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Implementation of a Model for Predicting the Size Distribution of Coal Particles in a Horizontal Fluid Pipeline

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Abstract

In this study, a model for the prediction of coal particle gradation in horizontal fluid pipeline was developed. The intent of this study is to develop a model that can predict the coal particle gradations in horizontal coal-liquid slurry pipelines. A semi-empirical model originally developed by Karabelas has been modified by applying the k-ε approach to model a dimensionless diffusivity, originally assumed to be constant. The mechanistic model developed shows that dimensionless diffusivity is a function of root mean square turbulent velocity fluctuations and many other parameters which include fluid velocity, pipe diameter, carrier fluid and suspended particles densities, particle size, and the efflux concentration. The modified model was compared with four different sets of experimental data and a CFD model, and was found to have overall good agreement. It was employed to analyze particle gradations and the results show that larger coal particles tend to drift to the pipe bed, leaving the upper section of the pipes for smaller particles. The results show that concentration profile of coal particles in single-sized homogenous slurry is different from that of the same particle concentration in multi-sized slurry flow for each particle size and the same efflux concentration. The modified model was applied to analyze the d50 concentration profile and the result shows that d50 concentration profile represents the actual profile accurately within low and moderate range of particle sizes in the slurry. The mechanistic model presented in this study will be useful for coal and other mining industries worldwide, for accurate prediction of concentration profiles and choice of particle sizes as well as flow velocity of slurry in horizontal pipelines to avoid deposition and clogging of particles along the pipeline, thereby reducing associated danger in the pipelines.

Keywords: Mechanistic model, dimensionless diffusivity, turbulent velocity fluctuation, concentration profile, particles gradation.

1. Introduction

One of the ways of transporting coal from the mining location to the consumption location is by slurry flow in horizontal pipelines. Slurry flow is liquid-solid two-phase flow, usually turbulent in nature. This enables the particles disperse within the fluid as they are carried along the pipeline. Coal particles in the slurry may sediment if appropriate flow conditions are not met, leading to economic damage. Slurry pipelines are used in mining and chemical industries for long distance transportation of coal, iron, copper, phosphate concentrates, oil-sand mixture and other mineral ores [1, 2, 3]. Solid particles in the slurry flow are capable of settling to form sliding bed if the velocity is below critical deposition. As the velocity drops further, the sliding bed eventually becomes stationary. Dangers such as increase wear rate of pipeline internal wall, increase pressure drop, pitting initiation and increase corrosion rate of pipe internal wall and pipeline total blockage are associated with allowing sliding bed in slurry flow. In an attempt to provide solution to the above outlined dangers, many researches have been undertaken to investigate conditions that will eliminate particles deposition in pipe bed. It is a known fact that high turbulence with swirling flow enhances particles and fluid mixing [4]. Detailed study on swirling flow can be found in [5,6,7,8]. Many attempts have also been made to model the minimum velocity of slurry flow at which solid particles in the flow will remain suspended in the flowing fluid stream (Critical Velocity) and below which the particles start depositing to form sliding bed (Critical deposition velocity). Durand developed a correlation for critical velocity prediction in 1953 from his experimental result. According to Yan [9], Durand's correlation shows that critical velocity is a function of pipe diameter and the particle-fluid density ratio. Wasp et al. [10] modifies Durand's correlation by adding terms that account for the effect of particle size and concentration. Oroskar and Turian [11] developed a correlation for critical velocity using data obtained from literature using fluid-particle balance method. Kaushal and Tomita [12] used laboratory experimental data to modify Wasp correlation. Many other correlations and experimental researches for predicting critical velocity has been developed in recent

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times by many researchers, some of which are the works of [13, 14, 15, 16, 17, 18].

Mechanistic models have also been developed to predict critical deposition velocity. For instance, Davies, [19] applied the force balance and turbulent theory to model the critical deposition velocity of particles in flowing fluid. Davies showed how the critical deposition velocity is a function of concentration, particle sizes, pipe diameter and specific gravity. Doron and Barnea, [20] applied the torque balance on the particles to model the minimum bed velocity defined as the minimum velocity at which coarse particles start rolling along the bed in the flow direction. The model shows that the minimum bed velocity is a function of bed concentration, particles and carrier fluid density. Obaseki et al. [3] modeled the critical suspending velocity of sand particles in crude oil using the force balance and Lagrangian multiphase approach. The model competes favorably with other models on critical velocity. Particle distribution and gradation in slurry flow through pipelines is another area of interest. Normally, it is expected that particles of higher density and larger particles should occupy the bed while tinier and less dense particles should occupy the middle and the upper layers in the flow. Particle distribution and gradation in pipelines is a function of many parameters and when the parameters are not optimum, the particle distribution can have negative effect on pipe walls. These effects could range from flow complexities, high erosional rate to pitting corrosion initiation, and can lead to pressure drops. Few analytical models have been developed to analyze the vertical concentration profile in pipelines, some of which are discussed here. O'Brien [21] and Rouse [22] presented a diffusion equation for sediment transportation in rivers. They concluded that the rate of upward transfer of sediments due to turbulence is equal to downward sedimentation due to gravity. The model was found to be inaccurate for wide range of particle sizes. Ismail [23] modified the model of O'Brien and Rouse to predict concentration profile in close rectangular channel and validated it with experimental data. Wasp et al. [24] developed a logarithmic base model to predict vertical concentration profile in pipes using experimental data from their study and Ismail's work. The model predicted concentration profile for coal slurry with good agreement with their experimental data. Karabelas [25] developed a semi-empirical model for vertical concentration profile and particles distribution in pipes and rectangular duct. The model was originally developed for diluted suspension. Karabelas model was validated with experimental data using Kerosene as carrier fluid. Seshadri et al. [26] modified the Karabelas model by striking out the assumption of constant dimensionless particles diffusivity and formulated a correlation for diffusivity which is a function of efflux concentration and static settle concentration. He validated the modified model with experimental data with good agreement. Kaushal et al. [27] also modified the Karabelas model to predict concentration profile for rectangular duct using experimental data and following the same approach of Seshadri et al. [26]. Kumar et al. [28] applied the Kaushal et al. [27], and modified Karabelas model to determine fly ash and bottom ash particles distribution in pipes and their effect on pressure drop. Wu et al. [29] applied the probability density function (PDF) to modify O'Brien [21] model for prediction of vertical concentration profile of suspended sediments in rivers. His results show good agreement with Rouse [22] model. The major drawback in the empirical correlation and semi-empirical correlation models reviewed so far is that they are most times not general in application. For instance, Karabelas model shows good agreement with Karabelas experimental data obtained using Kerosene as carrier fluid but could not predict accurately, the concentration profile of zinc tailings slurry flow. In recent times generalized models for particles distribution and gradation in fluid flow is studied using Computational Fluid Dynamics executed on ANSYS-CFX and ANSYS FLUENT software. Ekambara et al. [30] studied the horizontal slurry pipeline flow using ANSYS-CFX. The simulation results were compared to experimental data obtained from literatures and it was found to have better agreement compared to empirical correlations. Messa and Malavasi [2] used CFD_PHOENICS software to simulate particle distribution in slurry flow in straight and bent pipelines using the Eulerian Two-fluid Inter-Phase Slip Algorithm (IPSA). The results of the model were validated using experimental data from literatures and better agreement was observed. Messa et al. [31] used CFD_PHOENICS software to simulate particle volume fraction and velocity distribution in slurry flow in straight using the Eulerian Two-fluid IPSA. Experimental data from literatures were also used to validate the simulation results. Kumar et al. [32] investigated experimentally and numerically high concentrate iron ore slurry flow in pipelines. He used theRNGk-ε model for the CFD simulation and the result shows good agreement with data from his experiment. CFD simulation of particle distribution in slurry flow were also carried out by Ofei and Ismail [33], Tarodiya et al. [34], Kaushal et al. [35], Kumar et al. [36]. Messa et al. [37] undertook a comparative review study of the state of the art of CFD simulation and experimental methods of analyzing slurry flow in pipelines. The authors noted that Eulerian-Eulerian approach is the most preferred choice as it gives more information than the experimental method. Jawarneh et al. [38] studied the analytical approximate solution for decaying laminar swirling flows within a narrow annulus. Due to wall friction, the Hagen-Poiseuille flow profiles of the swirl velocity diminish steadily as they move downstream. As the Reynolds number grows, the tangential velocity increases, and the profile flattens near mid-gap. The dimensionless pressure is nonlinear and sensitive to swirl and Reynolds numbers at the inlet. It declines rapidly downstream and is steeper at small gaps.Ahmed.et al. [39] conduct a numerical study of two-phase glass bead-water slurry flow. The turbulence phase of the flow was modeled using the Eulerian two phase modeling approach. The results show that the solid particles were distributed asymmetrically along the vertical plane of the pipe-cross section. The interaction of solid particles with the pipe wall becomes increasingly pronounced as flow velocity increases. Khlaifat et al. [40] examined the flow of dead sea mud slurry in a horizontal pipe. The shear stress power law equation was used in a mathematical model method. The created model's results suggested that it may be used to predict the flow rate of dead sea mud slurry via a conduit under pressure drop conditions.

Nowadays, models aimed at investigating underlining parameters that influence particles distribution in slurry flow in pipelines are being investigated. Some of these investigations were carried out experimentally while others are done with the aid of CFD software. Few examples of such CFD investigation can be found in [41, 42, 43]. For instance, Chen et al. [44] studied the hydraulic transport of non-spherical particles in pipeline. The results obtained from the study compares well with experimental data. Similar research was carried out by Zheng et al. [45], this time; coarse particles transportation in pipeline was considered using both computational and experimental approach. Very good agreement was observed between the computational and the experimental results. The effect of partially filled pipe on slurry transport was investigated experimentally by Cunliffe et al. [46]. In this study, settling behaviours of slurries in partially filled pipelines was investigated and a correlation for predicting the settling rate of particles was developed based on the experimental data. Shi et al. [47] investigated the impact of swirls on slurry flow in horizontal pipelines. The author concluded that increasing the intensity swirls flow from pipe inlet is beneficial to multi-sized slurry transportation because it hinders particles deposition.

Though, Numerical models generally give better results and excellent agreement with experimental data. Empirical, semi-empirical and mechanistic models that can compete or perform better than numerical models are very few in literatures. This work will attempt to modify the existing semi empirical models to give accurate results comparable to data from experiments and numerical simulations. The main aim of this study is to develop a mechanistic model for calculating a varying dimensionless diffusivity, which will be applied to the semi-empirical Karabelas model in order to improve its performance. The incorporation of the newly developed model for dimensionless diffusivity into Karabelas model will result in a modified Karabelas model in which the prediction accuracy and the universal applicability have been improved. The modified model presented in this study will be useful in coal and other mining industries worldwide for accurate prediction of concentration profiles and choice of particle sizes as well as flow velocity of slurry in horizontal pipelines, to avoid deposition and clogging of particles along the pipelines, which will in turn reduce associated danger in the pipelines.

2. MODELING FORMULATION

2.1. Model Implementation

In still water, coal particles will sediment and form layers in the pipeline bed, causing inner corrosion. When the velocity of the coal slurry is increased, the coal particles start sliding along the bed gradually and eventually get picked up by the stream. The mass concentration of the coal particles in the flowing stream depends on the flow velocity and the turbulence fluctuations in the pipe. The increase in velocity will lead to increase in turbulence fluctuations which will lead to particle dispersion, reduced chances of particle-settling and then eventually increases the concentration of coal particles in the stream. In summary, the distribution of particles along the vertical axis in the stream of flowing fluid in pipes is a function of many parameters namely the flow velocity, mean square velocity fluctuation, particles size and specific gravity, fluid density, pipe diameter, and volume of particles in the flowing fluid.

$$C(y) = f(V, u', d, \rho_s, \rho, D, C) \tag{1}$$

Where $C(y) =$ concentration of particles at point y along the vertical axis in the fluid (v/v); $V =$ Velocity of stream (m/s); $u' =$ root mean square turbulent velocity fluctuation (m/s); $d =$ particles size (m); $\rho_s =$ particle density (Kg/m^3); $\rho =$ fluid density (Kg/m^3); $D =$ pipe diameter (m) and $C =$ particles volume concentration in the fluid (v/v). The model originally proposed by Karabelas [25] captured all the independent variables in equation (1) except the root mean square velocity fluctuation. In this work, the parameter considered as a constant was modified to include the effect of velocity fluctuation and to predict coal particle gradation in coal slurry flow in horizontal pipelines.

2.2. The Karabelas Model

The original Karabelas model is given as;

$$E\frac{\partial C_j}{\partial y} - C_j\left(w_j - V_y\right) = 0; \quad j = 1,2, \ldots n \tag{2}$$

Where; $V_y = \sum_{i=1}^{n} w_i c_i$, $C_j =$ concentration of jth particles fraction(v/v), $w_j =$ settling velocity of jth particle fraction, and $E =$turbulent diffusivity (m^2/s).For uniform particle size in fluid, the equation can be written as;

$$E\frac{\partial C}{\partial y} - Cw(1 - C) = 0 \tag{3}$$

The solution to (equ. 2) is given as;

$$C_j(y) = \left[\frac{\overline{v}_j e^{-K_j y'}}{E(K_j)}\right]\left[1 + \sum_{i=1}^{n}\frac{\overline{v}_j e^{-K_j y'}}{E(K_j)}\right]^{-1} \tag{4}$$

The terms in the (equ.4) are expressed as shown.

$$E(K_j) = 1 + \frac{K_j^2}{8}\left(1 + \frac{K_j^2}{24}\right) + 0K_j^6 \tag{5}$$

$$K_j = \frac{w_j}{\in v^*} \tag{6}$$

$$\overline{v}_j = \frac{\overline{c}_j}{1 - \overline{C}} \tag{7}$$

$$\in = \frac{E}{Rv^*} \tag{8}$$

Where; $\in =$ dimensionless diffusivity (assumed to have a constant value of 0.25), $v^* = V\sqrt{f/2} =$ friction velocity, $R =$ Radius of pipe, $y' = y/R =$dimensionless vertical distance, $\overline{c}_j =$ average volumetric concentration of jth particle fraction, $\overline{C} =$ average concentration of particles in the pipe (v/v). The assumption in the development of the model is that the dimensionless diffusivity is constant and the particle concentration is a function of only vertical coordinate.

2.3. Modifications of the Karabelas Model

The assumption that the dimensionless diffusivity is constant has been contested. For instance, Seshadri et al. [26] proposed a correlation for dimensionless diffusivity through zinc tailings slurry flow experiment for three different velocities. This empirical correlation is given as;

$$\in = 0.07\left[+0.576\exp\left(\frac{3.29C_v}{C_{vss}}\right)\right] \tag{9}$$

Where; $C_v =$ volume concentration, $C_{vss} =$ static settled concentration. The empirical correlation does not show whether or not the fluid velocity has effect on the dimensionless diffusivity. Kaushal et al. [12] also came up with similar correlation for open channel slurry flow. Kaushal's correlation is given as;

$$\in = \epsilon_1 \beta \quad (10)$$

Where;

$$\beta = 1 + 0.09322\exp\left(\frac{5.5423C_{vf}}{C_{vss}}\right) \quad (11)$$

C_{vf} =efflux concentration, ϵ_1 =Liquiddiffusivity which was also shown to be varying with vertical coordinate and partly with friction velocity [48].

2.4. Modeling Methodology

This study modifies the calculation method of the dimensionless diffusivity going through mechanistic approach. The$k - \varepsilon$ model was employed to develop a mechanistic model that will include the effect of velocity fluctuation for predicting diffusivity and which was applied to Karabelas model to predict coal particle gradations in horizontal pipelines. There is so far no such development of a mechanistic model for predicting varying dimensionless diffusivity as this. The flow chart in Fig.1 shows the approach employed in this study to determine coal particle gradation in horizontal pipelines.

2.4.1. Root Mean Square Turbulent Velocity Fluctuation (u')

According to Davies [19], the eddy length for turbulent flow is related to turbulent velocity fluctuation for a single-phase flow and is represented as;

$$u'^3 = Pl \quad (12)$$

Where $l =$ eddy length (m) and $P =$ Power dissipated per unit mass (m^2/s^3). The eddy length is a function of pipe diameter calculated as;

$$l = \eta D \quad (13)$$

Where η is constant which can take any of the values viz 0.022, 0.038 and 0.07 depending on the flow channels. For the purpose of this study, we developed a correlation for η as a function of pipe diameter based on this range: $\eta = 0.07D for D \leq 150mm$, $\eta = 0.038\ for\ 150mm < D \leq 300mm$ and $\eta = 0.022 for\ 300mm < D \leq 600mm$. The newly proposed correlation is given as $\eta = 0.095e^{-3D}$. This correlation was used to evaluate η in this study.

and P is given as [9];

$$P = 2f\frac{V^3}{D} \quad (14)$$

Where $f =$ friction factor (-), $D =$ Pipe internal diameter (m), $V =$ flow stream velocity (m/s). The friction factor for turbulence flow in pipes is given as [49];

$$f = \begin{cases} \frac{0.079}{Re^{0.25}} for\ 4 \times 10^3 \leq Re \leq 10^5 \\ 0.0008 + \frac{0.05525}{Re^{0.237}} for\ 5 \times 10^5 \leq Re \leq 4 \times 10^7 \end{cases} \quad (15)$$

Where; $Re = \frac{\rho V D}{\mu}$ is the Reynolds' number, $\mu =$ fluid viscosity (m^2/s).Combining equation (12) and (13) gives the root mean square velocity fluctuation as;

$$u' = V\left(\frac{2fl}{D}\right)^{1/3} \quad (16)$$

When the stream velocity is equal or greater than critical velocity, particles are suspended in the fluid flow. Therefore, we set $V = V_d$, where V_d is the critical velocity calculated from the critical deposition velocity model of Wasp et al. [5] as;

$$V_d = 4\left(\frac{d}{D}\right)^{1/6} C_{vf}^{1/5}\sqrt{2gD(s-1)} \quad (17)$$

Where; $d =$ particle diameter (m), g =acceleration due to gravity (m/s^2), $s = \frac{\rho_s}{\rho} =$ particles to fluid density ratio (-) and $C_{vf} =$ particles efflux concentration [-].For a solid-liquid two phase flow, the hindered settling due to particles leads to higher velocity fluctuations. In this study, for a solid-liquid two phase flow, the function, ψ, which accounts for turbulent dissipation due to turbulent fluctuation is added in equation (16) to give the final expression for turbulent fluctuation velocity as;

$$u' = \psi V_d\left(\frac{2fl}{D}\right)^{1/3} \quad (18)$$

ψ is defined by equation (18) as;

$$\psi = \frac{1}{1+3.64C_{vf}} \quad (19)$$

2.4.2. Turbulent Kinetic Energy (k), Turbulent Dissipation Rate (ε) and Eddy Viscosity (μ_t)

The turbulent kinetic energy and the dissipation rate have most times been determined by the two-equation model normally solved using Computational Fluid Dynamics (CFD) software. In this study, the turbulent kinetic energy is calculated in terms of the root mean square velocity fluctuation as;

$$k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \quad (20)$$

Where; $\overline{u'}$, $\overline{v'}$ and $\overline{w'}$ are root mean square velocity fluctuation vectors in the x, y and z direction. For pipe flow it is assumed that $\overline{u'} = \overline{v'} = \overline{w'}$. For fluid velocity $V \geq 3.4\ m/s$, u' is calculated from equation (18). Therefore, equation (20) becomes;

$$k = \frac{3}{2}u'^2 \quad (21)$$

Equation (21) can still be used for fluid velocity up to $3.0\ m/s$. However, for greater accuracy, fluid velocity $V < 3.4\ m/s$ and u' is calculated from turbulent intensity as;

$$u' = VI_t \quad (22)$$

Therefore, k is calculated as;

$$k = \frac{3}{2}(VI_t)^2 \quad (23)$$

Where; $I_t = \frac{u'}{V}$ is the turbulent intensity calculated as a function of Reynold's number for pipeline slurry flow;

$$I_t = 0.16R_e^{-1/8} \quad (24)$$

The turbulent dissipation rate is calculated in terms of turbulent kinetic energy and eddy length using the relation;

$$\varepsilon = \frac{C_\mu k^{3/2}}{l} \quad (25)$$

Where; $C_\mu = 0.09$

The eddy viscosity is also determined from k and ε as;

$$\mu_t = \frac{C_\mu k^2}{\varepsilon} \quad (26)$$

2.4.3. Stokes Number (St) and Turbulent Schmidt Number (SC_t)

The Stokes number is the ratio of particle times scale, τ_s, and turbulent turnover,τ_t, Mathematically, given as;

$$St = \frac{\tau_s}{\tau_t} \quad (27)$$

The particle time scale is calculated as;

$$\tau_s = \frac{\omega_s}{g(1-{}^1/_s)} \quad (28)$$

and the turbulent turnover is given as;

$$\tau_t = \frac{\gamma k}{\varepsilon} = \frac{\gamma \mu_t}{C_\mu k} \quad (29)$$

Where; $\omega_s =$ particle settling velocity (m/s), g =acceleration due to gravity (m/s^2), $s = \frac{\rho_s}{\rho} =$ Particles to fluid density ratio (-) and $\gamma =$ constant (taken as 1.0).Absi et al. [50] derived the turbulent Schmidt number from the two-fluid model and Kinetic model and it is given as;

$$SC_t = \left(\frac{St}{1-1/s} + \frac{1}{1+St}\right)^{-1} \quad (30)$$

The Particle settling velocity for single particle for Reynold's number (R_p) is within the range;

$1 \leq R_p \leq 1000$, is given by Yan, [4] as;

$$\omega_o = \left[\frac{2g}{27}\left(\frac{\rho_s-\rho}{\rho}\right)\right]^{5/7} \frac{\rho^{3/7} d^{8/7}}{\mu^{3/7}} \quad (31)$$

The actual settling velocity due to hindered settling is obtained by considering the hindered settling term and is represented as;

$$\omega_s = \omega_o\left(1 - C_{vf}\right)^z \quad (32)$$

$z$is a function of particle and pipe size originally defined by Richardson and Zaki [28] for a range of particle Reynolds' number, R_p. It is represented as;

$$z = 4.65 + 19.5\left(\frac{d}{D}\right) for R_p < 0.2 \quad (33a)$$

$$z = 4.35 + 17.5\left(\frac{d}{D}\right) for\ 0.2 < R_p < 1.0 \quad (33b)$$

$$z = 4.45 + 18\left(\frac{d}{D}\right) R_p^{-0.1} for R_p > 1.0 \quad (33c)$$

$$R_p = \frac{\omega_o d}{\nu} \quad (34)$$

Where; $d =$ particle diameter (m), $\nu =$ the kinematic viscosity of fluid, $\rho =$ fluid density (Kg/m^3), and $\rho_s =$ particle density (Kg/m^3),

2.4.4. Turbulent Diffusivity (ϵ) and Dimensionless Turbulent Diffusivity (ξ)

The turbulent diffusivity, ϵ, is modeled as;

$$\epsilon = \frac{\mu_t}{SC_t} \quad (35)$$

The dimensionless turbulent diffusivity, ξ, is determined from the turbulent diffusivity and the radius of pipe by the formula;

$$\xi = \frac{\epsilon}{Rv^*} \quad (36)$$

Substituting equations 25-30 and 35 into (36) with $v^* = V\sqrt{f/2}$ and simplifying gives the mathematical model for calculating dimensionless diffusivity as;

$$\xi = \frac{2\eta}{V}\left(\frac{2k}{f}\right)^{1/2}\left[\frac{\varepsilon\omega_s}{\gamma g k(1-\rho/\rho_s)^2} + \frac{\gamma g k(1-\rho/\rho_s)}{\varepsilon\omega_s + \gamma g k(1-\rho/\rho_s)}\right] \quad (37)$$

Equation (37) shows that the dimensionless diffusivity ξ, is not constant. Dimensionless diffusivity can be calculated directly from equation (37) or by performing step-by-step calculations from equation (13) to (36).

3. RESULTS AND DISCUSSION

3.1. Results Verification and Model Validation

To validate the modified Karabelas model presented in this work, experimental data of Roco and Shook, [51], Schaan et al. [52], Gillies et al. [53] and Kaushal and Tomita, [54] were used. The carrier fluid used in these experiments is water (density, $\rho = 998\ Kg/m^3$) and the suspended particles (disperse phase) is sand. Data from CFD analysis of sand-water multiphase flow in pipeline carried out by Ekambara *et al.* [30] was also employed to further validate the performance of the modified model. Details of CFD data used in this study can be found in [30]. The experimental data are presented in Table 1. The modified model of this work uses the pipe center as the origin. From the center upward is a positive vertical distance and downward is negative. Therefore, the vertical distance is given as $-R \leq y \leq R$. For dimensionless vertical distance, it is given as $-1 \leq y/R \leq 1$. In order to compare our results with experimental data and with the pipe bed as the reference point, the dimensionless vertical distance is converted as follows; $0 \leq y/D \leq 1 = \frac{1}{2}(1 + (-1 \leq y/R \leq 1))$.

Fig.2. shows the comparison of this work with the data of Roco and Shook, [51]. This work predicted the concentration profile which is in good agreement with the experimental data and compares well with CFD results. There are deviations in Fig 2, R3 and R4. This is because, the flow velocity is much closer to the transition velocity, 3.4 m/s; the velocity which determines whether the root mean square velocity fluctuation will be calculated from equation (18) or equation (22). The original Karabelas model with $\xi = 0.25$ could not predict the concentration profile of Roco and Shook's, [48] experimental data. This is because, ξ is not varying with the parameters influencing the concentration profile (Eq 1).Validation with Schaan et al. [52] experimental data is presented in Fig. 3. The plots show good agreement of the model developed in this work and experimental data. The original Karabelas model also predicts the concentration profile. For this experimental data, it is observed that the fixed value of $\xi = 0.25$ coincides with the varying value of ξ obtained from the modified model presented in this work.Fig 4 shows the validation of the modified model presented in this work with the experimental data of Gillies et al. [53]. Reasonable agreement between the data obtained from experiment and the model in this work is observed. The developed model prediction of the concentration profile competes favorably with the CFD model of Ekambara et al. [30] in all the six data of Gillies et al. [53].The mechanistic model was also compared with the experimental data of Kaushal and Tomita, [54] and the results are shown in Fig 5. The figures K1, K2, K3 and K4 of Fig 5 show good agreement between the modified model and experimental data. Apart from K2 (of Fig 5) which deviated at larger particles diameter. The deviation of the new model in Fig 5 K2 is similar that of Fig 2 R3 and both have larger particles diameter. This phenomenon can be explained as follows: First, some of the larger particles may not have been peaked up by the fluid stream due higher inertia, rather they may have rolled along the pipe bottom thereby occupying the bottom of the pipe. Secondly, larger particles that are picked up in the stream may not have settled exactly as predicted by equation (32-34), rather, they may have higher concentration in the region below the main stream (y/D < 0.5) due repeated peaking up by turbulence and settling due higher inertia and lower concentration in the bed.The model of the work captured the experimental data points better than the CFD results. The original Karabelas model also shows good agreement with experimental data of Schaan et al. [52] as shown in S1 and S2 of Fig 3.

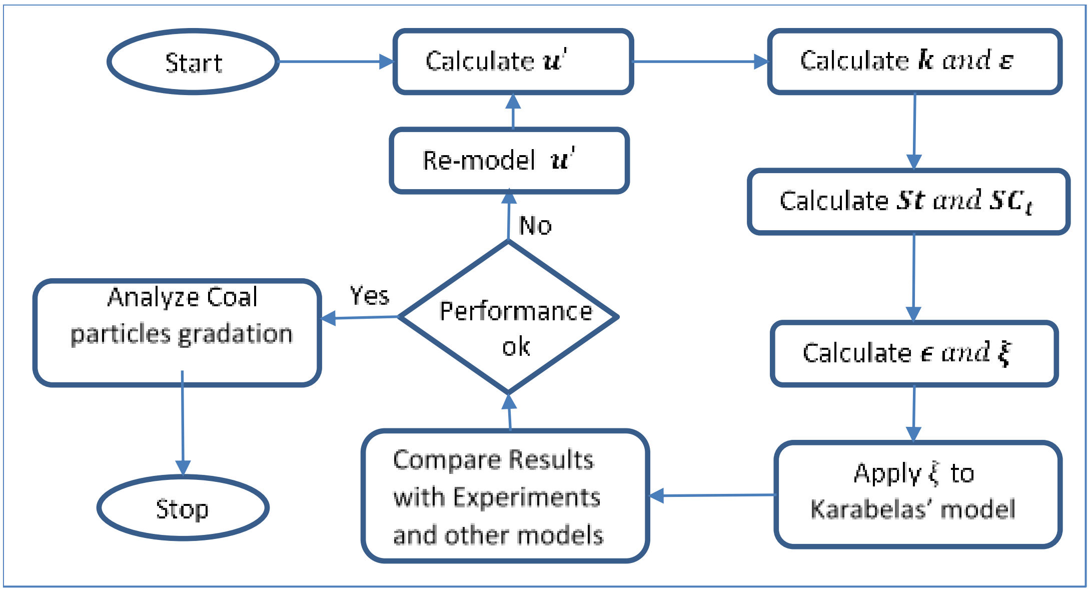

Figure 1. Modeling Flow Chart

Table 1. Experimental Data

Author	Designation In Plot	Pipe Diameter (mm)	Particle Density (Kg/m^3)	Particles Mean diameter (μm)	Average Efflux Concentration (-)	Fluid Velocity (m/s)
Roco & Shook (1983)	R1	51.5	2650	165	0.0918	3.78
	R2	51.5	2650	165	0.286	4.33
	R3	51.5	2650	480	0.203	3.44
	R4	263	2650	165	0.0995	3.5
	R5	495	2650	165	0.104	3.16
Schaan et al. (2000)	S1	150	2650	90	0.32	3.0
	S2	150	2650	90	0.39	3.0
Gillies et al. (2004)	G1	103	2650	270	0.10	5.4
	G2	103	2650	270	0.20	5.4
	G3	103	2650	270	0.30	5.4
	G4	103	2650	270	0.40	5.4
	G5	103	2650	90	0.19	3.0
	G6	103	2650	90	0.33	3.0
Kaushal & Tomita, (2007)	K1	54.9	2470	125	0.20	3.0
	K2	54.9	2470	440	0.20	3.0
	K3	54.9	2470	125	0.30	3.0
	K4	54.9	2470	125	0.40	3.0

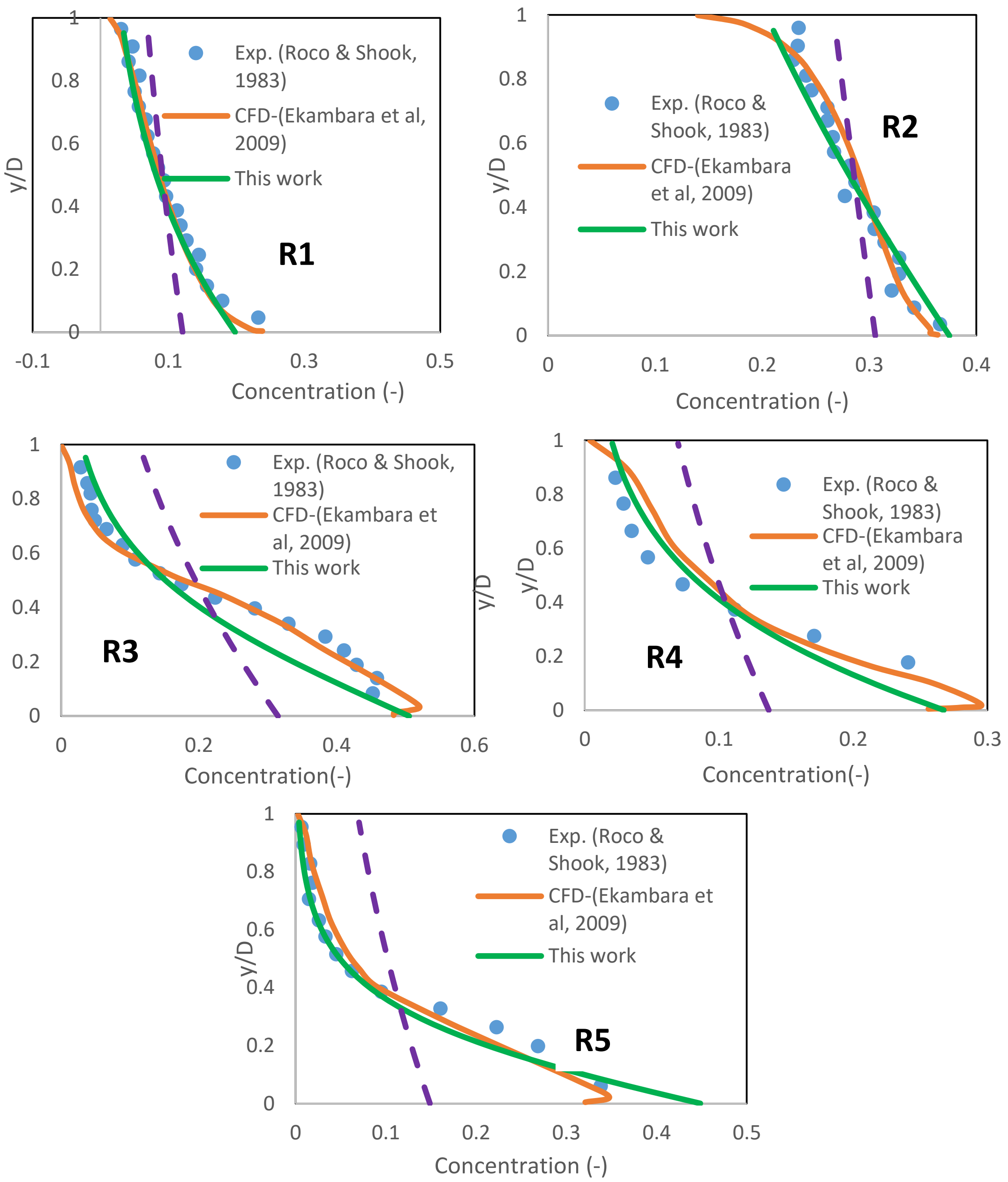

Figure 2. Validation with Roco and Shook [51] experimental data

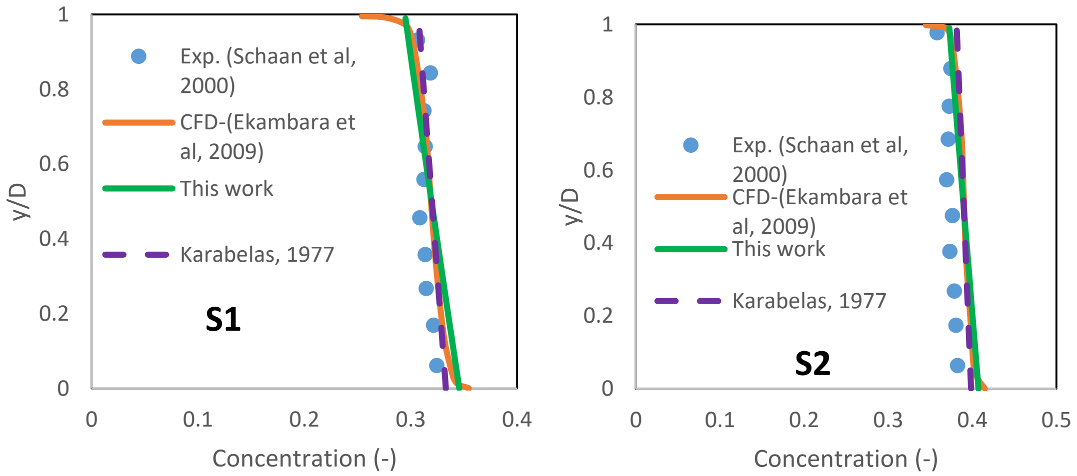

Figure 3. Validation with Schaan et al. [52] experimental data

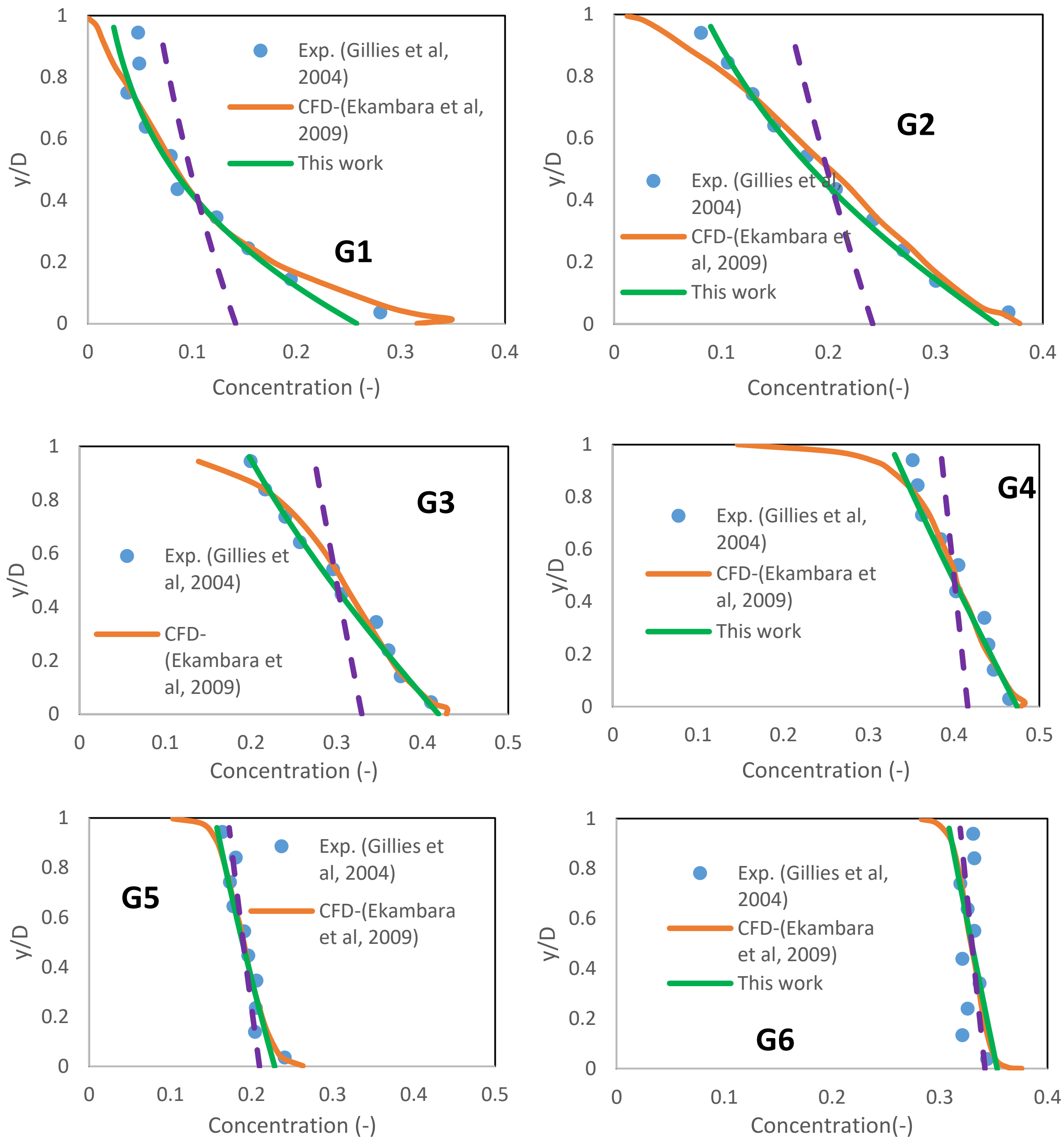

Figure 4. Validation with Gillies et al. [53] experimental data

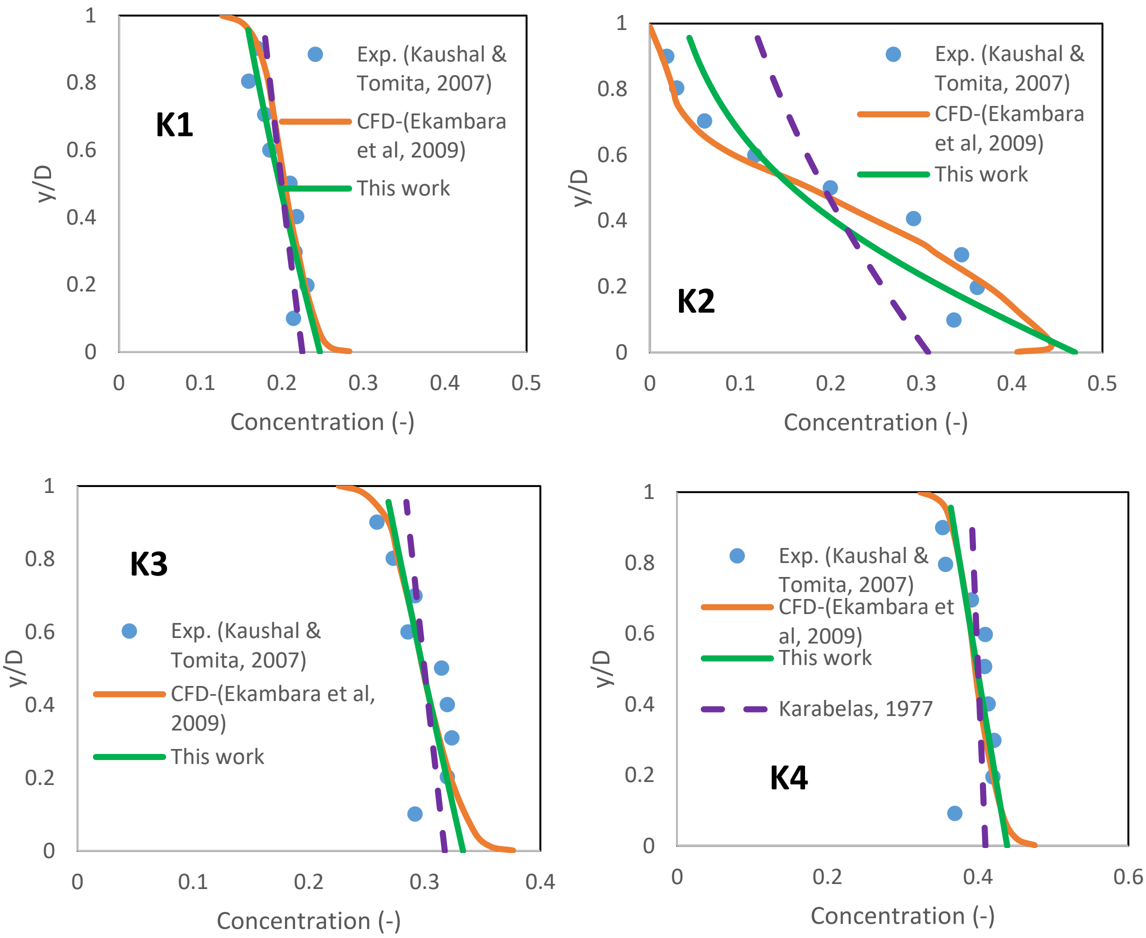

Figure 5. Validation with Kaushal and Tomita [54] experimental data

3.2. Coal Particles Gradation in Horizontal Pipeline

Coal particle concentration at different levels in horizontal slurry flow is examined in this section. The property of the prototype coal slurry is given in Table 2. Particles concentration at the different levels in Fig 6 is investigated to determine the grade of particles dominant in each level. Each coal particle size having a concentration value equal to 45% is examined. Also, the combined coal particles of different sizes each having an efflux concentration of 9% in a multi-sized particulate slurry flow (containing five different particle sizes) having a total concentration of 45 % is examined at different vertical levels of the pipe cross section. Here, we also checked if concentration profile determined from the median (or mean) particle size (**d_{50}**) actually represents the concentration profile in the pipe.

Fig 6 shows the different vertical levels in the pipe cross section. The vertical distance along the radius of the pipe cross section is given as $-R \leq y \leq R$, so point R represents the topmost inner wall of the pipe and -R represents the bottom inner wall. Point -0.9R best represents the pipe bed, the point of maximum concentration if all the particles sediment at V = 0.0 m/s. The concentration of particles of different sizes is examined at point -0.9R, -0.5R, 0.0R, 0.5R, and 0.9R, where 0.0R is the pipe center.

3.2.1. Homogenous Size of Coal Particles Gradation

In this analysis, the slurry is assumed to contain homogenous (single) particle size with efflux concentration of 45%. The variation of concentration of the coal particles at the different vertical levels for different velocities is examined. Particle diameter range of 1.0-800 μm was used for the analysis. The results of these analyses are shown in Fig 7. At the center of the pipe (R=0), concentration decreases with particle sizes at lower velocities and are almost constant at higher velocities. At the bed (-0.9R) and near bed (-0.5R), the concentration increases with increase in coal particle sizes at all the reference velocities, though the rate of increase in concentration reduces as the velocity increases, indicating that at higher velocity, the turbulent intensity increases causing the particles to be dispersed within the carrier fluid. At, the upper (0.5R and 0.9R) section of the pipe, the concentration decreases with increase in particles sizes at all the reference velocities showing that larger particles tend to occupy the lower section of the pipe cross section. One important observation from this analysis is that the concentration at all velocities and in all section of the pipe approaches the reference efflux concentration as the particle sizes approach 1.0 μm. That is, very small coal particle size (around 1.0 μm) will always have uniform concentration at all velocity.

Table 2. Coal Pipeline Data

Coal Type	Velocity (m/s)	Density (Kg/m^3)	Concentration (-)	Particles Mean diameter (μm) Case 1	Particles Mean diameter (μm) Case 2	Pipe Diameter (mm)
Anthracite and Semi-anthracite	0.5 to 5.4	1470	0.09	90	150	495
		1470	0.09	165	160	495
		1470	0.09	270	170	495
		1470	0.09	480	180	495
		1470	0.09	800	190	495

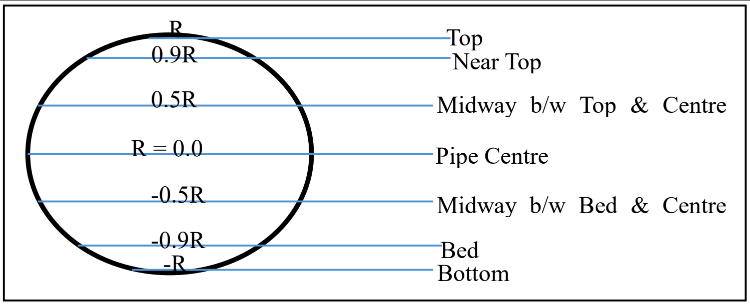

Figure 6. Cross Section of Coal Pipeline

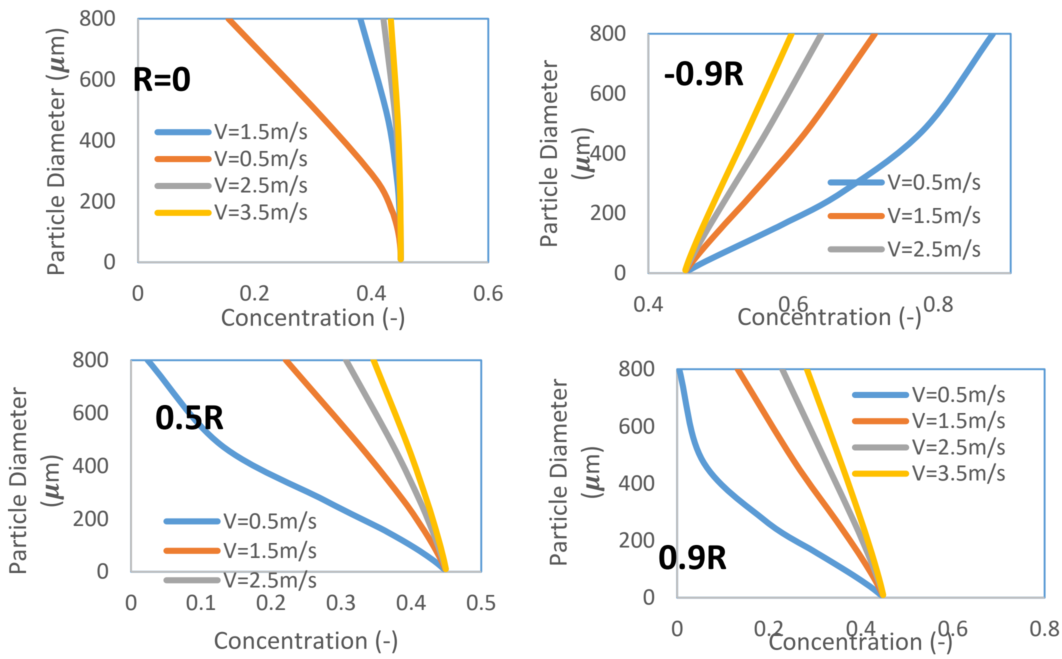

Figure 7. Isolated coal particles grade at different vertical levels in the pipeline

3.2.2. Multi-sized Coal Particles Gradation

In this analysis, the slurry is assumed to contain multi-sized particles, each having an efflux concentration of 9.0% in a mixture with total efflux concentration of 45%. Two mixture slurry containing five (5) different particles sizes are examined. The first size mixture (Case 1) contains a wide particle size range of 710 μm; the second size mixture (Case 2) has a narrow particle size range of 40μm. The five particles' sizes in mixture case1 and case2 and their concentration are tabulated in Table 1. Fig 8 shows that in case 1 mixture, the larger particles have concentration profile which are concave and closer to the x-axis, indicating that larger particles occupy the lower part (-0.5R and -0.9R) of the pipe vertical cross section. The smaller sized coal particles have slightly convex profile which shows that the smaller sized particles occupy the middle and the upper level in the pipe cross section. For case 2 mixture, the concentration profiles of the particles are much closer and have similar shape. This is because the particle sizes are much closer to each other. At higher velocity, the concentration profile will likely be the same for all the particle sizes. Table 2, 3, 4 and 5 show the percentage of the total concentration occupied by each particle sizes at the different vertical levels of the pipe cross section for mixture slurry flow case 1 and case2. For case 1 mixture slurry flow (Table 2 and Table 3), the larger particles mostly occupied the lower section with about 40-70% of total concentration at the level depending on the flow velocity, while the smaller sized particles occupied the upper section with similar percentages. For mixture case 2 (Table 4 and Table 5), the different particle sizes compete for equal space, meaning that there is not much difference in their concentrations in all the levels in the pipe section, though, there is an indication that larger sized particles occupied the lower section and smaller sized particles occupied the upper section of the pipe cross section.

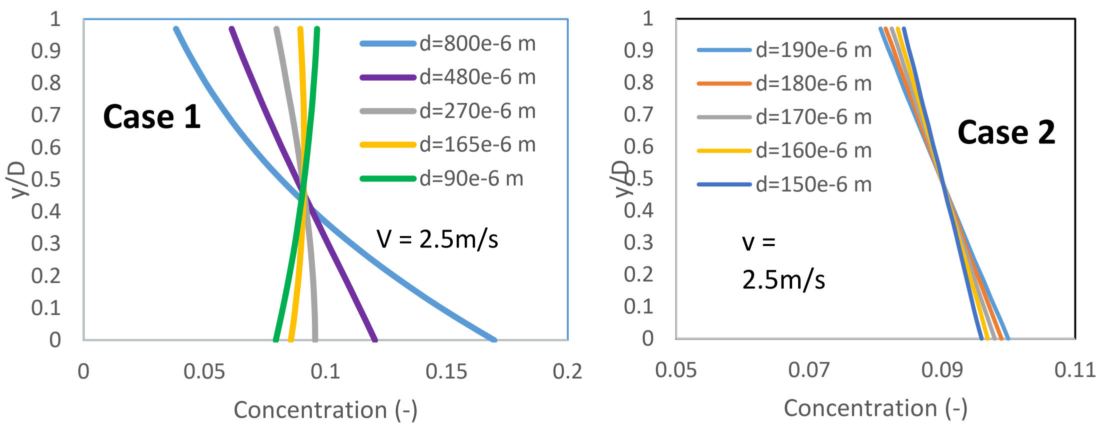

Figure 8. Concentration profile for different coal particle grades in a mixture slurry flow

Table 3. Coal Particle grades concentration at different vertical levels in the pipeline for particle size variation of Case 1 @ flow speed of 0.5m/s

Particle Size (μm)	Average Efflux Conc.	% of Total Conc. at -0.9R	% of Total Conc. at -0.5R	% of Total Conc. at R=0	% of Total Conc. at 0.5R	% of Total Conc. at 0.9R
	Case 1 @ v = 0.5m/s					
90	0.09	7.4972	15.6130	27.1837	37.2703	44.0670
165	0.09	9.6193	17.6334	26.1766	30.6001	31.8476
270	0.09	13.4014	20.2578	23.6312	21.7076	18.6303
480	0.09	23.4803	23.3995	16.2152	8.8485	5.0065
800	0.09	46.0018	23.0962	6.7934	1.5735	0.4485
Total	0.45	100.0000	100.0000	100.0000	100.0000	100.0000

Table 4. Coal Particle grades concentration at different vertical levels in the pipeline for particle size variation of Case 1 @ flow speed of 2.5m/s

Particle Size (μm)	Average Efflux Conc.	% of Total Conc. at -0.9R	% of Total Conc. at -0.5R	% of Total Conc. at R=0	% of Total Conc. at 0.5R	% of Total Conc. at 0.9R
	Case 1 @ v = 2.5m/s					
90	0.09	15.0097	17.4644	20.6089	23.7483	26.2057
165	0.09	16.0641	18.1173	20.5620	22.7883	24.3743
270	0.09	17.7488	19.0958	20.4325	21.3492	21.7836
480	0.09	21.7587	21.1444	19.9215	18.3284	16.8914
800	0.09	29.4188	24.1780	18.4752	13.7859	10.7451
Total	0.45	100.0000	100.0000	100.0000	100.0000	100.0000

Table 5. Coal Particle grades concentration at different vertical levels in the pipeline for particle size variation of Case 2 @ flow speed of 0.5m/s

Particle Size (μm)	Average Efflux Conc.	% of Total Conc. at -0.9R	% of Total Conc. at -0.5R	% of Total Conc. at R=0	% of Total Conc. at 0.5R	% of Total Conc. at 0.9R
	Case 2 @ v = 0.5m/s					
150	0.09	18.7100	19.3988	20.2776	21.1755	21.9068
160	0.09	19.3373	19.7014	20.1483	20.5852	20.9267
170	0.09	19.9824	20.0023	20.0095	19.9970	19.9730
180	0.09	20.6451	20.3009	19.8611	19.4119	19.0463
190	0.09	21.3253	20.5967	19.7035	18.8305	18.1472
Total	0.45	100.0000	100.0000	100.0000	100.0000	100.0000

Table 6. Coal Particle grades concentration at different vertical levels in the pipeline for particle size variation of Case 2 @ flow speed of 2.5m/s

Particle Size (μm)	Average Efflux Conc.	% of Total Conc. at -0.9R	% of Total Conc. at -0.5R	% of Total Conc. at R=0	% of Total Conc. at 0.5R	% of Total Conc. at 0.9R
	Case 2 @ v = 2.5m/s					
150	0.09	19.6301	19.8015	20.0168	20.2333	20.4073
160	0.09	19.8123	19.9000	20.0091	20.1176	20.2039
170	0.09	19.9973	19.9992	20.0007	20.0009	20.0002
180	0.09	20.1850	20.0993	19.9916	19.8833	19.7963
190	0.09	20.3754	20.2000	19.9819	19.7649	19.5923
Total	0.45	100.0000	100.0000	100.0000	100.0000	100.0000

3.2.3. Hindered Settling due to Multi-sized Coal Particles in Slurry Flow

In the course of the analysis of particle gradation, it was observed that the concentration profile of any particular coal particle size when in mixture with other particle sizes in multi-sized particulate flow and when the same sized coal particles is in homogenous slurry flow, is not the same even when their concentration is the same. Fig 9 shows that coal particles with $90\mu m$ and $180\mu m$ diameter in homogenous slurry flow (isolated) and multi-sized slurry flow mixture (case 1 and 2 for $90\mu m$ and $180\mu m$ respectively) and 9.0% efflux concentration in both cases have different concentration profiles. From Fig 9 and Fig 8, at flow velocity of 0.5m/s, Coal particles in multi-sized particulate flow occupy more of the upper section of the pipe, leaving the lower section for larger particles. Thus, the concentration profile shows a slight increase in concentration with increase in vertical distance. In homogenous (single-sized) flow, the particles are seen to have a concave concentration profile that is closer to the x-axis, indicating that the particles occupy more of the lower section of the pipe, that is, the particles settle with little resistance. This can be explained by assuming that the settling of the smaller sized particles is hindered by the larger sized particles and vice versa. The same is observed at a velocity of 2.5 m/s, however, the variations in the concentration profiles diminishes with an increase in the flow velocity. For instance, at 5.4m/s, the concentration profiles almost assume the same shape and slope.

3.2.4. d_{50} Concentration Profile of Multi-sized Coal Particulate Slurry Flow

Most times, it is not convenient to determine all the different particle sizes that constitute a slurry flow. Researchers use the median (or mean) size (d_{50}) of the particles to represent the bulk mixture. In this analysis, the accuracy of median size concentration profile is examined using the case1 and case 2 variations of particle sizes in mixture as shown in Table 2. The actual (Total) concentration profile is the sum of the concentration profile of each (jth) particle size. Then the d_{50}concentration profile is the profile determined by representing all the different particle sizes by the median (or mean) size. That means, the different particles are assumed to have one size (the median or mean size) and a concentration equal to the concentration of the entire mixture. The mean or the median size of particles can be determined by sieve analysis. For case 1, the median and mean size of coal particles are $270\mu m$ and $361\mu m$respectively and $170\mu m$ for case 2 mixture. The results of this analysis are shown in Fig 10. The d_{50}concentration profile represents the actual concentration at all the reference velocities accurately, especially for case 2 mixture where the range of particle sizes is $40\mu m$. From Fig 10, it can be deduced that the wider the range of sizes, the less accurate d_{50}concentration profile will represent the actual concentration profile because case 2 mixture shows some noticeable difference between the concentration profiles.

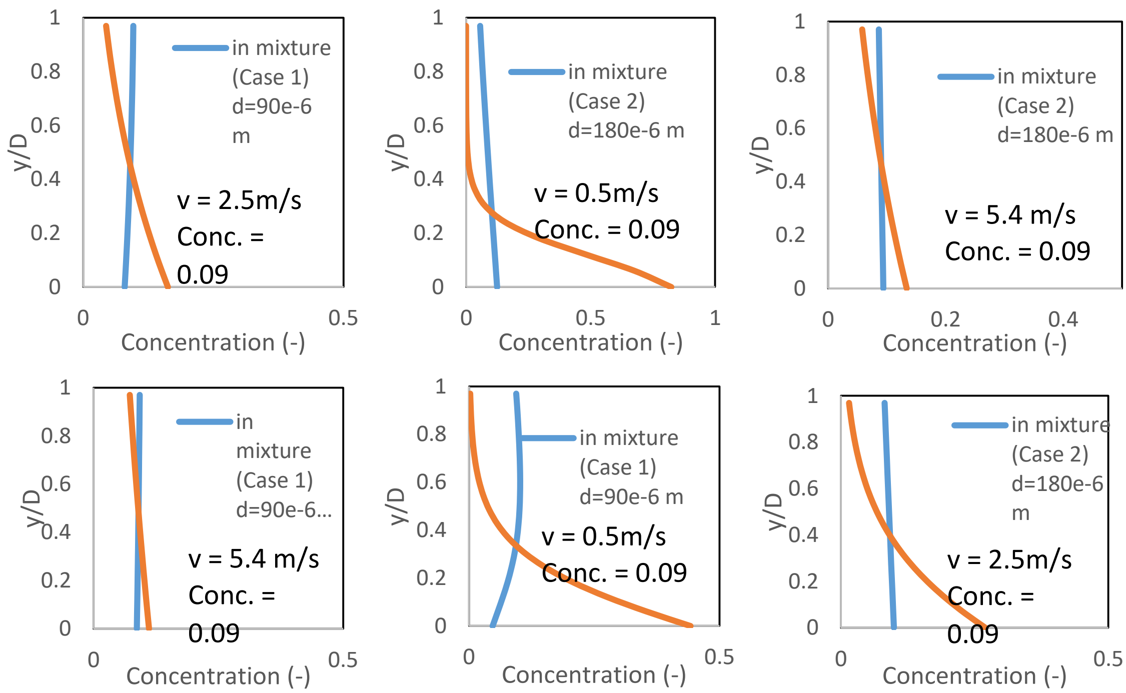

Figure 9. Effect of hindered settling due variation of particles'sizes in slurry flow on concentration profile

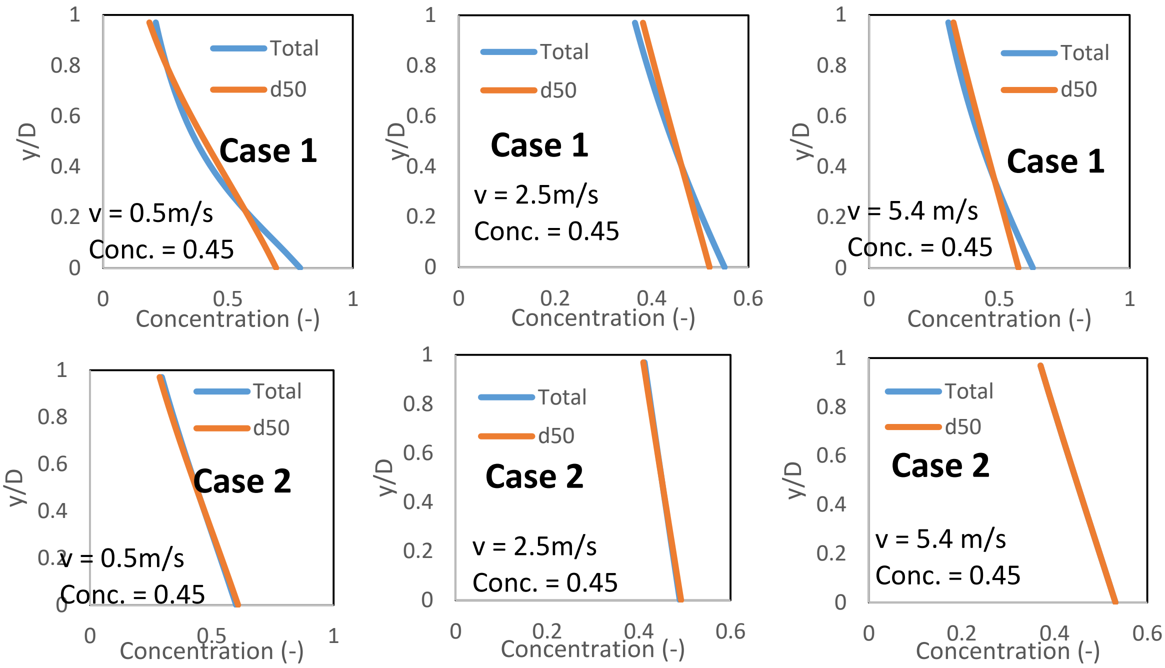

Figure 10. Comparing d_{50} concentration profile with actual profile

3.2.5. Effect of Velocity Fluctuation

The dimensionless diffusivity is more constant but is dependent on the several key parameters of the solid-liquid two phase flow. Therefore, the modified model is expected to be much responsive to changing parameters. In this section, the dynamic responses of the modified model with respect to varying mean square turbulent fluctuation velocity is evaluated. Velocity fluctuation values range from 0.5-1.5m/s to 0.001-0.01m/s and were used in the evaluation. The result is presented in Fig 11. From Fig 11, the velocity fluctuation increase leads to more uniform distribution of particles. This result is expected because velocity fluctuation is a function of fluid velocity, that is, increase in the slurry velocity leads to increase in velocity fluctuation and vice versa. Fig 11(A) shows that the concentration profile tilted closer to vertical as the velocity fluctuation increases. Vertical concentration profile means absolute uniform particle distribution. Fig 11(B) shows that the concentration profile tilted closer to horizontal as the velocity fluctuation dropped to 0.001m/s indicating that the particles occupied the lower half of the pipeline diameter.

3.2.6. Effect of Pipe Diameter

The effect of pipe diameter on the concentration was investigated using the modified model. Particle sizes of 90µm and 480µm were used to investigate the concentration profile of sand particles in water at concentration of 0.45 (-) and 0.09 (-). Pipe diameter of 51.5mm, 103mm, 263mm and 495mm were analyzed. The results of the analysis were shown in Fig 12. From the results, it was observed that larger pipe diameters have higher concentration margin between the top and the bed of the pipe due to non-uniform particle distribution. More particles are found in the near bed region of the pipeline for pipe diameter of 495mm at a concentration of 0.09 (-) as shown in Fig 12(B). According to the modified model prediction in Fig 12, particles will tend to settle in the bed of larger diameter pipeline than in smaller diameter pipeline. This observation is consistent with the CFD analysis result of Zhang et al. [55]. Turbulent kinetic energy, which is the force that causes particles to remain suspended and dispersed in the carrier fluid, decreases with increase in pipe sizes at constant velocity.

3.2.7. Effect of Particles' Density

Particle density influence on the concentration profile in slurry pipeline is also analyzed using the modified model. Particle density range of 1470kg/m^3, 1950kg/m^3, 2650kg/m^3 and 3650kg/m^3, pipe diameter of 263mm, efflux concentration of 0.09 (-) and 0.45 (-), particle diameter of 480µm and velocity of 3.0m/s were considered in the analysis. The results of the analysis show that particle density also influences the concentration profile. Highly dense particles tend to sink to the pipe bed as shown in Fig 13. The effect of density is significant. For instance, in Fig 13(A), the concentration profile is observed to have tilted toward the horizontal as the density increases and vice versa. The higher the particle density, the higher the turbulent kinetic energy required to lift and transport it as suspended particles in the pipeline. The modified model captured this in the prediction of effect of density on the concentration profile.

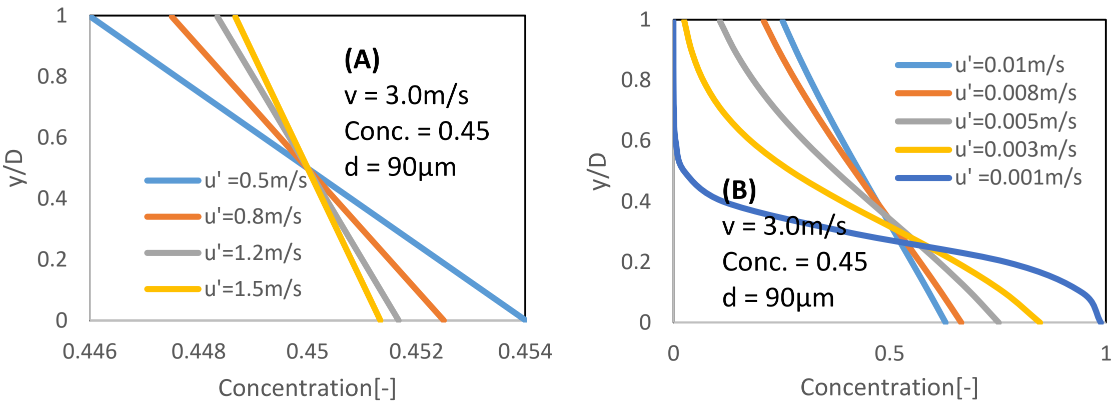

Figure 11. Effect of mean square velocity fluctuation on concentration profile

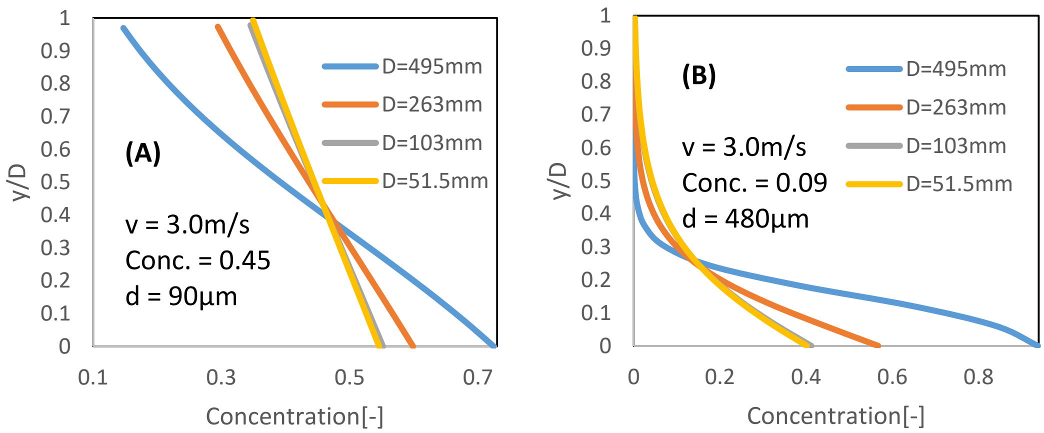

Figure 12. Effect of pipe diameter on concentration profile

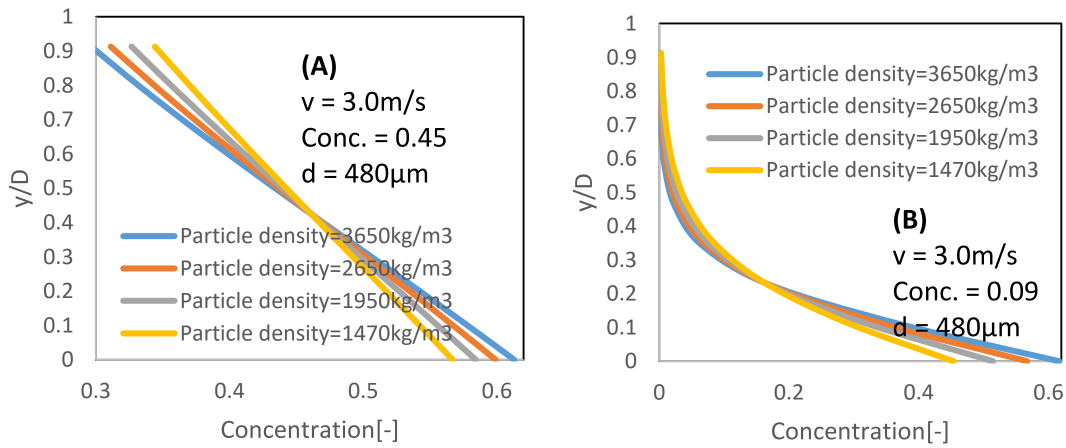

Figure 13. Effect of particle density on concentration profile

4. CONCLUSIONS

In this study, a mechanistic model for calculating a varying dimensionless diffusivity was developed and applied to modify Karabelas model. The modified model has been tested against experimental data of four different articles and found to have excellent agreement. Also, CFD model's result was also used to validate the modified model presented in this study. The mechanistic model having being validated to be accurate was employed to study coal particle gradation in slurry flow and the results are in line with the findings of others and with what is scientifically expected.

The following conclusions were drawn from this study.

1. The dimensionless diffusivity originally assumed to be constant in Karabelas model depends on several parameters such as the flow velocity, pipe diameter, turbulent fluctuation velocity, particle size, density and efflux concentration. This study incorporated the effect of velocity fluctuation for predicting diffusivity which was also applied to Karabelas model to predict coal particle gradations in horizontal pipelines.
2. Findings proved that the modified model (Mechanistic) presented in this study is responsive to the changes in each of the parameters in (i), thus giving excellent

prediction of concentration profile at various conditions of flow.

3. Findings show that the particles of coal in a slurry flow is graded such that the concentration of the particles increase with increase in sizes at the lower section of the pipe. The trend reverses at the upper section of the pipe.
4. Findings show that the particle concentration profile when in multi-sized particulate slurry flow is not the same as when in homogenous (single-sized) slurry flow; the reason being the existence of hindered settling due to multiple sizes of particles in the slurry flow in which the smaller particles tend to occupy the upper levels leaving the lower levels of the pipe for the larger particles.
5. The d_{50}concentration profile accurately represents the actual concentration profile of the range of particle sizes: narrow and moderately narrow. The study establishes that accuracy of d_{50}concentration profile diminishes as the range of particle sizes increase

Conflicts of interest

The authors said, to the best of their knowledge there was no conflict of interest at the time of this study.

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