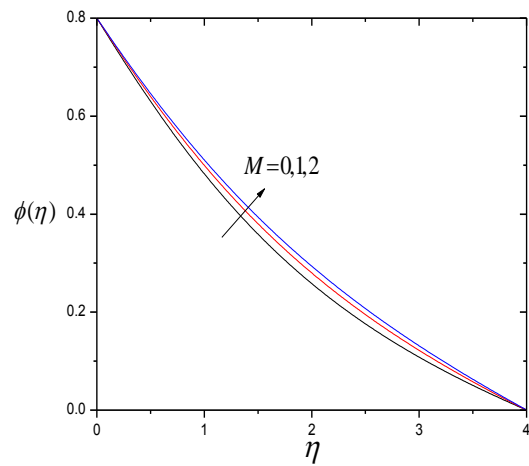
Figure 5. Concentration profile for various values of magnetic parameter M
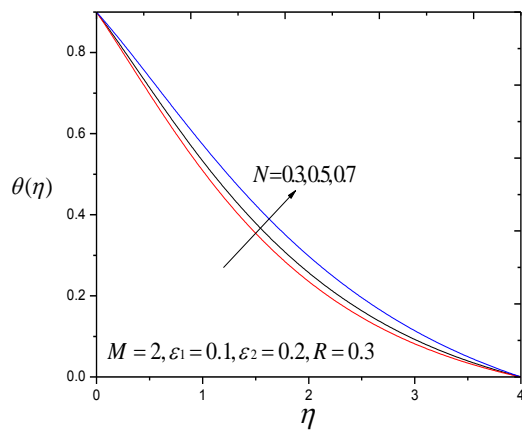
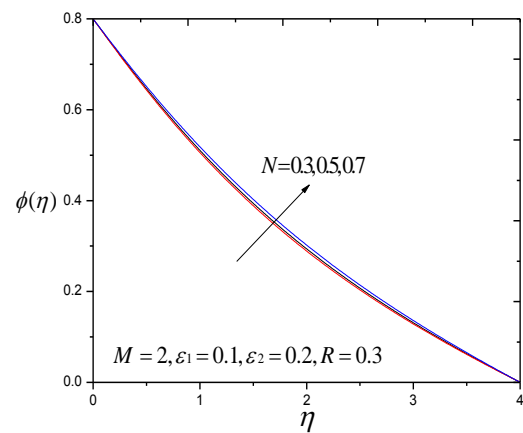**Figure 8.** Temperature profile for various values of coupling number N.

Figure 9. Concentration profile for various values of coupling number N.

Figures 10-13 depict the variation of thermal stratification parameter ε_1 on the non-dimensional velocity, microrotation, temperature and concentration with $M = 2$, $N = 0.5$, $\varepsilon_2 = 0.2$ and $R = 0.3$. It is seen from Figure 10 that the velocity decreases with the increasing value of thermal stratification ε_1. This is because the thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. It is noticed from Figure 11 that the values of microrotation changes sign from negative to positive within boundary layer. Also, it is observed that the

magnitude of the microrotation increases with the increasing value of thermal stratification parameter ε_1. Figure 12 demonstrates that the non-dimensional temperature of the fluid decrease with the increasing value of thermal stratification parameter. When the thermal stratification effect is taken into consideration, the effective temperature difference between the plate and the free boundary fluid will decrease; hence, the thermal boundary layer is thickened and the temperature is reduced. It is observed from Figure 13 that the concentration of the fluid increases with the increasing value of the thermal stratification parameter.
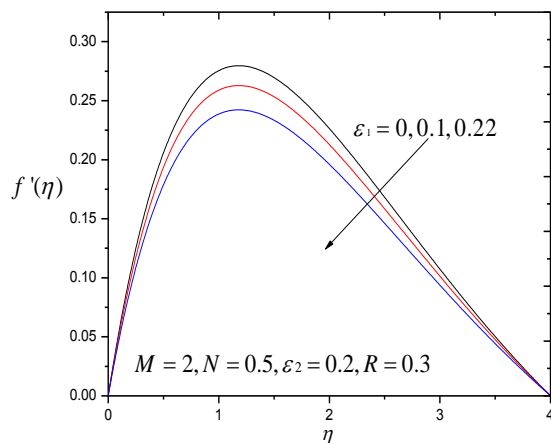

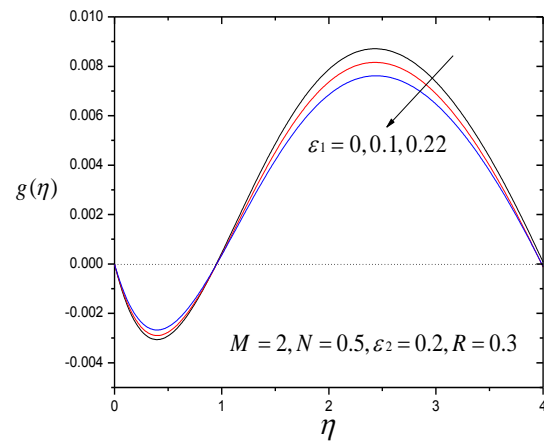**Figure 10.** Velocity profile for various values of thermal stratification parameter ε_1.

Figure 11. Microrotation profile for various values of thermal stratification parameter ε_1.
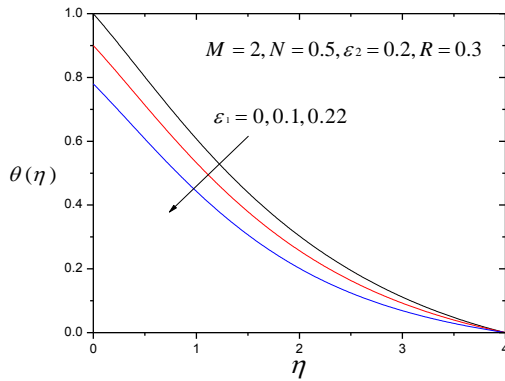
Figure 12.   Temperature profile for various values of thermal stratification parameter ε₁.
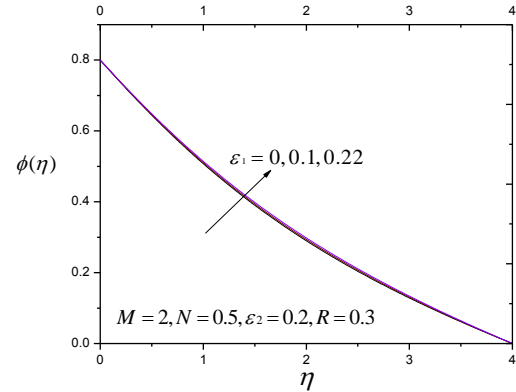

Figure 13.   Concentration profile for various values of thermal stratification parameter ε₁.

The effect of the solutal stratification parameter on the non-dimensional velocity component, microrotation, temperature, and concentration is shown in Figures 14-17 with $M = 2$, $N = 0.5$, $\varepsilon_1 = 0.1$ and $R = 0.3$. It is clear from Figure 14 that the velocity of the fluid decreases with the increase in the solutal stratification parameter. It is noticed from Figure 15 that the microroatation values changed from negative to positive within the boundary layer. Also,

it is observed that the magnitude of the microrotation increases with the increase in the solutal stratification parameter. From Figure 16, we observe that the temperature increases with the increasing value of solutal stratification parameter. It is seen from Figure 17 that the concentration of the fluid decreases with the increase of the thermal stratification parameter.
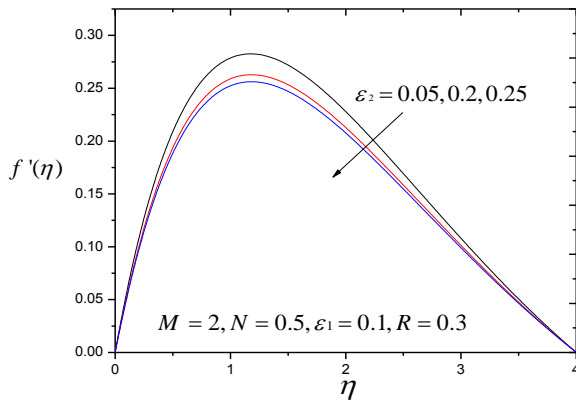

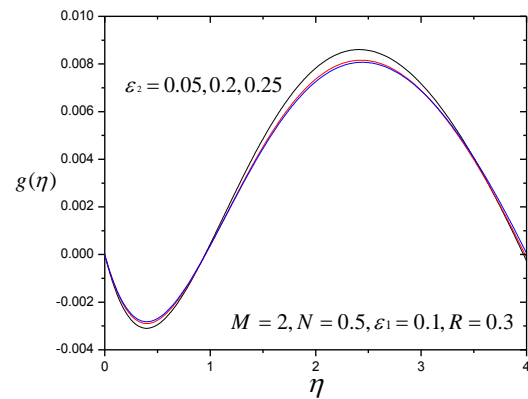**Figure 14.**   Velocity profile for various values of solutal stratification parameter ε₂.

Figure 15.   Microrotation profile for various values of solutal stratification parameter ε₂.
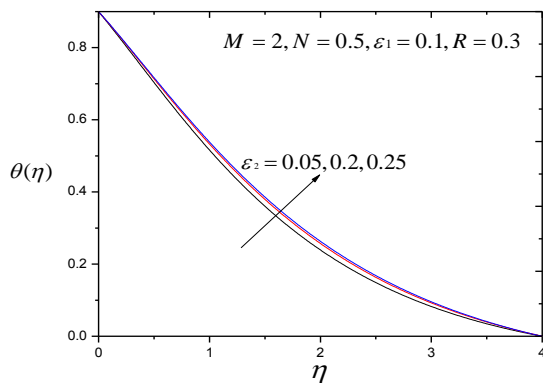

Figure 16. Temperature profile for various values of solutal stratification parameter ε₂.
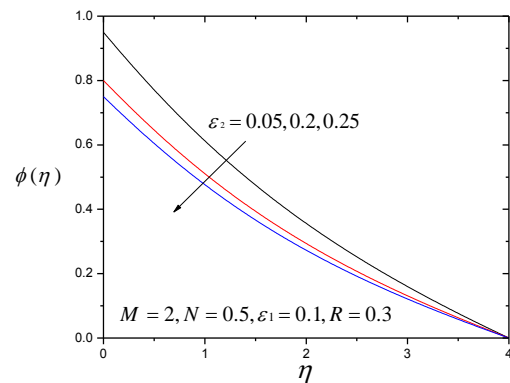

Figure 17. Concentration profile for various values of solutal stratification parameter ε₂.

The effect of the chemical reaction parameter on the non-dimensional velocity component, microrotation, temperature, and concentration with $M = 2$, $N = 0.5$, $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.2$ are depicted in Figures 18-21. It is clear from Figure 18 that the velocity of the fluid decreases with increasing value of chemical reaction parameter R. It is seen from Figure 19 that the microroatation values changed from negative to positive within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in the chemical reaction parameter R. From Figure 20, we observed that temperature increases with the increasing value of the chemical reaction parameter R. It is noticed from Figure 21 that the concentration of the fluid decreases with the increasing values of the chemical reaction parameter R.

Table 2 shows the effects of the magnetic parameter M, coupling number N, thermal stratification parameter ε_1, solutal stratification parameter ε_2 and chemical reaction parameter R on the skin friction coefficient C_f, couple stress coefficient Mw, local Nusselt number Nu and Sherwood number Sh. It is clear from this table that the skin friction, Nusselt number and Sherwood number decrease and the wall couple stress increases as M increases. It demonstrates that the skin friction coefficient, Nusselt number, Sherwood number and wall couple stress decrease with the increasing value of the coupling number N. For increasing the value of N, the effect of microstructure becomes significant; hence, the wall couple stress decreases. The effect of the thermal stratification parameter ε_1 is to decrease the skin friction coefficient, Nusselt number and Sherwood number whereas it increases the wall couple stress. It is seen that the skin friction coefficient, Nusselt number and Sherwood number decrease and the wall couple stress increases as ε_2 increases. The skin friction coefficient decreases and the wall couple stress increases with the increasing value of the chemical reaction parameter R, and it is interesting to note that the opposite situation occurs in the case of heat and mass transfer coefficient, i.e, the Nusselt number decreases whereas the Sherwood number increases as the chemical reaction parameter R increases.
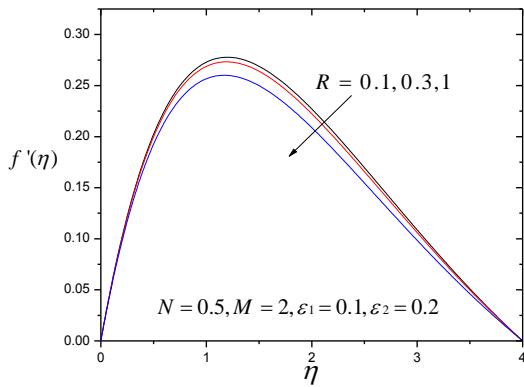

Figure 18. Velocity profile for various values of chemical reaction parameter R
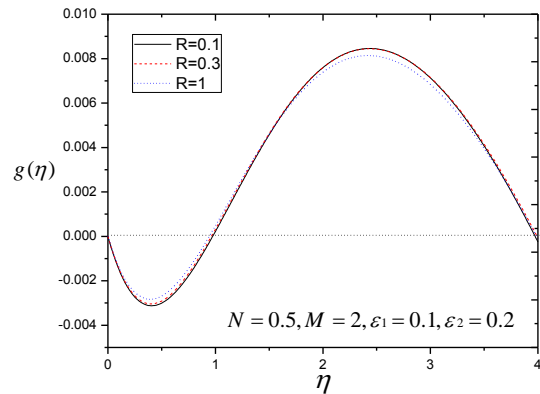

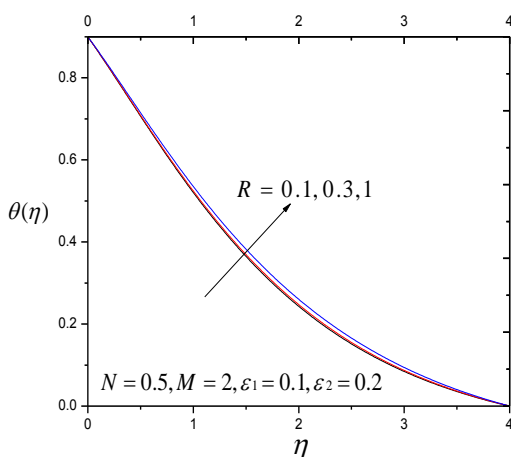**Figure 19.** Micro rotation profile for various values of chemical reaction parameter R.

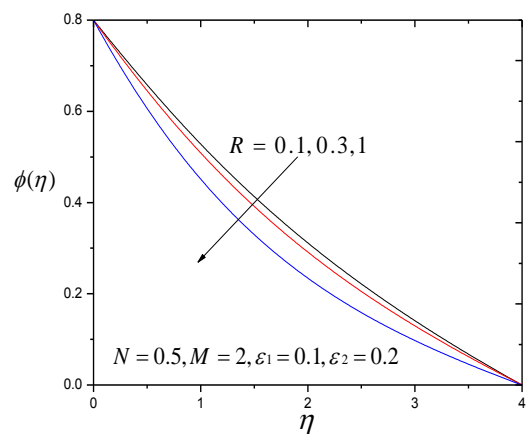**Figure 20.** Temperature profile for various values of chemical reaction parameter R.

Figure 21. Concentration profile for various values of chemical reaction parameter R.

Table 2. Values of $f''(0)$, $-\theta'(0)$, Nu and Sh for various values of M, N, ε_1, ε_2, and R.

M	N	ε_1	ε_2	R	$f''(0)$	$-\theta'(0)$	Nu	Sh
0	0.5	0.1	0.2	0.3	0.77428	0.02626	0.49192	0.35622
1	0.5	0.1	0.2	0.3	0.64684	0.02016	0.40952	0.33552
2	0.3	0.1	0.2	0.3	0.70211	0.00760	0.38490	0.38490
2	0.5	0.1	0.2	0.3	0.56721	0.01646	0.34932	0.32250
2	0.7	0.1	0.2	0.3	0.40492	0.03242	0.29280	0.31185
2	0.5	0.0	0.2	0.3	0.60251	0.01745	0.36882	0.32630
2	0.5	0.22	0.2	0.3	0.52459	0.01520	0.32585	0.31800
2	0.5	0.1	0.05	0.3	0.61349	0.01770	0.37095	0.37530
2	0.5	0.1	0.25	0.3	0.55174	0.01602	0.34199	0.30496
2	0.5	0.1	0.2	0.1	0.58597	0.01738	0.36440	0.29162
2	0.5	0.1	0.2	1.0	0.56535	0.01626	0.34655	0.42755

5. Conclusions

Free convection heat and mass transfer in an electronically conducting and viscous dissipating micropolar fluid over a vertical plate with magnetic, thermal and solutal stratification and chemical reaction effects are considered. The non-linear partial differential equations are transformed into a system of coupled non-linear ordinary differential equations by using similarity variables and then solved numerically using the Runge-Kutta fourth-fifth order method along with the shooting method. From the numerical calculations of the skin friction coefficient, couple stress coefficient, Nusselt number and Sherwood number it is concluded that:

- An increase in magnetic parameter decreases the velocity, skin friction coefficient, heat and mass transfer rates but enhances the temperature, concentration and wall couple stress.
- An increase in the coupling number decreases the velocity, skin friction coefficient, couple stress coefficient, heat and mass transfer rates but increases the temperature and concentration distributions.
- The higher value of the thermal (solutal) stratification parameter results in lower velocity, temperature (concentration), skin friction and heat and mass transfer rate but higher concentration (temperature) and wall couple stress.
- It is also found that the microrotation changes sign from negative to positive values within the boundary layer in the presence of stratification.
- An increase in the chemical reaction parameter decreases the velocity, concentration, skin friction coefficient and heat transfer rate but enhances the temperature, wall couple stress and mass transfer rate.

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