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Design Optimization of Selected Mechanical Engineering Components using Variants of Rao Algorithms

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Abstract

The present paper proposes self-adaptive multi-population elitist (SAMPE) Rao algorithms and chaotic Rao algorithms for design optimization of selected mechanical engineering components. The proposed algorithms are applied to 25 benchmark problems and 15mechanical engineering design optimization problems to examine their performance. The Friedman rank test is utilized to demonstrate the significance of the proposed algorithms and the algorithms are ranked according to their performance. The results obtained using the proposed algorithms are compared with the results obtained using other advanced optimization algorithms to demonstrate the effectiveness of the proposed algorithms. It is observed that the performance of the proposed algorithms is either better or competitive to the basic Rao algorithms and the other advanced optimization algorithms.

Keywords: Design optimization, Rao algorithms, Self-adaptive multi-population elitist Rao algorithms, Chaotic Rao algorithms.

1. Introduction

Engineering design is a process of decision-making to achieve certain goals while satisfying the constraints and human needs[1]. The optimum engineering design topic is very important to achieve the best product in terms of performance parameters related to the product. In the field of Mechanical engineering design optimization problems, the designers consider specific objectives, such as deflection, wear, strength, weight, corrosion, etc. A complete mechanical system design optimization leads to a complex mathematical formulation with many design variables. Thus, the optimization techniques are applied to individual components than a complete mechanical system[2].The mathematical model formulation in engineering design optimization is a complex task for researchers and designers. The design optimization problems include design variables (continuous, integer and discrete variables)and design constraints. For the good performance of engineering elements, design constraints must be satisfied.

In the present study, the proposed algorithms are applied to 25 benchmark problems and 15 standard design optimization problems. The design optimization problems are difficult to solve as they depend on many design variables and must satisfy certain design constraints. The design optimization problems solved in the present work are from mechanical engineering. To solve any design optimization problems, first the mathematical model of that problem is formulated. The mathematical model consists of the design objective, design variables and design constraints associated with the problem. In the present work, the mathematical models of design optimization problems are taken from the literature and the proposed algorithms are applied to those problems.

The researchers have developed different metaheuristic algorithms to solve design optimization problems. However, any single optimization algorithm cannot solve all the problems efficiently. So, researchers try to develop new algorithms or modify the existing algorithms to get more efficient results. They have proposed multi-population optimization algorithms based on different advanced optimization algorithms. The sub-population-based algorithms divide the population into the number of sub-population and thus increase the diversity of the search process. The number of sub-populations changes adaptively according to the improvement in solution after each iteration. The diversity of the search process can further be improved by the inclusion of elitism approach in multi-population based Rao algorithms[3]. So, in the present study the concept of elitism is integrated with multi-population based Rao algorithms.

In recent times, researchers have found that the solution's quality can be improved by combining optimization algorithms. Chaos is one of the techniques that can be combined with different optimization algorithms to solve optimization problems of various engineering fields. Chaotic systems are nonlinear dynamical systems that are very sensitive to their initial conditions. Chaos search is a powerful technique for hybridization because of its dynamic characteristics. Chaos

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can be included in advanced optimization algorithms in three ways (1)by substituting the random numbers of the algorithm with chaotic numbers generated using chaotic maps,(2)by using the chaotic map function for the local search approaches, (3)by chaotically generating the control parameters of algorithms. In the present work, chaos is incorporated in Rao algorithms by replacing the random numbers with the chaotic sequence that is produced by the Chebyshev chaotic map.

The rest of the paper has the following structure: section 2 presents the previously proposed advanced optimization algorithm with the concept of multi-population and hybridization to solve optimization problems. Section 3 describes the Rao, SAMPE Rao and Chaotic Rao algorithms. Section 4 describes the application of the proposed algorithms to solve benchmark problems and design optimization problems. Section 5 discusses the results obtained after applying the proposed algorithms to different problems. Section 6 presents the conclusions of the present study.

2. Literature Review

Rao and Saroj [3]presented self-adaptive multi-population elitist (SAMPE) Jaya algorithm to solve engineering optimization problems. The elitism concept was used to improve the search mechanism of the proposed algorithm. SAMPE Jaya algorithm was applied to benchmark problems, mechanical design optimization problems and micro-channel heat sink design optimization problem to evaluate its performance. Alatas[4]proposed Harmony Search (HS) algorithm that uses chaotic maps for parameter adaptation. The classical HS algorithm was integrated with seven chaotic maps and they were applied to benchmark function to test their performance. The results demonstrated that the quality of the solution was improved for some optimization problems and in some cases global searching capability of the algorithm was enhanced. Zhang and Ding[5]presented a Multi-Swarm Self-Adaptive and Cooperative Particle Swarm Optimization (MSCPSO)algorithm. MSCPSO divides PSO into four sub-swarms. All sub-swarms update the records adaptively and cooperatively. It enhances the diversity of the search method and prevents the premature convergence of the algorithm. Gandomi et al.[6]presented a firefly algorithm(FA) with chaos. FA with 12chaotic maps was employed to find the optimum results for benchmark functions. FA mimics the social behavior of fireflies based on flashing and attraction characteristics of fireflies. FA with chaos increases the effectiveness of the search process to find the global optimum.

Rao and Patel [7]presented a modified teaching learning based optimization (TLBO) algorithm. They applied it to solve the multi-objective optimization of shell and tube heat exchanger and plate-fin heat exchanger. The cost minimization of the exchanger and maximization of heat exchanger effectiveness are two objectives. The results proved the effectiveness and accuracy of the proposed algorithm are better than other algorithms. Rao and Saroj [8]presented a self-adaptive multi-population based Jaya (SAMP-Jaya) algorithm to solve optimization problems. The exploration and exploitation rates were controlled by dividing the population into sub-populations. The proposed algorithm was applied to benchmark problems and plate-fin heat exchanger optimization problem. The results indicated the effectiveness of the SAMP Jaya algorithm. Farah and Belazi [9] presented chaotic Jaya algorithm by implementing three new mutation strategies. The chaotic sequence generated using the proposed 2D chaotic map was integrated with Jaya algorithm. The performance of the proposed algorithm was evaluated using sixteen benchmark functions. The Comparisons of results with other algorithms showed the enhancement in results by using proposed algorithm.

Yu et al. [10]proposed multi-population chaotic Jaya algorithm (MP-CJAYA) to solve the economic load dispatch. In the proposed algorithm, the method of multi-population and chaotic optimization algorithm were applied on the original Jaya algorithm. The comparisons of results with other algorithms indicated that MP-CJAYA performs better than all the other algorithms. Arora et al. [11]presented a modified butterfly optimization algorithm to solve mechanical design optimization problems and validated the algorithm for three engineering design problems.Chakraborty et al. [12]proposed an enhanced whale optimization algorithm and solved six engineering optimization problems.

Rao and Pawar [13]proposed Self-adaptive multi population Rao algorithms and investigated the algorithms on 14 design optimization problems. Rao and Pawar [14]solved the chosen mechanical system components design optimization problems using Rao algorithms.Talatahari et al. [15]developed a material generation algorithm and implemented itfor the optimum design of engineering problems. Azizi et al. [16]proposed an atomic orbital search metaheuristic optimization algorithm. The performance of the proposed algorithm was tested on constrained design problems from different engineering fields.

The metaheuristic algorithms have been used since last few decades to solve various engineering optimization problems and these algorithms have been found successful [17-19]. The metaheuristic algorithms have their benefits, but most of them rely on algorithm-specific parameters besides the general controlling parameters like the population size and the number of iterations. These algorithm-specific parameters must be tuned correctly to get better results, otherwise it adversely affects the algorithm's performance. The perfect tuning of the algorithm-specific control parameters is tiresome process, and it increases the computational efforts. So, considering the above points, Rao[20]introduced three simple, algorithm-specific parameter less and metaphor-less optimization algorithms, known as Rao algorithms.

From the above literature, it is observed that by using subpopulation-based elitist optimization algorithms and by using chaos in metaheuristic algorithms, the performance of an optimization algorithm can be improved. So, the objectives of the present study are:

- To propose SAMPE Rao and Chaotic Rao algorithms.
- To examine the performance of the proposed algorithms on benchmark problems.
- To examine the performance of the proposed algorithms on 22 engineering optimization problems.

3. Rao, SAMPE Rao and Chaotic Rao algorithms

3.1. Rao algorithms

Rao algorithms are recently developed advanced optimization algorithms[20].There are three versions of Rao algorithms, Rao1, Rao2 and Rao3 algorithm. They utilize the best and the worst candidate solutions from the whole population to get an optimal solution during the search process. In Rao2 and Rao3 algorithms, the candidate solutions interact randomly during thesearch process. Let f(x) is the objective function,the number of design variables is'm' and the number of candidate solutions (population size) is 'n'. Let us assume, for any iteration number 'q', the variable 'p' corresponding to the best and worst solutions are$x_{p,best,q}$ and $x_{p,worst,q}$ respectively. The value of the variable 'p' for the candidate 'r' during the iteration 'q' is $x_{p,r,q}$.Then the following equation are used to modify the value of $x_{p,r,q}$,

$$x'_{p,r,q}=x_{p,r,q}+r_{1,p,q}(x_{p,best,q}-x_{p,worst,q}) \quad (1)$$

$$x'_{p,r,q}=x_{p,r,q}+r_{1,p,q}(x_{p,best,q}-x_{p,worst,q})+r_{2,p,q}(|x_{p,t,q} \text{ or } x_{p,s,q}|-|x_{p,s,q} \text{ or } x_{p,t,q}|) \quad (2)$$

$$x'_{p,r,q}=x_{p,r,q}+r_{1,p,q}(x_{p,best,q}-|x_{p,worst,q}|)+r_{2,p,q}(|x_{p,t,q} \text{ or } x_{p,s,q}|-(x_{p,s,q} \text{ or } x_{p,t,q})) \quad (3)$$

where, $x'_{p,r,q}$is the updated value of $x_{p,r,q}$.$r_{1,p,q}$ and $r_{2,p,q}$ are random numbers in the range [0, 1] for thevariable 'p' during the iteration 'q'.

$$x_{new}=x_{old}+r_1(x_{best}-x_{worst}) \quad (4)$$

$$x_{new}=x_{old}+r_1(x_{best}-x_{worst})+r_2(|x_{old} \text{ or } x_{random}|-|x_{random} \text{ or } x_{old}|) \quad (5)$$

$$x_{new}=x_{old}+r_1(x_{best}-|x_{worst}|)+r_2\big(|x_{old} \text{ or } x_{random}|-(x_{random} \text{ or } x_{old})\big) \quad (6)$$

3.2. Self-Adaptive Multi Population Elitist (SAMPE) Rao algorithms

In SAMPERao algorithms, the whole population is divided into the number of sub-populations based on the fitness function value. Each sub-population takes charge for the exploration or exploitation of the search space. Then the concept of elitism is used to further enhance the diversity of the search process.The number of sub-populations is changed based on the improvement of the fitness valueafter every iteration. The flowchart of SAMPE Rao algorithms is shown in Fig. 1.

The steps of the SAMPERao algorithms are as follows:

1. Decide the number of design variables (m), populationsize (n), elite size (ES) and termination criterion. The maximum number of function evaluations or the number of iterations orrequired accuracy may be considered as a termination criterion.
2. Generate the random initial candidate solutions.
3. Divide thepopulation into the number of sub-populations according to the fitness function value of candidate solutions(initially number of sub-populations =2). Then substitute the 'ES' number of the worst solutions of the inferior group by thebest solutions of the superior group.
4. Modify the solutions in each sub-populationsusing equations of Rao algorithms independently. The modified solutions are kept if there is an improvement from the old solutions.
5. Combine all sub-populations. If there is an improvement in the current best solution compared to the previous best solution, then increase the number of sub-population by 1 for exploration, otherwise decrease the number of sub-population by 1 for exploitation.
6. Check the termination criterion. If it is fulfilled, then report the best optimum solution. Otherwise, repeat steps 2 to 6 again.

3.3. Chaotic Rao algorithms

The working of the Chaotic Rao algorithms is like that of the Rao algorithms. In the present study, the initial population is generated randomly and then the chaotic sequence generated using the mathematical equation of a chaotic map is used to update the candidate solutions in each iteration by replacing random numbers with chaotic numbers. For example, if Chebyshev map chaotic function is used, then Eq. (7) is used for the chaotic random number generation.

$$x_{i+1} = |cos(k\,cos^{-1}(x_i))|,\ x_i \in [0,1],\ k > 1 \quad (7)$$

where, x_{i+1} and x_i are (i+1)th and ith term of a chaotic sequence, respectively.

Many chaotic maps are available for chaotic number generation like logistic map, Bernoulli shift map, sine map, Chebyshev map, etc. The performance of chaotic Rao algorithms is observed using the above-mentioned maps for benchmark functions and design optimization problems. Chaotic Rao algorithms using Chebyshev chaotic map give better results than those using other maps. So, in the present study Chebyshev chaotic map is used for benchmark functions and design optimization problems in chaotic Rao algorithms. The flowchart of chaotic Rao algorithms is shown in Fig. 2.

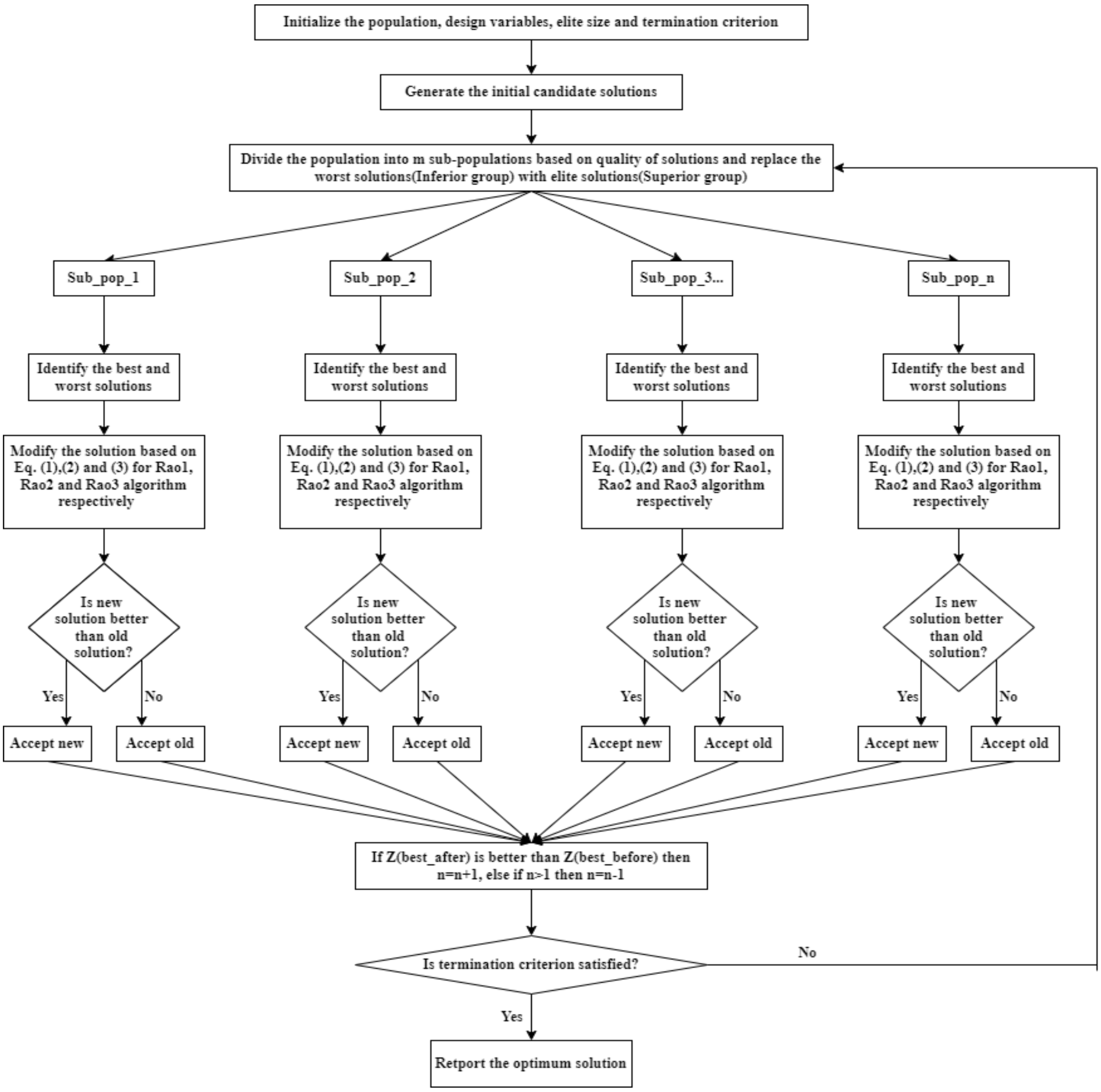

Figure 1. Flowchart of the SAMPE Rao algorithms

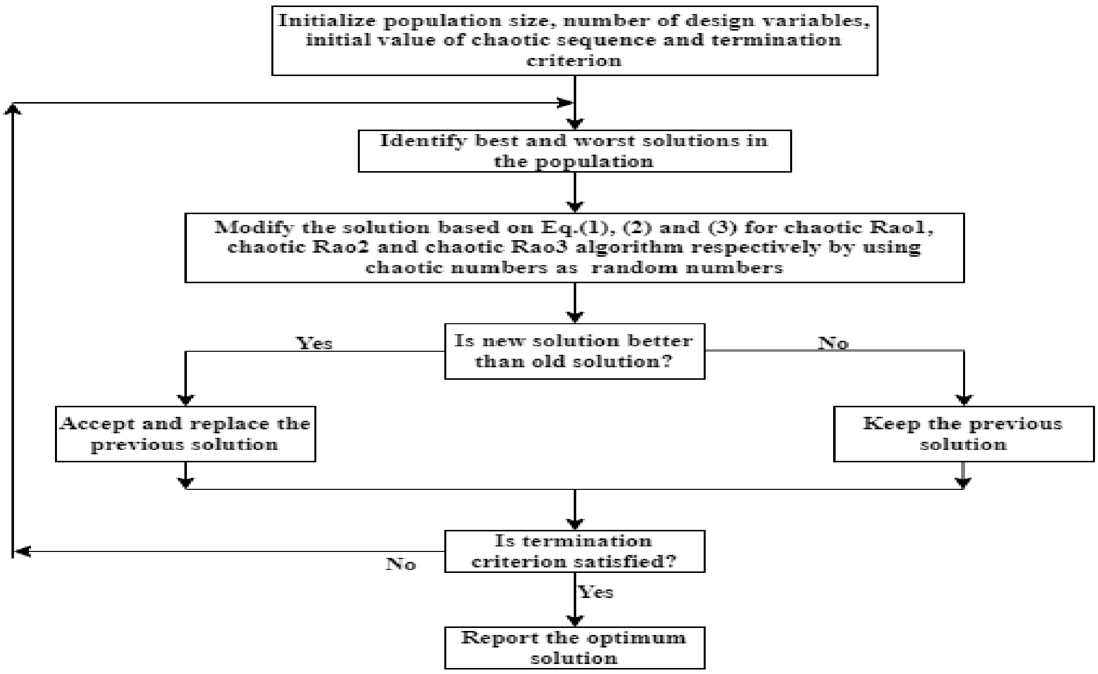

Figure 2. Flowchart of chaotic Rao algorithms

4. Application of the proposed algorithms

4.1. Benchmark problems

The mathematical formulations of benchmark problems are given in appendix A [20].The performance results for 25 unconstrained benchmark problems are obtained using the proposed modified Rao algorithms. The proposed algorithms are run for 30 times with 500000 function evaluations for each benchmark function and the statistical results are presented in Table 1. The optimization results for the same benchmark functions using Rao algorithms presented in[20] are compared with the results attained using the proposed algorithms.The improvement in the results using the proposed Rao algorithms is highlighted in bold for each benchmark function. In terms of best (B) and mean (M) results of the benchmark problems, the proposed algorithms have obtained either the same or better results compared to the corresponding Rao algorithms. In terms of the mean function evaluations (MFE), it is observed that the performance of the proposed algorithms is better except the performance of SAMPE Rao1 algorithm for functions f_8, f_{10} and f_{11}, SAMPE Rao 2 algorithm for functions f_7 and f_{11}, SAMPE Rao3 algorithm for function f_7 and Chaotic Rao2 algorithm for functionsf_7 and f_{22}.

Table 1. Statistical results of proposed algorithms for 25 benchmark problems

Function		**Rao1[20]**	**Rao2[20]**	**Rao3[20]**	**SAMPE Rao1**	**SAMPE Rao2**	**SAMPE Rao3**	**Chaotic Rao1**	**Chaotic Rao2**	**Chaotic Rao3**
f_1	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	499976	499791	277522	**499968**	**499631**	**264202**	**499913**	**499435**	**277174**
	NP,ES/CS_1	NA	NA	NA	30,2	30,2	20,1	30,0.2	30,0.2	12,0.2
f_2	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	499975	499851	276556	**499968**	**499674**	**242812**	**499895**	**499538**	**273111**
	NP,ES/CS_1	NA	NA	NA	60,2	60,2	20,2	30,0.2	30,0.2	12,0.2
f_3	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	9805	7612	7325	**2998**	**2472**	**2218**	**7927**	**3729**	**5275**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	20,0.8	10,0.6	15,0.6
f_4	**B**	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00
	M	-5.67E-01	-1.00E+00	-1.00E+00	**-1.00E+00**	-1.00E+00	-1.00E+00	**-7.00E-01**	-1.00E+00	-1.00E+00
	W	0.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.00E+00	0.00E+00	-1.00E+00	-1.00E+00
	SD	5.04E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.66E-01	0.00E+00	0.00E+00
	MFE	3010	11187	14025	992	**919**	**842**	2607	**3222**	**2860**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	20,0.8	10,0.2	10,0.2
f_5	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	77023	110544	143088	**19863**	**76908**	**99996**	**26514**	**43283**	**49149**
	NP,ES/CS_1	NA	NA	NA	10,2	20,2	20,2	10,0.2	10,0.2	10,0.2
f_6	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	1.80E-24	7.87E-27	0.00E+00	**0.00E+00**	**0.00E+00**	0.00E+00	**0.00E+00**	**0.00E+00**
	W	0.00E+00	5.35E-23	1.32E-25	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	9.76E-24	2.61E-26	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	385066	477753	488127	**164446**	238352	239369	**59000**	80542	78871

	NP,ES/CS$_1$	NA	NA	NA	15,2	15,2	15,2	15,0.2	15,0.2	15,0.2
f_7	**B**	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01
	M	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01
	W	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01	-5.00E+01
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	17485	37209	34796	**15535**	50948	48648	**11470**	55908	**31416**
	NP,ES/CS$_1$	NA	NA	NA	15,2	15,2	15,2	15,0.2	6,0.2	6,0.2
f_8	**B**	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02
	M	-2.10E+02	-3.09E+01	-2.10E+02	-2.10E+02	**-2.10E+02**	-2.10E+02	-2.10E+02	**-1.38E+02**	-2.10E+02
	W	-2.10E+02	1.17E+03	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	-2.10E+02	9.30E+02	-2.10E+02
	SD	0.00E+00	4.13E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.74E+02	0.00E+00
	MFE	48231	144156	142253	89054	194142	**130167**	**15966**	92515	**138122**
	NP,ES/CS$_1$	NA	NA	NA	20,2	20,2	20,2	10,0.2	10,0.7	15,0.2
f_9	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	345615	499767	258451	**304884**	**347959**	**153980**	**318355**	**35444**	**242684**
	NP,ES/CS$_1$	NA	NA	NA	25,2	15,2	15,2	20,0.2	10,0.3	12,0.2
f_{10}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	301513	499849	144367	499996	**499724**	**127782**	**232622**	**499469**	**144354**
	NP,ES/CS$_1$	NA	NA	NA	20,1	30,2	20,2	15,0.2	30,0.2	12,0.4
f_{11}	**B**	8.95E-26	1.86E-16	1.40E-14	2.17E-10	5.93E-12	3.87E-15	**1.66E-28**	**3.60E-18**	**1.97E-18**
	M	6.64E-01	7.40E-01	7.40E-01	4.07E+00	3.99E-01	6.82E-03	1.33E+00	1.33E-01	4.64E-08
	W	3.99E+00	2.22E+01	2.22E+01	6.98E+01	4.01E+00	1.78E-01	3.99E+00	3.99E+00	1.39E-06
	SD	1.51E+00	4.05E+00	4.05E+00	1.26E+01	1.22E+00	3.25E-02	1.91E+00	7.28E-01	2.54E-07
	MFE	489811	478410	478420	491910	492311	491894	462326	496479	498527
	NP,ES/CS$_1$	NA	NA	NA	20,1	20,1	20,1	20,0.2	30,0.2	30,0.2
f_{12}	**B**	6.67E-01	2.81E-30	6.67E-01	**1.86E-26**	**1.82E-30**	**2.42E-30**	**4.38E-30**	**1.90E-30**	6.67E-01
	M	6.67E-01	2.89E-01	6.67E-01	5.78E-01	3.56E-01	6.00E-01	6.44E-01	4.89E-01	**6.67E-01**
	W	6.67E-01	6.67E-01	6.67E-01	6.67E-01	6.67E-01	6.67E-01	6.67E-01	6.67E-01	6.67E-01
	SD	0.00E+00	3.36E-01	7.39E-05	2.30E-01	3.38E-01	2.03E-01	1.22E-01	3.00E-01	0.00E+00
	MFE	75427	113638	159231	208609	294330	139129	92647	274361	344089
	NP,ES/CS$_1$	NA	NA	NA	20,2	30,2	30,2	30,0.2	30,0.2	30,0.2
f_{13}	**B**	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	M	3.98E-01	3.98E-01	3.98E-01	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	W	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	SD	1.05E-05	1.03E-05	1.44E-07	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	102785	41263	80683	2059	4673	71397	1188	2456	46560
	NP,ES/CS$_1$	NA	NA	NA	10,2	10,2	15,2	10,0.2	10,0.2	10,0.2
f_{14}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	3129	4751	3425	**1166**	**3573**	**1289**	**1127**	**1849**	**1512**

	NP,ES/CS_1	NA	NA	NA	15,2	15,2	15,2	10,0.2	10,0.2	10,0.2
f_{15}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	2963	4272	3191	**1096**	**1457**	**1126**	**1082**	**1673**	**1254**
	NP,ES/CS_1	NA	NA	NA	15,2	15,2	15,2	10,0.2	10,0.2	10,0.2
f_{16}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	4725	12337	6821	**1886**	**3658**	**2843**	**1572**	**4422**	**2352**
	NP,ES/CS_1	NA	NA	NA	15,2	15,2	15,2	10,0.2	10,0.2	10,0.2
f_{17}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	W	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	5583	4485	4312	**1348**	**1207**	**1168**	**2031**	**2362**	**2289**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	10,0.2	10,0.2	10,0.2
f_{18}	**B**	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00
	M	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00
	W	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00	-1.80E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	3863	2694	2751	**1091**	**1037**	**1239**	**1397**	**2704**	**2587**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	10,0.2	10,0.2	10,0.2
f_{19}	**B**	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00	-4.69E+00
	M	-4.67E+00	-4.43E+00	-4.49E+00	**-4.68E+00**	**-4.54E+00**	**-4.57E+00**	**-4.68E+00**	**-4.63E+00**	**-4.64E+00**
	W	-4.54E+00	-3.12E+00	-3.50E+00	-4.65E+00	-3.91E+00	-3.66E+00	-4.54E+00	-4.48E+00	-4.50E+00
	SD	3.09E-02	3.60E-01	2.79E-01	1.27E-02	1.63E-01	1.89E-01	3.05E-02	6.01E-02	5.14E-02
	MFE	39710	67252	58401	158680	109073	91618	23677	131485	99287
	NP,ES/CS_1	NA	NA	NA	20,2	20,2	30,2	30,0.2	30,0.2	30,0.2
f_{20}	**B**	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	M	3.00E+00	5.70E+00	3.00E+00	3.00E+00	**3.00E+00**	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	W	3.00E+00	8.40E+01	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	SD	0.00E+00	1.48E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	180121	180121	353893	**91753**	87284	**326782**	**55361**	**103526**	**333562**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	10,0.2	15,0.2	15,0.2
f_{21}	**B**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	M	1.45E-10	0.00E+00	0.00E+00	**0.00E+00**	0.00E+00	0.00E+00	**0.00E+00**	0.00E+00	0.00E+00
	W	3.71E-09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SD	6.78E-10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	82792	3139	4453	2065	**2047**	**1809**	2349	**1671**	**2947**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	10,2	10,0.2	10,0.2	15,0.2
f_{22}	**B**	1.51E-14	7.99E-15	4.44E-15	**7.99E-15**	7.99E-15	4.44E-15	**7.99E-15**	7.99E-15	4.44E-15
	M	5.67E-01	1.04E-14	6.69E-15	1.20E-01	**1.02E-14**	**6.34E-15**	7.04E-01	1.33E+01	**5.63E-15**
	W	2.22E+00	1.51E-14	1.51E-14	2.66E+00	2.22E-14	1.51E-14	2.00E+01	2.00E+01	7.99E-15
	SD	7.41E-01	3.14E-15	2.38E-15	5.09E-01	3.79E-15	2.42E-15	3.64E+00	9.57E+00	1.70E-15
	MFE	129392	417741	76352	410787	358564	161576	443238	325673	173332

	NP,ES/CS_1	NA	NA	NA	90,2	40,2	120,2	80,0.2	25,0.2	60,0.2
f_{23}	**B**	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	M	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	W	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	18839	95983	243748	**3812**	**58484**	**236463**	**3711**	**63048**	**239722**
	NP,ES/CS_1	NA	NA	NA	10,2	10,2	8,2	15,0.2	15,0.2	40,0.2
f_{24}	**B**	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	M	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	W	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	SD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MFE	4459	3022	3271	**3077**	**2811**	**2280**	**2303**	**2198**	**3185**
	NP,ES/CS_1	NA	NA	NA	15,2	20,2	15,2	15,0.2	10,0.6	15,0.2
f_{25}	**B**	1.35E-32	1.35E-32	1.35E-32	1.35E-32	1.35E-32	1.35E-32	1.35E-32	1.35E-32	1.35E-32
	M	1.47E-03	5.79E-02	1.60E-02	**7.32E-04**	**4.71E-03**	**9.42E-03**	**1.10E-03**	**1.35E-32**	**2.17E-03**
	W	1.10E-02	1.60E+00	1.41E-01	1.10E-02	9.74E-02	9.74E-02	1.10E-02	1.35E-32	2.10E-02
	SD	3.80E-03	2.91E-01	3.50E-02	2.79E-03	1.79E-02	2.44E-02	3.35E-03	0.00E+00	5.20E-03
	MFE	173661	115593	55637	422550	363520	195872	308780	390500	214620
	NP,ES/CS_1	NA	NA	NA	80,2	80,2	80,2	60,0.2	60,0.2	60,0.2

B-Best solution, M-Mean solution, W-Worst solution, SD-Standard deviation, MFE-Mean function evaluations, NP-Population size, ES-Elite size, CS_1-Initial value of chaotic sequence, NA- Not available

The results attained by applying the proposed algorithms to the benchmark functions are used to perform the Friedman rank test[21].The Friedman rank test examines the statistical performance of one algorithm against other algorithms. The statistical results achieved using different algorithms for a function are compared and the algorithm which gives the best result gets rank 1 and then other algorithms are ranked in ascending order according to their performance. Similarly, the ranks are given to algorithms for every function and then the mean rank of each algorithm is computed. Table 2 shows the results of the Friedman rank test. From the average ranking, it is seenthat the Chaotic Rao1 algorithm performs better than the other algorithms. The rank of each algorithm according to its performance is also presented in Table 2. From the χ^2 score value of 56.128, the p value is obtained, and it is much less than 0.05, which validates the better performance of the proposed algorithms.

4.2. Mechanical engineering design optimization problems

The proposed algorithms are appliedto15mechanical engineeringdesign optimization problems to test their performance. The mathematical models of these problems are described in the Appendix B. The detailed description of these problems is given in [16] and [22].

4.2.1. Four stage gear box:

This problem is designed to minimize the total weight of gear box. The material of gears is aluminum–bronze, so the aim is considered as the minimization of the total gear volume. 22 design variables (eight integer and other discrete variables) consist of the blank thickness, the number of teeth and the gear and pinion position. The 86 design constraints are related to the contact ratio, strength of the gears, size of gears, pitch, assembly of gears, and kinematics.

4.2.2. Rolling element bearing:

Fig. 3 shows schematic diagram of rolling element bearing. This problem is designed to maximize the load carrying capacity of a rolling element bearing. The ball diameter (D_b), pitch diameter (D_m), total number of balls (Z), the raceway curvature coefficient (f_i and f_o) are five design variables and the design parameters of the rolling element bearing problem (β, ε, e, K_{Dmin}, K_{Dmax},) are other five design variables. All design variables excluding Z are continuous variables. The design constraints are related to kinematic and manufacturing considerations.

Table 2. Friedman rank test results

Algorithm	**Rao1**	**Rao2**	**Rao3**	**SAMPE Rao1**	**SAMPE Rao2**	**SAMPE Rao3**	**Chaotic Rao1**	**Chaotic Rao2**	**Chaotic Rao3**
Friedman average rank	6.52	7.32	6.68	3.92	4.72	3.68	3.4	4.44	4.32
Rank for algorithm	7	9	8	3	6	2	1	5	4

4.2.3. Gas transmission compressor:

This problem is designed to minimize the cost of gas pipeline transmission system to deliver 100 million cubic feet of gas per day. The distance between the compressor stations (x_1, in miles) and the compressor ratio (x_2, division of discharge pressure and flow rate), the pipe diameter (x_3, in inches) and flow rate (x_4, in ft^3 /sec) are four design variables. This problem has one inequality constraint.

4.2.4. Speed reducer:

Fig. 4 shows schematic diagram of speed reducer. This problem is designed to minimize the total weight of the speed reducer. The face width (b), the number of teeth on pinion (Z, integer variable), the gear module (m), shaft 1 diameter (d_1), and the shaft 2 diameter (d_2), the length of shaft 1 (l_1), the length of shaft 2 (l_2) are seven design variables. Except Z, all other design variables are continuous variables. This problem has eleven constraints associated with the surface stress, the bending stress in the gear teeth, the stresses in the shafts, and transverse deflections of the shafts.

4.2.5. Pressure vessel:

Fig. 5 shows schematic diagram of pressure vessel. This problem is designed to minimize the total cost of Material, welding and forming. The design variables are the shell thickness as T_s (x_1), head thickness as T_h (x_2), shell inner radius as R (x_3), and cylindrical section length of the vessel without considering the head as L(x_4). Due to the constraint in the availability of rolled steel plates, T_s and T_h must be multiple of 0.0625 in. The remaining two variables are continuous. This problem has four design constraints.

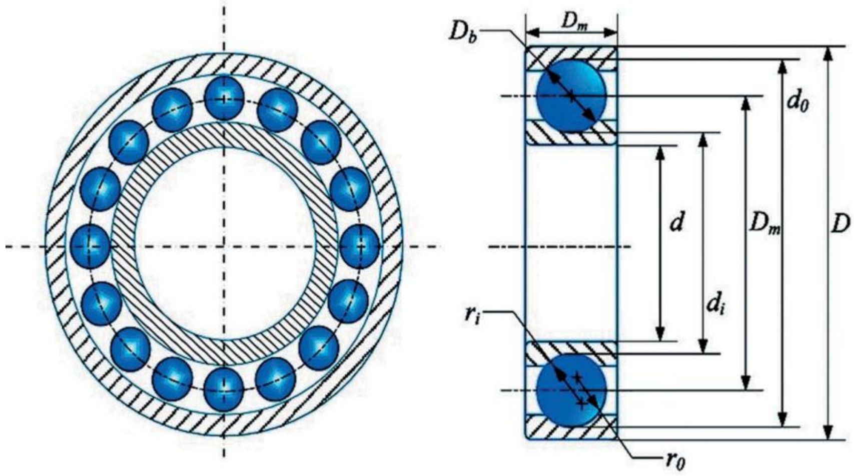

Figure 3. Rolling element bearing[13]

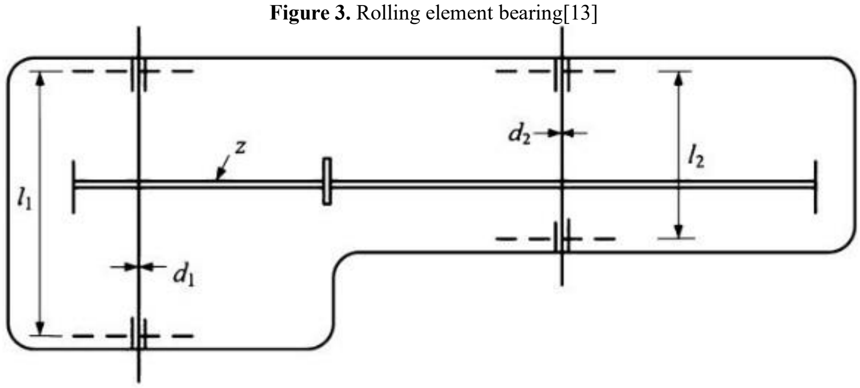

Figure 4. Speed reducer [25]

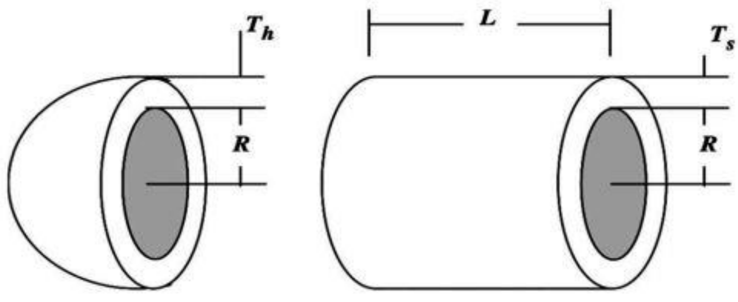

Figure 5. Pressure vessel [13]

4.2.6. Tension/compression spring – case 1:

Fig. 6 shows schematic diagram of tension/compression spring. This problem is designed to minimize the weight of a tension/compression spring. The wire diameter d (x_1), mean coil diameter D (x_2) and the number of active coils N (x_3) are three continuous design variables. Four design constraints are associated with the shear stress, minimum deflection, surge frequency and limits on outside diameter and design variables.

4.2.7. Tension/Compression spring – case 2:

This problem is designed to minimize the volume of a compression spring under static loading. The mean coil diameter D (x_2, continuous), the wire diameter d(x_1, discrete), and the number of active coils N (x_3, integer) are three design variables. The value of variable d is taken from Table 3.

4.2.8. Piston lever:

Fig. 7 shows schematic diagram of piston leaver. This problem is designed to minimize the oil volume by locating the piston components' position (H, D, B, and X) when the piston lever is raised from 0^o to 45^o. The inequality constraints imposed are related to equilibrium of forces, maximum bending moment of the lever, minimum piston stroke and geometrical conditions.

4.2.9. Gear train:

Fig. 8 shows schematic diagram of gear train. This problem is designed to minimize the gear ratio in the gear train. The number of teeth on the gears are four design variables. The given range for the number of teeth on each gear is considered as the constraints.

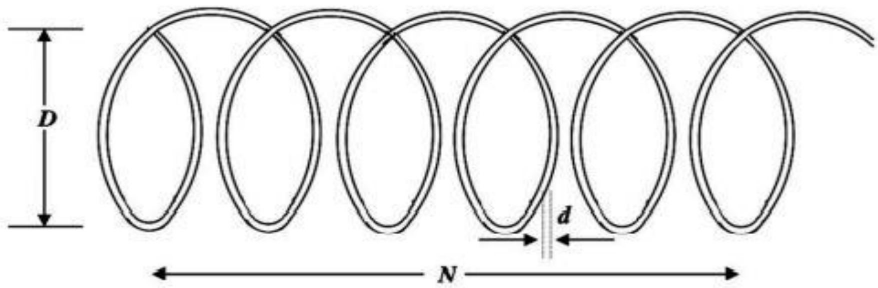

Figure 6. Compression spring [13]

Table 3. Wire diameter [24]

Wire diameters (in)						
0.009	0.0095	0.0104	0.0118	0.0128	0.0132	0.014
0.015	0.0162	0.0173	0.018	0.020	0.023	0.025
0.028	0.032	0.035	0.041	0.047	0.054	0.063
0.072	0.080	0.092	0.105	0.120	0.135	0.148
0.162	0.177	0.192	0.207	0.225	0.244	0.263
0.283	0.307	0.331	0.362	0.394	0.4375	0.500

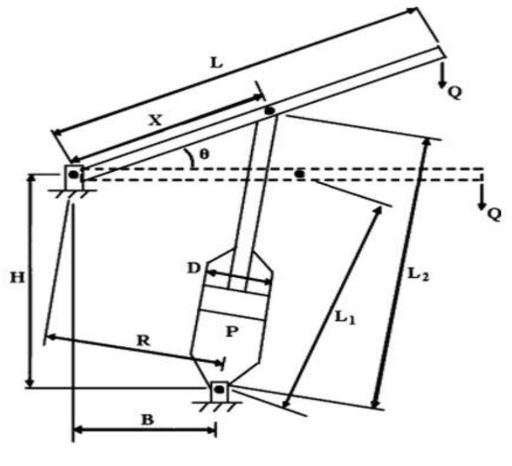

Figure 7. Piston lever [25]

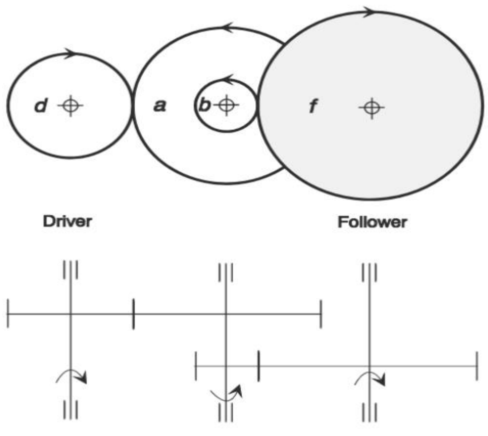

Figure 8. Gear train[16]

4.2.10. Corrugated bulkhead:

This problem is designed to minimize the weight of a corrugated bulkhead used in tankers. The length (l), width (w), depth (h) and thickness (t) of the bulkhead are four design variables considered with six inequality constraints.

4.2.11. Planetary gear train:

Fig. 9 shows schematic diagram of planetary gear train. This problem is designed to minimize the maximum errors in the gear ratio of a planetary gear train. The number of teeth in the gears(Z_1, Z_2, Z_3, Z_4, Z_5 and Z_6- integer variables), the number of planet gears(Z_p), module of the first gear(m_1) and module of the second gear(m_2) are nine design variables.P, m_1, and m_2 are discrete design variables. The design constraints (ten inequality constraints and one equality constraint) are related to various assembly and geometric restrictions.

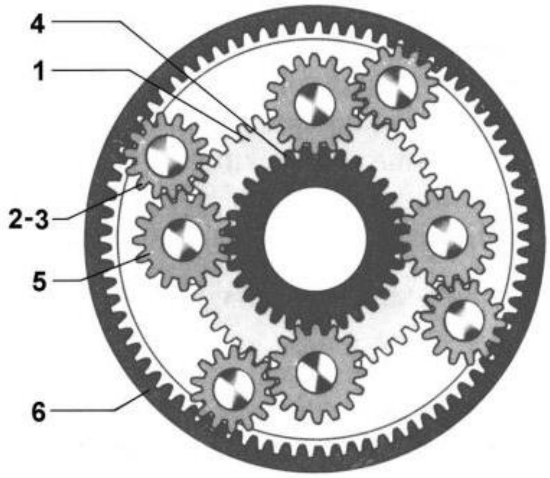

Figure 9. Planetary gear train [23]

1: Small sun gear, 2,3: Broad planet gear, 4: Large sun gear, 5: Narrow planet gear, 6: Ring gear

4.2.12. Step cone pulley:

Fig. 10 shows schematic diagram of step cone pulley. This problem is designed to minimize the weight of the step cone pulley. The pulley diameter in each step(d_1, d_2, d_3, d_4) and width of the pulley(w) are five design variables. Eight inequality constraints and three equality constraints ensure that the belts have the same tension ratios, same belt length for every step and transmit same power. The step pulley transmits minimum 0.75 hp.

4.2.13. Hydrostatic thrust bearing:

Fig.11 shows schematic diagram of hydrostatic thrust bearing. This problem is designed to minimize the power loss related to bearing. Oil viscosity (μ),flow rate (Q),Bearing step radius (R) and recess radius (R_0) are four design variables. Total seven constraints are related to oil film thickness, load-carrying capacity, rise in oil temperature, inlet oil pressure and some physical requirements.

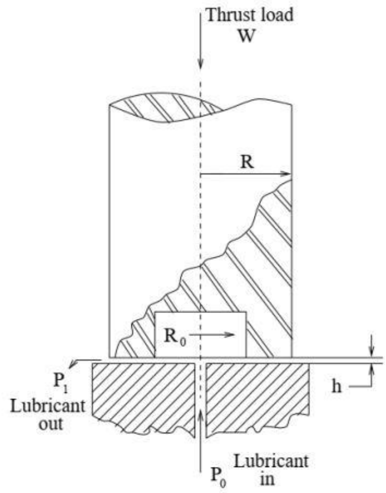

Figure 11. Hydrostatic thrust bearing[16]

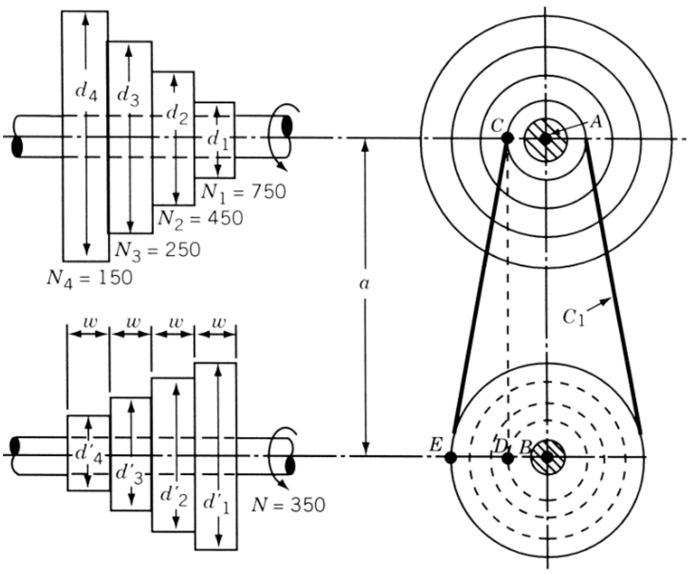

Figure 10. Step cone pulley [16]

4.2.14. Welded beam:

Fig. 12 shows schematic diagram of welded beam. This problem is designed to minimize the fabrication cost of the welded beam. The weld thickness as h(x_1), weld length as l(x_2), the bar height as t(x_3) and bar thickness as b(x_4) are four design variables. This design problem has seven constraints associated with the shear stress (τ), the bending stress in the beam (σ), the bucking load on the beam (P_c), the end deflection of the beam (δ), and the side constraints.

4.2.15. Multiple disk clutch brake:

Fig. 13 shows schematic diagram of multiple disk clutch brake. This problem is designed to minimize the total mass of the clutch brake. The actuating force (F), outer radius (r_o), inner radius (r_i), disc thickness (t) and the number of contact surfaces (Z) are five discrete design variables. This problem has eight non-linear design constraints.

5. Results and discussion

In the present work, the results are obtained with the R2019a version of the MATLAB tool. The laptop with the 1.80-GHz Intel Core i7-8550U processor and 8GB RAM is used for computation. Each design optimization problem is solved using Rao, SAMPE Rao and Chaotic Rao algorithms for25 times. The statistical results achieved using Rao, SAMPE Rao, Chaotic Rao algorithms and other advanced optimization algorithms presented in [16] and [22]are compared for each design optimization problem. [16] and [22] considered maximum functions evaluations as 200000 and 60000, respectively for design optimization problems presented in their respective papers. The best fitness function values and the statistical results achieved using the proposed algorithms are compared with thoseachieved using other advanced optimization algorithms in the previous studies. Different combinations of population size and elite sizes for SAMPE Rao algorithms and different combinations of population size and initial value of chaotic sequence are tested for all the problems, the combination which gives the best result for different algorithms is mentioned in the statistical results comparison tables.

5.1. Four stage gear box:

The maximum FEs in this problem is considered as 50000. Table 4 compares the statistical results of this problem over 25 runs. The best fitness value obtained is $f(\overline{x}) = 36.26590769$ cm^{3}at $\overline{x} =$ (19, 41, 19, 39, 18, 38, 18, 38, 3.175, 3.175, 3.175, 3.175, 101.6, 63.5, 63.5, 88.9, 88.9, 88.9, 76.2, 76.2, 50.8, 50.8) using SAMPE Rao1 algorithm. Table 5 compares the best fitness value obtained in the present study and the same obtained by ABC-GA(Artificial Bee Colony-Genetic Algorithm), ABC-DE (Artificial Bee Colony-Differential Evolution), ABC-BBO(Artificial Bee Colony-Biogeography-based optimization), TLBO, ABC, and AOS. The best fitness value is obtained by SAMPE Rao1 algorithm.

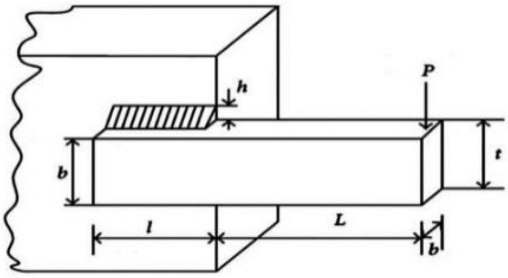

Figure 12. Welded beam [13]

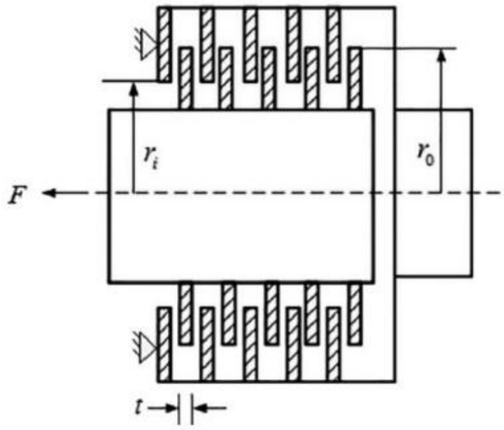

Figure 13. Multiple disk clutch brake [14]

Table 4. Statistical results comparison for a four stage gear box design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
AOS[a]	37.4042245	52.83708891	90.81422082	1.190E+01	NA	NA
Rao1	37.35797821	72.20780915	156.2196704	3.320E+01	41396	25
Rao2	36.5289327	103.3275184	240.5116744	4.620E+01	26237	25
Rao3	37.28608556	68.17318794	151.9493665	3.100E+01	35015	25
SAMPE Rao1	36.26590769	56.27290804	85.24647758	1.340E+01	40910	25,2
SAMPE Rao2	36.443023	60.79267004	113.2822656	2.260E+01	42548	50,6
SAMPE Rao3	36.29036038	54.07181791	106.5927524	1.750E+01	44856	50,6
Chaotic Rao1	36.47490382	74.97541999	130.3131561	2.890E+01	38794	25,0.7
Chaotic Rao2	36.48867434	168.5684829	618.2785095	1.350E+02	30084	29,0.6
Chaotic Rao3	36.46666447	138.4370045	275.2995689	6.550E+01	30737	30,0.6

[a] Results are taken from [16]

Table 5. Comparison of best fitness value for a four stage gear box design problem

Algorithm	ABC-GA[a]	ABC-DE[a]	ABC-BBO[a]	TLBO[a]	ABC[a]	AOS[b]	Present study
Best result	55.494494	59.763563	46.623205	43.792433	49.836165	37.4042245	**36.26590769**

[a] Results are taken from [2]; [b] Results are taken from [16]

5.2. Rolling element bearing:

The maximum FEs in this problem is considered as 25000. Table 6 compares the statistical results of this problem over 25 runs for TLBO, AOS, and the proposed algorithms. The best fitness value obtained is $f(\overline{x}) =$ 81859.741597 N at $\overline{x} =$ (125.719056, 21.42559, 0.515, 0.515, 11, 0.462578, 0.7, 0.809635, 0.3, 0.064782) by all the proposed algorithms and TLBO. The result given by AOS algorithm is inferior and it is an infeasible solution because the number of rolling elements was considered by [16] as a continuous variable.

5.3. Gas transmission compressor:

The maximum FEs in this problem is considered as 200000. Table 7 compares the statistical results of this problem over 25 runs. The best fitness value obtained is $f(\overline{x}) =$ 2964895.417 at $\overline{x} =$ (50, 1.178281, 24.592121, 0.388346) using all the proposed algorithms and the AOS algorithm.

5.4. Speed reducer:

The maximum FEs in this problem is considered as 25000. Table 8 compares the statistical results of this problem over 25 runs obtained by CGO, SEA (Simple Evolutionary Algorithm), OASBS (Optimization Algorithm based on Socio Behavioural Simulation Model), CSA, and DSS-MDE (Dynamic Stochastic Selection-Multimember Differential Evolution). The best fitness value obtained is $f(\overline{x}) =$ 2994.424466 kg at $\overline{x} =$ (3.5, 0.7, 17, 7.3, 7.715320, 3.350541, 5.286655) by all the proposed algorithms. The SEA, OASBS, and CSA have obtained inferior values. The Chaotic Rao1 algorithm requires the minimum FEs to reach an optimal solution.

Table 6. Statistical results comparison for a rolling element bearing design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
TLBO [a]	81859.74	81438.987	80807.8551	6.600E-01	NA	50
AOS [a*]	83918.49253	82175.21266	83826.38337	2.338E+01	NA	NA
Rao1	81859.741597	81564.87429	80807.82664	3.603E+02	10997	30
Rao2	81859.741597	81817.666137	80807.855077	2.104E+02	11306	30
Rao3	81859.741597	81817.66614	80807.85508	2.104E+02	11045	30
SAMPE Rao1	81859.741597	81790.21112	80807.85508	2.090E+02	9909	40,2
SAMPE Rao2	81859.741597	81859.741597	81859.741597	0.000E+00	9515	35,2
SAMPE Rao3	81859.741597	81859.741597	81859.741597	0.000E+00	9588	35,2
Chaotic Rao1	81859.741597	81859.741597	81859.741597	0.000E+00	8726	20,0.2
Chaotic Rao2	81859.741597	81859.741597	81859.741597	0.000E+00	16201	30,0.3
Chaotic Rao3	81859.741597	81859.741597	81859.741597	0.000E+00	15760	30,0.3

[a] Results are taken from [16]; [*] Infeasible solution because the number of rolling elements was considered as a continuous variable

Table 7. Statistical results comparison for a gas transmission compressor design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS1
AOS [a]	2964895.417	2965102.327	2966483.832	2.518E+02	NA	NA
Rao1	2964895.419	2964895.444	2964895.52	2.667E-02	162163	10
Rao2	2964895.419	2999784.479	3626159.761	1.356E+05	153950	10
Rao3	2964895.419	2995303.489	3147941.927	6.822E+04	127464	10
SAMPE Rao1	2964895.417	2994126.234	3147941.927	6.340E+04	148446	10,2
SAMPE Rao2	2964895.417	3026809.386	3358534.842	1.044E+05	121613	10,2
SAMPE Rao3	2964895.417	3005984.840	3147941.927	7.315E+04	135558	10,2
Chaotic Rao1	2964895.417	2964895.424	2964895.517	1.954E-02	151003	10,0.4
Chaotic Rao2	2964895.417	4012200.787	12061229.75	2.653E+06	113224	10,0.3
Chaotic Rao3	2964895.417	3119110.378	3771891.386	2.296E+05	110822	10,0.9

[a] Results are taken from [16]

Table 8. Statistical results comparison for a speed reducer design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
SEA [a]	3025.005	3088.7778	3078.5918	NA	NA	100
OASBS [a]	3008.08	3012.12	3028.28	NA	NA	100
CSA [a]	3000.981	3007.1997	3009	4.963E+00	NA	50
DSS-MDE [a]	2994.471066	2994.471066	2994.471066	3.580E−12	NA	20
CGO [a]	2994.443649	2994.465397	2995.504933	1.102E-01	NA	NA
Rao1	2994.424466	2994.424466	2994.424466	0.000E+00	6373.2	15
Rao2	2994.424466	2994.424466	2994.424466	0.000E+00	12848	30
Rao3	2994.424466	2994.424466	2994.424466	0.000E+00	12897	30
SAMPE Rao1	2994.424466	2994.424466	2994.424466	0.000E+00	6270	15,2
SAMPE Rao2	2994.424466	2994.424466	2994.424466	0.000E+00	9108	25,2
SAMPE Rao3	2994.424466	2994.424466	2994.424466	0.000E+00	9540	25,2
Chaotic Rao1	2994.424466	2994.424466	2994.424466	0.000E+00	3958	10,0.2
Chaotic Rao2	2994.424466	2994.424466	2994.424466	0.000E+00	12621	25,0.8
Chaotic Rao3	2994.424466	2994.424466	2994.424466	0.000E+00	12409	25,0.8

[a]Results are taken from[22]

5.5. Pressure vessel:

The maximum FEs in this problem is considered as 10000. Table 9 compares the statistical results of this problem over 25 runs obtained by CPSO (Coevolutionary Particle Swarm Optimization), QPSO (Quantum behaved PSO), NM-PSO (Nelder–Mead PSO), MBA (Mine Blast Algorithm), and CGO (Chaos Game Optimization) algorithm. The best fitness value obtained is $f(\overline{x}) =$ 6059.714335 \$ at $\overline{x} =$ (0.8125, 0.4375, 42.098446, 176.636596) by all the proposed algorithms. The results obtained by CPSO, QPSO, NM-PSO, and MBA algorithms violate the requirement that the variables $T_s(x_1)$ and T_h (x_2) are to be integer multiples of 0.0625 inch. The best mean fitness value is attained using the Chaotic Rao1 algorithm.

5.6. Tension/compression spring – case 1:

The maximum FEs in this problem is considered as 60000. Table 10 compares the statistical results of this problem over 25 runsfor CGA (Co-evolutionary Genetic Algorithm), OASBS, STA (State Transition Algorithm), BA (Bat Algorithm) and CGO (Chaos Game Optimization). The best fitness value obtained is $f(\overline{x}) =$ 0.01266525 in^{3}at $\overline{x} =$ (0.0516173, 0.3549922, 11.3910097) using Chaotic Rao1 algorithm. Even though the best fitness value obtained by BA and CGO seem to be slightly better, but these are to be considered infeasible due to violation of a constraint.

5.7. Tension/Compression spring – case 2:

The maximum FEs in this problem is considered as 10000. Table 11 compares the statistical results of this problem over 25 runsfor AOS and the proposed algorithms. The best fitness value obtained is $f(\overline{x}) =$ 2.625281953 in^{3}at $\overline{x} =$ (0.263, 0.9048483122, 15) by all the proposed algorithms. Even though the best fitness value obtained by AOS algorithm seems to be slightly better, but this is to be considered infeasible due to violation of a constraint. The best mean fitness value is attained using Chaotic Rao3 algorithm.

Table 9. Statistical results comparison for a pressure vessel design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS1
CPSO [a*]	6061.0777	6147.1332	6363.8041	8.645E+01	NA	NA
QPSO [a*]	6059.7208	6440.3786	7544.4925	4.484E+02	NA	20
NM-PSO [a*]	5930.3137	5946.7901	5960.0557	9.161E+00	NA	NA
MBA [a*]	5889.3216	6200.6477	6392.5062	1.603E+02	NA	50
CGO [a]	6247.672819	6250.957354	6330.958685	1.075E+01	NA	NA
Rao1	6059.714335	6088.684774	6128.630739	2.020E+01	6413.0	30
Rao2	6059.714335	6065.997125	6103.736305	1.240E+01	8943.0	30
Rao3	6059.714335	6062.390819	6090.579595	6.600E+00	9142.0	30
SAMPE Rao1	6059.714335	6073.607190	6097.912054	1.220E+01	5579.0	25,2
SAMPE Rao2	6059.714335	6064.985476	6093.022428	1.030E+01	8850.0	30,2
SAMPE Rao3	6059.714335	6061.806949	6090.526208	6.500E+00	9422.0	30,2
Chaotic Rao1	6059.714335	6061.583912	6072.623805	3.810E+00	8345.0	20,0.3
Chaotic Rao2	6059.714335	6063.910489	6095.724076	1.080E+01	9454.0	30,0.8
Chaotic Rao3	6059.714335	6061.721305	6090.526309	6.290E+00	9435.6	30,0.8

[a] Results are taken from [22]; [*] Infeasible solution because the variables $T_s(x_1)$ and T_h (x_2) are not integer multiples of 0.0625 inch.

Table 10. Statistical results comparison for a tension/compression spring case-1 design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS$_1$
CGA [a]	0.01270478	0.0127692	0.01282208	3.939E−05	NA	NA
OASBS [a]	0.0126692	0.0129227	0.0167172	5.198E−05	NA	30
STA [a]	0.01266534	0.01266534	0.01272968	2.167E−05	NA	30
BA [a*]	0.01266522	0.01266522	0.0168954	1.420E-03	NA	10
CGO [a*]	0.01266524	0.012670085	0.012719055	1.090E−05	NA	NA
Rao1	0.012665712	0.012677199	0.012718743	1.168E-05	44440	10
Rao2	0.012665495	0.012986421	0.017773158	1.014E-03	17214	10
Rao3	0.012666085	0.012838545	0.01319258	2.262E-04	12070	10
SAMPE Rao1	0.012665361	0.012682374	0.012719054	2.044E-05	45396	10,2
SAMPE Rao2	0.012665459	0.012710172	0.012719054	1.879E-05	21958	10,2
SAMPE Rao3	0.012665564	0.012702301	0.012721539	2.346E-05	15390	10,2
Chaotic Rao1	0.01266525	0.012665568	0.012666418	3.000E-07	46446	10,0.2
Chaotic Rao2	0.01266527	0.012803096	0.01319258	1.999E-04	10463	10,0.2
Chaotic Rao3	0.012665349	0.012862042	0.01319258	2.322E-04	6939	10,0.2

[a] Results are taken from [22]; [*] Infeasible solution due to violation of a constraint.

Table 11. Statistical results comparison for a tension/compression spring case-2 design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS$_1$
AOS [a*]	2.615360373	2.64371161	2.863796184	4.285E-02	NA	NA
Rao1	2.625281953	2.626052428	2.63408739	1.93E-03	5788	10
Rao2	2.625281953	2.983025419	5.39453905	7.62E-01	1669	10
Rao3	2.625281953	2.88088446	6.720428845	8.54E-01	1554	10
SAMPE Rao1	2.625281953	2.62577663	2.631345549	1.39E-03	5573	15,2
SAMPE Rao2	2.625281953	2.648636546	3.142985801	1.04E-01	2160	15,2
SAMPE Rao3	2.625281953	2.648636546	3.142985801	1.04E-01	3195	15,2
Chaotic Rao1	2.625281953	2.625281994	2.625282751	1.64E-07	4649	15,0.2
Chaotic Rao2	2.625281953	2.785515745	5.104522609	5.64E-01	4648	15,0.7
Chaotic Rao3	2.625281953	2.625281953	2.625281953	0.00E+00	4759	15,0.7

[a] Results are taken from [16]; [*] Infeasible solution because number of active coils in a spring is considered as continuous variable.

5.8. Piston lever:

The maximum FEs in this problem is considered as 20000. Table 12 compares the statistical results of this problem over 25 runs obtained by HPSO (Hierarchy Particle Swarm Optimization), GA (Genetic Algorithm), DE, CS, MGA, and AOS algorithm. The best fitness value obtained is $f(\overline{x}) = 8.412698$ in^{3}at $\overline{x} =$ (0.05, 2.04151359, 4.08302718, 120) by all the proposed algorithms. The best mean fitness value is attained using SAMPE Rao1 algorithm.

5.9. Gear train:

The maximum FEs in this problem is considered as 500. Table 13 compares the statistical results of this problem over 25 runs. The best fitness value obtained is $f(\overline{x}) =$ 2.70086E-12 at $\overline{x} =$ (49, 19, 16, 43) by all the proposed algorithms. The best mean fitness value is attained using SAMPE Rao3 algorithm. Table 14 compares the best fitness value obtained in the present study and the same obtained by CS (Cuckoo Search), ALO, MFO (Moth Flame Optimization), MVO (Multi Verse Optimizer), and AOS algorithms. Even though the best fitness value obtained by AOS algorithm seems to be slightly better, but this is to be considered infeasible because the number of gear teeth was considered as a continuous variable in [16].

5.10. Corrugated bulkhead:

The maximum FEs in this problem is considered as 15000. Table 15 compares statistical results of this problem over 25 runs. The best fitness value obtained is $f(\overline{x}) =$ 6.84295801 at $\overline{x} =$ (57.69230769, 34.14762035, 57.69230769, 1.05) by all the proposed algorithms except CSA. The best fitness value obtained by CSA is infeasible as the corresponding values of $\overline{x}$violates a constraint. The Chaotic Rao1 algorithm requires the minimum FEs to reach an optimal solution.

Table 12. Statistical results comparison for a piston lever design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
HPSO [a]	162	187	197	1.340E+01	NA	NA
GA [a]	161	185	216	1.820E+01	NA	NA
DE [a]	159	187	199	1.420E+01	NA	NA
CSA [a]	8.4271	40.2319	168.592	5.906E+01	NA	50
AOS[b]	8.419142742	33.7412759	60.66498628	9.347E+01	NA	NA
Rao1	8.412698	84.76151	167.4727	8.111E+01	6944	10
Rao2	8.412698	103.8487	167.4727	7.953E+01	11207	15
Rao3	8.412698	97.48632	167.4727	8.058E+01	9935	20
SAMPE Rao1	8.412698	**73.0191**	167.4727	7.884E+01	8184	20,2
SAMPE Rao2	8.412698	**84.76151**	167.4727	8.111E+01	7778	15,2
SAMPE Rao3	8.412698	**84.76151**	167.4727	8.111E+01	7578	15,2
Chaotic Rao1	8.412698	**78.39911**	167.4727	8.058E+01	5113	20,0.3
Chaotic Rao2	8.412698	**91.12391**	167.4727	8.111E+01	11543	15,0.3
Chaotic Rao3	8.412698	**91.12391**	167.4727	8.111E+01	9575	11,0.2

[a]Results are taken from [15]; [b]Results are taken from [16]

Table 13. Statistical results comparison for a gear train design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
AOS [a*]	2.29E-19	6.25E-15	9.06E-14	1.260E-14	NA	NA
Rao1	2.70086E-12	2.44995E-08	2.38608E-07	4.734E-08	225	10
Rao2	2.70086E-12	5.74147E-08	8.949E-07	1.781E-07	220	10
Rao3	2.70086E-12	8.81838E-08	8.949E-07	2.437E-07	212	10
SAMPE Rao1	2.70086E-12	1.36555E-08	8.92118E-08	2.005E-08	256	10,2
SAMPE Rao2	2.70086E-12	1.68745E-08	1.76128E-07	3.482E-08	267	15,2
SAMPE Rao3	2.70086E-12	1.14852E-08	7.80223E-08	1.732E-08	199	10,2
Chaotic Rao1	2.70086E-12	2.08542E-08	1.5244E-07	3.778E-08	238	10,0.2
Chaotic Rao2	2.70086E-12	5.30282E-08	5.04146E-07	1.223E-07	251	10,0.6
Chaotic Rao3	2.70086E-12	6.0809E-08	8.949E-07	1.773E-07	244	15,0.4

[a]Results are taken from [16];

Table 14. Best fitness value comparison for a gear train design problem

Algorithm	CS[a]	ALO[a]	MFO[a]	MVO[a]	AOS[b*]	Present study
Best result	2.701e-12	2.7009e-12	2.7009e-12	2.7009e-12	2.29e-19	**2.70086e-12**

[a]Results are taken from [13]; [b]Results are taken from [16]; [*]Infeasible solution because number of gear teeth was considered as a continuous variable.

Table 15. Statistical results comparison for a corrugated bulkhead design problem

Algorithm	B	M	W	SD	MFE	NP, ES/CS_1
CSA [a*]	5.894331	5.988257	6.126749	6.436E-02	NA	NA
AOS [a]	6.84295801	7.060808377	7.066936186	6.491E-04	NA	NA
Rao1	6.84295801	6.84295801	6.84295801	0.000E+00	4290	10
Rao2	6.84295801	6.84295801	6.84295801	0.000E+00	4573	10
Rao3	6.84295801	6.84295801	6.84295801	0.000E+00	4260	10
SAMPE Rao1	6.84295801	6.84295801	6.84295801	0.000E+00	**4231**	14,2
SAMPE Rao2	6.84295801	6.84295801	6.84295801	0.000E+00	**4326**	10,2
SAMPE Rao3	6.84295801	6.84295801	6.84295801	0.000E+00	**4066**	10,2
Chaotic Rao1	6.84295801	6.84295801	6.84295801	0.000E+00	**3442**	10,0.2
Chaotic Rao2	6.84295801	6.84295801	6.84295801	0.000E+00	**4277**	10,0.9
Chaotic Rao3	6.84295801	6.84295801	6.84295801	0.000E+00	**4132**	8,0.6

[a] Results are taken from [16]; [*] Infeasible solution due to violation of a constraint

5.11. Planetary gear train:

The maximum FEs in this problem is considered as 200000. Table 16compares the statistical results of this problem over 25 runs using the proposed algorithms, PSO (Particle Swarm Optimization), ABC(Artificial Bee Colony), PVS (Passing Vehicle Search), AOS, and QSA (Queuing Search Algorithm). The best fitness value obtained is $f(\overline{x}) =$ 0.52325 at $\overline{x} =$ (40,21,14,19,14,69,5,1.75,2) using all the proposed algorithms and AOS algorithm. The mean fitness values attained using the proposed algorithms are slightly higher than the same obtained using AOS algorithm.

5.12. Step cone pulley:

The maximum FEs in this problem, is considered as 15000. Table 17compares the statistical results of this problem over 25 runs for TLBO (Teaching Learning Based Optimization), WOA (Whale Optimization Algorithm), WCA (Water Cycle Algorithm), MBA (Mine Blast Algorithm), and AOS. The best fitness value obtained is $f(\overline{x}) =$ 16.63450485 kg at $\overline{x} =$ (85.98624253, 40, 54.76430076, 73.01317694, 88.42841982) by all the proposed algorithms.. Even though the result obtained by AOS appears to be better, but it is to be considered as an infeasible solution because the value of one design variable is out of the given range. The best mean fitness value is attained using the Chaotic Rao2 algorithm.

5.13. Hydrostatic thrust bearing:

The maximum FEs in this problem is considered as 25000. Table 18compares the statistical results of this problem over 25 runs for TLBO, AOS, and the proposed algorithms. Table 19 compares the best fitness value obtained in present study and those obtained by PSO, GASO (Genetic Algorithm aided Stochastic Optimization), GeneAS (Genetic Adaptive Search), ABC, TLBO, and AOS algorithm. Even though the result obtained by AOS appears to be better, but it is to be considered as an infeasible solution because of the violation of a constraint. The best fitness value is obtained by SAMPE Rao1 algorithm, and the value is $f(\overline{x}) =$ 1625.031214 btu/s at $\overline{x} =$ (5.955330, 5.388607, 2.269248, 0.000005358845).

Table 16. Statistical results comparison for a planetary gear train design problem

Algorithm	B	M	W	SD	MFE	$NP,ES/CS_1$
PSO[a]	0.53	0.5361934	NA	NA	NA	50
ABC[a]	0.525769	0.5272922	NA	NA	NA	50
QSA[a]	0.525589	0.525589	NA	NA	NA	30
PVS[a]	0.525588	0.53063	NA	NA	NA	NA
AOS[b]	0.52325	0.529848233	0.537058824	3.894E-03	NA	NA
Rao1	0.52325	0.534683372	0.553181818	6.143E-03	12459	30
Rao2	0.52325	0.588581422	1.87875	2.689E-01	9880.8	30
Rao3	0.52325	0.545598896	0.805652174	5.460E-02	14033	35
SAMPE Rao1	0.52325	0.531301937	0.549705882	6.245E-03	10611	20,2
SAMPE Rao2	0.52325	0.53330837	0.549849108	5.618E-03	9283	20,2
SAMPE Rao3	0.52325	0.532603237	0.545064935	5.561E-03	11022	40,2
Chaotic Rao1	0.52325	0.53293644	0.549705882	5.719E-03	11623	25,0.6
Chaotic Rao2	0.52325	0.54700953	0.805652174	5.436E-02	9379.2	40,0.2
Chaotic Rao3	0.52325	0.536461805	0.549311686	5.899E-03	14137.6	40,0.4

[a]Results are taken from [15]; [b]Results are taken from [16]

Table 17. Statistical results comparison for a step cone pulley design problem

Algorithm	B	M	W	SD	MFE	$NP,ES/CS_1$
TLBO[a]	16.63451	24.0113577	74.022951	3.400E-01	NA	50
WOA[a]	16.6345213	20.93829477	24.8488259	3.349E+00	NA	20
WCA[a]	16.63450849	17.53037682	18.83302997	9.229E-01	NA	20
MBA[a]	16.6345078	16.702535	18.3237145	2.627E-01	NA	20
AOS[a]*	16.08558875	16.29548945	16.80334816	1.772E-01	NA	NA
Rao1	16.63450485	16.72640748	17.59405649	2.198E-01	12845	20
Rao2	16.63450485	16.74252202	17.51387727	2.471E-01	12888	20
Rao3	16.63450485	16.78532149	18.01364527	3.673E-01	13249	20
SAMPE Rao1	16.63450485	16.69646353	17.25819841	1.400E-01	12812	30,2
SAMPE Rao2	16.63450485	16.70155067	17.30303274	1.598E-01	13647	25,2
SAMPE Rao3	16.63450485	16.7270869	17.4362292	2.204E-01	12669	30,2
Chaotic Rao1	16.63450485	16.67806324	16.88845818	8.562E-02	10208	15,0.3
Chaotic Rao2	16.63450485	16.64720778	16.95205707	6.351E-02	14751	20,0.2
Chaotic Rao3	16.63450485	16.74985978	19.24078858	5.204E-01	14552	20,0.2

[a]Results are taken from [16]; *Infeasible solution because the value of one design variable is out of the given range

5.14. Welded beam:

The maximum FEs in this problem is considered as 15000. Table 20compares the statistical results of this problem over 25 runs obtained by CDE (Co-evolutionary Differential Evolution), WCA (Water Cycle Algorithm), IAPSO (Improved accelerated PSO), STA (State Transition Algorithm), and CGO.The best fitness value obtained is $f(\overline{x})$ =1.724852309 $ at $\overline{x}$ =(0.20573, 3.470489, 9.036624, 0.20573) by all the algorithms. The best fitness values given by algorithms STA and CGO algorithms are not feasible as the corresponding values of the design variables violate a constraint. The Chaotic Rao1 algorithm requires the minimum FEs to reach an optimal solution.

5.15. Multiple disk clutch brake:

The maximum FEs in this problem is considered as 500. Table 21compares the statistical resultsof this problem over 25 runs obtained by CGO, WCA, TLBO, and C-ITGO (constrained Iterative Topographical Global Optimization) and the proposed algorithms. The best fitness value obtained is $f(\overline{x})$ =0.235242458 kgat $\overline{x}$ =(70, 90, 1, 1000, 2) by all the proposed algorithms. The results obtained by WCA, TLBO, and C-ITGO are inferior. The Chaotic Rao2 algorithm requires the minimum FEs to reach an optimal solution.

Table 18. Statistical results comparison for a hydrostatic thrust bearing design problem

Algorithm	B	M	W	SD	MFE	NP,ES/CS$_1$
TLBO[a]	1625.44276	1797.70798	2096.8012	1.900E-01	NA	50
AOS[b*]	1621.926212	1752.413561	1831.449755	2.362E+01	NA	NA
Rao1	1625.143383	1643.604394	1808.117508	3.670E+01	23984	10
Rao2	1625.346729	1815.285715	3386.590418	4.763E+02	21576	10
Rao3	1625.221851	1730.441329	3388.349231	3.491E+02	21469	10
SAMPE Rao1	1625.031214	1840.052283	3385.845191	4.331E+02	22935	10,2
SAMPE Rao2	1625.089866	1744.303738	3386.979117	3.741E+02	22990	10,2
SAMPE Rao3	1625.084662	1744.362308	3385.814478	3.502E+02	22386	10,2
Chaotic Rao1	1625.10655	1637.083563	1692.563375	1.974E+01	23530	10,0.3
Chaotic Rao2	1625.087311	1836.948829	3386.439864	5.049E+02	22797	10,0.3
Chaotic Rao3	1625.171166	1949.270479	3385.588751	6.433E+02	22051	10,0.4

* Infeasible solution due to violation of a constraint; [a] Results are taken from [13];[b]Results are taken from [16]

Table 19. Comparison of best fitness value for a hydrostatic thrust bearing design problem

Algorithm	GASO[a]	GeneAS[a]	ABC[a]	TLBO[a]	AOS[b]*	Present study
Best result	1950.286	2161.4215	1625.44276	1625.44276	1621.9262	**1625.031214**

[a] Results are taken from [13];[b]Results are taken from [16]; *Infeasible solution due to violation of a constraint

Table 20. Statistical results comparison for a welded beam design problem

Algorithm	B	M	W	SD	MFE	NP,ES/CS$_1$
CDE [a]	1.733461	1.768158	1.824105	2.219E-02	NA	NA
WCA [a]	1.724856	1.726427	1.744697	4.290e−03	NA	NA
IAPSO [a]	1.724852	1.724853	1.724862	2.020e−06	NA	50
STA [a*]	1.6956397	1.6956397	1.7530472	1.830E−02	NA	30
CGO [a*]	1.670335792	1.670378098	1.670902785	9.300E−05	NA	NA
Rao1	1.724852309	1.724852309	1.724852309	0.000E+00	14631	10
Rao2	1.724852309	1.724852309	1.724852309	0.000E+00	14888	20
Rao3	1.724852309	1.724852309	1.724852309	0.000E+00	14844	20
SAMPE Rao1	1.724852309	1.724852309	1.724852309	0.000E+00	10260	15,2
SAMPE Rao2	1.724852309	1.724852309	1.724852309	0.000E+00	14171	20,2
SAMPE Rao3	1.724852309	1.724852309	1.724852309	0.000E+00	14477	20,2
Chaotic Rao1	1.724852309	1.724852309	1.724852309	0.000E+00	6650	10,0.2
Chaotic Rao2	1.724852309	1.724852309	1.724852309	0.000E+00	14881	20,0.6
Chaotic Rao3	1.724852309	1.724852309	1.724852309	0.000E+00	14821	20,0.6

[a] Results are taken from [22]; *Infeasible solution due to violation of constraints.

Table 21. Statistical resultscomparison for a multiple disk clutch brake design problem

Algorithm	B	M	W	SD	MFE	NP,ES/CS$_1$
WCA [a]	0.313656	0.313656	0.313656	1.690E−16	NA	NA
TLBO [a]	0.313657	0.3271662	0.392071	6.700E-01	NA	20
C-ITGO [a]	0.313656	0.313656	0.313656	1.130E−16	NA	20
CGO [a]	0.235242458	0.235242458	0.23524246	1.950E−10	NA	NA
Rao1	0.235242458	0.235242458	0.235242458	0.000E+00	132.0	10
Rao2	0.235242458	0.235242458	0.235242458	0.000E+00	119.0	20
Rao3	0.235242458	0.235242458	0.235242458	0.000E+00	123.0	20
SAMPE Rao1	0.235242458	0.235242458	0.235242458	0.000E+00	128.0	15,4
SAMPE Rao2	0.235242458	0.235242458	0.235242458	0.000E+00	113.0	15,2
SAMPE Rao3	0.235242458	0.235242458	0.235242458	0.000E+00	99.0	10,2
Chaotic Rao1	0.235242458	0.235242458	0.235242458	0.000E+00	113.0	10,0.2
Chaotic Rao2	0.235242458	0.235242458	0.235242458	0.000E+00	93.0	15,0.2
Chaotic Rao3	0.235242458	0.235242458	0.235242458	0.000E+00	98.0	15,0.2

[a] Results are taken from [22]

6. Conclusions

The Self-adaptive Multi-population Elitist (SAMPE) Rao algorithms and Chaotic Rao algorithms are proposed in the present work. These algorithms are based on the recently developed Rao algorithms. The SAMPE Rao algorithms increase the exploration and exploitation rate of the search process in finding the optimal solution. The Chaotic Rao algorithms help to find the optimal solution without getting stuck at the local optimum. 25 unconstrained benchmark functions and 15 constrained mechanical engineering design optimization problems are solved using the proposed algorithms to test their performance. The Friedman rank test is used to validate the proposed algorithms' superior performance. The proposed algorithms are ranked according to their performance in the Friedman rank test; the chaotic Rao1 algorithm is ranked first. The best fitness value and the statistical results achieved by the proposed algorithms and previously reported results using other advanced optimization algorithms for engineering optimization problems are compared. This comparison shows that the proposed algorithms effectively solve most of the benchmark problems and the constrained engineering design problems. In this paper, design optimization problems of mechanical engineering components are considered but the proposed algorithms can also be applied to more complex design optimization problems. The proposed algorithms can also be used to solve multi-objective optimization problems.

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Appendix A:

Table A.1 Unconstrained benchmark functions considered[20]

No.	Function	Formulation	D	Search range	C
f_1	Sphere	$f_1(x) = \sum_{i=1}^{D} x_i^2$	30	[-100, 100]	US
f_2	SumSquares	$f_2(x) = \sum_{i=1}^{D} i x_i^2$	30	[-10, 10]	US
f_3	Beale	$f_3(x) = (2.25 - x_1 - x_1 x_2^2)^2 + (1.5 - x_1 + x_1 x_2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$	5	[-4.5, 4.5]	UN
f_4	Easom	$f_4(x) = -cos(x_1)cos(x_2)exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$	2	[-100, 100]	UN
f_5	Matyas	$f_5(x) = 0.26(x_1^2 + x_2^2) - 0.48 x_1 x_2$	2	[-10, 10]	UN
f_6	Colville	$f_6(x) = 100(x_1^2 - x_2)^2 + (x_3 - 1)^2 + (x_1 - 1)^2 + 90(x_3^2 - x_4)^2 + 19.8(x_2 - 1)(x_4 - 1) + 10.1((x_2 - 1)^2 + (x_4 - 1)^2)$	4	[-10, 10]	UN
f_7	Trid 6	$f_7(x) = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=2}^{D} x_i x_{i-1}$	6	[-D^2 , D^2]	UN
f_8	Trid 10	$f_8(x) = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=2}^{D} x_i x_{i-1}$	10	[-D^2 , D^2]	UN
f_9	Zakharov	$f_9(x) = \sum_{i=1}^{D} x_i^2 + \left(\sum_{i=1}^{D} 0.5 i x_1\right)^2 + \left(\sum_{i=1}^{D} 0.5 i x_1\right)^4$	10	[-5, 10]	UN
f_{10}	Schwefel 1.2	$f_{10}(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j^2\right)^2$	30	[-100, 100]	UN
f_{11}	Rosenbrock	$f_{11}(x) = \sum_{i=1}^{D} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	[-30, 30]	UN
f_{12}	Dixon-Price	$f_{12}(x) = (1 - x_1)^2 + \sum_{i=2}^{D} i(x_{i-1} - 2x_i^2)^2$	30	[-10, 10]	UN
f_{13}	Branin	$f_{13}(x) = \left(x_2 - \frac{5.1 x_1^2}{4\pi^2} + \frac{5x_1}{\pi - 6}\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cos(x_1) + 10$	2	[-5, 10] [0, 15]	MS
f_{14}	Bohachevsky 1	$f_{14}(x) = x_1^2 + x_2^2 - 0.3cos(3\pi x_1) - 0.4cos(4\pi x_2) + 0.7$	2	[-100, 100]	MS
f_{15}	Bohachevsky 2	$f_{15}(x) = x_1^2 + 2x_2^2 - 0.3cos((3\pi x_1)(4\pi x_2)) + 0.3$	2	[-100, 100]	MN
f_{16}	Bohachevsky 3	$f_{16}(x) = x_1^2 + 2x_2^2 - 0.3cos(3\pi x_1 + 4\pi x_2) + 0.3$	2	[-100, 100]	MN
f_{17}	Booth	$f_{17}(x) = (x_1 - 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	2	[-10, 10]	MS

f18	Michalewicz 2	$f_{18}(x) = -\sum_{i=1}^{D} sin\, x_1 \left(sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20}$	2	[0, π]	MS
f19	Michalewicz 5	$f_{19}(x) = -\sum_{i=1}^{D} sin\, x_1 \left(sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20}$	5	[0, π]	MS
f20	GoldStein-Price	$f_{20}(x) = [1 + (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)(x_1 + x_2 + 1)^2]$ $[30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	[-2, 2]	MN
f21	Perm	$f_{21}(x) = \sum_{k=1}^{D}\left[\sum_{i=1}^{D}(i_k + \beta)\left(\left(\frac{x_i}{i}\right)^k - 1\right)\right]^2$	4	[-D, D]	MN
f22	Ackley	$f_{22}(x) = -20\, exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D} cos\, 2\pi x_i\right) + 20 + e$	30	[-32, 32]	MN
f23	Foxholes	$f_{23}(x) = \left[\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right]^{-1}$	2	[-65.536, 65.536]	MS
f24	Hartman 3	$f_{24}(x) = -\sum_{i=1}^{4} c_i\, exp\left[-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right]$	3	[0, 1]	MN
f25	Penalized 2	$f_{25}(x) = 0.1\left[sin^2(\pi x_1) + \sum_{i=1}^{D-1}(x_i - 1)^2\{1 + sin^2(3\pi x_{i+1})\} + (x_D - 1)^2 + (1 + sin^2(2\pi x_D))\right] + \sum_{i=1}^{D} u(x_i, 5, 100, 4)$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \le x_i \le a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$	30	[-50, 50]	MN

D: Dimension, C: Characteristics, U: Unimodal, M: Multimodal, S: Separable, N: Non-separable.

Appendix B: Mathematical formulations of the design optimization problems considered [16]

Problem 1: Planetary gear train:

Design variables:

$\overline{x} = [Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_p, m_1, m_2]$ B.1

Objective function:

Minimize,

$f(\overline{x}) = max\,|\,i_k - i_{0k}|, k = \{1,2,R\}$ B.2

$i_1 = \frac{Z_6}{Z_4}, i_{01} = 3.11, i_2 = \frac{Z_6(Z_1Z_3 + Z_2Z_4)}{Z_1Z_3(Z_6 - Z_4)}, i_{02} = 1.84, i_R = -\frac{Z_2Z_6}{Z_1Z_3}, i_{0R} = -3.11$ B.3

Design constraints:

$C_1(\overline{x}) = \mathrm{D_{max}} - m_3(Z_6 + 2.5) \geq 0$ B.4

$C_2(\overline{x}) = \mathrm{D_{max}} - m_1(Z_1 + Z_2) - m_1(Z_2 + 2) \geq 0$ B.5

$C_3(\overline{x}) = \mathrm{D_{max}} - m_2(Z_4 + Z_5) - m_2(Z_5 + 2) \geq 0$ B.6

$C_4(\overline{x}) = m_1 + m_2 - |m_1(Z_1 + Z_2) - m_2(Z_6 - Z_3)| \geq 0$ B.7

$C_5(\overline{x}) = 2 + \delta_{22} + Z_2 - (Z_1 + Z_2)\sin(\pi/Z_p) \geq 0$ B.8

$C_6(\overline{x}) = Z_3 + 2 + \delta_{33} - (Z_6 - Z_3)\sin(\pi/Z_p) \geq 0$ B.9

$C_8(\overline{x}) = (Z_3 + Z_5 + 2 + \delta_{35})^2 - (Z_6 - Z_3)^2 - (Z_4 + Z_5)^2 + 2(Z_6 - Z_3)(Z_4 + Z_5)cos(2\pi/Z_p - \beta) \geq 0$ B.10

Where,

$\beta = \frac{cos^{-1}((Z_6 - Z_3)^2 + (Z_4 + Z_5)^2 - (Z_3 + Z_5)^2)}{2(Z_6 - Z_3)(Z_4 + Z_5)}$ B.11

$C_9(\overline{x}) = 4 + 2\delta_{34} - Z_6 + 2Z_3 + Z_4 \geq 0$ B.12

$C_{10}(\overline{x}) = 4 + 2\delta_{56} - Z_6 + Z_4 + 2Z_5 \geq 0$ B.13

$h(\overline{x}) = \frac{Z_6 - Z_4}{Z_p} = integer$ B.14

Where,

$Z_p = (3,4,5), m_1, m_2 = (1.75, 2, 2.25, 2.5, 2.75, 3)mm$ B.15

$17 \leq Z_1 \leq 96, 14 \leq Z_2 \leq 54, 14 \leq Z_3 \leq 51, 17 \leq Z_4 \leq 46, 14 \leq Z_5 \leq 51, 48 \leq Z_6 \leq 124,$ B.16
$Z_i = integer, \quad i = 1,2, \dots 6$

$D_{\max}$=220 mm, δ_{22}=0.5, δ_{33}=0.5, δ_{55}=0.5, δ_{35}=0.5, δ_{56}=0.5, δ_{34}=0.5

Problem 2: Step-cone pulley

Design variables:

$\overline{x} = [d_1, d_2, d_3, d_4, w]$ B.17

Objective function:

Minimize, $f(x) = \frac{\pi}{4}\rho w\left\{d_1^2\left[1 + \left(\frac{N_1}{N}\right)^2\right] + d_2^2\left[1 + \left(\frac{N_2}{N}\right)^2\right] + d_3^2\left[1 + \left(\frac{N_3}{N}\right)^2\right] + d_4^2\left[1 + \left(\frac{N_4}{N}\right)^2\right]\right\}$ B.18

Design constraints:

$C_1(x) = c_2 - c_1 = 0$ B.19

$C_2(x) = c_3 - c_1 = 0$ B.20

$C_3(x) = c_4 - c_1 = 0$ B.21

$C_{4,5,6,7}(x) = 2 - R_i \leq 0$ B.22

$C_{8,9,10,11}(x) = (0.75 \times 745.6998) - P_i \leq 0$ B.23

Where c_i: belt length, N_i: Speed to be achieved

$c_i = \frac{\pi d_i}{2}\left(\frac{N_i}{N}+1\right)+\frac{\left(\frac{N_i}{N}-1\right)^2 d_i^2}{4a}+2a$ B.24

R_i: tension ratio

$R_i = exp\left\{\mu\left[\pi - 2\,sin^{-1}\left\{\frac{d_i}{2a}\left(\frac{N_i}{N}-1\right)\right\}\right]\right\}$ B.25

P_i: power transmitted at each step

$P_i = stw\left[1 - exp\left\{-\mu\left[\pi - 2\,sin^{-1}\left\{\frac{d_i}{2a}\left(\frac{N_i}{N}-1\right)\right\}\right]\right\}\right] \times \left(\frac{\pi d_i N_i}{60}\right)$ B.26

$\rho = 7200 kg/m^3, \mu = 0.35, a = 3\ m, t = 8\ mm, s = 1.75 MPa,$

$16 \le w(mm) \le 100,\ 40 \le d_i(mm) \le 100$, i=1, 2, 3, 4 B.27

$N = 350\ rpm,\ N_1 = 750\ rpm,\ N_2 = 450\ rpm, N_3 = 250\ rpm,\ N4 = 150\ rpm$

Problem 3: Hydrostatic thrust bearing:

Design variables:

$\overline{x} = [R, R_0, Q, \mu]$ B.28

Minimize, $f(\overline{x}) = \frac{1}{12}\left(\frac{QP_o}{0.7}+E_f\right)$ B.29

Design constraints:

$C_1(\overline{x}) = W_s - W \le 0$ B.30

$C_2(\overline{x}) = P_o - P_{max} \le 0$ B.31

$C_3(\overline{x}) = \Delta T_{max} - \Delta T \le 0$ B.32

$C_4(\overline{x}) = h_{min} - h \le 0$ B.33

$C_5(\overline{x}) = R_0 - R \le 0$ B.34

$C_6(\overline{x}) = \left(\frac{\gamma}{gP_o}\right)\left(\frac{Q}{2\pi Rh}\right)^2 - 0.001 \le 0$ B.35

$C_7(\overline{x}) = \left(\frac{W}{\pi(R^2 - {R_0}^2)}\right) - 5000 \le 0$ B.36

Where,

γ= $weight\ density\ of\ oil = 0.0307\ \frac{lb}{in^3}$, B.37

C=$specific\ heat\ of\ oil = 0.5$ BTU/lb^{o}F, n=-3.55; C_1=10.04, W_s=100982.74 lb

P_{max}=1000 psi, ΔT_{max}=50^{o}F, h_{min}=0.001 in, g=386.4 in/s^2, N=750 rpm B.38

P= $\frac{(\log_{10}(\log_{10}(8.122 \times 10^6\mu+0.8)))\text{-c1}}{n}$ B.39

$\Delta T = 2((10^P) - 560)$ B.40

E_f= 9336Q$\gamma C \Delta T$ B.41

$h = \left(\frac{2\pi N}{60}\right)^2\left(\frac{2\pi\mu}{E_f}\right)\left(\frac{R^4 - R_0^4}{4}\right)$ B.42

$P_o = \left(\frac{6\mu Q}{\pi h^3}\right) ln\left(\frac{R}{R_0}\right)$ B.43

$W = \left(\frac{\pi Po}{2}\right)\left(\frac{R^2 - R_0^2}{ln(R/R_0)}\right)$ B.44

$1 \le R, R_0(in), Q\ (in^3/s) \le 16,\ 10^{-1} \le \mu \le 16 \times 10^{-6}$ B.45

Problem 4: Four stage gearbox

Design variables:

$\overline{x} = [N_{p1}, N_{g1}, N_{p2}, N_{g2}, N_{p3}, N_{g3}, N_{p4}, N_{g4}, b_1, b_2, b_3, b_4, x_{p1}, x_{g1}, x_{g2}, x_{g3}, x_{g4}, y_{p1}, y_{g1}, y_{g2}, y_{g3}, y_{g4}]$ B.46

Objective function:

Minimize, $f(\overline{x}) = \frac{\pi}{1000}\left[\frac{b_1c_1^2(N_{p1}^2+N_{g1}^2)}{(N_{p1}+N_{g1})^2}+\frac{b_2c_2^2(N_{p2}^2+N_{g2}^2)}{(N_{p2}+N_{g2})^2}+\frac{b_3c_3^2(N_{p3}^2+N_{g3}^2)}{(N_{p3}+N_{g3})^2}+\frac{b_4c_4^2(N_{p4}^2+N_{g4}^2)}{(N_{p4}+N_{g4})^2}\right]$ B.47

Design constraints:

$$C_1(\overline{x}) = \left(\frac{366000}{\pi\omega_1}+\frac{2c_1N_{p1}}{N_{g1}+N_{p1}}\right)\left(\frac{(N_{p1}+N_{g1})^2}{4b_1c_1^2N_{p1}}\right) \le \frac{\sigma_N J_R}{0.0167WK_oK_m} \quad \text{B.48}$$

$$C_2(\overline{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1N_{p1}}+\frac{2c_2N_{p2}}{N_{g2}+N_{p2}}\right)\left(\frac{(N_{p2}+N_{g2})^2}{4b_2c_2^2N_{p2}}\right) \le \frac{\sigma_N J_R}{0.0167WK_oK_m} \quad \text{B.49}$$

$$C_3(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1N_{p1}N_{p2}}+\frac{2c_3N_{p3}}{N_{g3}+N_{p3}}\right)\left(\frac{(N_{p3}+N_{g3})^2}{4b_3c_3^2N_{p3}}\right) \le \frac{\sigma_N J_R}{0.0167WK_oK_m} \quad \text{B.50}$$

$$C_4(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1N_{p1}N_{p2}N_{p3}}+\frac{2c_4N_{p4}}{N_{g4}+N_{p4}}\right)\left(\frac{(N_{p4}+N_{g4})^2}{4b_4c_4^2N_{p4}}\right) \le \frac{\sigma_N J_R}{0.0167WK_oK_m} \quad \text{B.51}$$

$$C_5(\overline{x}) = \left(\frac{366000}{\pi\omega_1}+\frac{2c_1N_{p1}}{N_{p1}+N_{g1}}\right)\left(\frac{(N_{p1}+N_{g1})^3}{4b_1c_1N_{g1}N_{p1}^2}\right) \le \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin\phi\cos\phi}{0.0334WK_oK_m}\right) \quad \text{B.52}$$

$$C_6(\overline{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1N_{p1}}+\frac{2c_2N_{p2}}{N_{p2}+N_{g2}}\right)\left(\frac{(N_{p2}+N_{g2})^3}{4b_2c_2N_{g2}N_{p2}^2}\right) \le \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin\phi\cos\phi}{0.0334WK_oK_m}\right) \quad \text{B.53}$$

$$C_7(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1N_{p1}N_{p2}}+\frac{2c_3N_{p3}}{N_{p3}+N_{g3}}\right)\left(\frac{(N_{p3}+N_{g3})^3}{4b_3c_3N_{g3}N_{p3}^2}\right) \le \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin\phi\cos\phi}{0.0334WK_oK_m}\right) \quad \text{B.54}$$

$$C_8(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1N_{p1}N_{p2}N_{p3}}+\frac{2c_4N_{p4}}{N_{p4}+N_{g4}}\right)\left(\frac{(N_{p4}+N_{g4})^3}{4b_4c_4N_{g4}N_{p4}^2}\right) \le \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin\phi\cos\phi}{0.0334WK_oK_m}\right) \quad \text{B.55}$$

$$C_{9-12}(\overline{x}) = N_{pi}\sqrt{\frac{\sin^2\phi}{4}+\frac{1}{N_{pi}}+\left(\frac{1}{N_{pi}}\right)^2} + N_{gi}\sqrt{\frac{\sin^2\phi}{4}+\frac{1}{N_{gi}}+\left(\frac{1}{N_{gi}}\right)^2} - \frac{\sin\phi\,(N_{pi}+N_{gi})}{2} \ge CR\cos\phi_{min} \quad \text{B.56}$$

$$C_{13-16}(\overline{x}) = d_{min} \le \frac{2c_iN_{pi}}{N_{pi}+N_{gi}} \quad \text{B.57}$$

$$C_{17-20}(\overline{x}) = d_{min} \le \frac{2c_iN_{gi}}{N_{pi}+N_{gi}} \quad \text{B.58}$$

$$C_{21}(\overline{x}) = x_{p_1} + \left(\frac{(N_{p1}+2)c_1}{N_{p1}+N_{g1}}\right) \le L_{max} \quad \text{B.59}$$

$$C_{22-24}(\overline{x}) = \left[x_{g(i-1)} + \left(\frac{(N_{pi}+2)c_i}{N_{pi}+N_{gi}}\right)\right]_{i=2,3,4} \le L_{max} \quad \text{B.60}$$

$$C_{25}(\overline{x}) = -x_{p_1} + \left(\frac{(N_{p1}+2)c_1}{N_{p1}+N_{g1}}\right) \le 0 \quad \text{B.61}$$

$$C_{26-28}(\overline{x}) = \left[-x_{g(i-1)} + \left(\frac{(N_{pi}+2)c_i}{N_{pi}+N_{gi}}\right)\right]_{i=2,3,4} \le 0 \quad \text{B.62}$$

$$C_{29}(\overline{x}) = y_{p_1} + \left(\frac{(N_{p1}+2)c_1}{N_{p1}+N_{g1}}\right) \le L_{max} \quad \text{B.63}$$

$$C_{30-32}(\overline{x}) = \left[y_{g(i-1)} + \left(\frac{(N_{pi}+2)c_i}{N_{pi}+N_{gi}}\right)\right]_{i=2,3,4} \le L_{max} \quad \text{B.64}$$

$$C_{33}(\overline{x}) = -y_{p_1} + \left(\frac{(N_{p1}+2)c_1}{N_{p1}+N_{g1}}\right) \le 0 \quad \text{B.65}$$

$$C_{34-36}(\overline{x}) = \left[-y_{g(i-1)} + \left(\frac{(N_{pi}+2)c_i}{N_{pi}+N_{gi}}\right)\right]_{i=2,3,4} \leq 0$$ B.66

$$C_{37-40}(\overline{x}) = \left[x_{gi} + \left(\frac{(N_{gi}+2)c_i}{N_{pi}+N_{gi}}\right)\right]_{i=1,2,3,4} \leq L_{max}$$ B.67

$$C_{41-44}(\overline{x}) = -x_{g_i} + \left(\frac{(N_{gi}+2)c_i}{N_{pi}+N_{gi}}\right) \leq 0$$ B.68

$$C_{45-48}(\overline{x}) = y_{g_i} + \left(\frac{(N_{gi}+2)c_i}{N_{pi}+N_{gi}}\right) \leq L_{max}$$ B.69

$$C_{49-52}(\overline{x}) = -y_{g_i} + \left(\frac{(N_{gi}+2)c_i}{N_{pi}+N_{gi}}\right) \leq 0$$ B.70

$$C_{53-56}(\overline{x}) = (0.945c_i - N_{pi} - N_{gi})(b_i - 5.715)(b_i - 8.255)(b_i - 12.70)(-1) \leq 0$$ B.71

$$C_{57-60}(\overline{x}) = (0.646c_i - N_{pi} - N_{gi})(b_i - 3.175)(b_i - 8.255)(b_i - 12.70)(+1) \leq 0$$ B.72

$$C_{61-64}(\overline{x}) = (0.504c_i - N_{pi} - N_{gi})(b_i - 3.175)(b_i - 5.715)(b_i - 12.70)(-1) \leq 0$$ B.73

$$C_{65-68}(\overline{x}) = (c_i - N_{pi} - N_{gi})(b_i - 3.175)(b_i - 5.715)(b_i - 8.255)(+1) \leq 0$$ B.74

$$C_{69-72}(\overline{x}) = (N_{pi} + N_{gi} - 1.812c_i)(b_i - 5.715)(b_i - 8.255)(b_i - 12.70)(-1) \leq 0$$ B.75

$$C_{73-76}(\overline{x}) = (N_{pi} + N_{gi} - 0.945c_i)(b_i - 3.175)(b_i - 8.255)(b_i - 12.70)(+1) \leq 0$$ B.76

$$C_{77-80}(\overline{x}) = (N_{pi} + N_{gi} - 0.646c_i)(b_i - 3.175)(b_i - 5.715)(b_i - 12.70)(-1) \leq 0$$ B.77

$$C_{81-84}(\overline{x}) = (N_{pi} + N_{gi} - 0.504c_i)(b_i - 3.175)(b_i - 5.715)(b_i - 8.255)(+1) \leq 0$$ B.78

$$C_{85}(\overline{x}) = \omega_{min} \leq \frac{\omega_1(N_{p1}N_{p2}N_{p3}N_{p4})}{N_{g1}N_{g2}N_{g3}N_{g4}}$$ B.79

$$C_{86}(\overline{x}) = \frac{\omega_1(N_{p1}N_{p2}N_{p3}N_{p4})}{N_{g1}N_{g2}N_{g3}N_{g4}} \leq \omega_{max}$$ B.80

where,

$$CR_{min} = 1.4, d_{min} = 25.4\ mm, \phi = 20^o, W = 55.9, J_R = 0.2,$$
$$K_M = 1.6, K_o = 1.5, L_{max} = 127\ mm,$$ B.81

$$\sigma_H = 3290\frac{kg}{cm^2}, \sigma_N = 2090\frac{kg}{cm^2}, \omega_1 = 5000\ rom,$$ B.82

$$\omega_{min} = 245\ rpm, \omega_{max} = 255\ rpm, C_p = 464$$

$$x_{p1}, y_{p1}, x_{gi}, y_{gi} = (12.7, 25.4, 38.1, 50.8, 63.5, 76.2, 88.9, 101.6, 114.3)mm$$ B.83

$$b_i = (3.175, 5.715, 8.255, 12.7)mm$$ B.84

$$7 \leq N_{pi}, N_{gi} \leq 76, N_{pi}, N_{gi} = integer$$ B.85

Problem 5: Rolling element bearing

Design variables:

$$\{x\} = [D_m, D_b, f_i, f_o, Z, K_{Dmax}, K_{Dmin}]$$ B.86

Objective function:

Maximize,

$$f(\overline{x}) = \begin{cases} f_c Z^{2/3} {D_b}^{1.8} & , if\ D_b \leq 25.4\ mm \\ 3.647 f_c Z^{2/3} {D_b}^{1.4} & , if\ D_b \leq 25.4\ mm \end{cases}$$ B.87

where

$$f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_o-1)}{f_o(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3}\left[\frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}}\right]\left[\frac{2f_i}{2f_i-1}\right]^{0.41}$$ B.88

$\gamma = \frac{D_b \cos\alpha}{D_m} \quad (Here\ \alpha = 0)$ B.89

Design constraints:

$$C_1(x) = \frac{\varphi_0}{2\sin^{-1}(D_b/D_m)} + 1 \geq Z$$ B.90

$C_2(x) = 2D_b - (D-d)K_{Dmin} \geq 0$ B.91

$C_3(x) = (D-d)K_{Dmax} - 2D_b \geq 0$ B.92

$C_4(x) = \beta w - D_b \geq 0$ B.93

$C_5(x) = D_m - (D+d)(0.5-e) \geq 0$ B.94

$C_6(x) = (D+d)(0.5+e) - D_m \geq 0$ B.95

$C_7(x) = 0.5(D - D_b - D_m) - (\varepsilon \times D_b) \geq 0$ B.96

$C_8(x) = f_i - 0.515 \geq 0$ B.97

$C_9(x) = f_o - 0.515 \geq 0$ B.98

where

$\varphi_0 = 2\pi - 2\cos^{-1}\left[\frac{\left(\frac{D}{2}-T-D_b\right)^2 - \left(\frac{d}{2}+T\right)^2 + U^2}{2\left(\frac{D}{2}-T-D_b\right)U}\right]$ B.99

$T = \frac{D-d-2D_b}{4}, \qquad U = \frac{D-d}{2} - 3T$ B.100

$D = 160\ mm, d = 90\ mm, \ \ w = 30\ mm,$ B.101

$0.15(D-d) \leq D_b(mm) \leq 0.45(D-d), \ \ 0.5(D+d) \leq D_m(mm) \leq 0.6(D+d),$ B.102

$0.515 \leq f_o \leq 0.6, \quad 0.515 \leq f_i \leq 0.6, \quad 4 \leq Z \leq 50,$ B.103

$0.4 \leq K_{Dmin} \leq 0.5, 0.6 \leq K_{Dmax} \leq 0.7, 0.3 \leq \varepsilon \leq 0.4,$ B.104

$0.02 \leq e \leq 0.1, 0.6 \leq \beta \leq 0.85$ B.105

Problem 6: Gas transmission compressor:

Design variables:

$\overline{x} = [x_1, x_2, x_3, x_4]$ B.106

Objective function:

Minimize, $f(\overline{x}) = 8.61 \times 10^5 \times {x_1}^{1/2} x_2 {x_3}^{-2/3} {x_4}^{-1/2} + 3.69 \times 10^4 \times x_3 + 7.72 \times 10^8 \times {x_1}^{-1} {x_2}^{0.219} - 765.43 \times 10^6 \times x_1$ B.107

Design constraints:

$C_1(\overline{x}) = x_4 {x_2}^{-2} + {x_2}^{-2} - 1 \leq 0$ B.108

$20 \leq x_1\ (miles) \leq 50, 1 \leq x_2 \leq 10, 20 \leq x_3\ (in) \leq 50, 0.1 \leq x_4\ (ft^3/s) \leq 60$ B.109

Problem 7: Tension/Compression spring Case-1

Design variables:

$\overline{x} = [x_1, x_2, x_3]$=[*d,D,N*] B.110

Objective function:

Minimize $f(\overline{x}) = (x_3 + 2) x_2 {x_1}^2$ B.111

Design constraints:

$C_1(\overline{x}) = 71785 {x_1}^4 - {x_2}^3 x_3 \leq 0$ B.112

$$C_2(\overline{x}) = -\frac{4{x_2}^2 - x_1 x_2}{12566(x_2 {x_1}^3 - {x_1}^4)} - \frac{1}{5108 {x_1}^2 - 1} \geq 0$$ B.113

$C_3(\overline{x}) = 140.45 x_1 - {x_2}^3 x_3 \geq 0$ B.114

$C_4(\overline{x}) = x_1 + x_2 - 1.5 \leq 0$ B.115

$0.05 \leq x_1(in) \leq 2, 0.25 \leq x_2(in) \leq 1.3, 2 \leq x_3 \leq 15$ B.116

Problem 8: Tension/Compression spring Case-2

Design variables:

$\overline{x} = [x_1, x_2, x_3]$=[*d,D,N*] B.117

Objective function:

Minimize $f(\overline{x}) = \frac{\pi^2 x_2 {x_1}^2 (x_3+2)}{4}$ B.118

Design constraints:

$C_1(\overline{x}) = \frac{8c_f F_{max} x_2}{(\pi {x_1}^3) - S} \leq 0$ B.119

$C_2(\overline{x}) = l_f - l_{max} \leq 0$ B.120

$C_3(\overline{x}) = d_{min} - x_1 \leq 0$ B.121

$C_4(\overline{x}) = x_2 - D_{max} \leq 0$ B.122

$C_5(\overline{x}) = 3 - \frac{x_2}{x_1} \leq 0$ B.123

$C_6(\overline{x}) = \sigma_p - \sigma_{pm} \leq 0$ B.124

$C_7(\overline{x}) = \sigma_p + \frac{F_{max} - F_p}{k} + 1.05(x_3 + 2)x1 - l_f \leq 0$ B.125

$C_8(\overline{x}) = \sigma_w - \frac{F_{max} - F_p}{k} \leq 0$ B.126

Where,

$F_{max} = 1000\ lb, l_{max} = 14\ in, d_{min} = 0.2\ in, S = 189000\ psi, D_{max} = 3\ in, F_p = 300\ lb, \sigma_{pm} = 6\ in,$
$\sigma_w = 1.25\ in, G = 11.5 \times 10^6\ psi$ B.127

$c_f = \frac{4(x_2/x_1)\text{-}1}{4(x_2/x_1)\text{-}4} + \frac{(0.615x1)}{x_2}$ B.128

$k = \frac{G {x_1}^4}{8 x_3 {x_2}^2}, \sigma_p = \frac{F_p}{k}$ B.129

$l_f = \frac{\text{Fmax}}{k} + 1.05(x_3+2)x_1$ B.130

$0.009 \leq x_1(in) \leq 0.5, 0.6 \leq x_2(in) \leq 3, 1 \leq x_2 \leq 70$ B.131

Problem 9: Gear train

Design variables:

$\overline{x} = [x_1, x_2, x_3, x_4] = [n_A, n_B, n_C, n_D]$ B.132

Objective function:

Minimize $f(\overline{x}) = \left(\frac{1}{6.931} - \frac{x_3 x_2}{x_1 x_4}\right)^2$ B.133

$12 \leq x_1, x_2, x_3, x_4 \leq 60$ B.134

Problem 10: Piston lever:

Design variables:

$\overline{x} = [H, B, D, X]$ B.135

Objective function:

Minimize,

$$f(\overline{x}) = \frac{\pi D^2(L_2 - L_1)}{4}$$ B.136

Design constraints:

$C_1(\overline{x})$=QLcos(45)-RF≤0 B.137

$C_2(\overline{x})$=Q(L-X)-$M_{max} \leq 0$ B.138

$C_3(\overline{x})$=1.2(L_2-L_1)-$L_1 \leq 0$ B.139

$C_4(\overline{x}) = \frac{D}{2}$-B ≤ 0 B.140

Where,

$$R = \frac{|-X(Xsin(45) + H) + H(B - Xcos(45))|}{\sqrt{(X - B)^2 + H^2}}$$ B.141

$$P = 1500\ psi, F = \frac{\pi P D^2}{4}; L_1 = \sqrt{(X - B)^2 + H^2}; L_2 = \sqrt{(X * sind(45) + H)^2 + (B - X * cosd(45))^2}$$ B.142

$L = 240\ in, M_{max} = 1.8 \times 10^6\ lbs.in,\ Q = 10000\ lb$ B.143

$0.05 \leq H, B, D\ (in) \leq 500, 0.05 \leq X\ (in) \leq 120$ B.144

Problem 11: Corrugated bulkhead

Design variables:

$\overline{x} = [w, h, l, t]$ B.145

Objective function:

Minimize,

$$f(\overline{x}) = \frac{5.885t(w + l)}{w + sqrt(l^2 - h^2)}$$ B.146

Design constraints:

$C_1(\overline{x}) = -th(0.4w + l/6) + 8.94(w + \sqrt{l^2 - h^2}) \leq 0$ B.147

$C_2(\overline{x}) = -th^2(0.2w + l/12) + 2.2(8.94(w + \sqrt{l^2 - h^2}))^{4/3} \leq 0$ B.148

$C_3(\overline{x}) = -t + 0.0156w + 0.15 \leq 0$ B.149

$C_4(\overline{x}) = -t + 0.0156l + 0.15 \leq 0$ B.150

$C_5(\overline{x}) = -t + 1.05 \leq 0$ B.151

$C_6(\overline{x}) = -l + h \leq 0$ B.152

$0 \leq w, h, l\ (cm) \leq 100, 0 \leq t\ (cm) \leq 5$ B.153

Problem 12: Speed reducer

Design variables:

$\overline{x} = [b, m, Z, l_1, l_2, d_1, d_2]$ B.154

Objective function:

Minimize,

$$f(x) = 0.7854bm^2(3.3333Z^2 + 14.9334Z - 43.0934) - 1.508b(d_1^2 + d_2^2) + 7.4777(d_1^3 + d_2^3) + 0.7854(l_1 d_1^2 + l_2 d_2^2)$$ B.155

Design constraints:

$C_1(x) = 27 - bm^2 Z \leq 0$ B.156

$C_2(x) = 397.5 - bm^2 Z^2 \leq 0$ B.157

$C_3(x) = 1.93l_1^3 - mZd_1^4 \le 0$ B.158

$C_4(x) = 1.93l_2^3 - mZd_2^4 \le 0$ B.159

$C_5(x) = \sqrt{(745l_1/mZ)^2 + (16.9 \times 10^6)} - 110d_1^3 \le 0$ B.160

$C_6(x) = \sqrt{(745l_2/mZ)^2 + (157.5 \times 10^6)} - 85d_2^3 \le 0$ B.161

$C_7(x) = mZ - 40 \le 0$ B.162

$C_8(x) = 5m - b \le 0$ B.163

$C_9(x) = b - 12m \le 0$ B.164

$C_{10}(x) = 1.5d_1 - l_1 + 1.9 \le 0$ B.165

$C_{11}(x) = 1.1d_2 - l_2 + 1.9 \le 0$ B.166

$2.6 \le b \le 3.6, 0.7 \le m \le 0.8, 17 \le Z \le 28, 7.3 \le l_1 \le 8.3,$ B.167

$7.8 \le l_2 \le 8.3, 2.9 \le d_1 \le 3.9, 5 \le d_2 \le 5.5$ B.168

Problem13: Pressure vessel

Design variables:

$\overline{x} = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$ B.169

Objective function:

Minimize

$f(\overline{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$ B.170

Design constraints:

$C_1(\overline{x}) = x_1 - 0.0193x_3 \ge 0$ B.171

$C_2(\overline{x}) = x_2 - 0.00954x_3 \ge 0$ B.172

$C_3(\overline{x}) = \pi x_3^2x_4 + \frac{4}{3}\pi x_3^3 - 1296000 \ge 0$ B.173

$C_4(\overline{x}) = 240 - x_4 \ge 0$ B.174

$x_1, x_2 \in [0.0625, 0.125, \ldots\ldots\ldots, 1.1875, 1.25]\ (in)$, $10 \le x_3, x_4 (in) \le 200$ B.175

Problem 14: Welded beam

Design variables:

$\overline{x} = [x_1, x_2, x_3, x_4] = [h, l, t, b]$ B.176

Objective function:

Minimize $f(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$ B.177

Design constraints:

$C_1(\overline{x}) = \tau(\overline{x})_{max} - \tau(\bar{x}) \ge 0$ B.178

$C_2(\overline{x}) = \sigma(\overline{x})_{max} - \sigma(\bar{x}) \ge 0$ B.179

$C_3(\overline{x}) = P_c(\overline{x}) - P \ge 0$ B.180

$C_4(\overline{x}) = \delta(\overline{x})_{max} - \delta(\bar{x}) \ge 0$ B.181

$C_5(\overline{x}) = x_4 - x_1 \ge 0$ B.182

$C_6(\overline{x}) = x_1 - 0.125 \ge 0$ B.183

$C_7(\overline{x}) = 5.0 - 0.10471x_1^2 - 0.04811x_3x_4(14.0 + x_2) \ge 0$ B.184

$0.1 \le x_1 \le 2, 0.1 \le x_2 \le 10, 0.1 \le x_3 \le 10, 0.1 \le x_4 \le 2$ B.185

where,

$\tau(\vec{x}) = \sqrt{(\tau')^2 + 2\tau'\tau"\frac{x_2}{2R} + (\tau")^2}, \quad \tau' = \frac{P}{\sqrt{2}x_1x_2}; \qquad \tau" = \frac{MR}{J}$ B.186

$M = P\left(L + \frac{x_2}{2}\right)$ B.187

$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}$ B.188

$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}$ B.189

$\sigma(\vec{x}) = \frac{6PL}{x_4x_3^2}, \quad \delta(\vec{x}) = \frac{4PL^3}{Ex_3^3x_4}$ B.190

$P_c(\vec{x}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$ B.191

$$P = 6000\ lb, L = 14\ in, E = 30 \times 10^6\ psi, G = 12 \times 10^6\ psi,$$ B.192

$$\sigma_{max} = 30000\ psi\tau_{max} = 13600\ psi\delta_{max} = 0.25\ in$$ B.193

Problem 15: Multiple disc clutch brake:

Design variables:

$\overline{x} = [x_1, x_2, x_3, x_4, x_5] = [r_i, r_o, t, F, Z]$ B.194

Objective function:

Minimize

$f(\overline{x}) = \pi({x_2}^2 - {x_1}^2)x_3(x_5 + 1)\rho$ B.195

Design constraints:

$C_1(\overline{x})$ = Δr+$x_1 - x_2 \leq 0$ B.196

$C_2(\overline{x})$ = $-l_{max} + (x_5+1)(x_3 + \delta) \leq 0$ B.197

$C_3(\overline{x})$ = P_{r_z}-$P_{max} \leq 0$ B.198

$C_4(\overline{x})$ = $P_{rz}v_{sr}$-$v_{srmax}P_{max} \leq 0$ B.199

$C_5(\overline{x}) = v_{sr}$-$v_{srmax} \leq 0$ B.200

$C_6(\overline{x}) = T - T_{max} \leq 0$ B.201

$C_7(\overline{x})$ = sM_s-$M_h \leq 0$ B.202

$C_7(\overline{x})$ = -T ≤ 0 B.203

Where,

Δr=20 mm, l_{max}=30 mm, μ=0.6, v_{srmax}=10000 mm/s, δ=0.5 mm, s=1.5, B.204

T_{max}=15s, N=250 rpm, I_z=55 kg.m², M_s=40 Nm, M_f=3 Nm, P_{max}=1 MPa, ρ=0.0000078 kg/mm^3 B.205

$M_h = \frac{2}{3}\mu x_4 x_5 \frac{{x_2}^3 - {x_1}^3}{{x_2}^2 - {x_1}^2}$ B.206

$\omega = \frac{\pi N}{30}, \quad A = \pi({x_2}^2 - {x_1}^2), \quad P_{rz} = \frac{x_4}{A}, \quad R_{sr} = \frac{2}{3}\frac{{x_2}^3 - {x_1}^3}{{x_2}^2 - {x_1}^2},$ B.207

$v_{sr} = \frac{\pi R_{sr} N}{30}, \quad T = \frac{I_z\omega}{M_h + M_f}$ B.208

$x_1 = (60, 61, 62, \ldots, 80)mm, \quad x_2 = (90, 91, 92, \ldots, 110)mm, \quad x_3 = (1, 1.5, 2, 2.5, 3)N,$ B.209

$x_4 = (600, 610, 620, \ldots, 1000)N, \quad x_5 = (2, 3, 4, 5, \ldots, 9)$ B.210