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# Development of A New Technique for Modeling and Optimizing Manufacturing Errors for Cn Machine Tools

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**Abstract**

This paper presents a new technique for optimizing manufacturing tolerances. This technique is based on the combination of two methods, the goal programing method and the genetic algorithm. Firstly, cubic splines interpolation is used to describe machining errors by a set of cubic polynomials. Tool path error, table motion error and tool wear error are considered in this study. Then, based on the goal programming method, the optimization problem is established. In order to avoid weighting effects in the objective functions, we used a genetic Non-dominated Sorting Genetic Algorithm (NSGA) for the resolution of the objective programming problems. A description of optimization processes based on NSGA is presented, and some of the genetic operators are explained. As a result, zero percent rejection of machining parts are obtained by this method. In this study, only three type of machining errors are considered.



*Keywords:* Manufacturing tolerances, Goal programming, Machining errors, Cubic spline interpolation, Genetic algorithm;

## 1. Introduction

Currently, Multi-axis machine tools are used in various fields of advanced manufacturing, for instance the aerospace, the military, the automobile manufacturing and other fields. Improving the tools' machining precision is one of the major significant pursuits for industry manufactures. Machining errors are investigated and divided into two categories, systematic and random errors. Then, the compensation method is performed by tool path modification [1]. However, only systematic errors are compensated in this study. Systematic errors are determined [2] based on three stage experimental study. A new methodology for error compensation of free-form surface is offered [3]. The machined surface is obtained under the base of the T-spline surface reconstruction, using the online inspection data. Using mirror symmetry model, a compensate surface is constructed and used in the CAM process for error compensation. However, the impact of sampling point's distributions on the machining precision are not showed. By using an adaptive neuro-fuzzy inference system and a neural network system, an intelligent system for machine condition monitoring is developed [4].

Thermal errors can have significant effects on CNC machine tool accuracy. The errors come from thermal deformations of the machine elements affected by heat sources within the machine structure or from ambient temperature change [5]. A test piece is considered in order to evaluate thermal errors of five axis machine tools. Also, the R-Test measurement system is used in order to inspect the thermal comportment of the machine tools which were used in thermal test piece machining [6]. However, the R-Test measurement instruments require experienced operators to avoid collusion. In [7] a review of all experimental techniques used for temperature measurement of machine tools and workpiece, a novel approach for an adaptive learning control for thermal error estimation and compensation for 5-axis machine tools was adopted [8]. The sources of thermal error are presented and discussed, then the temperature monitoring technology and thermal deformation monitoring technology are presented. Finally, a new measurement technology called the "fiber bragg grating distributed sensing technology" is introduced for heavy-duty CNC machine tools. A technique has been developed for the error's compensation under temperature stress on five-axis machine tools [9].

Tool error is an important factor that affects the quality and tolerance of machined parts. Tool errors are classified into static and dynamic errors, then error identification method based on shape mapping are presented [10]; a new prediction model of tool errors is developed based on back propagation neural network and genetic algorithm. Based on tool error parameters, the machining error are determined by using the prediction model, then by adjusting the NC code, the tool errors compensation method is carried out. However, machine tool errors and lubrication effect are not considered. A novel technique to estimate the volumetric accuracy of five axis machine tools is presented [11]. A spherical deviation measurement method based on double ball-bar is proposed. An adaptive machining approach by using measured free form deformation data is developed [12]. Template tool positions are revised based

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on information from the real part geometry. A Bezier surface is used to project the novel tool positions. However, this approach can be more improved by taken into account cutting forces. In order to minimize tool deflection errors, genetic algorithm is used to determine the optimized machining parameters [13]. Tool wear monitoring technique in real time is presented in [14] based on ultrasonic system and adaptive neuro-fuzzy inference system.

Many sources of errors contribute in the tool path deviation, kinematic and geometric errors contribute with large amount of deviations [15]. A new technique for three axis machine tools kinematic errors identification is proposed, by conducting a series of machining test in order to separate geometric errors from profile errors [16]. In order to identify geometric errors on three axis machine tools, thermo-invariant multi-features bar (MFB) is designed and developed [17]. A kinematic model for a 5-axis machine tool is built in order to identify geometric error and setup position error, the methodology to calculate these errors is presented [18]. Geometric errors of the two rotary axes are identified and compensated in [19] by adopting an artifact as the test piece and tough-trigger probe for indirect measurement. A new error compensation model is developed based on tool path modification. However; we notice deviation between the predicted and the measured error, this study can be more enhanced by considering other source errors, such as thermo-mechanical error. Adopting differential motion matrix, a new model to identify position independent geometric errors is built in [20]. A geometric errors compensation method for large five axis machine tools is presented. A laser tracker is used for the machine tool tip position measurement; then two position-dependent geometric error models of a machine tool are constructed based on tool tip measurement. Optimal machine tool compensation tables are generated for each model [21]. An efficient geometric errors identification method for non-orthogonal five axis machine tools is considered for the coupling relationship of the error parameters [22].

## 2. Modeling of machining errors using cubic spline

Modeling machining errors or any large data set can be a very challenging task. Generally, the higher the order of polynomial is, the more accurate it is. However; the computation operations on polynomials of high degree involve certain problems, it is suitable to use polynomials of low degree. In order to achieve the higher accuracy and minimize the complexity of computation operations, cubic spline is proposed in this paper for machining errors modeling instead of polynomials interpolation.

A spline is a set of polynomials of degree k that are smoothly connected at certain data points. At each data point, two polynomials connect, and their first derivatives (tangent vectors) have the same values. The definition also requires that all their derivatives up to the (k − 1)st be the same at the point [23].

For data set of N points; cubic spline $S_k(x)$ in $[x_k, x_{k+1}]$ with $k = 0, \dots, N-1$. Are defined with the following steps:

$$\begin{aligned} S_k(x_k) &= y_k \\ S_k(x_{k+1}) &= y_{k+1} \\ S'_k(x_k) &= S'_{k-1}(x_k) \\ S'_k(x_{k+1}) &= S'_{k+1}(x_{k+1}) \\ S''_k(x_k) &= S''_{k-1}(x_k) \\ S''_k(x_{k+1}) &= S''_{k+1}(x_{k+1}) \end{aligned} \tag{1}$$

In this work, path errors, tool wear and table movement errors are modeled by the cubic spline method. The data set used in this study is taken from Ph.D. thesis of (Rahou, 2010) [24] which include these error measurements. Evaluation between cubic spline and Polynomials interpolation is also presented to show the advantages of modeling by cubic spline in this section.

### *2.1. Tool path error modeling*

Figure (1) represents the cutting tool path errors. The cubic spline $f_1(x)$ is composed of a set of 24 cubic polynomials smoothly connected

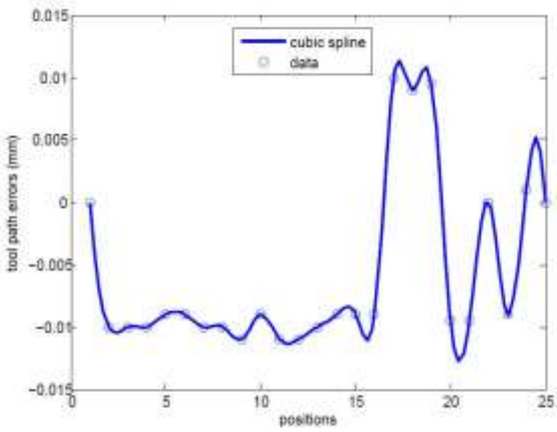


**Figure 1.** Cutting tool path error curve

$$f_1(x) = \begin{cases} S_0 & 0 \le x \le 1 \\ S_1 & 1 \le x \le 2 \\ \vdots & \\ S_{24} & 24 \le x \le 25 \end{cases} \tag{2}$$

### *2.2. Table motion errors*

Figure (2) represents the table motion errors. The cubic spline $f_2(x)$ of table motion errors is a set of 24 cubic equations

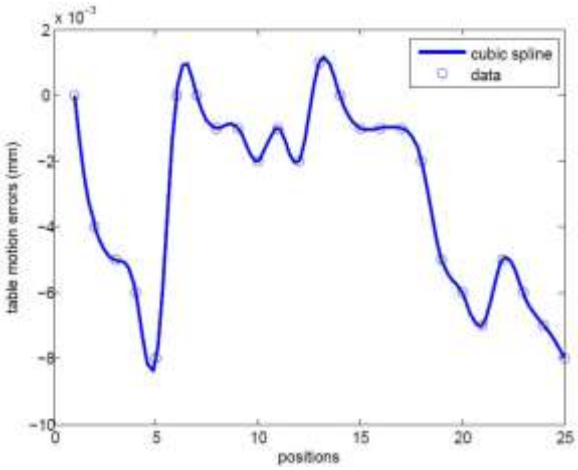


**Figure 2.** Table motion errors curve

$$f_2(x) = \begin{cases} S_0 & 0 \le x \le 1 \\ S_1 & 1 \le x \le 2 \\ \vdots & \\ S_{24} & 24 \le x \le 25 \end{cases} \quad (3)$$

*2.3. Tool wear error modeling*

As we can see in Figure (3), cubic spline curve and linear curve are the same, so it is more practical to work with the linear curve rather than the cubic spline curve, especially if we know that the spline curve is composed from 39 cubic polynomials.

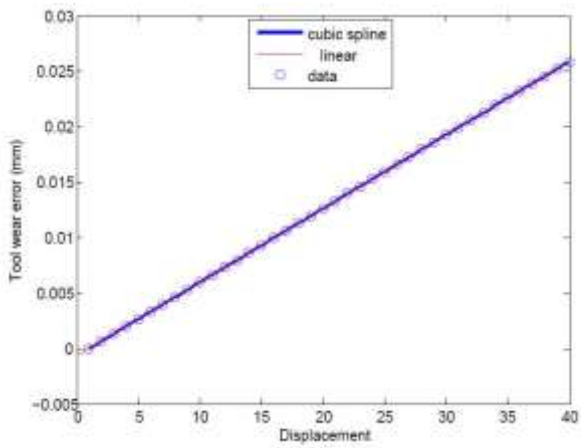


**Figure 3.** Tool wear error curve

Tool wear error is given by the linear equation (3)

$$f_3(x) = 0.00066 * x - 0.0006 \quad (4)$$

As we can see in fig (1) and fig (2) with data set of 25 points, using polynomials interpolation (for instance Lagrange polynomial) will produce a polynomial of 24 degree, contrary to cubic spline interpolation which produced a set of cubic polynomials smoothly connected. The highest precision is achieved, with low degree polynomials.

Manufacturing errors are given above by mathematical equations. In this paper, an investigation is presented, by seeking the optimum amount of each manufacturing error so, the sum of these manufacturing errors does not exceed the tolerance design equation. 5.

$$\sum_{1}^{3} f_i(x) \le IT \quad (5)$$

This technique will not only guarantee the conformity of all the machined parts; but can be used for errors prediction.

Goal programming is used to establish the problem, then based on genetic algorithm, the problems is solved.

## 3. Goal Programming problem formulation

Based on goal programming, a nonlinear programming problem is established. Then, a minimization of the weighted sum of deviations from the goals was carried out.

$$\begin{cases} \textbf{Minimize} \sum_{i=1}^{P} |\boldsymbol{f_i(x) - g_i}| \\ \boldsymbol{C(x) \le c} \quad (\textbf{constraints}) \end{cases} \quad (6)$$

$f_i(x)$ : Objective functions

$g_i$: The goal set for the i-th goal (for i = 1, 2, ..., p);

$C(x)$: Manufacturing tolerance interval

$c$: Design tolerance interval

By introducing the negative and positive deviations $N_i$ *and* $P_i$ respectivelly. The system (6) can be written as:

$$\min Z = \sum_{i=1}^{P} (w_i * N_i + w_i * P_i)$$

Subject to (7)

$$\begin{cases} \sum_{i=1}^{P} a_{ij} x_j - P_i + N_i = g_i \quad for\ each\ goal\ i \quad (i = 1,2, \dots \dots p) \\ x_j \ge 0 \quad (for\ j = 1,2, \dots \dots n) \\ P_i \times N_i = 0 \quad (for\ i = 1,2, \dots \dots n) \\ P_i \ge 0, \ N_i \ge 0 \end{cases}$$

In our case, positive deviations are minimized in order not to violate the tolerance interval. Which lead us to rewrite the equation (5) to the next system

$$\min \sum_{1}^{4} w_i P_i$$

Subject to

$$\begin{aligned} f_1(x) - P_1 &= IT \\ f_2(x) - P_2 &= IT \\ f_3(x) - P_3 &= IT \\ \sum_{1}^{3} f_i(x) - P_4 &= IT \\ P_i &\ge 0 \end{aligned} \quad (8)$$

Where

$w_i$ is the weight factors fixed by the user which represents the goals preference and the sum equal to one.

$N_i$*and* $P_i$ are the negative and the positives deviations from the goal respectively.

$IT$ is the tolerance interval

Note that in the system (8), the negative deviations $N_i$ in the constraints are eliminated by using a ($\le$) relation.

## 4. Non-dominated sorting genetic algorithm

The selection operation is the only difference between the non-dominated sorting genetic algorithm (NSGA) and simple genetic algorithm. The crossover operator and mutation operator work the same way. NSGA used in this study is a real parameters GA.

Classifying the population into $P_k$ classes based on non-domination is the first stage of NSGA.

$$P = \cup_{k=1}^{n} P_k \quad (9)$$

$n$: The total number of fronts

*4.1. Non-dominated sorting of a population:*

Considering a population P of N solutions, the following procedure could be used to deduce the classes non-dominated of solutions:

- Step 0: set all non-dominated sets $P_k = \emptyset$; (k=1, 2...). For non-domination level $k = 1$
- Step 1: set counter solution $i = 1$, $P' = \emptyset$.
- Step 2: for a solution $j \in P$ ($j \neq i$). If solution $j$ dominate solution $i$, go to step 4.
- Step 3: Increase $j$ and go to step 2. Otherwise set $P' = P' \cup \{i\}$
- Step 4: increment$i$. If $i \leq N$ go to step 2. Otherwise stop and declare $P'$ as the non-dominated set.
- Step 5: update $P_k = P'$ and $P = P/P'$
- Step 6: if $P \neq \emptyset$; increment $k$ and go to Stage1. If not, stop and announce all non-dominated sets $P_k$

This procedure can handle any numbers of objectives, maximization or minimization problems can be treated. The next step is to assign a value for each solution of these classes.

*4.2. Sharing method*

For a given front $k$, which contain $n_k$ solutions; each have a fitness value equal to $f_k$, the sharing method is explained as follow:

Firstly, the sharing functions value is calculated by using the following function [25]

$$Sh(d_{ij}) = \begin{cases} 1 - \left(\frac{d_{ij}}{\sigma_{share}}\right)^2, & if\ d_{ij} \leq \sigma_{share} \\ 0; & otherwise \end{cases} \quad (10)$$

The parameter $d_{ij}$ is the Euclidian distance between any two solutions in the populations in the same front. $d_{ij}$ Is determined as follows:

$$d_{ij} = \sqrt{\sum_{p=1}^{P} \left(\frac{x_k^i - x_k^j}{x_k^{max} - x_k^{min}}\right)^2} \quad (11)$$

$P$ : The problem variables. Then, a niche count $\eta c_i$ is calculated for the $i - th$ solution as follows:

$$\eta c_i = \sum_{j=1}^{N} Sh(d_{ij}) \quad (12)$$

The final step is to calculate the shared fitness value as follows

$$f'_i = {f_i}/{\eta c_i} \quad (13)$$

The minimum shared fitness in this class is noted $f'^{min}_k$. In order to proceed the next non-dominated class, the assigned fitness value is equal to:

$$f_{k+1} = f'^{min}_k - \epsilon \quad (14)$$

$\epsilon$: is small positive number.

*4.3. reproduction operator*

Making several copies of good solutions and removing bad solutions from the population is the main goal of the selection operator while keeping population size constant, Based on the dummy fitness value and stochastic remainder proportionate selection.

*4.4. Crossover and mutation operators:*

As mentioned above, selection operator cannot produce any new solutions, it only duplicate good solutions at the expense of bad solutions. Crossover and mutation operators are responsible to create new solutions.

For crossover operation, two solutions are picked (called parent solutions) from the mating pool at random and crossed with a probability $p_c = 0.9$. in this study simulated binary crossover (SBX) operator is used. The procedure of computing children solutions $x_1$ and $x_2$ from the parent solutions $y_1$ and $y_2$ is described below [25]:

Step 1: choose a random number $u \in [0; 1]$

Step 2: calculate $\beta_{qi}$ using the next equation:

$$\beta_{qi} = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}} \ ; & if\ u_i \leq 0.5 \\ \left(\frac{1}{2(1-u_i)}\right)^{\frac{1}{\eta_c+1}} \ ; & otherwise \end{cases} \quad (15)$$

$\eta_c$ : distribution index. In this study, $\eta_c$ is fixed 20.

Step 3: the children solutions are created using the following equations

$$x_1 = 0.5\left[(1 + \beta_{qi})\, y_1 + (1 - \beta_{qi})\, y_2\right] \quad (16)$$

$$x_2 = 0.5\left[(1 - \beta_{qi})\, y_1 + (1 + \beta_{qi})\, y_2\right] \quad (17)$$

Using the above step by step, the children solutions are created. Note that the two children solutions are symmetric about the parent solutions.

Polynomial mutation [25] is used in this study to create a new solution $z_i$ from the parent solution $x_i$; the following step by step is used for a mutation probability $p_m = 0.1$:

Step 1: create a random number $u \in [0; 1]$

Step 2: calculate the parameters $\delta_i$ as fallows:

$$\delta_i = \begin{cases} (2u_i)^{\frac{1}{\eta_m+1}} - 1 & \text{if}\ u_i < 0 \\ 1 - \left[2\,(1-u_i)^{\frac{1}{\eta_m+1}} - 1\right]^{\frac{1}{\eta_m+1}} & \text{if}\ u_i \geq ( \end{cases} \quad (18)$$

$\eta_m$ : distribution index for mutation. $\eta_m = 150$ is fixed in this study.

Step 3: calculate Zi by the relation (19):

$$z_i = x_i + \delta_i(x_i^u - x_i^l) \quad (19)$$

$x_i^u and x_i^l$ Are the superior and inferior bounds of parameter $x_i$.

## 5. Problem description and optimization problem

In this paper, the above multi objective GA is used in order to solve goal programming problem in system 8, however some changes are needed for this purpose.

Minimizing the positive deviations $P_i$ in system 8, with $P_i \geq 0$ generate two scenarios:

If $P_i = 0 \Rightarrow f_i(x) = IT \Rightarrow f_i(x) - IT = 0$.

Or $P_i > 0 \Rightarrow f_i(x) - P_i = IT \Rightarrow f_i(x) - IT = P_i$.

We can reformulate system 8 as:

$$min < f_i (x) - IT > \quad (20)$$

With the bracket operator $< >$ returns the operand value if it is positive, otherwise returns 0. By this way a GP problem is rewritten to multi objective problem, and we can use NSGA to solve it. The advantage is that we can get several solutions to the GP problem simultaneously which are not subjective to the user. We use a population of size 50 for a 50 generation.

RAHOU *et all* [2] consider the following part as a test piece Figure. 4. The tolerance interval is fixed at 0.02 (IT = 0.02). The results of the following system (21) are shown in Figure 5 and listed in the table. 1.

$$\begin{aligned} &min < f_1 (x_1) - 0.02 > \\ &min < f_2 (x_2) - 0.02 > \\ &min < f_3 (x_3) - 0.02 > \\ &min < f_1 (x_1) + f_2 (x_2) + f_3 (x_3) - 0.02 > \end{aligned} \quad (21)$$

Subject to

$1 \leq x_1 \leq 25,\ 1 \leq x_2 \leq 25, 1 \leq x_3 \leq 40.$

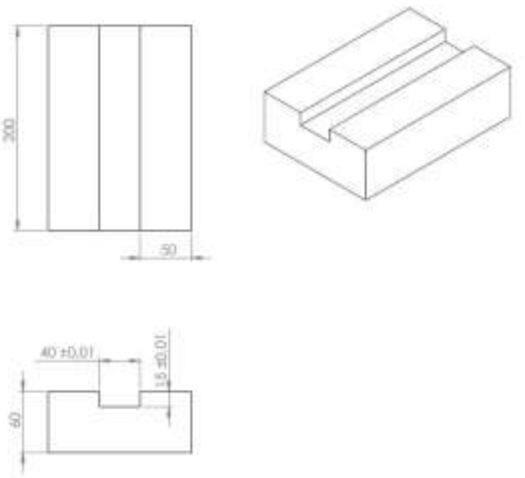


**Figure 4.** test piece

**Figure 5.** Optimization result

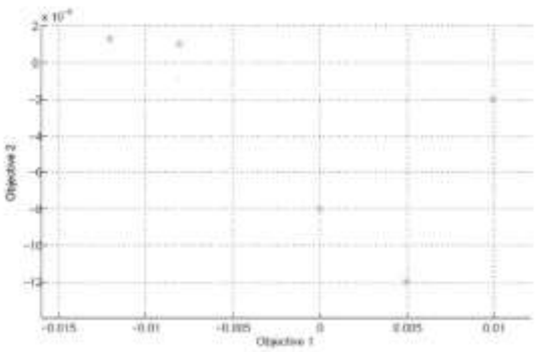


**Table 1.** Optimization result

| $x_1$ | $x_2$ | $x_3$ | $f_1 (x_1)$ | $f_2 (x_2)$ | $f_3 (x_3)$ | $\sum_1^3 f_i (x_i)$ |
|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 0 | -0.008 | 0 | -0.008 |
| 10 | 7 | 15 | -0.008 | 0.001 | 0.01 | 0.003 |
| 17 | 10 | 20 | 0.01 | -0.002 | 0.012 | 0.02 |
| 20 | 14 | 30 | -0.012 | 0.0013 | 0.017 | 0.0063 |
| 24 | 21 | 40 | 0.005 | -0.012 | 0.027 | 0.02 |

We can notice that all the result founded are less then IT= 0.02, more on that, the sum of the machining errors is less than the IT.

We can also fix some errors at the minimum, in order to get the max of the other errors with the condition that the sum of all the machining errors are less than IT. For example, we can set tool wear error at 0.02, which mean we change the cutting tool when the tool wear error exceeds 0.02 however, that mean we minimize in the tool life. But in the other hand we can permit to maximize the amount of the other errors like table motion error, considering the sum of all the machining errors are less than or equal to design tolerance IT.

Based on this study and the result listed in the table 1, we can see that the workpiece accuracy is more influenced by the tool wear error $f_3$ , approximately by 42% than the other errors.

## 6. Conclusion

In this paper, we simultaneously consider tool wear-, table motion-, and tool path errors and have addressed the accuracy – errors trade off problem for multi-axis machine tools. A new methodology is developed in this study for optimizing the machining errors of multi-axis machine tools and the procedure of reallocating of each machining error. This was achieved by optimizing the machining errors and taken the design tolerance as hard constraint in order to achieve zero percent rejection. The machining errors considered in this study mentioned above are modeled based on cubic spline interpolation, then based on goal programming the optimization problem is formulated. Finally, NSGA is used to solve this problem. The result obtained in this study are summarized here:

- Using cubic spline interpolation for modeling generate high accuracy model of machining errors with low degree polynomials.
- It is robust
- Zero percent rejection of machining parts are obtained by this method
- More complex models for 100% conformity rate by including other errors can easily be applied using the same framework.

In terms of shortcomings of the work and areas of future studies, the following issues would be recommended for further studies:

This work considers only tool path-, tool wear-, and table motion errors. Extension to the other errors such as thermal induced errors should be considered in future research.

 

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