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Simplified Mathematical Modeling of Temperature Rise in Turning Operation Using MATLAB

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Abstract

The problem of temperature rise at work piece and chip can be overcome to an extent by using already developed analytical models. But it is believed that these models comprise a very complicated equation and could not be evaluated easily in a very short (in seconds) time using any of the available mathematical softwares. For this, an attempt has been made to simplify an analytical model (capable of determining temperature rise distribution at chip side due to combined effect of deformation zones) by relating it to cutting parameters (basic machining parameters) followed by a step-by-step evaluation of the analytical equation using MATLAB® programming. It is seen that the developed coding can also be used to determine the temperature rise distribution at work piece due to deformation zones with small changes. Both codings are separately validated using previously obtained results by scientists.

The developed coding may be used for selecting optimum cutting parameters during machining. Moreover, developed coding may help industries to estimate the temperature rise at various points of chip and work piece for any set of machining parameters during the start of the operation itself and, thus, reducing planning and idle time.

Keywords: *Machining, Temperature Rise, Work Piece, Chip, Analytical Model, MATLAB*.

1. Introduction

Machining is a process for removal of material in the form of chips to give work piece desired dimensions and shape by using cutting tools. A lot of energy is consumed while machining and most of it gets converted into heat energy due to the generation of deformation zones (primary, secondary & tertiary), which directly affects the cost and quality of the product. Generated heat results in the rise of temperature at tool, chip and work piece, which if goes beyond the limit, directly affects the productivity by affecting tool (rapid wear, plastic deformation, thermal flaking, thermal fracture, formation of built up edge, and dimensional inaccuracy) and work piece (oxidation, rapid corrosion, internal cracks, dimensional inaccuracy, burning and poor surface finish) along with the induction of thermal stresses at their surfaces. Temperature rise generation also leads to the formation of undesirable quality of chip [1]; it was also seen that the temperature rise at chip is same as the temperature rise at tool [2]. So it is worth to have an estimation of temperature rise distribution at work piece, chip and tool for various cutting parameters so that the machinability can be improved by obtaining optimum cutting parameters.

There are basically three methods to determine the temperature rise distribution at tool, chip and work piece namely, experimental [3], numerical and analytical [4, 5], each of which has its respective merits and demerits. Numerical methods take a long time to develop the results for a particular set of cutting parameters and it gives accurate results. On the other hand, analytical models (developed by researchers since 1951) claimed to make close approximation of temperature rise distribution at tool, work piece and chip due to deformation zone(s)for any set of machining parameters in very small time[2, 4, 5].

R. Komanduri, Z. B. Hou [2,6,7] and Y. Hunag, S. Y. Liang [8] reported recent analytical models but used FEM to validate the work. A. G. Atkins [9], P. Mottaghizadeh, M. Bagheri [10], B. R. Ramji [11], T. M. El Hossainy, M. H. El-Shazly, M. Abd-Rabou [12] also used FEM for a similar study which is a time consuming method. To the best of the author's knowledge, a method to determine the temperature rise distribution in few seconds, which is usually required during machining, has not been developed so far. In the present paper, an analytical model (with a complex equation), developed by a researcher in the past to determine temperature rise distribution at chip side due to combined effect of primary and secondary deformation zone for turning process, is simplified and is related to

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cutting parameters with the help of MATLAB® software so that worker can have an idea about temperature rise generation at chip side in a few seconds during machining just by feeding basic machining parameters. It was further seen that developed coding can also be applied to simplify the modelling equation with small changes, which can determine the temperature rise distribution at work piece due to deformation zones. Both the coding is validated separately with previously obtained results of scientists. The developed coding acts as a generalized model which is applicable to any set of machining parameters (cutting parameters, tool material and geometry, work piece and geometry). These models are applicable for dry turning operations and are applicable for any set of parameters including any other environmental conditions.

Section 2 catalogues the analytical models used in the present work followed by the assumptions taken to solve the modelling equations in section 3. A step-by-step procedure to generate and feed input data, to be used to evaluate both modelling equations, is described in section 4. This procedure is performed by MATLAB® coding and presented in a flowchart. Section 5 deals with the systematic and easy coding developed in MATLAB® software to evaluate modelling equation of chip side followed by its validation with already developed results of scientists. Section 6 deals with changes in previously developed modelling equation for determining the temperature distribution at work piece side followed by its validation with previously obtained results. Sections 7 and 8 present the conclusion and the future scope of the present work.

2. Analytical Equations

Komanduri and Hou [6] scored the latest success at developing an accurate analytical model that is capable of determining temperature rise distribution at chip side due to combined effect of deformation zones. Pertaining equation developed from the model is given as equation (1).

To the best of the author's knowledge, they also scored the latest success at developing an analytical model that is capable of determining the temperature rise distribution at work piece due to primary deformation zone very close to accuracy [7]. These modelling equations are applicable to any set of machining parameters for dry turning operations. The pertaining equation developed from the model is given as equation (2). Abbreviations and required basic formulae used to generate input parameters of equation (1 and 2) are given in the appendix.

Now it can be seen that both equations are very complicated and could not be evaluated at any available mathematical software to obtain results easily and with lesser time. Moreover, it needs many input parameters; and an approach to obtain them is not predictive in the equations. Further sections deal with the generation of input parameters with the help of cutting parameters, and the simplification of both equations followed by the assumptions needed to evaluate them.

$$
\begin{aligned}
\theta_c = \frac{q_{se}}{\pi k}\Bigg\{ & (B_c - \Delta B)\int_{l_i=0}^{l} e^{-V_c(X-l_i)/2a}\left[K_0\left(R_i V_c/2a\right) + K_0\left(R_i' V_c/2a\right)\right] dl_i \\
& + 2\Delta B \int_{l_i=0}^{l} \left(\frac{l_i}{l}\right)^m e^{-V_c(X-l_i)/2a}\left[K_0\left(R_i V_c/2a\right) + K_0\left(R_i' V_c/2a\right)\right] dl_i \\
& + C\Delta B \int_{l_i=0}^{l} \left(\frac{l_i}{l}\right)^n e^{-V_c(X-l_i)/2a}\left[K_0\left(R_i V_c/2a\right) + K_0\left(R_i' V_c/2a\right)\right] dl_i \Bigg\} \\
& + \frac{q_s}{2\pi k}\int_{w_i=0}^{t_c/\cos(\emptyset-\alpha)} e^{-(X-X_i)V_c/2a}\left\{K_0\left[\frac{V_c}{2a}\sqrt{(X-X_i)^2+(Z-z_i)^2}\right]\right. \\
& \left. + K_0\left[\frac{V_c}{2a}\sqrt{(X-X_i)^2+(2t_c-Z-z_i)^2}\right]\right\} dw_i
\end{aligned}
\tag{1}
$$

where,

$X_i = L_{ab}$ - $w_i \sin(\emptyset-\alpha)$, $z_i = w_i\cos(\emptyset-\alpha)$, $R_i = \sqrt{(X-l_i)^2+Z^2}$ and $R_i' = \sqrt{(X-l_i)^2+(2t_c-Z)^2}$

$$
\begin{aligned}
\theta_w = \frac{q_s}{2\pi k}\int_{l_i=0}^{L_{ab}} e^{-(X+l_i\cos\emptyset)\frac{V}{2a}}\Bigg\{ & K_0\left[\frac{V}{2a}\sqrt{((X+l_i cos\emptyset)^2+(z-l_i sin\emptyset)^2)}\right] + \\
& K_0\left[\frac{V}{2a}\sqrt{((X+l_i cos\emptyset)^2+(z+l_i sin\emptyset)^2)}\right]\Bigg\} dl_i
\end{aligned}
\tag{2}
$$

3. Assumptions for Computing Modeling Equation

In order to solve the modeling equations, the following assumptions are to be considered:

a) Cutting process is orthogonal.
b) There is no heat loss to surrounding along the primary / secondary heat zones.
c) Preheating effect of the tool and work piece is negligible.
d) Model is applicable for turning process only.
e) Thermal conductivity of work piece and chip is the same and does not change with the change in the temperature.
f) Deformation zones are considered as plane heat source. But in reality, it is basically a zone.
g) Surface of the tool does not worn out during machining and machining is considered to be dry.
h) Heat generated due to tertiary deformation zone can be neglected. Hence, tool insert tip is considered to be sharp.
i) Lower limit of integral (given in equation 1) is taken as 0.000001 instead of zero in MATLAB® coding.

4. Theoretical Formulation of Analytical Solutions

The analytical equations to be computed are complex and are not related to cutting parameters. The layman can encounter problems to apply these modeling equations directly on shop floor. If these modeling equations can be simplified and related to cutting parameters then just by feeding the basic input variables, the operator can obtain results in seconds. To accomplish this objective, analytical models, developed by Komanduri and Hou [6, 7], are considered as a basis for study. Then various parameters of the modeling equations are related and evaluated through formulae given in Appendix which helped in relating equations with basic machining parameters. In order to solve the complex integrals in the equations, multiple Simpson's 1/3 rule is applied in coding. The systematic layout of the work is explained with the help of flow diagrams in the subsequent topics.

5. Generation of Input Parameters for Model

As mentioned earlier, many input parameters are required to solve the modeling equation. For this, cutting parameters are taken as first input to generate other required input parameters. This is done because the cutting parameters are the most initial input to machining. Moreover, by doing so, the temperature rise at chip and work piece can be controlled by controlling cutting parameters. A step-by-step approach, to generate required parameters, is done by MATLAB® programming and presented in Figure 1, in a flowchart. A brief discussion of Figure 1 is given below.

To solve the analytical equations, the input parameters to be considered are divided into three types namely 1st type, 2nd type and 3rd type. 1st kind of inputs is the obtained values from machining or measurement or they are direct data decided before machining, while 2nd and 3rd types of input are calculated data. 1st kind of input parameters serves as inputs to obtain 2nd type of parameters. Both types together are used to generate 3rd type of input data. Generated input parameters are used to solve modeling equations (1 & 2). In order to evaluate equation (1), MATLAB® coding is prepared as discussed in section 5.

6. Mathematical Computation and Validation of Thermal Modelling of Chip Side

This section discusses the coding used to evaluate equation (1) followed by its validation with the obtained results of Komanduri & Hou [6].

6.1. Computing Modelling Equation of Chip Side

This section deals with studying a step wise step procedure to evaluate the model in the form of flow diagrams based on MATLAB® coding.

The entire coding is divided into three parts. 1st part is used to feed 1st kind of input data followed by calculations of 2nd and 3rd kinds of input parameters. A step-by-step feeding and a calculation of data are shown in Figure 1. Formulae used for calculating the data is given in the appendix.

The 2nd division deals with the calculation of definite integrals of the complex functions involved in equation (1). Though MATLAB® software can directly integrate functions using "quad" command, it takes time (sometimes processing hangs the computer) to integrate complicated functions like the ones given in equation (1). To reduce time of calculation, Multiple Segment Simpson's 1/3rd rule is applied and its coding is presented here in the form of a flowchart (Figure 2). The value of the integration of each function is stored in software in "integral" variable.

The 3rd part emphasizes on coding used to evaluate the modelling equation (1) by using Multiple Segment Simpson's 1/3rd rule from part 2 and inputs from part 1 and display temperature rise distribution contour graphs with respect to co-ordinates. A step-by-step procedure for solving the modelling equation is shown in Figure 3.

The coding can be used to calculate the temperature rise distribution at various points of chip for any tool-work combination for turning operation and for any combination of cutting parameters, tool geometry and other cutting conditions in few seconds.

Five functions to be evaluated to solve equation (1) by Multiple Segment Simpson's 1/3rd rule are given by equation (3), (4), (5), (6) and (7), respectively. Values of constants used in modeling equation (1) i.e. *Bchip, ΔB*, C, m, & n are approximate values and estimated from Komanduri and Hou's [2, 4, 5, 6] functional analysis. Value of lower and upper limit for 1st and 2nd functions to be integrated are 0.000001 (≈ 0) & shear plane length respectively. Also, the lower and upper limits of 3rd, 4th and 5th functions to be integrated are 0.000001(≈ 0) & tool-chip contact length.

$$f(w_i) = \left(e^{-(X-X_i)^{V_c}/2a}\right) * K_0\left[\frac{V_c}{2a}\sqrt{(X-X_i)^2+(Z-z_i)^2}\right] \tag{3}$$

$$g(w_i) = \left(e^{-(X-X_i)^{V_c}/2a}\right) * K_0\left[\frac{V_c}{2a}\sqrt{(X-X_i)^2+(2t_c-Z-z_i)^2}\right] \tag{4}$$

$$h(l_i) = (e^{-V_c(X-l_i)}/_{2a}) * [K_0(^{R_iV_c}/_{2a}) + K_0(^{R_i^{'}V_c}/_{2a})] \quad (5)$$

$$j(l_i) = \left(\frac{l_i}{l}\right)^m e^{-V_c(X-l_i)}/_{2a} * [K_0(^{R_iV_c}/_{2a}) + K_0(^{R_i^{'}V_c}/_{2a})] \quad (6)$$

$$s(l_i) = \left(\frac{l_i}{l}\right)^n e^{-V_c(X-l_i)}/_{2a} * [K_0(^{R_iV_c}/_{2a}) + K_0(^{R_i^{'}V_c}/_{2a})] \quad (7)$$

$X_i \& z_i = function\ of\ (w_i)$,
$R_i \& R_i^{'} = function\ of\ l_i$

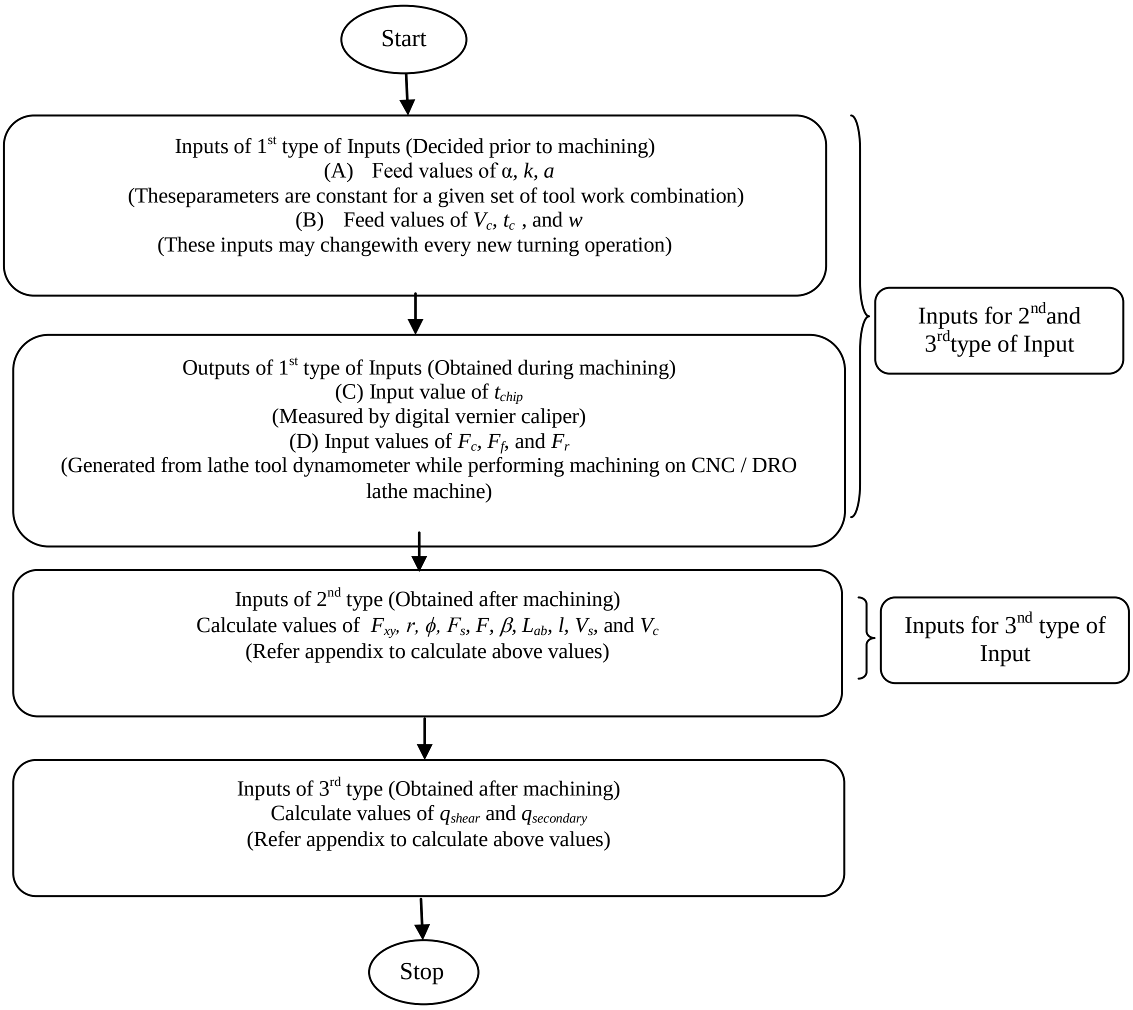


Figure 1. Flowchart showing step by step procedure to feed and calculate input data of all kinds needed to evaluate equation (1)

6.2. Validation of Program

Komanduri and Hou used Chao and Trigger's input parameters [12] (Table 1) for evaluating equation (1) and obtaining temperature rise distribution on chip side due to combined effect of primary and secondary heat source. MATLAB® coding is tested using same input parameters and temperature rise contours obtained from the program are depicted in Figure 4.

Comparing Figure 4 and results obtained in [6], it is observed that output results of both approaches match with a close proximity (maximum temperature value difference is 2°C). Therefore, the program can be considered for validation.

7. Mathematical Computation and Validation of Thermal Modelling of Work Piece Side:

This section emphasises the discussion of coding used to evaluate equation (2) followed by its validation with the results obtained by Komanduri & Hou [6].

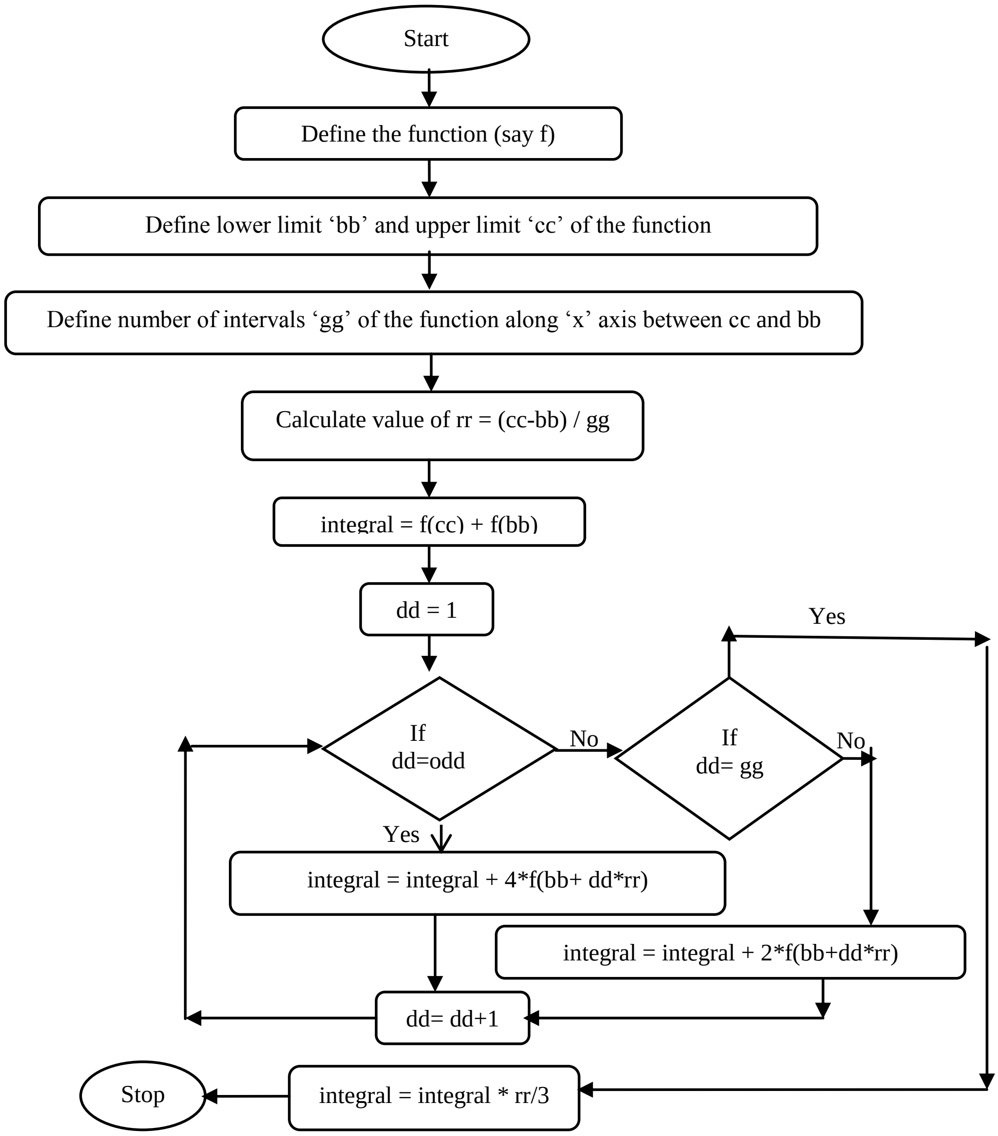


Figure 2. Flowchart showing step by step procedure to calculate definite integrals in MATLAB® by using Multiple Segment Simpson’s 1/3rd rule

7.1. Computing Modelling Equation of Work Piece Side

It was noticed that the coding developed in section 5.1 can be used to evaluate equation (2) by changing functions (f, g, h, j, & s) used in equations (3,4,5,6,7), respectively. This can be done by replacing function f by f_1, g by g_1, h by h_1 and so on. These changed functions are given in equations (8,9,10).

$$f_1(l_i) = e^{-(X+l_i cos\phi)\frac{V}{2a}} K_0 * \left[\frac{V_c}{2a}\sqrt{((X+l_i cos\phi)^2 + (Z - l_i sin\phi)^2)}\right] \quad (8)$$

$$g_1(l_i) = e^{-(X+l_i\, cos\phi)\frac{V}{2a}} K_0 * \left[\frac{V_c}{2a}\sqrt{((X+l_i cos\phi)^2 + (Z + l_i sin\phi)^2)}\right] \quad (9)$$

$$h_1(l_i) = 0 = j_1(l_i) = s_1(l_i) \quad (10)$$

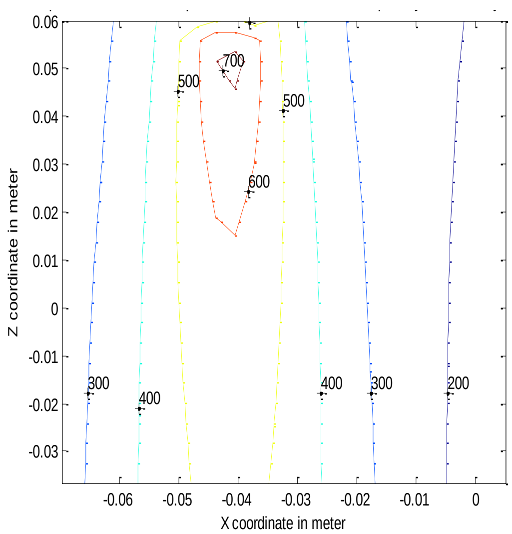
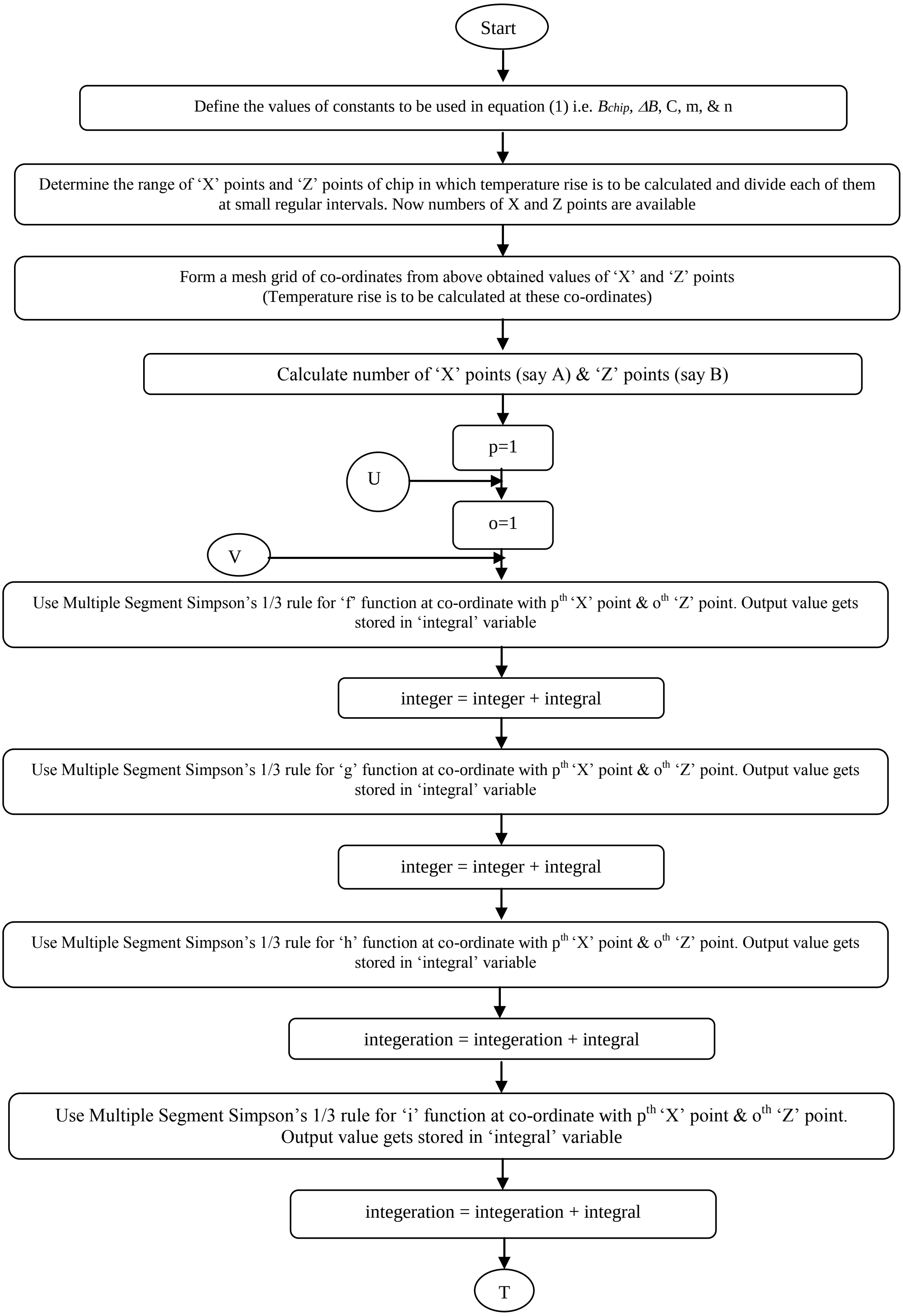


Figure 4. Temperature rise contours at various points of chip due to combined effect of two deformation zones using input parameters of Table 1 and developed MATLAB®coding


Start
Define the values of constants to be used in equation (1) i.e. B_{chip}, ΔB, C, m, & n
Determine the range of 'X' points and 'Z' points of chip in which temperature rise is to be calculated and divide each of them at small regular intervals. Now numbers of X and Z points are available
Form a mesh grid of co-ordinates from above obtained values of 'X' and 'Z' points (Temperature rise is to be calculated at these co-ordinates)
Calculate number of 'X' points (say A) & 'Z' points (say B)
p=1
U
o=1
V
Use Multiple Segment Simpson's 1/3 rule for 'f' function at co-ordinate with pth 'X' point & oth 'Z' point. Output value gets stored in 'integral' variable
integer = integer + integral
Use Multiple Segment Simpson's 1/3 rule for 'g' function at co-ordinate with pth 'X' point & oth 'Z' point. Output value gets stored in 'integral' variable
integer = integer + integral
Use Multiple Segment Simpson's 1/3 rule for 'h' function at co-ordinate with pth 'X' point & oth 'Z' point. Output value gets stored in 'integral' variable
integeration = integeration + integral
Use Multiple Segment Simpson's 1/3 rule for 'i' function at co-ordinate with pth 'X' point & oth 'Z' point. Output value gets stored in 'integral' variable
integeration = integeration + integral
T

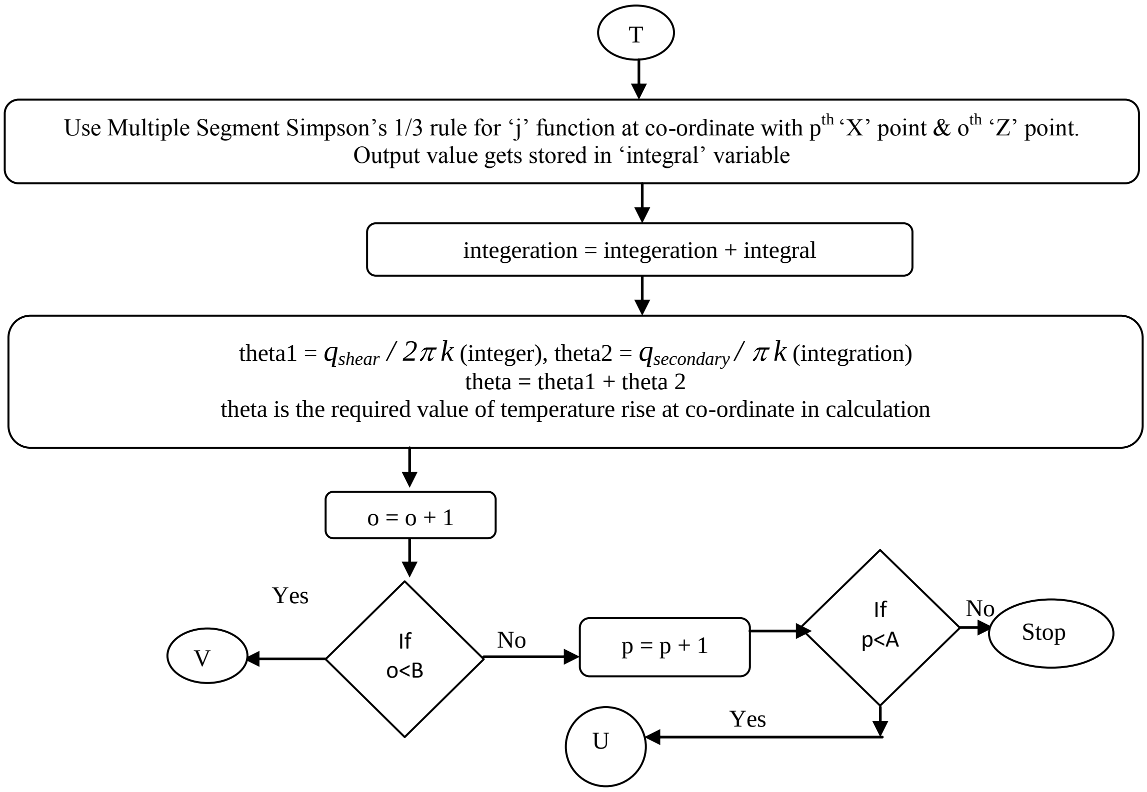


Figure 3. Step by step procedure to evaluate modeling equation (1) using Multiple Simpsons 1/3rd Rule (refer section 5) and input parameters to be used (refer section 4).

Table 1. Chao and Trigger's machining input parameters [12]

Work material	Steel NE 9445
Range of X co-ordinate	-700 µm to 50 µm
Range of Z co-ordinate	- 400µm to 600 µm
Tool	Triple carbide 4° rake
Cutting Velocity	152.4 cm/sec.
Depth of Cut	0.02489 cm
Width of Cut	0.2591 cm
Chip Contact Length	0.023 cm
Main Cutting Force	1681.3 N
Feed Force	854 N
Passive Force	Zero N
Chip Thickness	0.06637
Thermal Diffusivity	0.3777 cm^2/sec.
Thermal Conductivity	0.08234 Watt/cm °C
B_c= 0.652, ΔB = 0.312, C= 2.2, m= 0.26, & n= 16	

7.2. Validation of Coding for work piece Side

Komanduri and Hou used Lowen and Shaw's input parameters [9] (refer to Table 2) for validating equation (2) by obtaining temperature rise distribution on work piece due to primary heat source. Changed MATLAB® coding is tested using the same input parameters and temperature rise contours obtained from the program are depicted in Figure 5.

Comparing both the figures 5 and results obtained in [7], it can be noted that both results give almost the same output with a very little variation of temperature rise (maximum temperature rise value variation is 10°). Therefore, the program can be considered for validation. In the model, moving a co-ordinate system has been considered. Komanduri and Hou calculated the temperature rise distribution at an instantaneous co-ordinate system which is different from the author's consideration. Hence, the temperature rise distribution, in our result, is shifted from Komanduri and Hou's results.

8. Conclusion and Future Scope

By using the coding, industries can estimate the temperature rise distribution at various points of chip and work piece due to deformation zones for any machining parameters during the turning operation itself and, thus, reducing planning and the idle time. Moreover, by generating temperature rise distribution at chip side for particular machining parameters, one can also predict distribution of temperature at tool side since the temperature rise distribution is almost the same at both chip & tool for a particular tool-work combination. Coding shows how the basic machining parameters are connected with complex modeling equations so that by feeding them one can get the output results. Also, MATLAB® software is cheap and readily available. Moreover, the flowchart developed for the coding can act as a study material for other programmers to develop a similar coding on other software and, thus, increasing the compatibility of the work.

Apart from the work conveyed in the present article, further work can be done in this area in the future:

- The MATLAB® coding developed in the work can be used to calculate thermal stresses developed during machining at various points of chip and work piece.
- The coding can be modified for other simple operations (like facing) and complicated processes (like milling, grinding, drilling, etc.)
- The coding can be used to determine optimum cutting parameters for a particular tool-work combination. For this, various combinations of cutting parameters (using full factorial) can be tested to generate temperature rise distribution from a range of cutting parameters.

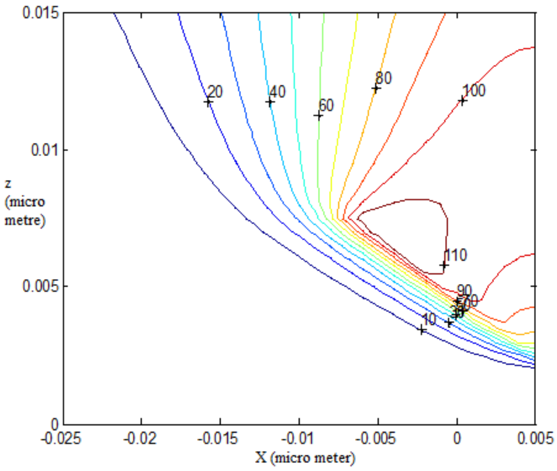


Figure 5. Temperature rise contours at various points of work piece due to primary deformation zone developed from MATLAB coding

Table 2. Cutting data for machining steel from Loewen & Shaw [13]

Work material	SAE B1113 steel
Tool	K2S carbide 20°rake, 5° clearance
Cutting Velocity	232cm/sec.
Depth of cut	0.006
Width of Cut	0.384cm
Chip Contact Length	0.023cm
Main Cutting Force	356 N
Feed Force	125N
Passive Force	Zero N
Chip Thickness Ratio	0.51
Thermal Diffusivity	0.1484 sq.cm/sec.
Thermal Conductivity	0.567Watt/cm deg. cel.
Range of X co-ordinate	-250 to 50 µm
Range of Z co-ordinate	-150 to 0 µm

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Appendix

Abbreviations	Details	Source	Unit
θ_c	Temperature rise at chip due to primary and secondary deformation zone	Refer equation (1)	˚C
θ_w	Temperature rise at work piece due to primary deformation zone	Refer equation (2)	˚C
α	Rake angle	Tool specifications	°
t_d	Undeformed chip thickness	= depth of cut	cm
t_c	Deformed chip thickness	Digital Vernier Calliper	cm
r	Chip thickness ratio	t_c/t_{chip}	--
ϕ	Shear angle	$tan^{-}\dfrac{r\ cos \propto}{1 - r\ sin \propto}$	°
w	Width of chip	= feed rate	cm
X, Z	X and Z co-ordinate at which temperature rise is to be calculated	---	µm
F_c	Cutting force	Experimental (dynamometer)	N
F_f	Feed force	Experimental (dynamometer)	N
F_r	Radial force	Experimental (dynamometer)	N
F_{xy}	Resultant of feed force and radial force	$\sqrt{(F_f^2 + F_r^2}$	N
F_t	Shear force	$F_c cos\emptyset - F_{xy} sin\emptyset$	N
F	Friction force	$F_c sin \propto + F_{xy} cos \propto$	N
N	Normal to Friction force	$F_c cos \propto - F_{xy} sin \propto$	N
V	Cutting velocity	Input Cutting Parameter	cm/s
V_c	Chip velocity	$\dfrac{V_c\ sin\emptyset}{\cos(\emptyset - \propto)}$	cm/s
V_s	Shear velocity	$\dfrac{V_c\ cos \propto}{\cos(\emptyset - \propto)}$	cm/s
L	Length of shear plane	$t_c/sin\emptyset$ or ${}^{t_{chip}}\!/_{cos(\emptyset - \alpha)}$	cm
β	Friction angle	$\tan^{-1} {}^{F}\!/_{N}$	°
l	Tool chip contact length	$\dfrac{t_c \sin(\emptyset + \beta - \alpha)}{sin\emptyset\ cos\beta}$	cm
k	Thermal conductivity of chip	Data book	J/cms˚C
a	Thermal diffusivity of chip	Data book	cm^2/s
q_s	Heat intensity of the primary heat source	$\dfrac{F_s V_s}{L_{AB} w}$	J/cm^2s
q_{se}	Heat intensity of the secondary heat	$\dfrac{F V_{chip}}{lw}$	J/cm^2s
B_c, ΔB, C, m, & n	Constants	Komanduri & Hou [3] functional analysis	--