JJMIE

Volume 16, Number 2, March. 2022
ISSN 1995-6665
Pages 301 - 307

Jordan Journal of Mechanical and Industrial Engineering

Optimal Response of a Quarter Car Vehicle Model with Optimal Skyhook Damper Based on Preview Control

L.V.V. Gopala Rao [a *], Rakesh Chandmal Sharma[b], V.S.V. Satyanarayana[a]

[a]*Vignan's Institute of Information Technology, Duvvada, Visakhapatnam, India*

[b]*Maharishi Markandeshwar (Deemed to be University) Mullana Ambala*

Received 4 JUNE 2020 *Accepted 3 FEB 2022*

Abstract

In this paper, a two degree of freedom(dof) quarter car vehicle model with skyhook damper passing through a rough road is considered. The skyhook damper control parameters namely the spring constant and the damping coefficient which determine the optimal performance of the skyhook damper. The optimal parameters of the skyhook damper are obtained by equating the control force of LQR with preview stochastic optimal control to that of skyhook damper. The parameters of the skyhook damper suspension are optimized to improve the vehicle performance to the level of active suspension system with preview control given by a performance index which is a weighted integral of the mean square acceleration, road holding, suspension stroke and control force.

Keywords: Active suspension, Preview control, Optimization, Quarter car, Random road.

1. Introduction

Road inudced excitation is the mazor source of vibration of an automobile which causes the discomfort to the occupants. Researchers have been working to achieve a better ride comfort for the passengers (1-3). Mostafizur Rahman Md and Arafat Rahman (4) applied potential energy theorem to design a driver seat suspension for improving the ride comfort. They added a double negative suspension spring to the suspension system and obtained a significant reduction of seat suspension magnitude. The active suspension system enhances vehicle performance by generating control force through an actuator to counteract road stimulation. Semi-active systems are being developed in view of overcoming the limitations, like low robust, time lag and high cost, associted with the active systems. In this context, Crosby and Karnopp[5] introduced the concept of skyhook damper based semi active suspensions. The sky-hook logic is based on an ideal design of a passive damper connected between the suspension mass and a notional point fixed in the sky. In the case of a passive suspension of a spring and damper in parallel between the sprung mass and the un-sprung mass corresponding to a two degree of freedom vehicle model, or between the sprung mass and the wheel corresponding to a single degree of freedom vehicle model increases the passive damping ratio that leads to a harsher ride. Karnopp et al.,[6] used the skyhook concept in a moving vehicle and compared the performance of skyhook damper with conventional passive suspension system. In subsequent years, many authors have done research work in the area of skyhook damper design, but the performance of skyhook damper depends on the damper parameters. Sammier et al.,[7] used the skyhook damper type suspension to improve the road holding and ride comfort of vehicle model. Hamrouni et al.,[8] compared the performance of skyhook controller with the performance of CRONE Controller and simulation results and showed better performance compared to the skyhook controller. In the design of skyhook damper, selection of damper parameter values are important to enhance the performance of skyhook damper suspension system.

Different optimization methods have been developed to improve the vehicle suspension performance and applied to quarter car, half car and full car vehicle models. Vladimir and Marian[9] optimized the half car model suspension parameters, such as the spring stiffness and damping coefficients using Genetic Algorithm optimization method. Quantum- behaved particle Swarm optimization method has been used by Lee and Cheng[10] to optimize the 14-dof nonlinear railway suspension parameters. Xu et al.,[11] used Artificial Fish Swam Algorithm to optimize the hydro pneumatic and Mechanical Elastic Wheel suspension of quarter car vehicle model and results shown the improvement of vehicle ride comfort. Mustafa et al.,[12] introduced a new model-free fuzzy logic controller based on particle swarm optimization (PSO-MFFLC) for the nonlinear active suspension systems and compared the performance of PSO-MFFLC with the time-delay estimation control, intelligent PID and classical PID. Rajagopal and Ponnusamy[13] improved the performance of active vehicle suspension system using the hybrid DEBBO algorithm by tuning the parameters of PID controller.

Even the LQR control gives satisfactory performance, it can be further improved using preview control. In addition to the potential improvements in performance, preview control requires lower power, reduced requirements for internal sensors, and simplified feed back control structure. The objective of the preview control in

* Corresponding author e-mail: lvvgrao@gmail.com.

  - Volume 16, Number 2 (ISSN 1995-6665)

the vehicle suspension system is to get the oncoming road undulations information, and to use this measured information in the active control strategy. Preview control is based on the feedback force that depends on system state, and it also includes feed forward force that requires the information of the road ahead of the vehicle. The performance of *rms* stroke and *rms* road holding improve with increasing preview distance up to a particular distance. The preview time depends on vehicle velocity and preview distance. As the velocity of the vehicle increases the preview distance is covered in quick time.

Vehicle suspension with active elements can give better results in case of ride comfort. But design and maintenance costs are more compared to those used in passive suspension elements like spring and viscous damper. Optimization of the parameters of the vehicle passive suspension and skyhook damper for all road conditions and speeds is a difficult task. So far many researchers have optimized vehicle suspension parameters for a particular speed for better performance.These optimal suspension parameters may not give uniformly better performance at all speeds.

In this work, the steady state random response control of a quarter vehicle model moving on a uneven road is considered. The parameters of the skyhook damper are obtained in an optimal way by equating the control forces of skyhook damper and the optimal preview control in a statistical sense and minimizing the square root of the sum of mean square difference between the relevant control responses of the LQR - preview and skyhook damper.

2. Problem Formulation

2.1. Mathematical modeling

In this section, an active suspension quarter car model with LQR control is presented. The road input information is assumed to be available by measurement at a preview distance of "L" in front of the vehicle. The direct calculation of the optimal control force for LQR model with preview is presented. The model is stated as the LQR model without preview when the preview distance is taken as zero.

Equations of motion of quarter car model with preview control shown in Figure 3 are,

$$m_1\ddot{y}_1 + k_1(y_1 - y_2) - U = 0 \quad (1)$$

$$m_2\ddot{y}_2 + k_1(y_2 - y_1) + k_2(y_2 - h) + U = 0, \quad (2)$$

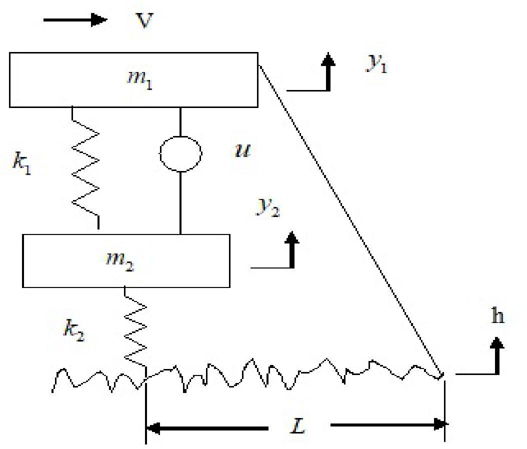

Figure 1. Quarter-car with preview suspension

where m_1 and m_2 are sprung and unsprung masses, k_1 and k_2 are the stiffnesses of the primary suspension and the tyre respectively, h is the random road input and U is the control force. The random road is approximated as Power spectral density of the is given as [14],

$$S_h(\omega) = \frac{\sigma^2}{\pi}\frac{\alpha_r V}{(\omega^2 + (\alpha_r V)^2)}, \quad (3)$$

where σ^2 is variance of the road undulations, V is forward velocity and α_r is road surface coefficient. The random road profile equation [15] is given by

$$\dot{h}(t) + \alpha_r V h(t) = w(t), \quad (4)$$

where $w(t)$ is a Gaussian white noise,

$$E[w(t)w^T(t+\tau)] = 2\sigma^2 V\alpha_r \delta(\tau),$$

where system state varibles are, $x_1 = y_1, x_2 = \dot{y}_1, x_3 = y_2, x_4 = \dot{y}_2, x_5 = h$ and equations (1), (2) and (4) are written as,

$$\dot{x} = Fx + gU(t) + dw(t), \quad (5)$$

U, F, x, g, d are control force vector, system matrix, state vector, control distribution vector and excitation distribution vector respectively.

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & 0 & \frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{k_1}{m_2} & 0 & -\frac{k_1+k_2}{m_2} & \frac{k_2}{m_2} & 0 \\ 0 & 0 & 0 & 0 & -\alpha_r V \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{-1}{m_2} \\ 0 \end{bmatrix}; \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (6)$$

2.2. Performance Criterion

The optimal suspension is designed based on stochastic optimal control.

The overall performance index,

$$J = \rho_1 J_1 + \rho_2 J_2 + \rho_3 J_3 + \rho_4 J_4, \quad (7)$$

where $J_1 = E[\ddot{y}_1{}^2]$; $J_2 = E[(y_1 - y_2)^2]$; $J_3 = E[(y_2 - h_1)^2]$; $J_4 = E[U^2]$ where $E[.]$ denotes expectation operator. ρ_1, ρ_2, ρ_3 and ρ_4 being the weighting factors. Equation (7) can be written as

$$J = E\left[\begin{bmatrix} x(t)^T & U(t)^T \end{bmatrix} \begin{bmatrix} A & n \\ n^T & B \end{bmatrix} \begin{bmatrix} x(t) \\ U(t) \end{bmatrix}\right], \quad (8)$$

where A and B are positive semi definite and definite matrices respectively, and are given by

$$A = \begin{bmatrix} \frac{\rho_1 k_1^2}{m_1^2} + \rho_2 & 0 & -\frac{\rho_1 k_1^2}{m_1^2} - \rho_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{\rho_1 k_1^2}{m_1^2} - \rho_2 & 0 & \frac{\rho_1 k_1^2}{m_1^2} + \rho_2 + \rho_3 & 0 & -\rho_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_3 & 0 & \rho_3 \end{bmatrix};$$

$$n = \begin{bmatrix} \frac{\rho_1 k_1}{m_1^2} \\ 0 \\ \frac{\rho_1 k_1}{m_1^2} \\ 0 \\ 0 \end{bmatrix}; \quad B = \left[\frac{\rho_1}{m_1^2} + \rho_4\right]$$

The optimal control force is,

$$u(t) = -c_1(t)x(t) + c_2 r(t), \tag{9}$$

where $c_1(t) = B^{-1}[n^T(t) + g^T S(t)]$, $c_2 = B^{-1}g^T$ are feed back and feed forward control gain vectors resepctively, and S matrix can be solved using the following equation,

$$SF_N + F_N^T S - SgB^{-1}g^T S + A_N = 0 \tag{10}$$

and $r(t)$ is given by

$$r(t) = \int_0^{t_p} exp[F_G^T \sigma] S dw(t+\sigma) d\sigma, \tag{11}$$

where $A_N = A - nB^{-1}n^T$; $F_N = F - gB^{-1}n^T$ and $F_G = F - gB^{-1}(n^T + g^T S)$.

Substituting equation (9) in equation (5) yields

$$\dot{x} = F_G\, x + gc_2 + d\, w(t). \tag{12}$$

The response of system described by covariance matrix as $P(t) = E[x\, x^T]$ which is obtained by solving the below Liapunov equation.

$$[F - gc_1]P(t) + P(t)[F - gc]^T + gc_2 E[r(t)x^T(t)] + E[x(t)r^T(t)](gc_2)^T + dE[w(t)x^T(t)] + E[x^T w^T(t)]d^T = 0 \tag{13}$$

where

$$E[x(t)r^T(t)] = P_1 + P_2 \tag{14}$$

P_1 and P_2 can be obtained as,

$$P_1 = \int_0^{t_p}\int_0^{t_p} \phi\big(t, t-(\sigma-\sigma_1)\big) gc_2\, exp[F_G\sigma]\, Sd\left(\frac{Q}{2}\right) d^T S\, exp[F_G^T\sigma_1]\, d\sigma\, d\sigma_1; \quad \sigma \geq \sigma_1 \tag{15}$$

otherwise $P_1 = 0$; for $\sigma < \sigma_1$

$$P_2 = \int_0^{p} \phi(t, t+\sigma) d(Q/2) d^T S^T exp[F_G^T \sigma]^T d\sigma \tag{16}$$

and

$$E[x(t)w^T(t)] = d(Q/2) + \int_0^{t_p} \phi(t, t-\sigma) gc_2 exp[F_G^T\sigma] Sd(Q/2) d\sigma. \tag{17}$$

By substituting equation (14) and equation (17) in equation (13), we get P(t) which is the response of system. Since the steady state closed loop system matrix F_g is stable, F_g^T is also stable and the exponential function in equation (11) also decreases with time. Hence the effect of knowledge of the future inputs on $r(t)$ will diminish with time and the knowledge of inputs from the distant future will not effect on the system performance. This concept is first observed by Tomizuka[16] and later by Hac [17].

3. Skyhook control

3.1. Quarter car model-skyhook damper

Passive suspension systems are restricted to generate forces in response to local relative motions, that is, between attachment points of contiguous bodies. In this section, a sky hook damper control quarter car model (Figure. 2) is presented. The equations of motion are given by

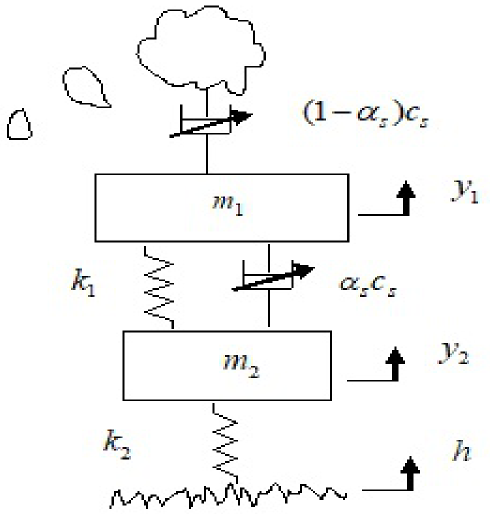

Figure 2. Quarter car model with skyhook damper

$$m_1\ddot{y}_1 + k_1(y_1 - y_2) + c_s(\dot{y}_1) - \alpha c_s \dot{y}_2 = 0, \tag{18}$$

$$m_2\ddot{y}_2 + k_1(y_2 - y_1) + \alpha_s c_s(\dot{y}_2 - \dot{y}_1) + k_2(y_2 - h) = 0, \tag{19}$$

where α_s and c_s are the skyhook damper parameters. For α_s=1, equations ((18)) and ((19)) reduce to passive suspension equations. The skyhook control force can be defined as,

$$U_s = c_s(\dot{y}_2 - \dot{y}_1) - (1 - \alpha_s)c_s\dot{y}_2. \tag{20}$$

Equations ((4)), ((18)) and ((19)) can be written as matrix differential equations using state space form,

$$\dot{x} = Fx + Dw(t), \tag{21}$$

where $x = [\,x_1\, x_2\, x_3\, x_4\, x_5]^T$ and $x_1 = y_1$, $x_2 = \dot{y}_1$, $x_3 = y_2$, $x_4 = \dot{y}_2$, $x_5 = h$,

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{-k_1}{m_1} & \frac{-(c_s)}{m_1} & \frac{k_1}{m_1} & \frac{\alpha c_s}{m_1} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{k_1}{m_2} & \frac{\alpha_s c_s}{m_2} & \frac{-(k_1+k_2)}{m_2} & \frac{-(\alpha_s c_s)}{m_2} & \frac{k_2}{m_2} \\ 0 & 0 & 0 & 0 & -\alpha_r V \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{22}$$

The vehicle response described by covariance matrix $P(t) = E[xx^T]$ is obtained by solving the below Liapunov equation.

$$\dot{P} = FP + PF^T + dQd^T \tag{23}$$

3.2. Optimal skyhook damper parameters

For obtaining the optimal skyhook damper control parameters α_s and c_s at a particular velocity the following new optimization strategy is adopted. In this strategy, the value of α_s is varied from 0 to 1 and for each value of α_s, the corresponding value of c_s is found by equating optimal control effort of the preview control algorithm as per the equation (9) to sky-hook damper control force as per the equation (20). This value of c_s is given by

$$c_s = \sqrt{\frac{E[U^2]}{E[\dot{y}_1 - \alpha_s\dot{y}_2]^2}}. \tag{24}$$

Thus for each value of α_s there is corresponding value of c_s for a specific vehicle velocity. Calculate the overall performance of vehicle with each set of skyhook damper parameters. Thus there is a need to chose one combination of α_s and c_s for sky hook damper which matches the vehicle performance as the LQR with preview control performance. This can be effected by minimizing the square root of the sum of the difference between the mean square vehicle responses obtained by LQR with preview control and the skyhook damper control. The values of α_s and c_s are thus obtained corresponding to the minimum rms difference given by

$$min\sqrt{\sum_{i=1}^{3} ((J_i)_{SH} - (J_i)_{LQRP})}, \quad (25)$$

the subscript LQRP denotes LQR with preview, SH denotes skyhook control and j_i's are obtained by equation (7).

In this process, one suitable set of (α_s, c_s) for a specific velocity is obtained in view of vehicle overall performance. Since the skyhook damper parameters, once chosen, cannot be varied, the optimal values α_s and c_s obtained for a specific velocity will not be optimal for other vehicle velocities, which is the reason behin choosing other sets of optimal values α_s and c_s for other vehicle velocities. It has been observed that the variation of α_s and c_s values is not very much and the average values of the optimal α_s and c_s are calculated for a velocity range. These average values are considered to be suitable for the skyhook damper to equal the performance of LQR with preview control. Best possible response of the vehicle and optimal control force are obtained by substituting the optimal values of α_s and c_s into equations (18) and (19) and solving the lyapunov equation (23).

The detailed step wise optimization procedure is given below.

- Step 1: Obtained optimal response of quarter car model using LQR with preview control.
- Step 2: Calculated control force of the LQR-preview control.
- Step 3: Control force term is taken from equations of motion of quarter vehicle model with LQR with preview control (Equation 9). This control force is reference force.
- Step 4: From equation of motion of sky-hook model, the control force is taken (Equation 20).
- Step 5: To obtain the optimal response of quarter car sky-hook model close to the optimal response of quarter car-LQR with preview control, both models suspension system forces must be equal.
- Step 6: Equated the control force (reference force) of the LQR model-preview (Equation 9) with control force of sky-hook damper quarter car model (Equation 20) as shown in Equation (24).
- Step 7: There are two unknowns (α_s and c_s) and one equation(Equation (24).
- Step 8: So for each velocity, fixed the range of α_s as 0 to 1 and corresponding c_s value is obtained using equation (24).
- Step 9: Calculated average of all the α_s and c_s values to find the optimum set of the parameters suitable for all vehicle velocities.

4. Results and Discussion

This section shows the response of the vehicle models, quarter car passive suspension, optimal sky-hook damper, LQR-preview and LQR without preview. The vehicle and road model parameters and the weights of performance index are $m_1 = 1000kg$, $m_2 = 100kg$, $k_1 = 36000N/m$, $k_2 = 360000N/m$, $\alpha_r = 0.15rad/m$, $\sigma^2 = 9 * 10^{-6}m^2$, $\rho_1 = 1$, $\rho_2 = 10^4$, $\rho_3 = 10^4$, $\rho_4 = 10^{-6}$. For the quarter car model, the vehicle parameters and the weighting factors are chosen as in [14]. The optimum values of the skyhook damper parameters obtained using the procedure explained in section 3.2 are $\alpha_s = 0.17$ and $c_s = 7518.5$Ns/m. The response statistics for the different control schemes, LQR control with a preview distance of P_d =2 m, LQR control with P_d =0, skyhook damper control with optimum parameters and the passive system are shown in Figures 3 to 6 respectively.

Figure 3 gives a picture of the variation of sprung mass acceleration with vehicle velocity for the passive suspension, LQR model with preview, LQR model without preview and optimum Sky hook damper model. The performamce of the Optimal sky hook model is equal to the performance of the LQR with preview control, marginally better than the LQR without preview, and substantially better than the passive suspension.

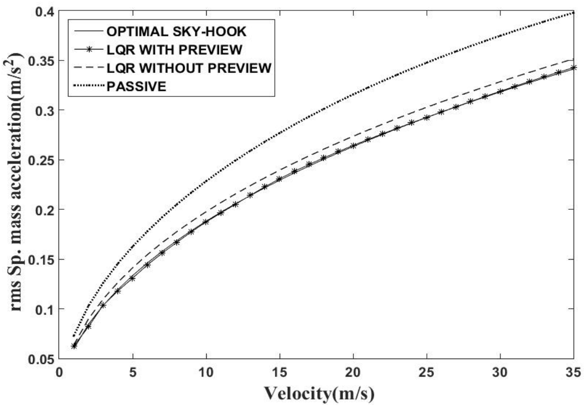

Figure 3. Variatin of sprung mass acceleration with velocity

In Figure 4 the *rms* stroke response is plotted for the four cases of passive, fully active(LQR) suspension without and with preview and semi-active suspension with skyhook damper with the optimal parameters. In this case, it is observed that the optimal skyhook damper performs almost same as the preview control at high velocity while at lower velocity its performance is not matched with that of the preview control. The suspension stroke performance of both of the fully active suspension with preview and the semi-active suspension with optimal skyhook damper is slightly better than the LQR without preview and obviously superior to the passive system. The stroke response of LQR-without preview is slightly worse, implying that the preview helps to improve the performance of the active suspension which is also reflected in the skyhook damper performance.

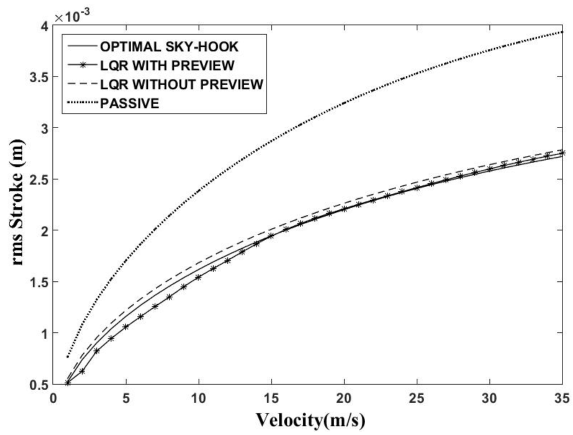

Figure 4. Suspension stroke response for different velocities

Figure 5 shows the road holding characteristics for the different vehicle suspension models. The active suspensions (LQR-preview and LQR-without preview control) performance is superior to the passive suspension. However, LQR with preview control shows better performance when compared to the LQR control without preview. The optimal skyhook damper performance is same as the performance of LQR-preview, and it is marginally superior to LQR model without preview.

From Figure 6, it is observed that the overall performance index, with respect to sprung mass acceleration, road holding and suspension stroke, the LQR with preview control performs the best and the performance in this case is significantly better than that corresponding to the passive suspension and marginally better than the LQR control without preview and the skyhook damper suspension with optimum parameters. The skyhook control with optimum parameters obtained by equating the control force corresponding to the LQR control with preview performs slightly improved when compared to the LQR control without preview.

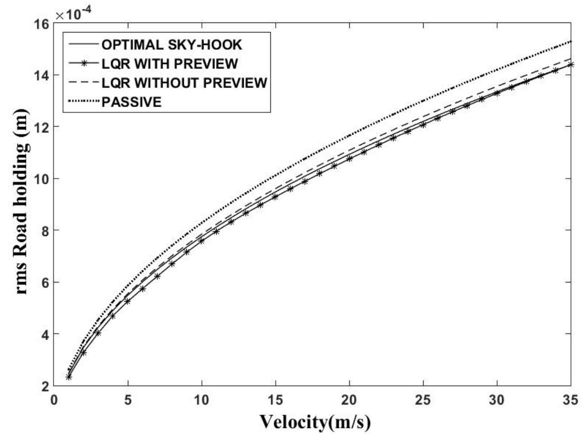

Figure 5. Road holding response for different velocities

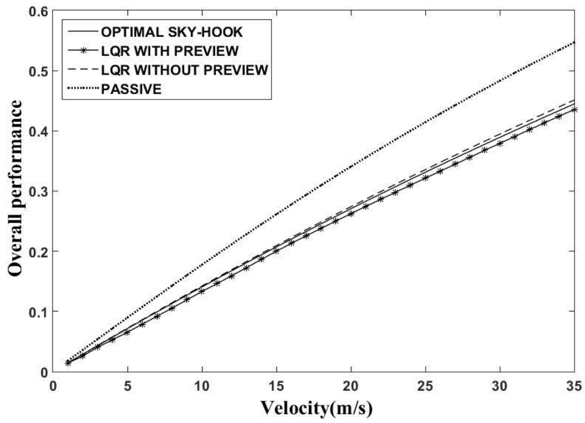

Figure 6. Overall response for different velocities

Figure 7 depicts the control effort required by the three different models. The rms control force of LQR- preview control matches with skyhook damper control and more than the LQR without preview control for most of the velocity range considered as expected.

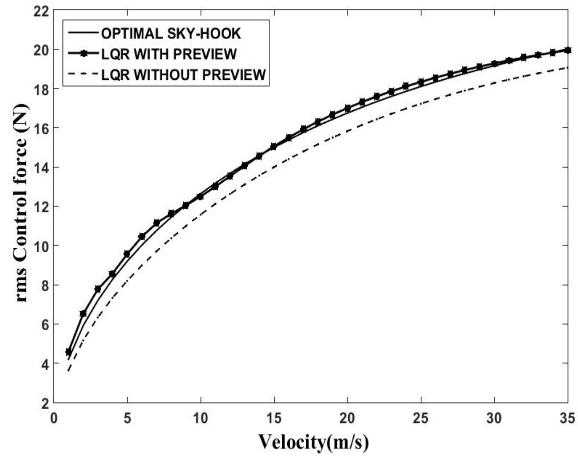

Figure 7. Variation of control force

As a further exercise, sprung mass acceleration, road holding, suspension stroke and overall performance index for different preview distances are plotted in Figures 8 to 11. It is seen from the figures that the performances of acceleration, road holding and suspension stroke improve with increasing preview distance up to a certain preview distance. Therefore, there is a limit preview distance beyond which the benefit of the preview control is not obtained.

From Figure 8 it is seen that the *rms* acceleration response improves with increasing preview distance up to certain preview distance say approximately 2 m beyond that the effect is only marginal. In case of stroke, road holding and overall performances also, as shown in figures 9,10 and 11, improvement is only up to a particular preview distance. Because of this saturation effect, the preview distance is taken as 2 m for the study.

As a further exercise, sprung mass acceleration, road holding, suspension stroke and overall performance index for different preview distances are plotted in Figures 8 to 11. It is seen from the figures that the performances of acceleration, road holding and suspension stroke improve with increasing preview distance up to a certain preview distance. Therefore, there is a limit preview distance beyond which the benefit of the preview control is not obtained.

From Figure 8 it is seen that the *rms* acceleration response improves with increasing preview distance up to certain preview distance say approximately 2 m beyond that the effect is only marginal. In case of stroke, road holding and overall performances also, as shown in figures 9,10 and 11, improvement is only up to a particular preview distance. Because of this saturation effect, the preview distance is taken as 2 m for the study.

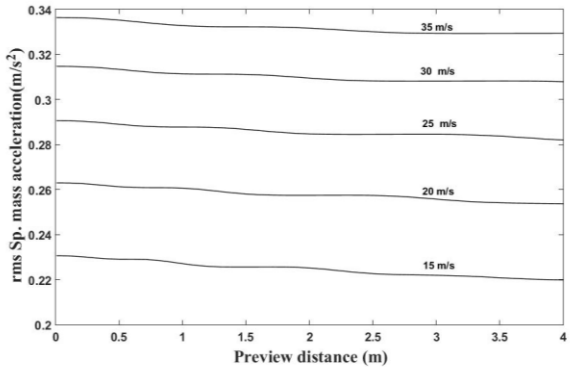

Figure 8. Sprung mass acceleration for different preview distances.

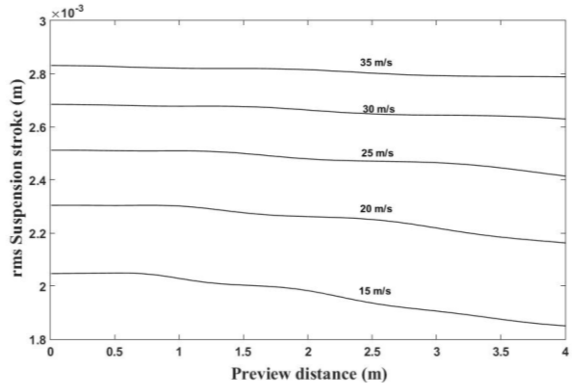

Figure 9. Suspension stroke for different preview distances.

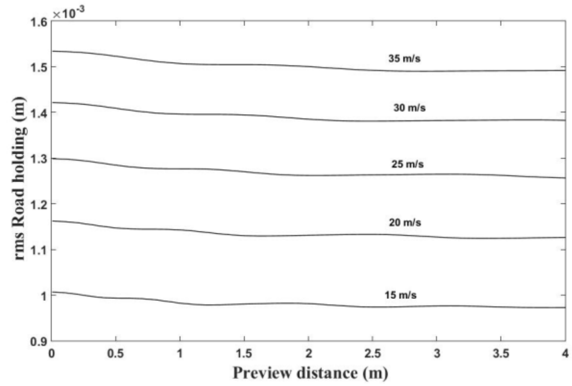

Figure 10. Road holding response for different preview distances.

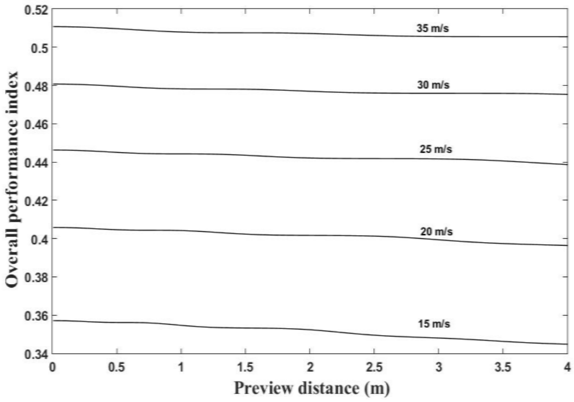

Figure 11. Overall response for different preview distances.

5. Conclusions

In this paper, a new optimization method is proposed for choosing optimal parameters of skyhook damper of quarter car model moving over a random path to obtain optimal response with respect to the ride comfort, road holding, suspension deflection and control force. The skyhook damper parameters are optimized on the basis of minimization of the total *rms* error which is the sum of errors of *rms* road holding, rms acceleration and *rms* stroke compared to LQR with preview control. This ensures that the mean square control force obtained by the skyhook damper is equal to the control force of LQR preview control. As a result, the sky hook damper's performance with optimal parameters is improved to the levels compared to the LQR with preview control. It can be concluded from the results of the skyhook damper suspension system and the LQR with preview control suspension system that the quarter car model performance with optimal skyhook damper is comparble to the vehicle model with preview optimal control and significantly superior to the passive suspension system. Error between the overall performance index of the optimal sky hook damper and the LQR control with preview is observed to be 1.5%. The sprung mass acceleration of the sky hook damper control exactly coordinates with the response of the system with LQR preview.

References

[1] Meng, Q., Chen, C.C., Wang, P., Sun, Z.Y. and Li, B., 2021. Study on vehicle active suspension system control method based on homogeneous domination approach. Asian Journal of Control, 23(1), 561–571. https://doi.org/10.1002/asjc.2242

[2] Ovalle, L., Ríos, H. and Ahmed, H., 2021. Robust Control for an Active Suspension System via Continuous Sliding-Mode Controllers. Engineering Science and Technology, an International Journal. https://doi.org/10.1016/j.jestch.2021.06.006

[3] Huang, Y., Na, J., Wu, X., Gao, G.B. and Guo, Y., 2018. Robust adaptive control for vehicle active suspension systems with uncertain dynamics. Transactions of the Institute of Measurement and Control, 40(4), pp.1237-1249. https://doi.org/10.1177%2F0142331216678312

[4] Rahman, M. and Rahman, M., 2021. Application of Potential Energy Method for Driver Seat Suspension System Using Quasi-Zero Stiffness: A Numerical and Experimental

Study. Jordan Journal of Mechanical & Industrial Engineering, 15(5). ISSN 1995-6665, pp. 431-439.

[5] Crosby,M.J., Karnopp, D., 1973. The Active damper- a new concept for shock and vibration control. Shock and Vibration Bulletin, 43.

[6] Karnopp,D., Crosby,M.J., Harward, R.A., 1974. Vibration control using semi-active force generators. ASME Journal of Manufacturing Science and Engineering, 96(2), 619–626. https://doi.org/10.1115/1.3438373

[7] Sammier,D., Oliver,S., Dugard,L., 2003. Skyhook and H_∞ control of semi-active suspensions: some practical aspects. Vehicle System Dynamics. 39(4), 279–308. http://dx.doi.org/10.1076/vesd.39.4.279.14149

[8] Hamrouni,E., Moreau,X., Benine-Neto,A., Hernette,V., 2019. Skyhook and CRONE active suspensions: A comparative study. IFAC-PapersOnLine. 52-5, 243–248. https://doi.org/10.1016/j.ifacol.2019.09.039

[9] Vladimir,G., Klucik,M., 2012. Optimization of vehicle suspension parameters with use of evolutionary computation. Procedia Engineering. 48, 174-179. https://doi.org/10.1016/j.proeng.2012.09.502

[10] Lee,C.K., Cheng,Y.C., 2014. Application of Uniform Design and Quantum-Behaved Particle Swarm Optimization in Solving the Sensitivity Problem a Railway Vehicle System. Procedia Engineering. 79, 427-436. https://doi.org/10.1016/j.proeng.2014.06.364

[11] Xu,H., Zhao,Q.Y., Ye,Chao., Lin,F., 2019. Integrated optimization for mechanical elastic wheel and suspension based on an improved artificial fish swarm algorithm. Advances in Engineering Software. 137, 1-8. https://doi.org/10.1016/j.advengsoft.2019.102722

[12] Mustafa,G.I.Y., Wang,H.P., Tian,Y., 2019. Vibration control of an active vehicle suspension systems using optimized model-free fuzzy logic controller based on time delay estimation. Advances in Engineering Software. 127, 141-149. https://doi.org/10.1016/j.advengsoft.2018.04.009

[13] Rajagopal, K., and Ponnusamy, L., 2015. Hybrid DEBBO Algorithm for Tuning the Parameters of PID Controller Applied to Vehicle Active Suspension System. Jordan Journal of Mechanical & Industrial Engineering. 9(2), 85-102.

[14] Hac,A., 1985. Suspension optimization of a two dof vehicle model using stochastic optimal control technique. Journal of Sound and Vibration. 100, 343-357. https://doi.org/10.1016/0022-460X(85)90380-3

[15] Narayanan,S., Senthil,S., 1998. Stochastic optimal active control of a 2-DOF quarter car model with non-linear passive suspension elements. Journal of Sound and Vibration, 211(3), 495-506. https://doi.org/10.1006/jsvi.1997.1396

[16] Tomizuka,M., 1976. Optimal linear preview control with application to vehi- cle su spension- revisited. ASME Journal of Dynamical Systems, Measurement and Control. 98(3), 309-315. https://doi.org/10.1115/1.3427040

[17] Hac,A., 1992. Optimal linear preview control of active vehicle suspension. Vehicle System Dynamics, 21, 167-195. https://doi.org/10.1080/00423119208969008