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Analytical and Graphical Optimal Synthesis of Crank-Rocker Four Bar Mechanisms for Achieving Targeted Transmission Angle Deviations

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Abstract

This paper presents a detailed methodology to optimally synthesize links' lengths of planar Crank-Rocker (C-R) mechanism to achieve a targeted design with definite transmission angle deviation. Analytical and graphical proposed methodologies are applied to three different case studies; each satisfies a definite case (task). The analytical methodology is based on deducing six design equations with equality constraints, which represent relations between the desired case conditions and the mechanism's lengths. Meanwhile, deflection and transmission angles; the time ratio limits or output angular stroke can be easily obtained. Furthermore, optimal synthesized results can fulfil any definite case requirements which can be represented using the corresponding six deduced equations. The optimal charts are presented to quickly obtain the optimal (C-R) mechanism's lengths, which are achieving the targeted transmission angles deviations. Consequently, the designers can easily select optimal synthesized crank-rocker mechanisms' lengths, instead of time consuming of optimization calculations. Also, this paper presented a fast-graphical methodology to directly obtain an optimal synthesized (C-R) mechanism's lengths. This methodology requires only identifying the design case related to the chosen mechanism class and the desired transmission angle deviations through giving the minimum and maximum transmission angles (γmin and γmax). Moreover, a direct relation between the mini-max transmission angle deviations, the (C-R) mechanisms classes and the performance parameters can be presented. Hence, this facilitates the specialists' mission in designing (C-R) mechanisms for special uses as driving conveying, screening and shaking mechanisms.

Keywords: Synthesis, Optimization, Crank-rocker mechanism, Transmission angle, Mechanism Design;

Nomenclatures

C-R	Crank-Rocker mechanism
J	Jacobian of the system
l	The longest link of the mechanism
R_1	The fixed link of the mechanism
R_2	The crank link of the mechanism
R_3	The coupler link of the mechanism
R_4	The rocker link of the mechanism
s	The shortest link of the mechanism
TR	Time ratio
p	Link is not the shortest or longest links
q	Link is not the shortest or longest links
x_n	The design variables
ε	A certain tolerance
δ	Deflection angle
γ	The transmission angle
γ_{min}	The minimum transmission angle
γ_{max}	The maximum transmission angle
γ_i	Angle γ at initial position of R_4
γ_f	Angle γ at final position of R_4
θ_1	The angular position of the fixed link
θ_2	The crank angular position
θ_3	The coupler angular position
θ_4	The rocker angular position
θ_{3i}	Angular position of R_3 at initial position
θ_{3f}	Angular position of R_3 at final position
$\theta_{3\mathrm{n}}$	Angular position of R_3 at 1st extreme position
θ_{3x}	Angular position of R_3 at 2nd extreme position
θ_{4i}	Initial angular position of R_4
θ_{4f}	Finial angular position of R_4
Δ_1	Deviation of γ_{min}
Δ_2	Deviation of γ_{max}
Δ_{cr}	Deviation of critical values of γ
Δ_l	Certain deviation of links

1. Introduction

The four-bar mechanism is commonly employed in different mechanical engineering applications. The main components of these planar mechanisms are links, joints, or pairs that satisfy the requirements of many practical engineering applications. Four bar Crank-Rocker (C-R) mechanism is the most applied type of planar mechanisms in mechanical systems and devices. Furthermore, one of the

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main effective design criteria of the planar mechanism is the transmission angle. The planar four bar mechanism's transmission angle is the one between the output and the coupler links. The optimal values of this angle variation are around 90^o. Generally, synthesis of the crank rocker four bar mechanisms has been discussed through the last decades. In earlier studies, the problem of optimizing the transmission angle of the crank-rocker mechanism is investigated in [1], which deals with a least square solution. In addition, a theoretical procedure for synthesizing four bar function generation with keeping the transmission angle in a specified range is proposed by Gupta in [2]. While graphical and analytical synthesizes of the crank rocker four bar mechanisms are introduced in [3], where the results are depending on maximizing the minimum of the transmission angle. Moreover, a synthesis procedure which satisfies a prescribed time ratio and rocker's angular swinging amplitude is presented in [4], which optimizes the required definite objective function. Also, synthesis equations are developed in [5] dealing with position, path, function generation and transmission angle constraints of four bar mechanisms to denote an approximation of the design region.

An analytical method for synthesizing the crank rocker mechanism with good quality motion and unit time ratio is proposed in [6], which introduces the synthesized results as a design chart. Furthermore, the transmission quality of planar and spherical linkages is discussed considering the zero mean linkages definition in [7]. While, a graphical method for designing the optimal links lengths of crank rocker mechanism is introduced in [8]. This method presents the synthesized mechanism's lengths depending on initial crank angle, minimum transmission angle, rocker link's amplitude and the two crank angles of the dead center positions of the rocker link. A synthesis algorithm for the planar four-bar mechanism with a single degree of freedom is investigated in [9]. This algorithm is dealing with the synthesized maximum deviation of transmission angle which is less than a certain specified bound. The graphical and analytical approaches of synthesizing (C-R) mechanisms considering the design parameters, such as rocker's swing angle, transmission angle and time ratio are developed in [10] associated with design charts via some specified parameters. The transmission angle's influence on the different parameters of a mechanism (for example; friction, mechanical advantage, pressure angle, transmission force, velocity, acceleration, input crank angle, tolerance and the performance sensitivity) is discussed in [11]. Furthermore, various mechanism's defects, such as branching, order, circuit and poor transmission angle are introduced in [12], in addition to present the rectification solution for successful synthesis. Synthesizing methodology of the planar four bar mechanism lengths for generating a certain motion is explained in [13], which depends on minimizing the maximum deviation of transmission angle. Furthermore, an analytical optimization of (C-R) mechanism through maximizing the minimum transmission angle is presented in [14]. Moreover, design nomograms for directly synthesizing the crank rocker mechanism links' ratios with a definite synthesized transmission angle range are given in [15]. Also, an approach dealing with the mechanism's lengths and transmission angle deviations is presented in [16]. On the other hand, a force transmissivity index is proposed for the planar mechanisms in [17], this index is based on the concepts of static force analysis, transmission angle deviation and power flow. Likewise, the influence of joint's clearance on path generation considering the transmission angle of four bar mechanism is investigated in [18]. Many published researches have been devoted to the optimal synthesis of (C-R) mechanism using suitable design optimization techniques. Some of these techniques are dealing with a specified path generation [19-26] and motion generation in addition to the design for finitely separated positions [27-30] considering the transmission angle as a design constraint.

In this paper, the transmission angle deviation is adopted as the desired task. Consequently, this paper presents a detailed methodology to optimally synthesize links' lengths of planar crank-rocker mechanism to achieve the targeted designs with definite transmission angle deviations. This suggested that the analytical methodology is based on deriving a set of nonlinear equations. The Newton-Raphson's iterative numerical technique can be employed using the MATLAB software in order to simultaneously solve these nonlinear equations. Three different case studies are discussed in this paper. The first case considers the deviations of minimum and maximum transmission angles, which are equal around 90^0 as mentioned in [15, 16]. While the second case exists when the total value of the minimum transmission angle's deviation from 90^0 can be increased more than the total value of the maximum transmission angle's deviation from 90^0 that is kept at a constant small value. Conversely, the reverse of the second case study represents the third case condition. This third case exists when the total value of the maximum transmission angle's deviation from 90^0 can be increased more than the total value of deviation of the minimum transmission angle from 90^0. Moreover, six design equality constraints equations can be deduced using some mathematical manipulation for each mechanism's desired condition. Also, these constraints clarify a direct relation between the mini-max transmission angle deviations, the (C-R) mechanisms classes and their performance.

Fortunately, if at least one of these deduced equations can be verified, the remaining conditions or equations can be also verified. This reveals the effectiveness of the proposed methodology. This methodology could be used directly to construct the required optimal (C-R) mechanism's lengths. Likewise, each case's optimal numerical solution can be compared with those of the other corresponding deduced equations of some approaches in the published literature. Finally, an effective simple graphical methodology is introduced in order to directly construct such optimal mechanisms through fast and simple steps.

2. Methodology

The methodology is organized in four steps. The first step is the identification of an initial feasible design domain, while the second one is selecting the main required parameters to synthesize the (C-R) mechanism's lengths satisfying a definite condition of transmission angle deviations. The third one is deducing six design equations as equality constraints for achieving the optimal (C-R) mechanism lengths. Finally, in the fourth step either the

Newton-Raphson's iterative numerical technique can be used for obtaining the optimal mechanism lengths or the graphical method to quickly obtain the optimal lengths.

2.1. Initial feasible design domain (F.D.D)

Crank-Rocker mechanism is shown in Fig. 1. This (C-R) mechanism contains an input link having a full rotation that is called a crank (R_2) and an output link that is called rocker (R_4) which oscillates between two dead-center positions as shown in Fig. 2. Also, links (R_2) and (R_4) are connected to the fixed link (R_1) by kinematics pairs O_2 and O_4, respectively. Coupler link (R_3) connects (R_2) with (R_4).

Clearly, the Newton-Raphson's iterative numerical method needs effective initial values for solving the deduced nonlinear equations. These initial values must be assumed within the following suitable feasible design domain.

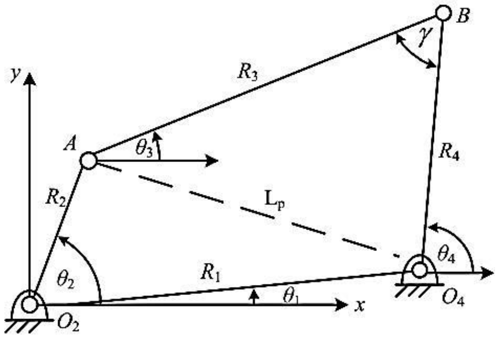

Figure 1: The crank-rocker mechanism

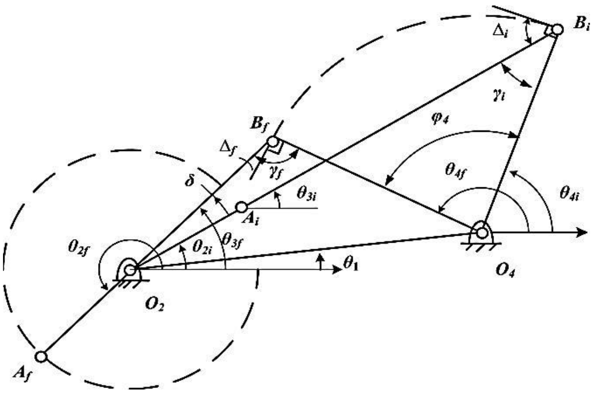

Figure 2: The two extreme positions of (C-R) mechanism

2.1.1. Mechanism Links' Lengths Domain

Planar kinematic chains have three inversions, as shown in Fig. 3. These three inversions of such planar kinematic chains can construct two different crank-rocker mechanisms when the Grashof 's criterion is valid as stated in [12] and [30], as; $s+l<p+q$.

Where, (s) denotes the shortest link of the mechanism, (l) denotes the longest link and (p, q) denote the lengths of the other two links. In addition, (s) is the input link of length (R_2), while the fixed link of length (R_1) is any link besides the input link (R_2). Furthermore, suggested limitations of these links (s, p, q and l), are;

$0.2 \leq s_{min} \leq s \leq s_{max}$, $s \leq p \leq p_{max}$, $p \leq q \leq q_{max}$ and $q \leq l \leq l_{max} \leq 1.1$m

Where, the maximum value of any link's length equals to its minimum value plus a suggested certain deviation which equals to (Δ_l). The numerical solution is presented for the inversions of the three different kinematic chain of links (s, p, q and l) arrangements, which realize six possible (C-R) lengths domain considering Δ_l = 0.04m as shown in Table 1. Generally, each mechanism can be represented as follows:

$M1_1$: $R_2<R_3<R_4<R_1$; $M1_2$: $R_2<R_1<R_4<R_3$
$M2_1$: $R_2<R_4<R_1<R_3$; $M2_2$: $R_2<R_4<R_3<R_1$
$M3_1$: $R_2<R_1<R_3<R_4$; $M3_2$: $R_2<R_3<R_1<R_4$

In addition, the mechanism lengths must verify the following inequality constraints as;

$$G_f = ((p+q)/(s+l)) - 1 > 0 \tag{1}$$

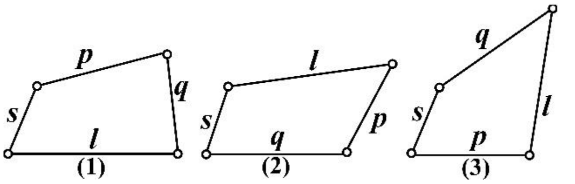

Figure 3: Three kinematic planar chains

Table 1. Six possible crank-rocker mechanisms lengths

R_i	Kinematic chain (1)		Kinematic chain (2)		Kinematic chain (3)	
	$M1_1$	$M1_2$	$M2_1$	$M2_2$	$M3_1$	$M3_2$
R_1	l	p	q	l	p	q
R_2	s	s	s	s	s	s
R_3	p	l	l	q	q	p
R_4	q	q	p	p	l	l

2.1.2. Prescribed Timing Domain

The frame (R_1) has a fixed angular position ($\theta_1 \geq 0$). In order to grantee a full mobility rotation of the input link (R_2) with avoiding order and branch defects, the following inequalities should be achieved as stated in [12] as follows;

$$0 \leq \theta_2 \leq 360^\circ \ , \ \theta_2^{\,i-1} \leq \theta_2^{\,i} \leq \theta_2^{\,i+1} \quad \text{and} \quad \sin\gamma > 0 \tag{2}$$

Where, (θ_2) describes the crank angular position and (i) denotes angular position number. Also, (γ) denotes the transmission angle, which is shown in Fig. 1. The transmission angle is given by the following equation;

$$\gamma = \theta_4 - \theta_3 \ , \ 40^\circ \leq \gamma \leq 140^\circ \text{, as presented in [11]} \tag{3}$$

Where, θ_3 and θ_4 are the coupler and rocker angular positions. If the obtained values of the mechanism synthesis are outside the prescribed domain, the selection of the initial values must be repeated using other modified values until the obtained results fall within the prescribed feasible domain. One or two of the six mechanisms tabulated in Table 1 represent the feasible design domain which can be considered for constructing the required optimal (C-R) mechanism.

2.2. The Optimal Synthesis of (C-R) Mechanism

The proposed technique for synthesizing the (C-R) mechanism is dealing with finding the optimal mechanism's lengths. These mechanism's lengths are synthesized to verify the definite minimum and maximum transmission angle deviations. Whereas, the optimum value of transmission angle (γ) is close to 90° as much as possible with a recommended maximum tolerance about ± 50° as mentioned in [11] and [30] for achieving smooth operation without jerky movements and maintaining a good quality of force transmission. Transmission angle (γ) can be obtained as stated in [1, 2] and [11] as follows;

$$\cos\gamma = \frac{R_3^{\,2} + R_4^{\,2} - R_1^{\,2} - R_2^{\,2} + 2R_1R_2\cos(\theta_2 - \theta_1)}{2R_3R_4} \quad (4)$$

The minimum and maximum transmission angles (γ_{min}, γ_{max}) are shown in Fig. 4. The values of angles (γ_{min}, γ_{max}) can be formulated using the first derivative of Eq. (4) with respect to θ_2 which equals to zero at ($\theta_2 - \theta_1$) = 0 and ($\theta_2 - \theta_1$) = 180°. Hence, γ_{min} and γ_{max} can be obtained as presented in [6] using the following equations;

$$\cos\gamma_{\min} = \frac{R_3^{\,2} + R_4^{\,2} - (R_1 - R_2)^2}{2R_3\,R_4} \quad (5)$$

$$\cos\gamma_{\max} = \frac{R_3^{\,2} + R_4^{\,2} - (R_1 + R_2)^2}{2R_3\,R_4} \quad (6)$$

The deviations Δ_1, Δ_2 and Δ_{cr} of the minimum, maximum and critical values of the transmission angles from 90° are respectively presented in [1] as follows;

$$\Delta_1 = 90^\circ - \gamma_{\min},\ \Delta_2 = \gamma_{\max} - 90^\circ \text{ and } \Delta_{cr} = Max[\Delta_1, \Delta_2] \quad (7)$$

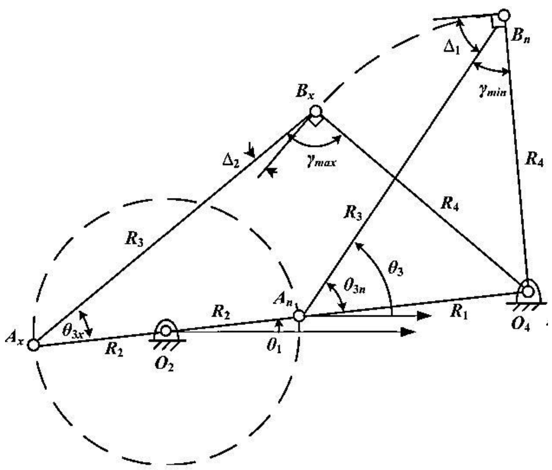

Figure 4: Minimum and maximum transmission angles of (C-R) mechanism

In addition, (γ_i) and (γ_f) are the transmission angles at the initial and final angular positions of the rocker link as shown in Fig. 2, both (γ_i) and (γ_f) can be respectively computed using Eq. (8) and Eq. (9) as follows;

$$\gamma_i = \cos^{-1}\left(\frac{R_4^{\,2} + (R_3 + R_2)^2 - R_1^{\,2}}{2R_4(R_3 + R_2)}\right) \quad (8)$$

$$\gamma_f = \cos^{-1}\left(\frac{R_4^{\,2} + (R_3 - R_2)^2 - R_1^{\,2}}{2R_4(R_3 - R_2)}\right) \quad (9)$$

Where Δ_i and Δ_f which are shown in Fig. 2 can be computed as follows;

$$\Delta_i = (90^\circ - \gamma_i) < \Delta_1 \quad \text{and} \quad \Delta_f = (\gamma_f - 90^\circ) < \Delta_2 \quad (10)$$

Applying sine and cosine law for two triangles ($A_nB_nO_4$ and $A_xB_xO_4$), which are shown in Fig. 4, the following equations can be obtained as follows;

$$\frac{R_1 - R_2}{\sin\gamma_{\min}} = \frac{R_4}{\sin\theta_{3n}} \quad (11)$$

$$\frac{R_4}{\sin\theta_{3n}} = \frac{R_3}{\sin(180^\circ - \theta_{3n} - \gamma_{\min})} \quad (12)$$

$$\frac{R_1 + R_2}{\sin\gamma_{\max}} = \frac{R_4}{\sin\theta_{3x}} \quad (13)$$

$$\frac{R_4}{\sin\theta_{3x}} = \frac{R_3}{\sin(180^\circ - \theta_{3x} - \gamma_{\max})} \quad (14)$$

$$\cos\theta_{3n} = \frac{(R_1 - R_2)^2 + R_3^{\,2} - R_4^{\,2}}{2R_3(R_1 - R_2)} \quad (15)$$

$$\cos\theta_{3x} = \frac{(R_1 + R_2)^2 + R_3^{\,2} - R_4^{\,2}}{2R_3(R_1 + R_2)} \quad (16)$$

Where $\theta_{3n}=\theta_3-\theta_1$ at ($\theta_2=\theta_1$) and $\theta_{3x}=\theta_3 - \theta_1$ at ($\theta_2=180^\circ + \theta_1$).

Thus, the synthesis can be performed by deriving six nonlinear equations in six desired mechanism's parameters; R_1, R_2, R_3 and R_4, in addition to coupler link positions (θ_{3n}, θ_{3x}) as unknowns.

The six nonlinear equations are derived as functions of (Δ_1, Δ_2) which can be solved using the Newton-Raphson iterative numerical method [30, 31].

The Eqs. (5-7) in addition to Eqs. (11-14) can be rewritten in the form of [$f_n(x_n)$] = [0] using mathematical manipulation as follows;

$$\left[f_n(x_n)\right] = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} R_3^{\,2} + R_4^{\,2} - (R_1 - R_2)^2 - 2R_3R_4\sin\Delta_1 \\ R_3^{\,2} + R_4^{\,2} - (R_1 + R_2)^2 + 2R_3R_4\sin\Delta_2 \\ (R_1 - R_2)\sin\theta_{3n} - R_4\cos\Delta_1 \\ R_4\cos(\theta_{3n} - \Delta_1) - R_3\sin\theta_{3n} \\ (R_1 + R_2)\sin\theta_{3x} - R_4\cos\Delta_2 \\ R_4\cos(\theta_{3x} + \Delta_2) - R_3\sin\theta_{3x} \end{bmatrix} = [0] \quad (17)$$

The design variables (x_n) are;

$$\left[x_n\right] = \left[R_1 \quad R_2 \quad R_3 \quad R_4 \quad \theta_{3n} \quad \theta_{3x}\right]^T \quad (18)$$

Furthermore, Jacobian (J) of the system is defined as; $J = [\partial f_n / \partial x_n]_{6\times 6}$. Also, the iterative formula is expressed as $[\Delta x_n] = -[J]^{-1}.\ [f(x_n)]$, thus, the determinate of the Jacobian matrix should not equal to zero (det[J]≠0).

The successive approximations for a solution can be obtained using the following form $x_n^{i+1} = x_n^i + \Delta x_n^i$, the first guess for the solution (x_n^i), which lies inside the initial values of the feasible design domain. Consequently, the Newton-Raphson's process takes less number of iterations and less computation time for obtaining the results. A convergence criterion of such system's solution could be achieved when the magnitude of the vector $f(x_n)$ is smaller than a certain tolerance (ε). Where $|f(x_n)| < \varepsilon$ and $\varepsilon = 10^{-5}$.

The obtained mechanism's lengths R_i (R_1, R_2, R_3 and R_4) using this solving technique are considered as the optimal mechanism's lengths. If one of these obtained results falls outside the prescribed range (F.D.D), the initial mechanism's lengths should be changed in order to repeat the solving procedure until the optimal mechanism lengths fall within the feasible design domain.

Consequently, these obtained mechanism's lengths are not unique, but these fall within the optimal design domain ($M1_1$ or $M2_2$).

2.2.1. Rocker Swing Angle of the (C-R) Mechanism

Figure 2 shows the swing angle (φ_4) of the output link (R_4) as an angle of oscillation between its angular position's limitations, which depends upon the mechanism's application. The rocker swing angle (φ_4) can be obtained as presented in [6] as follows;

$$\varphi_4 = \theta_{4f} - \theta_{4i} \quad (19)$$

Where θ_{4i} and θ_{4f} are the initial and finial angular positions of output rocker link (R_4). Values of θ_{4i} and θ_{4f} can be computed as follows;

$$\theta_{4i} = 180^\circ + \theta_1 - \cos^{-1}\left(\frac{R_1^{\,2} + R_4^{\,2} - (R_3 + R_2)^2}{2R_1R_4}\right) \quad (20)$$

$$\theta_{4f} = 180^\circ + \theta_1 - \cos^{-1}\left(\frac{R_1^{\,2} + R_4^{\,2} - (R_3 - R_2)^2}{2R_1R_4}\right) \quad (21)$$

2.2.2. Time Ratio of the (C-R) Mechanism

The time ratio (*TR*) between the forward and return angular strokes of the rocker link depends on the mechanism's lengths and the rotation's direction of the crank as shown in Fig. 2.

Time ratio becomes greater than one (*TR* >1), if the direction of the working (forward) stroke is the same as the rotation direction of the input link; where the value of deflection angle (δ) is positive. Otherwise, (*TR* <1) and (δ <0) if these directions are not the same as mentioned in [1, 6, 7]. The time ratio can be formulated as follows;

$$TR = (180^\circ + \delta)/(180^\circ - \delta) \quad (22)$$

Where, (δ) is called the deflection angle which can be formulated as follows;

$$\delta = \theta_{3f} - \theta_{3i} \quad (23)$$

Where, (θ_{3i}) and (θ_{3f}) are the angular positions of the coupler link at initial and final positions as shown in Fig. 2. These angular positions (θ_{3i}) and (θ_{3f}) can be formulated as follows;

$$\theta_{3i} = \cos^{-1}\left(\frac{R_1^{\,2} - R_4^{\,2} + (R_3 + R_2)^2}{2R_1(R_3 + R_2)}\right) + \theta_1 \quad (24)$$

$$\theta_{3f} = \cos^{-1}\left(\frac{R_1^{\,2} - R_4^{\,2} + (R_3 - R_2)^2}{2R_1(R_3 - R_2)}\right) + \theta_1 \quad (25)$$

2.3. Design Equality Constraints of the Synthesized (C-R) Mechanism Lengths

Using the previous analysis and some mathematical manipulation, the general constraint equations can be deduced as follows;

$$TR = \frac{180^\circ + \left(\varphi_4 - (\gamma_f - \gamma_i)\right)}{180^\circ - \left(\varphi_4 - (\gamma_f - \gamma_i)\right)} \quad (26)$$

$$\varphi_4 = \gamma_f - \gamma_i + \delta \quad (27)$$

$$\gamma_{\min} + \gamma_{\max} = 180^\circ - \lambda_d \quad (28)$$

$$\gamma_i + \gamma_f = 180^\circ - \lambda_{if} \quad (29)$$

$$S_s = \frac{2R_1R_2}{R_3R_4} \quad (30)$$

$$S_d = \frac{R_3^{\,2} + R_4^{\,2} - R_1^{\,2} - R_2^{\,2}}{R_3R_4} \quad (31)$$

The previous six deduced equality equations may be called the mechanism's characteristics, where;

$S_S = \sin\Delta_1 + \sin\Delta_2 = 2\sin(0.5\lambda_s)\,.\cos(0.5\lambda_d)$

$S_d = \sin\Delta_1 - \sin\Delta_2 = 2\cos(0.5\lambda_s)\,.\sin(0.5\lambda_d)$

$\lambda_d = \Delta_1 - \Delta_2$, $\lambda_s = \Delta_1 + \Delta_2$, $\lambda_{if} = \Delta_i - \Delta_f$, $\theta_1 = 0$

Equations (24) and (25) can be rewritten using some mathematical manipulation as follows;

$$\theta_{3i} = \cos^{-1}\left(\frac{2(R_3 + R_2) - R_4S_d}{2R_1 + R_4S_s}\right) = \cos^{-1}(c + c_i) \quad (32)$$

$$\theta_{3f} = \cos^{-1}\left(\frac{2(R_3 - R_2) - R_4S_d}{2R_1 - R_4S_s}\right) = \cos^{-1}(c + c_f) \quad (33)$$

Where; $c = \dfrac{R_3}{R_1}$, $c_i = \left(\dfrac{R_1^{\,2} - R_4^{\,2}}{2R_1(R_3 + R_2)}\right) - \left(\dfrac{R_3 - R_2}{2R_1}\right)$

and $c_f = \left(\dfrac{R_1^{\,2} - R_4^{\,2}}{2R_1(R_3 - R_2)}\right) - \left(\dfrac{R_3 + R_2}{2R_1}\right)$

So; $\theta_{4i} = \gamma_i + \theta_{3i}$, $\theta_{4f} = \gamma_f + \theta_{3f}$ (34)

The previous six deduced Eqs. (26-31) can be used as general equations for any case study or task of transmission angles deviations as follow;

2.3.1. The First Case Study: ($\Delta_1 = \Delta_2 = \Delta$)

For the first case, the previous six deduced Eqs. (26-31) can be rewritten after substituting the deviations $\Delta_1 = \Delta_2 = \Delta$ as follows;

$TR = 1$, as in [1, 3, 5, 6] (35)

$$\varphi_4 = 2(90^\circ - \gamma_i) = \gamma_f - \gamma_i \quad (36)$$

$\gamma_{\min} + \gamma_{\max} = 180^\circ$, as in [3, 6, 23] (37)

$$\gamma_i + \gamma_f = 180^\circ \quad (38)$$

$\sin\Delta = \dfrac{R_1R_2}{R_3R_4}$, i.e. $\cos\gamma_{\substack{\min\\ \max}} = \pm\dfrac{R_1R_2}{R_3R_4}$, as in [3] (39)

$R_3^{\,2} + R_4^{\,2} - R_1^{\,2} - R_2^{\,2} = 0$, as in [1-3] and [7, 23] (40)

Hence, these pervious equations imply the following necessary and sufficient conditions that must be verified as follows;

$$\theta_{3i} = \theta_{3f} = \cos^{-1}(R_3 / R_1) = \cos^{-1}(c) \quad (41)$$

Where; ($\Delta_i = \Delta_f$) and (λ_d ,λ_{if} , S_d , c_f , c_i, δ) are zeros values.

2.3.2. The Second Case Study: ($\Delta_1 > \Delta_2$)

For the second case, the six deduced Eqs. (26-31) can be rewritten after substituting the deviations $\Delta_1 = \Delta$ and $\Delta_1 > \Delta_2$ as follows;

$TR = \dfrac{180^\circ + \varphi_4 - \Delta_f - \Delta_i}{180^\circ - \varphi_4 + \Delta_f + \Delta_i}$, where, $(TR > 1)$ (42)

$$\varphi_4 = (\gamma_f - \gamma_i + \delta) > \gamma_f - \gamma_i \quad (43)$$

$$\gamma_{\min} + \gamma_{\max} = (180^\circ - \lambda_d) < 180^\circ \quad (44)$$

$$\gamma_i + \gamma_f = (180^\circ - \lambda_{if}) < 180^\circ \quad (45)$$

$$\cos\gamma_{\min} - \cos\gamma_{\max} = \frac{2R_1R_2}{R_3R_4} \quad (46)$$

$$R_3^{\,2} + R_4^{\,2} - R_1^{\,2} - R_2^{\,2} = (R_3R_4).(\cos\gamma_{\min} + \cos\gamma_{\max}) \quad (47)$$

The conditions $(R_3^{\,2} + R_4^{\,2} - R_1^{\,2} - R_2^{\,2}) > 0$ and $(TR > 1)$ are mentioned in [1] and [14]. Hence, these pervious equations imply the following necessary and sufficient conditions that must be verified as follows;

$$\theta_{3i} = \cos^{-1}(c + c_i) \ , \quad \theta_{3f} = \cos^{-1}(c + c_f) \quad (48)$$

Where; $\Delta_i > \Delta_f$, $c_f < c_i$, $\theta_{3f} > \theta_{3i}$ and (λ_d , λ_{if} , S_d, δ) are positive definite values.

2.3.3. The Third Case Study: ($\Delta_1 < \Delta_2$)

For the third case, the six deduced Eqs. (26-31) can be rewritten after substituting the deviations $\Delta_2 = \Delta$ and $\Delta_1 < \Delta_2$ as follows;

$TR = \dfrac{180^\circ + \varphi_4 - \Delta_f - \Delta_i}{180^\circ - \varphi_4 + \Delta_f + \Delta_i}$, where, $(TR < 1)$ (49)

$$\varphi_4 = (\gamma_f - \gamma_i + \delta) < \gamma_f - \gamma_i \quad (50)$$

$$\gamma_{\min} + \gamma_{\max} = (180^\circ - \lambda_d) > 180^\circ \quad (51)$$

$$\gamma_i + \gamma_f = (180^\circ - \lambda_{if}) > 180^\circ \quad (52)$$

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The conditions $(TR < 1)$ and $(R_3^2 + R_4^2 - R_1^2 - R_2^2) < 0$ are mentioned in [1] and [14]. Hence, these pervious equations imply the following necessary and sufficient conditions that must be verified Eqs. (46-48).

Where; $\Delta_i < \Delta_f$, $c_f > c_i$, $\theta_{3f} < \theta_{3i}$ and (λ_d , λ_{if} , S_d, δ) are negative definite values.

2.4. Graphical Synthesis Methodology

The graphical methods have been widely adopted in several fields such as mechanisms design and control for their simplicity and competency [32-34]. In this paper, the suggested graphical methodology can be easily and rapidly conducted according to six sequential steps as follows;

The first step is the identification of the synthesis case study (task) either by identifying the class of (C-R) mechanism or the desired transmission angle deviations through giving the desired values of Δ_1 and Δ_2. Hence, the minimum and maximum transmission angles (γ_{min} and γ_{max}) can be directly calculated.

The second step is assuming the initial angular position (θ_{3i}) of the coupler link (R_3), where (θ_{3i}) can be proportionally assumed within the values (30^0, 30.8^0, 34^0 and 37.5^0), which are respectively corresponding to the values (85^0, 75^0, 60^0 and 45^0) of (γ_{min}) for the first case ($\Delta_1 = \Delta_2$). Also, (θ_{3i}) can be assumed within the values (30^0, 32.8^0, 35.8^0 and 37.4^0), which are corresponding to the same previous values of (γ_{min}) for the second case ($\Delta_1 > \Delta_2$). Moreover, (θ_{3i}) can be proportionally assumed within the values (29^0, 28^0, 27.7^0 and 37.5^0), which are corresponding to the values (100^0, 105^0, 120^0 and 135^0) of (γ_{max}) for the third case ($\Delta_1 < \Delta_2$).

Assuming a value of (λ_{in}) is the third step. Where, (λ_{in}) is the difference between initial and minimum transmission angles (γ_i , γ_{min}), hence (γ_i) can be computed. Value of (λ_{in}) can be assumed like the previous step using the values (0.7^0, 2.2^0, 5.5^0 and10.9^0), (0.6^0, 1.6^0, 3.6^0 and 6.3^0) and (0.9^0, 1.2^0, 1.9^0 and 4.7^0) for the three cases, respectively.

The fourth step is assuming a value of (λ_{if}), where (λ_{if}) is the difference between final and maximum transmission angles (γ_f , γ_{max}), hence (γ_f) can be calculated. Value of (λ_{if}) can be assumed like the second step using the values (0.7^0, 2.2^0, 5.5^0 and 10.9^0), (0.6^0, 1.7^0, 4.1^0 and 8.5^0) and (0.9^0, 1.1^0, 1.7^0 and 3.4^0) for the three cases, respectively.

The fifth step is drawing the Cartesian coordinate XO_2Y. Thus, the xO_2y axis can be drawn by rotating XO_2Y with the angle (θ_1), as shown in Fig. 5. Locate the point (O_4) on the line (O_2x), where (O_2O_4) is a unit length represents the mechanism fixed link. Hence, the first or initial construction line "ICL" (ICL=$O_2A_iB_i$) can be drawn from point (O_2) with an inclination angle (θ_{3i}) with respect to the direction of the line (O_2O_4). Also, the line (O_4B_i) can be drawn from point (O_4) with an inclination angle ($\theta_{3i}+\gamma_i$) with respect to the direction of the line (O_2x) to intersect the direction of the line (ICL) in the point (B_i). Therefore, the length of (O_4B_i) represents (r_4). Also, the length of (O_2B_i) represents ($r_3 + r_2$). Where, the mechanism links proportions (r_2, r_3 and r_4) are based on the length of R_1 as; $r_2 = R_2/R_1$, $r_3 = R_3/R_1$ and $r_4 = R_4/R_1$.

The last step is the drawing of an arc with a radius (O_4B_i) from the center (O_4). Hence, two lines (O_4B_f) and ($B_f\ O_2$) can be drawn using the arc points with keeping the angle between these two lines equals to (γ_f).

Where, the line (O_2B_f) can be considered as the final construction line (FCL). Therefore, the length of (O_2B_f) represents (r_3 - r_2).

Finally, (r_3) equals to half of ($O_2B_i+O_2B_f$). Also, (r_2) equals to (O_2B_i - r_3). Hence, the desired given data (Δ_1 and Δ_2) can be checked via the obtained mechanism's ratios (r_2, r_3 and r_4).

If these mechanism's ratios satisfy both the desired Δ_1 and Δ_2, the six design constraint equations of the mechanism can be validated. Also, the position of the point (B_f) related to the line ICL can denote to the mechanism class number. Where, point (B_f) lies on ICL for the first case study of mechanism synthesis which can be named by class I, i.e.; inline or unit time ratio optimal mechanisms. While, the point (B_f) lies over ICL for the second case study of mechanism synthesis (class II mechanisms), while (B_f) lies down ICL for the third case study (class III mechanisms).

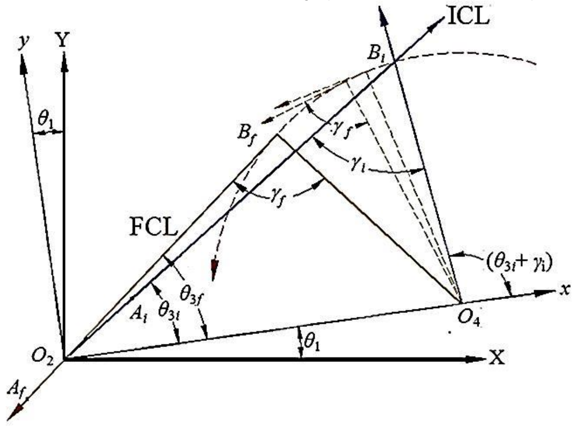

Figure 5: Graphical synthesis methodology

3. Results and Discussions

The presented analytical methodology can be used for achieving the targeted transmission angle deviations via the corresponding optimal mechanism links' lengths. These optimal links' lengths can be presented as lengths' proportions (r_2, r_3 and r_4) to facilitate the selection process for the proper mechanisms, which depends on a working area of various applications.

The optimal synthesized results concerned with the presented three case studies in the following sections.

3.1. Results of the First Case Study: ($\Delta_1 = \Delta_2 = \Delta$)

The optimal synthesized (C-R) mechanism's proportions in addition to the other important parameters of this case study are illustrated in Fig. 6 and Fig. 7. These are dealing with the equality deviations ($\Delta_1 = \Delta_2 = \Delta$) which increase from 5° to 60°.

The obtained results reveal the following significant observations;

Optimal mechanism's proportions r_2, r_4 of the type $M2_2$: $r_2 < r_4 < r_3 < 1$ increase as shown in Fig. 6 and the tabulated results in Table 2.

Obviously, the sum of the values ($\gamma_{min} + \gamma_{max}$) equals to (180°) and ($\gamma_i + \gamma_f$) equals to (180°). Also, $\Delta_i = \Delta_f$ where Δ_i increases from 4.3° to 45°.

Moreover, the deflection angle (δ) is zero i.e. $\theta_{3f} = \theta_{3i} = \theta_{2i}$ where θ_{3f} increases from 30° till 35.4°. These results indicate that $c_f = c_i = 0$ and $c = r_3$ for any mechanism's proportions in addition to $\theta_{3f} = \theta_{3i} = \theta_{2i} = \cos^{-1}(c)$ that verify the Eq. (41), hence, $TR = 1$.

Also, the output angular stroke ($\varphi_4 = \gamma_f - \gamma_i$) increases from 8.6° till 89.9° and the difference (λ_{in}) increases from 0.67° till 11.75° as (Δ) increases from 5° to 50°.

All these obtained results of the first case study concur with the six deduced design equality constraints in Eqs. (35-40), which may fall inside the required feasible design domain (F.D.D). Otherwise, other results that may appear through increasing (Δ) greater than 50° lies within the unfeasible design domain (U.F.D.D). This is due to the jamming and/or locking problems.

All the optimal synthesized results of (C-R) mechanism satisfying the conditions of this case study can be denoted by mechanisms of class I. Many of the previous literature as [1-3] and [7] are dealing with this kind of mechanisms which can be called zero deflection angles, zero mean, central, inline and unit time ratio.

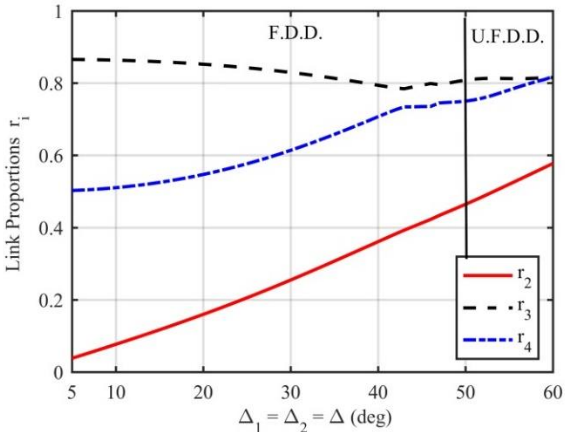

Figure 6: Optimal results of (C-R) mechanism's proportions of the first case study

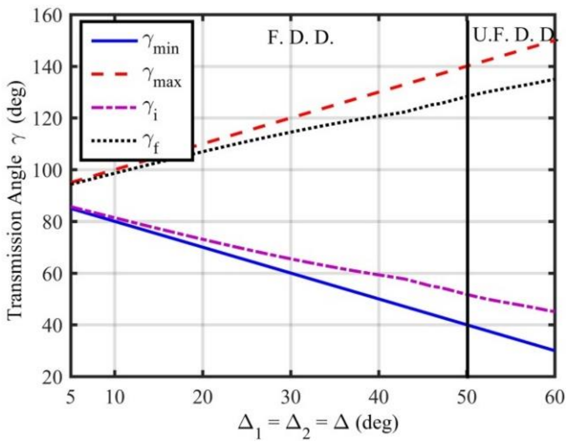

Figure 7: Transmission angles of optimal (C-R) mechanism of the first case study

3.2. Results of the Second Case Study: ($\Delta_1 > \Delta_2$, $\Delta_1 = \Delta$)

The results of the optimal synthesized mechanism's proportions, in addition to the other important parameters of this case study are illustrated in Figs. 8 in addition to Fig. 9. These are dealing with the first deviation (Δ_1=Δ) which increases from 6° to 60° while the second deviation (Δ_2) is kept at a fixed value of 5°.

The obtained results reveal the following important notes;

The optimal mechanism's proportions increase as shown in Fig. 8 in addition to the tabulated results in Table 3. Clearly, the sum of both values ($\gamma_{min} + \gamma_{max}$) and ($\gamma_i + \gamma_f$) are less than (180°). Furthermore, the value of (Δ_i) is greater than the value of (Δ_f) where Δ_i increases from (5.2°) to (49.3°) and Δ_f decreases from (4.2°) to (−13.5°).

Furthermore, the value of the deflection angle (δ) increases from 0.05° to 31.6°, i.e. $\theta_{3f} > \theta_{3i}$ where θ_{3f} increases from 30.4° till 70.9°. Also, θ_{3i} increases from 30.35° till 39.32°. These results indicate that $c_f < c_i$ and $c = r_3$ in addition to $\theta_{3f} = \cos^{-1}(c + c_f)$ as well as $\theta_{3i} = \cos^{-1}(c + c_i)$ which verifies the Eq. (48) for any mechanism's proportions. Besides, the output angular stroke $\varphi_4 = \gamma_f - \gamma_i + \delta$ increases from 9.5° till 67.36° and the time ratio range is $1 < TR \le 1.42$.

The obtained results of this case study concur with the six deduced design equality constraints in Eqs. (42-47) of M2$_2$: $r_2 < r_4 < r_3 < 1$ and may be to lie inside the required feasible design domain (F.D.D).

The optimal synthesized results of (C-R) mechanism satisfying this case's conditions can be named by mechanisms of class II. Some of the published literature are concerned with these kinds of mechanisms, which can be called positive off-central, positive off-line, more than unity time ratio and positive deflection angle mechanisms.

Table 2. Calculated results of the first case study

Mech. No.	Δ°	γ°_{min}	γ°_{max}	r_2	r_3	r_4	G_f	θ°_{3f}
1	5	85	95	0.0379	0.8654	0.5025	0.318	30.072
2	15	75	105	0.1167	0.8591	0.5249	0.239	30.783
3	30	60	120	0.2546	0.8295	0.6138	0.150	33.951
4	45	45	135	0.4125	0.7937	0.7350	0.082	37.469
5	60	30	150	0.5773	0.8154	0.8176	0.035	35.377
Mech. No.	θ°_{3i}	δ°	TR	φ°_4	γ°_i	γ°_f	Δ°_i	Δ°_f
1	30.072	0.0	1.0	8.651	85.674	94.326	4.326	4.326
2	30.783	0.0	1.0	25.695	77.153	102.848	12.847	12.847
3	33.951	0.0	1.0	49.008	65.496	114.504	24.504	24.504
4	37.469	0.0	1.0	68.281	55.860	124.140	34.140	34.140
5	35.377	0.0	1.0	89.840	45.080	134.920	44.920	44.920

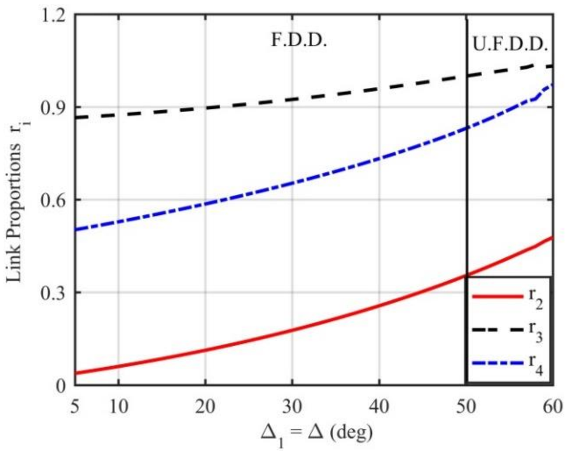

Figure 8: Optimal results of (C-R) mechanism's proportions of the second case study

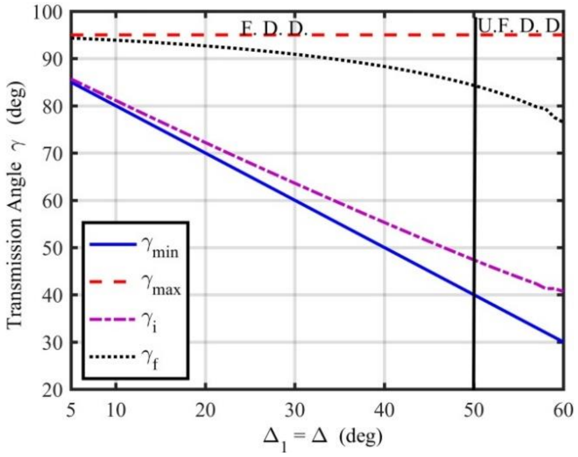

Figure 9: Transmission angles of optimal (C-R) mechanism of the second case study

Table 3. Calculated results of the second case study

Mech. No.	Δ°	γ°_{min}	γ°_{max}	r_2	r_3	r_4	G_f	θ°_{3f}
1	10	80	95	0.0602	0.8743	0.5284	0.323	31.813
2	15	75	95	0.0851	0.8846	0.5561	0.328	33.721
3	30	60	95	0.1772	0.9241	0.6532	0.340	40.773
4	45	45	95	0.3026	0.9786	0.7786	0.349	51.007
5	60	30	95	0.4781	1.0322	0.9718	0.306	70.898
Mech. No.	θ°_{3i}	δ°	TR	φ°_4	γ°_i	γ°_f	Δ°_i	Δ°_f
1	31.467	0.346	1.004	13.106	81.114	93.874	8.886	3.874
2	32.753	0.969	1.011	17.671	76.626	93.328	13.374	3.328
3	35.808	4.965	1.057	32.268	63.609	90.911	26.392	0.911
4	37.413	13.593	1.163	48.861	51.288	86.555	38.712	-3.455
5	39.327	31.571	1.425	67.366	40.703	76.498	49.297	-13.50

3.3. Results of the Third Case Study: ($\Delta_1 < \Delta_2$, $\Delta_2 = \Delta$)

The optimal results of synthesized mechanism's proportions in addition to the other essential parameters of this case study are illustrated in Fig. 10 in addition to Fig. 11. These results are dealing with the second deviation ($\Delta_2 = \Delta$) which increases from 6° to 60° while the first deviation (Δ_1) is kept at a fixed value of 5°.

The obtained results reveal the following significant observations;

The optimal results of mechanism's proportions concerning with the mechanism's type $M2_2$: $r_2<r_4<r_3<1$ and type $M1_1$: $r_2<r_3<r_4<1$ are shown in Fig. 10 and the tabulated results in Table 4.

Obviously, the sum of both values ($\gamma_{min} + \gamma_{max}$) and ($\gamma_i + \gamma_f$) are greater than (180°). Moreover, the value of (Δ_i) less than the value (Δ_f), where Δ_i decreases from (4.27°) to (−2.5°) and Δ_f increases from (5.7°) till (55.99°). Furthermore, the value of the deflection angle (δ) changes from (−0.05°) to (−20.9°), i.e. $\theta_{3f} < \theta_{3i}$. These results indicate that $c_f > c_i$ and $c = r_3$ in addition to $\theta_{3f} = \cos^{-1}(c + c_f)$ as well as $\theta_{3i} = \cos^{-1}(c + c_i)$ which verifies Eq. (55) for any mechanism's proportions. Moreover, the output angular stroke ($\varphi_4 = \gamma_f - \gamma_i + \delta$) increases from (9.5°) till (32.52°) and the time ratio relation is $1 > TR > 0.79$.

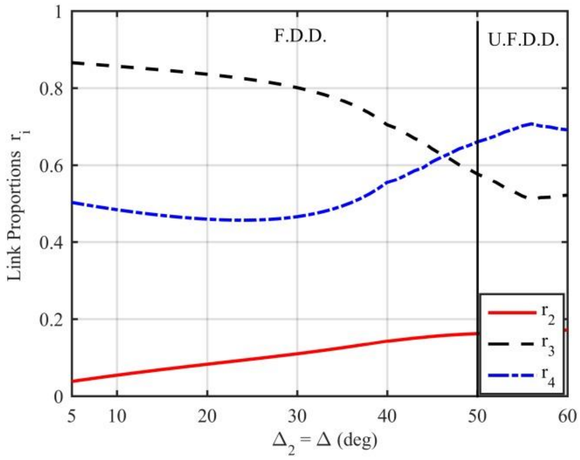

Figure 10: Optimal results of (C-R) mechanism's proportions of the third case study

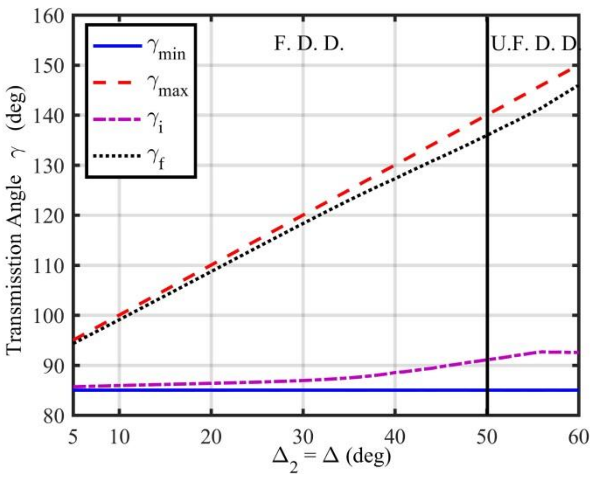

Figure 11: Transmission angles of optimal (C-R) mechanism of the third case study

All of these obtained optimal results of (C-R) mechanism concur with the six deduced design equality constraints in Eqs. (46-47) in addition to Eqs. (49-52), through considering the mechanism's proportions of $M2_2$ and $M1_1$, which are recommended to lie inside the required feasible design domain (F.D.D)

Mechanisms satisfying this case's conditions can be called mechanisms of class III. Some of previous literature are concerned with these kinds of mechanisms, which can be called negative off-central, negative off-line, less than unity time ratio and negative deflection angle mechanisms.

Table 4. Calculated results of the third case study

Mech. No.	Δ°	$\gamma^\circ_{\min}$	$\gamma^\circ_{\max}$	r_2	r_3	r_4	G_f	θ°_{3f}
1	10	85	100	0.0540	0.8562	0.4837	0.271	28.531
2	15	85	105	0.0687	0.8466	0.4689	0.231	27.079
3	30	85	120	0.1095	0.8008	0.4656	0.142	24.202
4	45	85	135	0.1551	0.6417	0.6084	0.082	27.076
5	60	85	150	0.1716	0.5212	0.6909	0.035	22.729
Mech. No.	θ°_{3i}	δ°	TR	φ°_4	γ°_i	γ°_f	Δ°_i	Δ°_f
1	28.847	-0.316	0.966	12.834	85.930	99.080	4.070	9.080
2	27.894	-0.816	0.991	16.907	86.155	103.878	3.845	13.878
3	27.708	-3.506	0.962	27.867	86.934	118.307	3.066	28.307
4	37.476	-10.40	0.891	31.472	89.703	131.574	0.297	41.574
5	43.648	-20.92	0.792	32.525	92.555	145.999	-2.555	55.999

The obtained results of each case concerning with the time ratio (T_r) are shown in Fig. 12. On the other hand, the results of each case dealing with the swing angle (φ_4) of rocker link are illustrated in Fig. 13. Clearly, the swing angle (φ_4) has the highest increasing rate with the first case study which falls inside the required feasible design domain (F.D.D) compared with other cases, where φ_4 increases till 76.4° for the first case, 55° for the second one and till 30.9° for the third one.

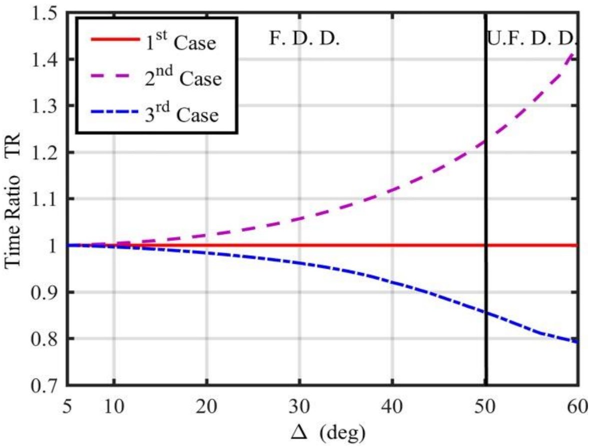

Figure 12: Time ratio of optimal (C-R) mechanism for three cases

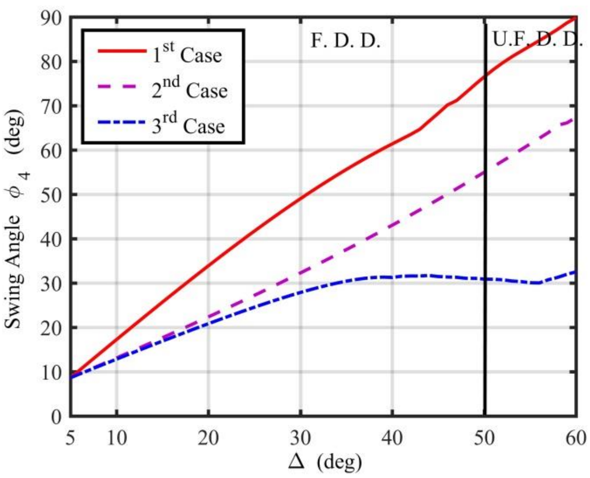

Figure 13: Rocker swing angle of optimal (C-R) mechanism of the three cases.

4. Validating the Optimal Results

Clearly, the obtained results facilitate the designer's work through selecting the appropriate mechanism's proportions for achieving the design requirements. It is very important to validate the calculated optimal synthesized (C-R) mechanism's proportions which satisfy the six design constraint equations. The optimal results are validated through comparisons with those of some earlier researches as [1, 2, 3, 6] and [14-16] which are tabulated in Table 5.

Table 5. Results validation through comparisons with earlier researches

Conditions	The Three Cases		
	$\Delta_1=\Delta_2$	$\Delta_1>\Delta_2$	$\Delta_1<\Delta_2$
$\gamma_{min}+\gamma_{max}=\pi-\lambda_d$	$=\pi$, as [3], [6], [15], [16]	$<\pi$	$>\pi$
$\gamma_i+\gamma_f=\pi-\lambda_{if}$	$=\pi$	$<\pi$	$>\pi$
$S_s=(2R_1R_2)/(R_3R_4)$	[16]		
$R_3^2+R_4^2-R_1^2-R_2^2=(R_3R_3)S_d$	[1], [2], [3], [15], [16]	[1], [14]	[1], [14]
$TR=(\pi+\delta)/(\pi-\delta)$	=1, as [1], [3], [6], [15], [16]	>1, as [1], [14]	<1, as [1], [14]

4.1 Solved Examples Using Graphical Synthesis Methodology

First example is dealing with given data; ($\Delta_1=\Delta_2=\Delta=30^\circ$, i.e., $\gamma_{min}=60^\circ$ and $\gamma_{max}=120^\circ$) of the first class I mechanism. This graphical synthesis methodology can be used to construct (C-R) mechanism's lengths via these given data through applying the sequential graphical steps. The second step is assuming the initial angular position ($\theta_{3i}=34^o$) of the link (R_3) related to the value of (γ_{min}). Also, the third and fourth steps are assuming values of ($\lambda_{in}=\lambda_{if}=5.5^o$) which are similar to the second step. The fifth step is drawing xO_2y axis which coincides with XO_2Y axis where ($\theta_1=0^o$), as shown in Fig. 14.

Locate the point (O_4) on the line (O_2x), where (O_2O_4) is a unit length. Hence, the first line ($O_2A_iB_i$) can be drawn from point (O_2) with inclination angle ($\theta_{3i}=34^o$) with respect to the direction of the line (O_2O_4). Also, the line (O_4B_i) can be drawn from point (O_4) with inclination angle ($\theta_{3i}+\gamma_i=99.5^o$) with respect to direction of line (O_2x) to intersect the direction of (O_2B_i) in (B_i). Therefore, the length of ($O_4B_i=0.615$) represents (r_4). Also, the length of (O_2B_i) represents ($r_3+r_2=1.082$).

Obtained optimal (C-R) mechanism's links ratios using the pervious steps are; $r_2=0.254$, $r_3=0.829$ and $r_4=0.615$ which, satisfy the six deduced design constraint equations. The mechanism's characteristics are approximately equal to: $\gamma_{\min}=60^o$, $\gamma_{\max}=119.9^o$, $\gamma_i=65.5^o$, $\gamma_f=114.5^o$, $\varphi_4=48.9^o$ and $TR\approx1.0$, which satisfy the desired design requirements.

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Also, a second example dealing with the class II of mechanism synthesis can obtain mechanism's links ratios via this graphical method using given data; (Δ_1=Δ= 25°, i.e., γ_{min}=65° and γ_{max}=95°) as shown in Fig. 15. Hence, θ_{3i}, λ_{in} and λ_{if} can be proportionally assumed as; (θ_{3i}=34.9°, λ_{in}=3° and λ_{if}=3.4°).

The mechanism's ratios using these data are; r_2=0.144, r_3=0.907 and r_4=0.614. The mechanism's characteristics are approximately equal to: $\gamma_{\min}$=65.2°, $\gamma_{\max}$=95.7°, γ_i=68°, γ_f =91.6°, φ_4=27.5° and TR ≈1.02, which satisfy the desired design requirements.

Moreover, this method can be used as a third example for synthesizing the mechanism of the third class III using given data; (Δ_2=Δ=25°, i.e., γ_{min}=85° and γ_{max}=115°) as shown in Fig. 16. Hence, θ_{3i}, λ_{in} and λ_{if} can be proportionally assumed as; (θ_{3i}=27.7°, λ_{in}=1.7° and λ_{if}=1.5°).

The mechanism's ratios using these data are; r_2=0.1, r_3= 0.818 and r_4=0.463. The mechanism's characteristics are approximately equal to: $\gamma_{\min}$=84.4°, $\gamma_{\max}$=115.6°, γ_i=86.7°, γ_f =113.5°, φ_4=25.3° and TR ≈0.93, which satisfy the desired design requirements.

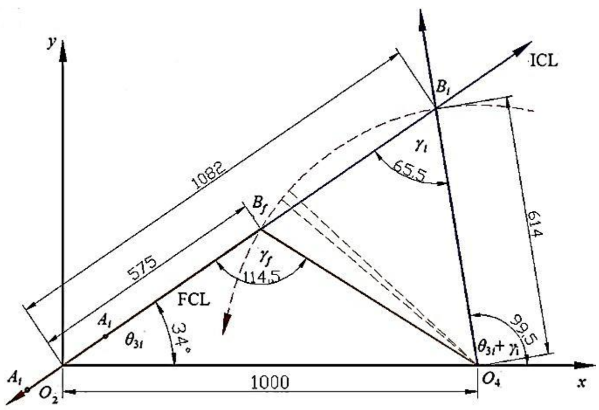

Figure 14: Graphical synthesis methodology for first case

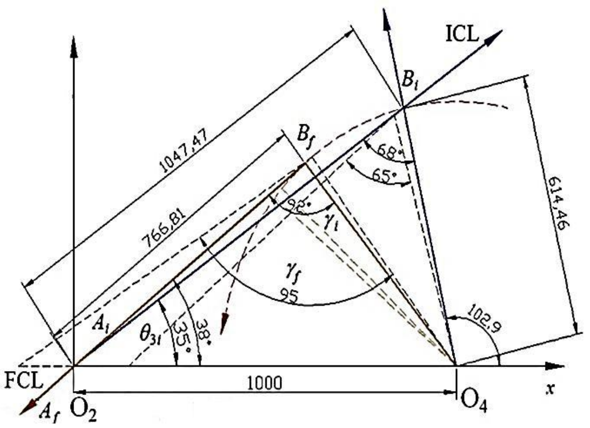

Figure 15: Graphical synthesis methodology for second case

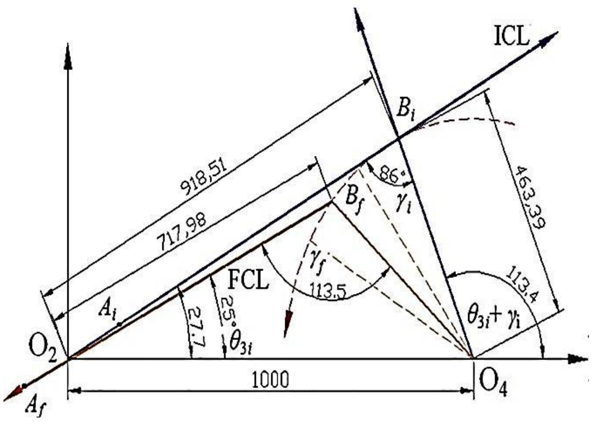

Figure 16: Graphical synthesis methodology for third case

5. Conclusion

This work proposed a detailed analytical methodology in addition to a fast-graphical methodology to optimally synthesize lengths' proportions of planar crank-rocker mechanism in order to accomplish targeted design with a definite transmission angle deviation. The analytical methodology deals with deducing six design equality constraint equations that satisfy three case studies. The discussion of the presented results reveals that the optimal synthesized (C-R) mechanisms are classified into three classes according to the three case studies for achieving targeted definite transmission angle deviations. The direct relation between the mini-max transmission angle deviations and the (C-R) mechanisms classes in addition to their six performance parameters are be presented.

If and only if the (C-R) mechanism's lengths verify the desired case conditions, the six deduced design constraint equality equations can be verified. The obtained optimal results using the presented methodology are concurring with those introduced in the previous literature using different approaches.

On the other hand, the suggested graphical synthesis methodology can be carried out to directly construct such optimal (C-R) mechanism's lengths. This graphical method is based on only choosing the design case related to the selected class of (C-R) mechanism beside the desired transmission angle deviations through giving the minimum and the maximum transmission angles in order to achieve an optimal synthesized crank-rocker mechanism's lengths.

The optimal charts are introduced to directly obtain the optimal (C-R) mechanism's lengths, which are achieving the targeted transmission angles deviations. Therefore, the designer can easily select optimal synthesized crank-rocker mechanisms' lengths which can be employed in several industrial applications. These applications may include using (C-R) mechanisms associated with a desired equal deviation of mini-max transmission angle for achieving vibrating motions in sieve conveyors. Also, these kinds of mechanisms with time ratio greater than one can be used for generating a required quick-return motion for shaper machines and the mechanisms with time ratio less than one can be used for generating a positive sliding stage of a conveyed mechanism and increasing the conveying capacity.

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