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# Application of Potential Energy Method for Driver Seat Suspension System Using Quasi-Zero Stiffness: A Numerical and Experimental Study

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**Abstract**

To improve comfort, a quasi-zero stiffness (QZS) drivers' seat suspension system is designed and fabricated based on potential energy method. At first, the mathematical model for the seat suspension is established. Thereafter, a model of seat suspension system with natural frequency of 2.45 Hz is fabricated to check validity of this method. A negative stiffness spring (NSS) is used as added system to reduce the natural frequency to 1.78 Hz. In addition, double NSS is added to suspension system to obtain QZS and natural frequency observed to near about 0.84 Hz. Hence, vibration magnitude of seat suspension is reduced to 27.3% in case of single NSS and 65.7% for double NSS compared with suspension system without NSS. Compared with the original seat suspension system, the new suspension system with NSS has better vibration isolation characteristics and can electively improve drivers' ride comfort.



*Keywords:* Vibration; Natural frequency; Transmissibility; Suspension system; Potential energy; Quasi-zero stiffness; Stability;

## 1. Introduction

While riding and driving, vehicles' drivers seem to be exposed to much vibration which causes uncomfortable and pain to body resulting in decreasing their working period as well as working efficiency[1]. An important requirement of vehicle design is to attenuate vibration level transmitted to driver's seat from chassis[2]. Vibration is unwanted and anticipated not only in human body but also in any engineering structure and in vehicle seat suspension system as well due to stability and life cycle concern[3, 4]. This type of unexpected vibration needs to be remedied by designing proper and an efficient seat suspension system. It is noted that purchasing vehicle is selective criterion which is not only limited by development of horse power or torque of the engine, fuel economy condition, and hundred km/h speed of vehicle[5]. Hence, an efficient, low natural frequency, and low vibration transmissibility seat suspension system is taken into consideration with great importance while designing vehicle[6]. Suspension designer can model seat suspension system by three categories, such as active, semi-active, and passive system [7]. Active system is capable of reducing natural frequency below 1 Hz using control units along with common suspension elements, such as mass, spring, and damper[8]. A renowned company 'BOSE' developed an active system of cost about $5000 which is very high with respect to commonly used passenger car [9]. In addition, 'ISUZU' company fabricated a semi-active system of natural frequency below 1 Hz of cost about $4000[10]. A seat suspension system of low natural frequency means the system is efficient in attenuating vibration magnitude. However, suspension designer needs to think about performance and cost at the same time when he/she designs. In contrast, active and semi-active system cannot function properly in very rough and tough environment if controlling unit is failed[11]. Passive system can be a better choice for high speed, heavy-duty, industrial, and agricultural vehicles in terms of performance and cost as well[12]. It is noted that natural frequency of passive suspension system is observed between 1-2 Hz which is used in most of the passenger's vehicles[13]. Hence, passive system needs to be modified in such way that it can attenuate vibration below natural frequency of 1 Hz. In addition, passive suspension system is modeled as single degree of freedom (SDOF) system consisting of mass, springs, and dampers as shown in Fig. 1. Mass of the driver seat includes mass of suspension frame, seat cushion, and mass of driver itself. This mass is supported by a spring with stiffness (K) and a damper with damping co-efficient (C).

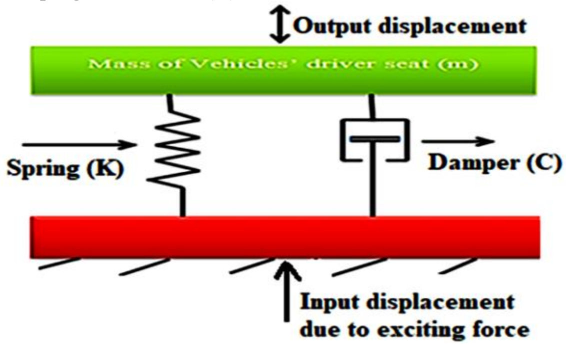


**Figure 1**. Schematic representation of SDOF damped passive seat suspension system.

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It is noted that un-damped natural frequency of the system can be given as follows

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \tag{1}$$

In addition, damped natural frequency is written as

$$f_d = f_n\sqrt{1-\zeta^2} \tag{2}$$

where damping ratio, $\zeta = \frac{C}{2\sqrt{mK}} = \frac{\text{Actual damping}}{\text{Critical damping}}$

It is worthy to mention that $f_d < f_n$ and it will obviously be better for comfort if un-damped natural frequency is near about or below damped natural frequency[14]. Because damper is a sophisticated and costly suspension element, for simplicity, damper in passive suspension system can be replaced by some approaches capable of reducing natural frequency and vibration transmissibility. Hence, un-damped passive system needs to be designed in such way that it can reduce level of vibration below 1 Hz. It is highly essential to understand the source of vibration, its nature and direction, the transmission path of the vibration energy to problem location, and natural frequency before designing un-damped passive system. Figure 2 shows vibration transmission model to vehicles' driver seat. As noted, vibration transmission path is the main point where different modifications are performed to increase vibration attenuation bandwidth. Design modifications on source and sink do not reduce vibration transmissibility whereas; vibration transmission path has received more attentions from automotive engineers for minimizing vibration. Several analytical, numerical, and experimental approaches have been developed for investigating the response of suspension system with added system, such as negative stiffness system (NSS).Hall et al. [15] reported vehicle seat vibration isolation and motion control through use of horizontal non-linear springs and slender beam structure. Kashdan et al. [16]designed and constructed a constrained bi-stable structure with negative stiffness behavior for providing extreme vibrational absorptive capacity. In addition, Meng et al. [17]designed and presented a novel quasi-zero stiffness vibration isolator by combining a disk spring as negative stiffness element with a vertical linear spring.

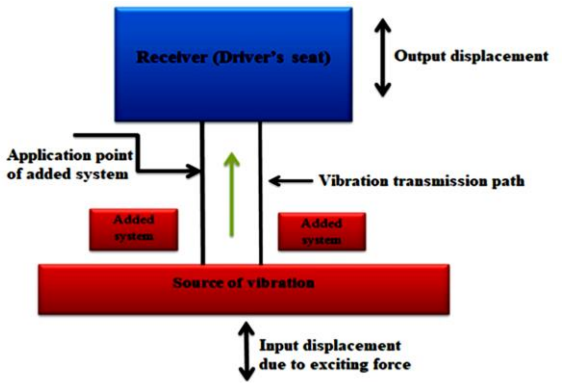


**Figure 2.**Added system to vibration transmission path (Reproduced from [13])

Palomares et al.[18]fabricated a suspension system consisting of vertical pneumatic spring with a damper and two pneumatic linear actuators for controlling vibro-isolation properties of suspension system. However, Rahman et al.[19]designed and proposed a vehicle drivers' seat suspension system totally in passive way using double negative stiffness system. In this study, un-damped passive seat suspension system is designed using double NSS as added system to vibration transmission path. Model of modified passive seat suspension system is mathematically analyzed by potential energy method. A new seat suspension system is fabricated and experimentally investigated for validation of mathematical model.

## 2. Mathematical Modeling by Potential Energy Method

Conventional un-damped passive seat suspension system consists of a payload mass and linear vertical spring and it is capable of reducing natural frequency below 1 Hz if the linear vertical spring is of at least 0.2 m which is practically unfeasible[20]. Equation (1) signifies that natural frequency is decreased by two ways: such as increasing mass and decreasing stiffness. However, those are not effective in actual practice since increasing mass of the system leads to increase the bulkiness of system[21]. In addition, stiffness of spring needs to be increased at the same time for reduction of vertical static displacement and stable support of as mass[22]. However, natural frequency is not reduced. In contrast, decreasing stiffness of vertical spring is not an effective way to reduce natural frequency due to large static deflection. This dichotomy relation of mass and spring with natural frequency of the suspension system is eliminated by implying quasi-zero stiffness. Here, NSS which is itself a unstable system, can be added with main system to obtain quasi-zero stiffness for reducing natural frequency. Figure 3 shows a new seat suspension system with double NSS. Here, mass (M) is moved downward due to externally applied force; vertical spring becomes compressed and gains potential energy. It tends to bring back the mass in equilibrium position very quickly due to restoring force resulting in much vibration. Hence, vertical spring faces resistance to going back to equilibrium position very quickly if NSS is added as supplementary system. NSS reduces functional stiffness of total system without reducing own stiffness of vertical spring. It will be clear from mathematical relationship obtained by potential energy method.

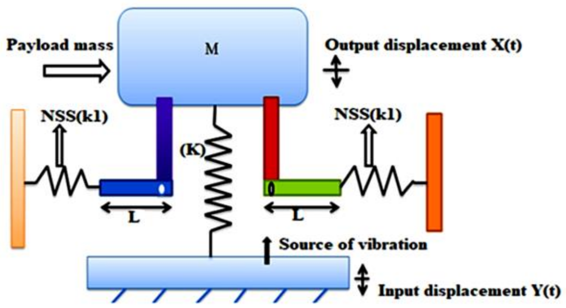


**Figure 3.**Schematic representation of un-damped passive seat suspension system with double NSS.

Potential energy of the system as shown in Fig. 3 without NSS can be given as follows

$$U_1 = \frac{1}{2}KX^2 \tag{3}$$

In addition, potential energy of the added double NSS system can be written as

$$U_2 = \frac{1}{2} \times 2k_1(\partial_O + \sqrt{L^2 - X^2} - L)^2$$

$$= k_1(\partial_O + \sqrt{L^2 - X^2} - L)^2 \tag{4}$$

where, $\partial_o$ is initial deflection of spring of NSS from equilibrium position when subjected to designed payload mass. Hence, total potential energy of the system can be reduced to, $U=U_1+U_2$

$$U(x) = \frac{1}{2} KX^2 + k_1(\partial o + \sqrt{L^2 - X^2} - L)^2 \tag{5}$$

It is noted that 1st derivative of potential energy function of an elastic system with respect to displacement is spring force[23]. From Equation (5), it can be written as

$$\frac{\partial U}{\partial X} = KX - 2k_1\left\{1 + \frac{\partial_o - L}{\sqrt{L^2 - X^2}}\right\}X \tag{6}$$

In addition, 2nd derivative of potential energy function of an elastic system with respect to displacement is stiffness of that elastic system[23].Hence, Equation (6)is reduced to

$$\frac{\partial^2 U}{\partial X^2} = K - 2k_1 - \frac{\partial}{\partial x}\left\{\frac{(\partial_o - L)X}{\sqrt{L^2 - X^2}}\right\} \tag{7}$$

Equation 7 represents that it is non-linear stiffness equation. It is noted that the only term K (stiffness) exists in this equation when it is treated as linear stiffness equation. It is worthy to mention that the other term is canceled out if we consider $\partial_o=L$ (initial deflection of NSS spring equal to bar length). Hence, the system will be linear. It is noted that natural frequency of linear system is lower than the natural frequency of non-linear system[24]. Hence, from Equation (5) potential energy function can be written at $\partial_o=L$ as follows,

$$U(x) = \frac{1}{2} KX^2 + k_1L^2 - k_1X^2 \tag{8}$$

From Equation (6), spring force can be written at $\partial_o=L$ as follows,

$$F(X) = KX - 2k_1X \tag{9}$$

From Equation (7), stiffness of the system can be shown at $\partial_o=L$ as follows,

$$\frac{\partial^2 U}{\partial X^2} = K_{Total} = K - 2k_1 \tag{10}$$

Natural frequency is as follows,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{Total}}{M}} = \frac{1}{2\pi}\sqrt{\frac{K - 2k_1}{M}} \tag{11}$$

In addition, displacement transmissibility can be reported as,

$$\frac{X}{Y} = \sqrt{\frac{1 + 4\zeta^2\left(\frac{f}{f_n}\right)^2}{\left(1 - \left(\frac{f}{f_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{f}{f_n}\right)^2}} \tag{12}$$

Displacement transmissibility becomes, $\frac{X}{Y} = \sqrt{\frac{1}{(1-r^2)^2}}$ as no damping element $\zeta$ is used in model as shown in Fig. 3, where frequency ratio, $r = \frac{f}{f_n} = \frac{\text{Exciting frequency}}{\text{Natural frequency}}$. It is noted that exciting frequency means the frequency at which the seat suspension system vibrates due to external disturbances. $\zeta$= damping ratio of seat suspension system $= \frac{\text{Actual damping}}{\text{Critical damping}}$.The NSS subtracts stiffness from vertical spring without reducing its own main stiffness. Furthermore, double NSS subtracts equal to its twice stiffness, and resultant stiffness should be as low as possible to lower natural frequency. Stiffness difference between vertical spring and NSS should always be positive, never can be equal to zero or negative for stability concern[25]. It is worthy to mention that considering stiffness of spring, seat suspension system needs to be designed in such way that stiffness difference tends to be zero, not exactly equal to zero, which is termed as Quasi-Zero stiffness[22-23]. Suspension elements should better be strong enough to withstand static load and at the same time, should be soft enough to absorb vibration, as it is required in designing suspension system[27]. It is contradictory, however, that seat suspension system having quasi-zero stiffness results in high static stiffness to carry payload and low dynamic stiffness to absorb vibration for reducing natural frequency.

Figure 3 shows three types of seat suspension using NSS as supplementary system. Figure 3(a) has been modified by using single NSS and Fig. 3(b) by another NSS.

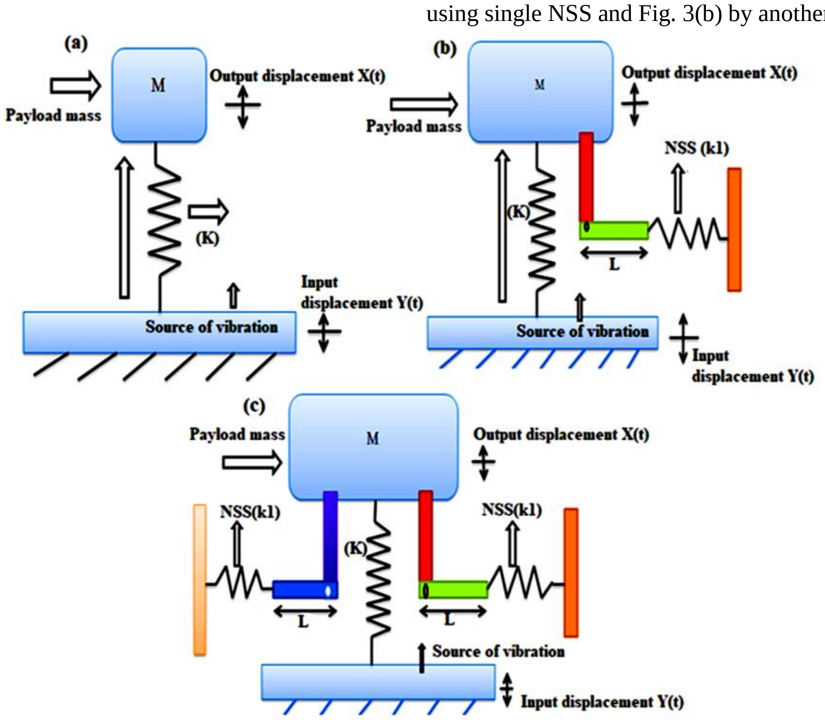


**Figure 3.**Schematic representation of Un-damped passive seat suspension system: (a) without NSS (b) with single NSS (c) with double NSS.

Effect of adding NSS with original system is to reduce natural frequency shown in Fig. 4. As noted natural frequency is obtained as 2.34 Hz against total mass, M=25 kg (Dead mass 15 kg, seat mass 7.5 kg and suspension seismic mass 2.5 kg) with only vertical spring of stiffness K=5500 N/m. Natural frequency is reduced to 1.70 Hz by adding single NSS of stiffness 2500 N/m and to 0.81 Hz adding double NSS of same stiffness. However, Wanget al.[28] reported natural frequency of below 1 Hz using damped suspension parameters. Hence, un-damped system shows better effectiveness than retrospectively damped one. In addition, vibration attenuation power of seat suspension with single NSS is more than suspension without NSS at high frequency. However, vibration attenuation power of seat suspension with double NSS is more than suspension system without NSS and with single NSS.

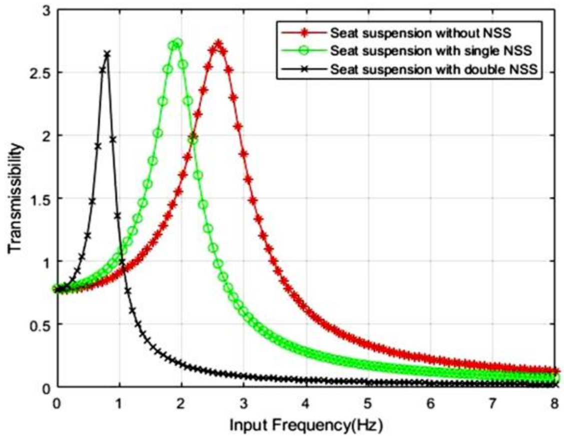


**Figure 4.** Variation of transmissibility with exciting input frequency for un-damped system during mathematical simulation.

Vibration transmission path is modified by using NSS replacing damper collaterally arranged with vertical spring, as this study is not dealing with damper used in seat suspension. A conventional damped system slightly reduces natural frequency and transmissibility as compared with un-damped seat suspension system with double NSS shown in Fig. 5. Figure 5 shows variation of natural frequency and vibration transmissibility of damped system for different damping co-efficient. Damped system consisting of vertical spring (stiffness 5500 N/m) collaterally arranged with a damper with damping co-efficient of 10 Ns/m, 20 Ns/m, and 30 Ns/shows natural frequency of 0.82 Hz. In addition, increasing damping co-efficient keeps natural frequency almost same. However, change of transmissibility is not significantly observeddue to changing viscous damping property. In contrast, 0.81 Hz natural frequency is obtained from seat suspension with double NSS, which is slightly lower than damped system.

Potential energy is a big factor for analysis vibration of vibrating system. If the change of slope of potential energy curve of SDOF with respect to displacement is reduced, then natural frequency and vibration transmissibility of that system is decreased[29]. Figure 6 shows that potential energy curve is obtained from upward and downward displacement of 3 mm from equilibrium position. It is noted that slope of the potential energy curves has relation such as a>b>c.Hence,itindicates slope of potential energy curve of seat suspension system with double NSS is lower than slope of potential energy curve of seat suspension system with single NSS and slope of potential energy curve of seat suspension system without NSS. It is concluded that lower slope of potential energy curve means slow releasing of restoring energy and more vibration attenuation bandwidth. In addition, the potential function is a convex one when an isolation system is stable [25]. Furthermore, the potential function of the added system is a concave function and it achieves the maximum value at the equilibrium point as shown in Fig. 6. Hence, the proposed new seat suspension with NSS is stable system as the potential function is a convex one as shown in Fig. 6.

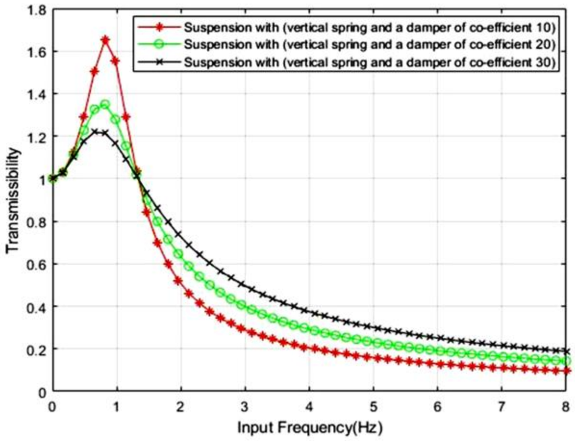


**Figure 5.** Variation of transmissibility with exciting input frequency for damped system during mathematical simulation.

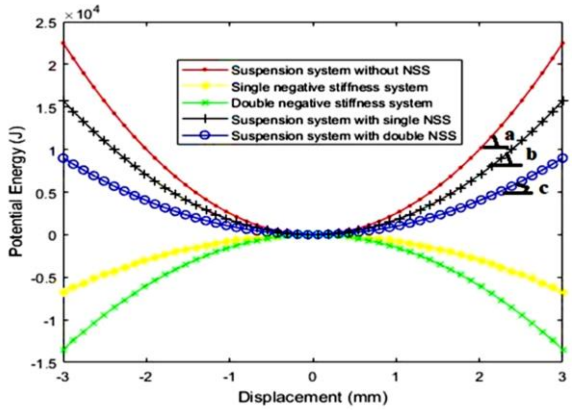


**Figure 6.** Change of potential energy of un-damped passive system during mathematical simulation.

In addition, the stability of seat suspension system shown in this study is further analyzed by varying damping co-efficient. The influences of suspension parameters are focused on improving vibration isolation performance neglecting the stability concern. Stability of suspension system is studied by varying exciting force frequency, exciting force acceleration, non-linearity of springs, and damping co-efficient [30].Varying damping co-efficient is an alternative way to check stability for an un-damped system [31]. Varying damping co-efficient has significant effect on displacement transmissibility ratio from Eqn. (12) and resonance frequency as shown in Fig.7. Amplitude of response and resonance frequency of seat suspension system without and with double NSS is increased and decreased in similar fashion with the increase and decrease of damping co-efficient as shown in Figs.7(a) and 7(b) respectively. However, it is reported that the stability conditions are independent of the excitation amplitude[32]. It is noted that any bifurcation of response and resonance frequency is not observed in Figs.7(a) and 7(b) with sudden jump down or jump up in unstable region[33].For a structurally unstable system, the bifurcation point is the critical value of a parameter that triggers a sudden or 'catastrophic' change in the response[33].In addition, increasing in damping co-efficient led to the reduction of transmissibility in resonant region with that in higher frequencies unaffected[35]. Furthermore, the unstable regions are decreased as the damping radio increases. This similar phenomenon also observed during analysis of stability of system with quasi-zero stiffness vibration isolator[36]. Hence, the solution of differential equation obtained from mathematical modeling of seat suspension system is stable[37]. Therefore, this feature can improve the isolation performance of the seat suspension system in a certain extent.

## 3. Working Principle of NSS

NSS is used as added system with main system in order to increase vibration isolation bandwidth of seat suspension system[38]. It is noted that NSS as an added system is unstable, but can be stable when combined with main system[39]. It consists of horizontal tensional springs, one link connected with upper frame of suspension system, and another link moves upward and downward. Furthermore, one end of horizontal spring is fixed at supporting stand, and other end with the moving shaft. Spring connected with shaft moves right to left and vice-versa through horizontal moving rail as shown in Fig. 8.

Negative stiffness springs are in equilibrium position as shown in Fig. 9(a). In contrast, when connecting, links move upward due to movement of system, upper links tend to move upward and the bar moves right to left and vice-versa as shown in Figs. 9(b) and 9(c). Negative stiffness springs reduce stiffness from vertical spring functionally not actually during movement and tend to make resistance at the time of sudden movement of vertical spring toward upward and downward direction. Total system is forced to move upward and downward direction slowly as a result the system experiences low magnitude vibration. This is the way NSS subtracts stiffness equal to its own from vertical spring as shown in Fig. 9. As seen, the system is in equilibrium position in Fig. 9(a), upward position in Fig. 9(b), and downward position in Fig. 9(c).

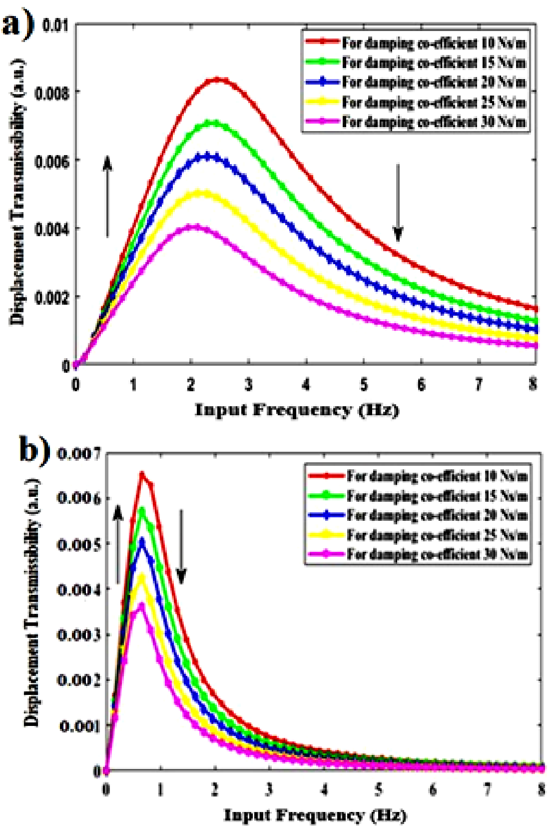


**Figure 7.** Effect of varying damping co-efficient on displacement transmissibility of seat suspension (a) without and (b) with double NSS.

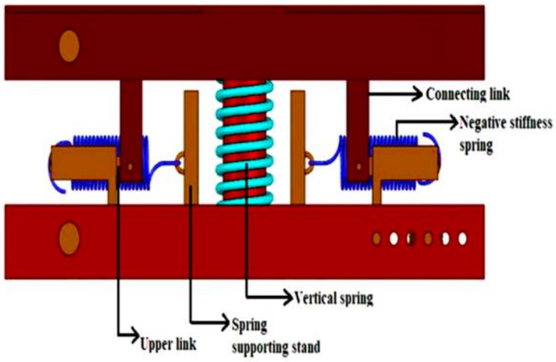


**Figure 8.** CAD model of structure of NSS.

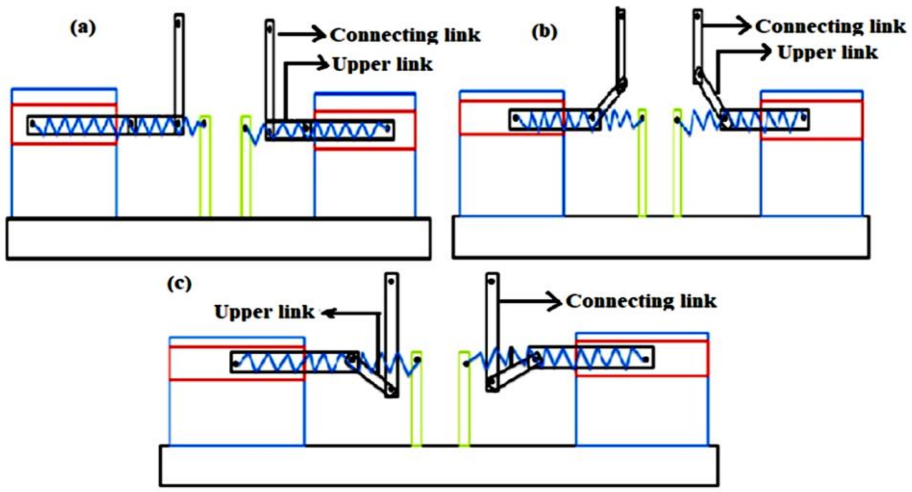


**Figure 9.**Working principle of NSS when system at (a) equilibrium position, (b) upward position, and (c) downward position.

 

## 4. Experimental Model

An efficient low natural frequency seat suspension system is a compromise between vibrating environment, working period, and working efficiency of people[40].Efficiency of seat suspension system depends on reducing displacement transmissibility, increasing vibration attenuation bandwidth and increased by modifying seat suspension parameters[41]. In this work, CAD model of experimental setup shown in Fig. 6consists of three major sections such as mechanical vibration exciter, suspension system, and seat with dead weight. Firstly, CAD model is developed considering design parameters. Furthermore, experimental model is fabricated exactly considering the design parameters. Figure10 shows the CAD model and Table 1 lists the design parameters used in fabricating suspension system. Stiffness of vertical spring and NSS are measured precisely both numerically and experimentally. Other geometries of different parts are measured very carefully.

In addition, four knuckles of dimension (length: 380mm, width: 25mm, thickness: 3mm), four horizontal and vertical bars of (length: 80mm, width: 10mm and thickness: 3mm), upper frame and lower frame of same dimensioning (length: 560mm, width: 460mm, material thickness of 3mm) are assembled to make experimental setup. A square shaped mechanical vibration exciter (72cm×72cm) consisting of six spring of same stiffness (1500 N/m), shaft-pulley bearing mechanism connected with motor shaft through a belt is used to create exciting force for giving input displacement.

**Table 1.**Parameters used in designing suspension system

| Serial No. | Name | Value |
|---|---|---|
| 01 | Mass (M) | 25 Kg |
| 02 | Stiffness of main spring (K) | 5500 N/m |
| 03 | Stiffness of negative spring ($k_1$) | 2500 N/m |
| 04 | Initial length of negative stiffness spring | 130 mm |
| 05 | Bar Length (L) | 70 mm |
| 06 | Initial deflection of spring ($\partial_o$) | 70 mm |

An inverter (Micro-processor/DSP, IGDP, 50 HZ, PF>0.9) is used to control motor speed for shaking exciter top surface at different electric frequency. Figure 11(a) shows single negative stiffness spring and Fig. 11(b) shows double negative stiffness spring added with main system to reduce natural frequency. An accelerometer (Brand: Lutron, Model: BVB-8207SD, SD card with data logger) having four magnetic probes is used for taking peak-to-peak displacement, peak velocity and acceleration reading. Exciting frequency of exciter top surface occurs using the above-mentioned readings as shown in Fig. 11(c). Driver seat is mounted on the top surface of the exciter as shown in Fig. 11(d), which excites at high frequency with increase of motor speed and at low frequency with decrease of motor speed.

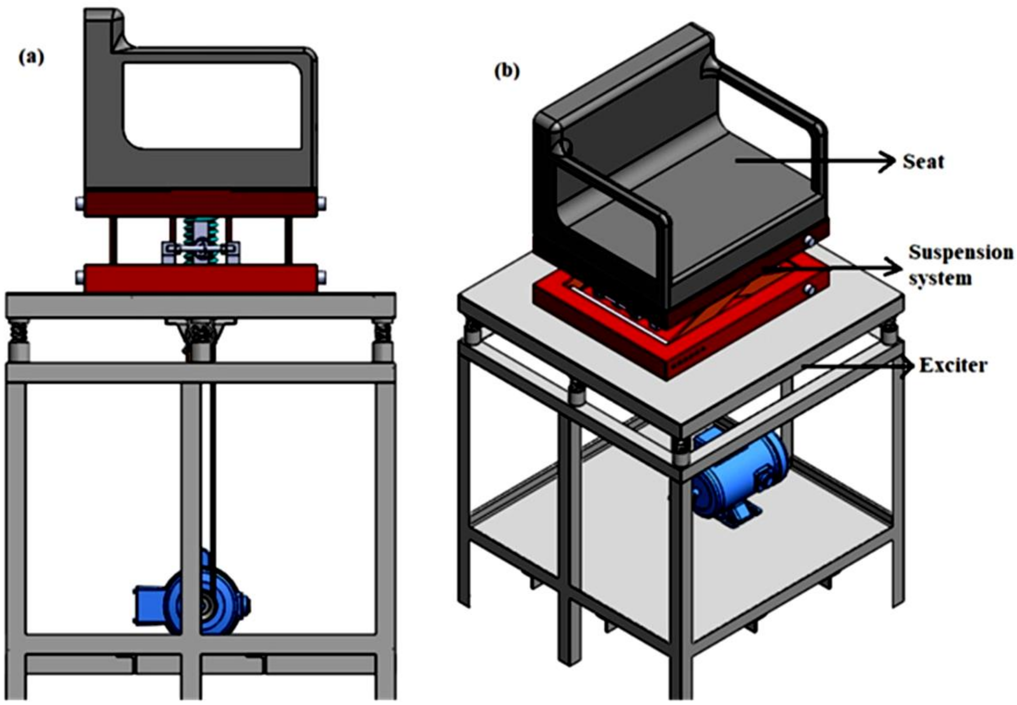


**Figure 10.**CAD model of experimental set up (a) front view (b) isometric view

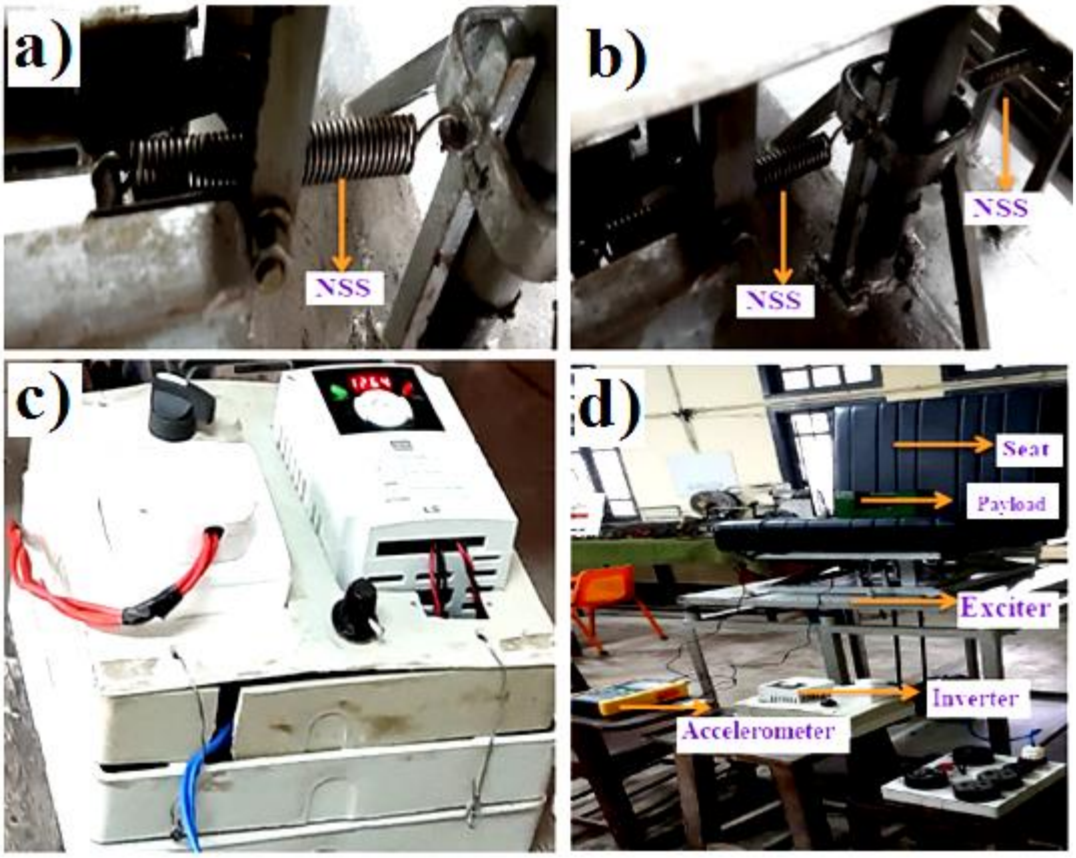


**Figure 11.**Images of (a) Single negative stiffness spring (b) Double negative stiffness spring (c) Inverter circuit (d) Experimental model.

In order to calculate displacement transmissibility, displacement reading of exciter top surface is taken as input signal whereas displacement of seat as output signal. Motor starts to run, and exciter tends to shake the system at about 7 Hz electric frequency. This process was continued up to 24 Hz of electric frequency with the increment of 1 Hz frequency consecutively, and peak-to-peak displacement, peak velocity and acceleration readings against each frequency are saved in external storage of accelerometer.

## 5. Experimental Results and Discussion

Effect of adding NSS with main system is shown in Fig. 12 obtained from experimental investigation. Four channels of vibration meter (Brand: Lutron, Model: BVB-8207SD) was used to take readings such as displacement, velocity, and acceleration with different exciting frequency. Natural frequency of 2.45 Hz is obtained for main system without NSS. However, natural frequency is reduced to 1.78 Hz for adding single NSS with main system.

In addition, 0.84 Hz natural frequency, a significant change is obtained from system with double NSS. It is worthy to mention that 26.3% and 65.88% vibration magnitude is reduced in case of using single NSS and double NSS with main system, respectively. In this study, there is some variation observed between mathematical simulation and experimental results. Main causes of this variation are due to the non-linearity, unbalancing of model in dynamic condition, dry friction among various parts, and improper machining of the parts assembled. In addition, linearity condition is considered for formulating mathematical equations. However, non-linearity exists in the experimental setup. Variation in natural frequency between mathematical simulation and experimental results of 4.6% for original seat suspension system without NSS, 4.9% for single NSS with main system and 3.96% for double NSS with main system is observed.

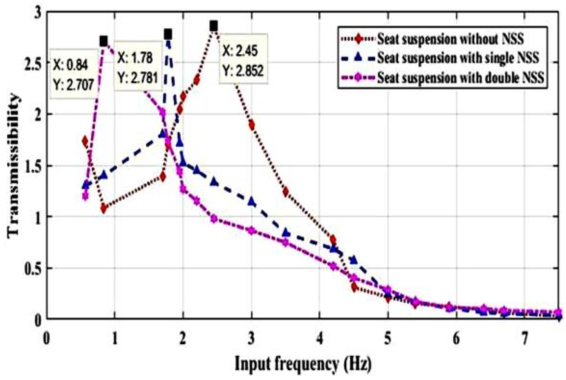


**Figure 12.**Comparison of natural frequency and transmissibility from experimental investigation.

## 6. Conclusions

This study represents an experimental and mathematical investigation on a driver seat suspension fabricated by using quasi-zero stiffness system. Adding of NSS with un-damped seat suspension system to decrease potential energy and natural frequency shows the ability of vibration attenuation bandwidth. At first, a setup is modeled with natural frequency of 2.34 Hz using mass-spring system without any NSS. Mathematical simulation results show that natural frequency of modified model can be reduced to 1.69Hz using single NSS and to 0.81 Hz using double NSS. In addition, natural frequency of 2.45 Hz for without NSS, 1.78 Hz for single NSS, and 0.84Hz for double NSS are observed from experimental results. Hence, vibration magnitude of seat suspension is reduced to 27.3% in case of single NSS and 65.7% for double NSS compared with suspension system without NSS. This type of un-damped passive seat suspension system will be costly economically compared to semi-active and active system. In addition, damper costis considered about $500 whereas replacing damper by adding double NSS with original seat suspension system costs about $70. Proposed seat suspension system is comparably suited in vehicles' driver seat in the context of vibration attenuation power, low cost, simplicity in design and installation. In addition, vibration attenuation mechanism using quasi-zero stiffness is applicable in machine having continuous vibrating parts, rotating shafts, and in machines such as brick breaking, concrete mixing, portable rice mill, sewing machine, and in agricultural machines.

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**Conflicts of Interest:** The authors declare no Conflict of interest.

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