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Design of a Vendor Managed Inventory Model for Impulse Purchase Products in a Two-level Supply Chain

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Abstract

Although there are multiple methodologies to carry out collaborative practices of inventory management, none are set up for impulse purchase products. This is a disadvantage because with the opening of new markets and the proliferation of consumer culture, the economic importance of buying products on impulse always remains relevant. In this paper, a Vendor Managed Inventory model was designed based on the direct participation of a vendor and a buyer (two-level supply chain), in order to agree on the procurement operations of a portfolio of impulse purchase products. For this proposal, a mathematical model based on classical optimization was designed to minimize inventory costs. Subsequently, a case study was conducted comparing the economic impact of the model with respect to a traditional supply agreement in a non-cooperative scenario. The results reflected positive economic effects in the implementation of the model related to the economies of scale to exploit fixed costs present in the agreement. Additionally, the conditions under which the implementation of this model grants individual and global benefits to the participating companies were validated.

Keywords: Vendor Managed Inventory, Supply chain management, Classical optimization, Impulse purchase products, Logistics;

1. Introduction

Proactive management and timely distribution in an organization's supply chain usually translate into greater savings and other benefits in its operational processes. Various inventory management and order preparation and delivery methodologies - e.g., JIT (Just-in-Time), ECR (Efficient Customer Response), or VMI (Vendor Managed Inventory)- have been proven to increase supply chain competitiveness through cost reduction. Foremost among these practices, the VMI often translates into a win-win situation for both parties: buyers save on storage costs by not having to allocate labor and space in managing overstocked inventories, and vendors save on distribution costs by coordinating shipments to different buyers [1], [2]. Therefore, once an integrated planning and inventory collaboration is established, buyers using a VMI supply chain are able to work together efficiently with their suppliers to optimize inventory replenishment [3]–[5].

The Council of Supply Chain Management Professionals (CSCMP) defines VMI as the practice of companies making suppliers responsible for determining order size and timing, usually based on receipt of retail point of sale (POS) and inventory data from their customer [6]. The evolution of VMI research has been directed towards an interdisciplinary environment where not only are the impacts on inventory policies quantified but also specialized models are designed for different types of products or business sectors, along with the possibility of including risk restrictions or preferences for each of the members of the agreement. These designs have used a wide range of technical tools. Consequently, the selection of an appropriate approach depends on the objectives set in a research project and the availability of necessary data and resources. The background is explored further in the literature review section.

Related to the above paragraph, although there are multiple methodologies to carry out this practice, none are set up for impulse purchase products. This is a disadvantage because with the opening of new markets and the proliferation of consumer culture, the economic importance of buying products on impulse always remains relevant [7]–[9]. This kind of merchandise can be defined as those products that a consumer acquires suddenly and immediately without a plan prior to purchase [10]. Impulse buying behavior has been described as a novelty or escape purchase that breaks the normal buying pattern[11]. Generally, these items are strategically displayed in hot spots (areas with a large circulation of people), such as near checkouts in retail stores. Along these lines, the previous research that has been carried out does not take into account the particularities of these products, leading to arbitrary or generalized models that are used for the management of the collaborative inventory of these goods.

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Therefore, the purpose of this project was to design a VMI inventory model for a two-level supply chain, which would represent an adequate logistics operations scheme for a portfolio of impulse purchase products between a vendor and a buyer. The model design was based on a three-stage methodology: involving gathering information, formulation, and validation of the model. For this purpose, a product characterization was carried out, collecting data, and diagnosing the key elements that this model should have. Subsequently, it was mathematically formulated using classical or unconstrained optimization; the choice of this technique among different alternatives was supported by an expert judgment. At last, in the validation stage, which is defined as a test stage where the model is executed to evaluate its response with respect to a real scenario; A case study was carried out with data from a retail company (buyer) and a sugar confectionery vendor, in order to evaluate the results for eight impulse purchase products in a period of five months.

In this way, this research contributes to the literature on supply chain management of impulse purchasing products and designs a VMI model for this purpose. This article is organized as follows: The related literature is reviewed in Section 2; Section 3 describes the methodology used in this model; Section 4 explains each of the stages of model design; Section 5 concludes this paper and give some recommendations for future works.

2. Literature review

From the review of past and recent literature, there were no studies related to VMI models focused on the management of impulse purchase product inventories. However, as for the study of variables related to the trend of impulse buying. Darrat et al. [12] and Badgaiyan et al.[7] studied impulsive buying tendency and validate it by examining its association with other relevant variables. Also, studies related to the role of the store in consumer decisions and psychology were consulted. Flamand et al.[13] studied the optimization of store-wide shelf-space allocation in order to maximize the visibility of products to consumers; Bellini et al.[10] explored the determinants of impulse buying in a context of more planning and preparation for shopping. In the same manner, Wei et al.[14] studied the relationship between flow experience, perceived transaction value, positive effect, shopping motivation, and impulse buying behavior. There are also authors, such as Leong et al.[15], Chen et al.[16], and Sundström et al.[17], who explored how online shopping affects impulsive buying behavior.

As for the design of VMI models, one of the most implemented techniques corresponds to mathematical modeling based on classical optimization. Lee et al.[18]examined VMI systems with stock out-cost sharing between a supplier and a customer using an EOQ model with shortages allowed under limited storage capacity. Additionally, Cai et al.[19] designed a two-echelon supply chain that markets two substitutable brands of a product with uncertain demand. Lee and Cho [3], examined (z, Z)-type contracts for VMI. A (z, Z) VMI contract stipulates minimum and maximum inventory levels and their corresponding under- and over-stocking penalties. Bai et al.[20] formulated an optimization model for the centralized system, in order to investigate the effects of carbon emission reduction on a supply chain with one manufacturer and two competing retailers for deteriorating products under VMI. Other authors proposed to integrate an optimization in transport costs or distribution routes. Rahim and Aghezzaf[21] optimized the inventory holding costs and the transportation costs for a two-stage supply chain. Similarly, Mateen and Chatterjee [22] developed analytical models for various approaches through which a single vendor-multiple retailer system may be coordinated through VMI. They also highlight the savings that can be derived in the transportation cost in a VMI setting. Stellingwerf et al.[1] quantified both the economic and environmental benefits of implementing cooperation via Joint Route Planning (JRP) and VMI, optimizing routing and inventory planning decisions simultaneously. Additionally, Saif-Eddine et al.[23] formulated a mathematical model to minimize the total supply chain cost considering the Inventory Location Routing Problem (ILRP) while adopting the VMI strategy.

Furthermore, it should be mentioned that linear and non-linear programming have played an important role in generating new research proposals for VMI in recent years. Park et al.[24] constructed a mixed-integer linear programming model for the vendor-managed inventory routing problem with lost sales, while maximizing the supply chain profit over a planning horizon. Other authors developed models based on non-linear programming (NLP). Hariga et al.[25] formulated a mixed integer nonlinear program that minimizes total supply chain costs and allows unequal shipment frequencies to the retailers. They considered a supply chain where a vendor manages its multiple retailers' stocks under a VMI contract. Diabat [26] addressed the issue of VMI by considering a two-echelon single vendor/multiple buyer supply chain network. The model finds the optimal sales quantity by maximizing profit, given as a nonlinear and non-convex objective function. In addition, Verma and Chatterjee [27] developed a nonlinear mixed-integer programming model to compute the optimal replenishment frequency and quantity for each of the retailer, such that the total system cost is minimized.

It is worth mentioning other approaches that may be considered. Sadeghi and Niaki[28] designed a bi-objective VMI model with a single vendor and multiple retailers, in which the demand is fuzzy, and the vendor manages the retailers' inventory in a central warehouse. Akbari Kaasgari et al.[29] formulated a VMI supply chain for perishable products by considering discount. Then, a genetic algorithm and a particle swarm optimization algorithm are developed for solving it. In the same way, Chen[30] considered a new decision issue for perishable products in production inventory with pricing and promotion for a single-vendor multi-buyer system comprising one manufacturer and multiple retailers. He developed a centralized decision model with VMI control system under a just-in-time shipment policy. Also, Filho et al.[31] presented a case study supported in the development of a system VMI attached to the philosophy of Customer Relationship Management whose goal was to map the buying behavior of customers who purchase low-volume products.

Finally, it is essential to mention the studies in the field of Game Theory. These add enormous value to the design of VMI models. Torres et al. [32] studied the evolution of individual strategies of the producer and the buyer by a formalism derived from the theory of evolutionary games. Tsao et al.[33] developed a multi-player retailer Stackelberg game to model the interaction between retailer and manufacturers. In this model, a retailer maximizes profit by taking the manufacturers' trade allowance response into account.Yang et al.[34] formulated joint configuration of a product catalogue and its supply chain as a leader-follower Stackelberg game that is enacted through a bi-level hierarchical optimization mechanism to model the coordination. Nishi and Yoshida [35] addressed the optimization of multi-period bilevel supply chains under demand uncertainty. The decentralized supply chain planning problem was modelled as a multi-period non-cooperative game. In a recent work, Chen and Xiao [36]developed game models for a two-echelon supply chain with one supplier and multiple competing retailers. They studied the pricing decision and the replenishment policy for each member.

3. Methodology

Three key stages were considered for the design of the model: information gathering, model formulation, and model validation. During the information gathering stage, the main logistic needs or requirements that companies have in relation to the inclusion of impulse purchase products within their VMI models were diagnosed. In addition, the nature of these products, the type of companies that commercialize them, and the ideal characteristics that the designed model should have were investigated. This first stage has, as its objective, analyzed the information that will determine the basic characteristics of the model and the industry to which it will be directed., based on current market trends. This stage included activities, such as the product characterization (Section 4.1) and the needs assessment (Section 4.2).

Subsequently, at the formulation stage, the model was designed using a technique that would allow for a better representation of the logistical operations of impulse purchase products. Relevant variables and assumptions were, therefore, established. This stage included activities, such as selecting the modeling technique (Section 4.3) and the mathematical formulation (section 4.4). Finally, the model was subjected to a validation stage. The objective of this stage was to evaluate the economic impact that would arise from its implementation. For this purpose, a real case study was carried out with the real data of a retail company and a supplier of sugar confectionery, to evaluate the results of the model for eight impulse purchase products in a period of five months. This stage included model validation (section 4.5). There is a more detailed explanation for each stage in the next section.

4. Model development

4.1. Product characterization

The Council of Supply Chain Management Professionals (CSCMP) defines product characterization as all of the elements that define a product's character, such as size, shape, weight, etc.[6] Therefore, a product characterization process can be defined as the establishment of the attributes for a given product. In this first design stage, impulse purchase products were characterized after various interactions with primary and secondary information sources. It is important to mention that two big retail companies supplied data from their product catalogs. Impulse buying products were characterized as products that cost little money, are quickly consumed, and require little time for purchasing decisions (i.e. chocolate bars, cookies, razors, chewing gum, candy, etc.). In addition, they are usually strategically located in the hot spots throughout a store [37], [38].

In terms of commercial and logistical characteristics, it was found that the main distribution channels for these products reproducers and wholesale distributors [39]. Given that the consumer sector generally requires greater intermediation to diversify the market. With regard to the marketing channel, the selection of the store is aimed at retailers that sell products directly to the customers, such as department stores, supermarkets, convenience stores, etc. Thus, although these products along the supply chain can be classified as retail or non-retail trade item, only those units that pass through the point of sale and are purchased by the final consumer can be denoted as impulse purchase products [40]. Consequently, the scope of this investigation takes into account those companies whose target market is represented by consumers who purchase these products through a retail channel. It is imperative to mention that other characteristics, such as types of packaging, transport, and storage management were also consulted.

Furthermore, companies generally resort to reducing dependence on forecasts and require increasing the frequency of delivery to reduce inventory shortage. Then, it is imperative to recognize that lead time analysis of this type of product over time establishes a specific analysis point for any particular SKU (Stock Keeping Unit). However, in general, these products have a short lead-time [41], since they are usually included in frequent orders, with a shorter, faster and less risky forecast horizon.

Finally, it was proposed that the model work with an aggregate demand as a fundamental input for its operation. This is in order to design a realistic model and minimize the statistical uncertainty that can arise when working with individual stores. Otherwise, it would be an impractical model because a company could have hundreds of stores distributed in a territory. This is explained in more detail within the formulation stage.

4.2. Needs assessment

A needs assessment stage was proposed to establish the elements that the model should have in order to avoid the mistake of failing to consider the current context of the research. This was carried out through a review of an academic and business landscape, where the current models of national and international companies such as Wal-Mart or Carrefour were consulted. In addition to the above, a review of academic articles from major scientific journals was conducted, taking into account journals from the last five years. The final synthesis constituted an extensive procedure in its own right. This was carried out through group consensus among the research team where it was decided that the following key points were to be considered:

- *A value chain vision.* It was necessary to have a proposal that helps to break down the existing barriers in the cooperation between the levels of the supply chain for this type of products, leaving aside individualistic benefits and seeking global efficiency as a value chain.
- *Order cycle.* Order cycle time is an important aspect, a VMI agreement must establish an appropriate balance related to costs and decision making on lot size.
- *Stockouts.* Stockouts must be considered in the model, due to the demand variability in relation to the customer service level, safety stocks, and customer service.
- *Case study*. In the literature review, most of the research carried out in recent years does not present a clear or real validation of their VMI models, most of them are only limited to numerical analysis. Therefore, it is necessary a contribution of a real case that illustrates the results of the designed model.

Additionally, the main factors that should be considered in the development of a model, and in the establishment of the guidelines of this collaborative agreement, were investigated. These can be seen as reflected in the set of variables and assumptions in the formulation of the model (section 4.4). It is important to mention that, in relation to the optimization of the model, it was recommended to orient it to minimize inventory costs.

4.3. Modeling technique selection

Due to the number of current modeling techniques and the nuances of each one, it was necessary to select the technique best suited to the characteristics that the model intended to achieve. The selection was made through an expert judgment based on an analytic hierarchy process (AHP).This activity had a group of 19 experts, whose professional research included the study of Production and Logistics Systems, and Operations Research. Having the most adept committee was crucial to reduce the error and uncertainty in the selection of the technique.

When multiple objectives are important to a decision maker, it may be difficult to choose between alternatives. Given a large number of judgments that can be made by expert staff, it was imperative to solve a Multi-Criteria Decision-Making problem. The analytic hierarchy process (AHP) is a method of measurement that relies on pair wise comparisons and the judgments of experts to derive priority scales. This method has been one of the most widely used decision-making techniques by decision-makers and researchers [42]–[44]. Thus, it was perfectly adjusted to the requirements of this problem, to carry out the selection of a modeling technique. The application of the AHP involves an extensive procedure for describing it[45], [46]. However, the fundamental aspects that were taken into account and the results obtained are detailed below.

A set of alternatives and a set of criteria are required to carry out an AHP. In the selection of alternatives, three modeling techniques were considered that were widely used by the authors of the scientific literature consulted, for the design of VMI models: 1) classical optimization, 2) non-linear programming, and 3) game theory. In respect to classical optimization, it is effective in obtaining the optimum solution of unconstrained and constrained continuous and differentiable functions. Besides, analytical methods make use of differential calculus in finding the optimum solution assuming that the function is differentiable concerning the design variables. Nevertheless, although analytical methods with essential and sufficient conditions are easier to use, these methods are difficult to apply for functions that are not continuous and/or not differentiable. Similarly, nonlinear programming continues to be an effective tool for supply chain modeling. The main advantage of a nonlinear programming approach is the guaranteed solution of a well-formulated problem and the ability to vary the supply chain parameters to understand the behavior of the system under various settings. Also, it make it possible to solve large-scale problems. However, nonlinear programming models are analytically hard to solve, and applying these methods to bigger cases can increase computation time. As a consequence, efficient algorithms and solution techniques could be adopted to find approximately optimal solutions and reduce the calculation times.

In addition, game theory has been recently applied to models for supply chain coordination. Given the current industry environment in which cooperative relations are becoming more prevalent in supply chains, a mutually beneficial approach addresses the coordination issues for vendor–buyer interactions. Thus, a VMI agreement can be modeled as either a dynamic cooperative or non-cooperative game concerning the overall supply chain. Also, once the whole game settles into an equilibrium, none of the chain members will be able to improve its payoff or profits by acting unilaterally without negatively affecting the performance of the other players. Despite these advantages, some drawbacks--mainly related to the fact that games that include multiple products that exist among multiple retailers are not easy to model--are present. As to which forms of games and roles are suitable for formulating coordinated decisions, these factors mainly depend on the competitive advantage of products in markets, as well as the organizational forms of their supply chain. In this respect, an individual entity's share of the market, and thus, negotiating power, also has significant effects as to the game's outcome.

Consequently, it was possible to select a recommended approach and contribute to aspects of it that had not yet been analyzed or explored. On the other hand, the criteria were selected by consensus, which was reached by defining the needs or requirements of the proposed model. The criteria for choosing a technique were as follows: 1) Ability to model complex systems, 2) Ease of replication, 3) Flexibility and 4) Variety of solutions. Each of these is explained in detail below:

- *Ability to model complex systems*. The selected technique should be useful to represent the study problem correctly, delineate the operational needs, and clearly define the expected outputs.
- *Ease of replication*. The selected technique should be easy to replicate, execute, and manipulate analytically. In order to reduce implementation costs, it is necessary to design a useful but not over-simplified model.
- *Flexibility.* As long as the established assumptions are met, the technique should allow the model to be applied in various scenarios
- *Variety of solutions*. The technique should serve to generate a variety of solutions that allow different

aspects or scenarios of the same case study to be analyzed.

In this way, Figure 1 shows the hierarchy proposed for this expert judgment. Note that the first level of the hierarchy is the goal (Modeling technique selection); The second level in the hierarchy is constituted by the criteria experts used to decide the modeling technique. The third level consists of the alternatives.

Thus, the second step in the AHP process was to derive the relative priorities or weights for the criteria. Evidently, the importance or weight of each criterion was different and because of this, experts first were required to derive by pairwise comparisons the relative priority of each criterion with respect to each of the others using a numerical scale for comparison. Although the exact method will not show in detail here, the general idea is simple. Next, once judgments were entered, it was necessary to check that they were consistent. For this purpose, AHP calculates a consistency ratio (CR) comparing the consistency index (CI) of the matrix in question (the one with our judgments) versus the consistency index of a random-like matrix (RI). In AHP, the consistency ratio is defined as CR = CI/RI.Saaty (1985)[47] has shown that a consistency ratio (CR) of 0.10 or less is acceptable to continue the AHP analysis. If the consistency ratio is greater than 0.10, it is necessary to revise the judgments to locate the cause of the inconsistency and correct it.

It is important to mention that the consistency index of the consensual valuations was calculated. This value did not reflect inconsistencies. Then, judgments about the alternatives were consistent and there was no contradiction in any of them. The result, called priority vector of alternatives constituted the solution of the expert judgment. This vector presented a preference percentage for each of the alternatives. Table 1 shows the results.

Table 1: Priority vector of alternatives

Alternatives	Ranking
Classical or unconstrained optimization	0.491
Game Theory	0.238
Nonlinear programming	0.271
Total	1.0

According to the results, it was clear from the results that the technique that best complied with the requirements – based on the consistency of all experts' judgments – was Classical Optimization. Its success rate was 49.1% higher than Game Theory and Nonlinear Programming, which has success rates of 23.8% and 27.1% respectively. Due to the above, it is possible to emphasize that execution process of expert judgment was correctly carried out for the selection of an appropriate technique that fulfilled a set of criteria and distinguished alternatives.

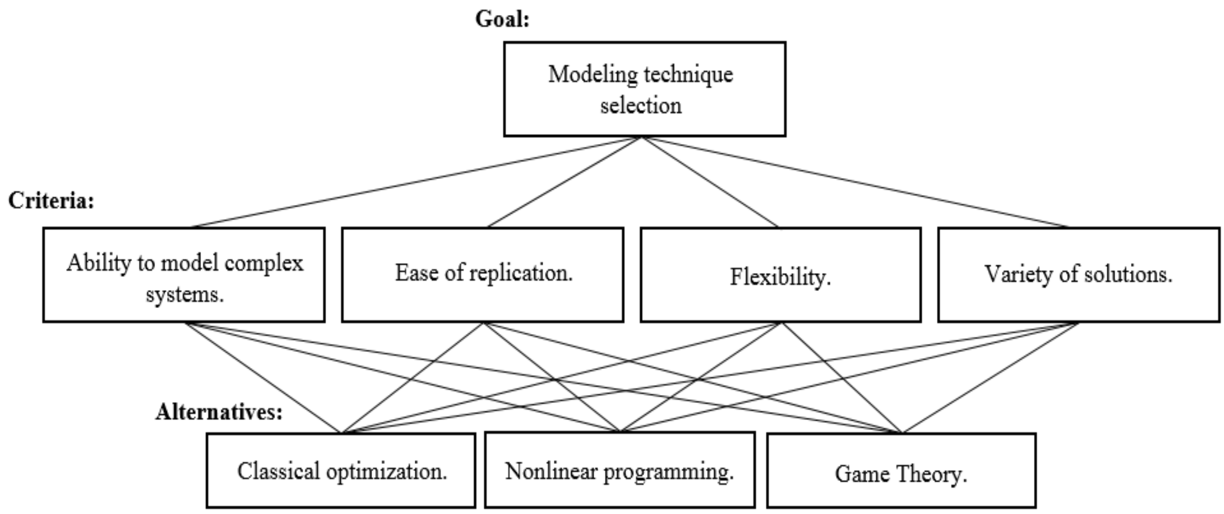

Figure 1: Hierarchical structure of AHP.

4.4. Model formulation and assumptions

A VMI model was designed for impulse purchase products in a two-level supply chain with a vendor and a buyer. The development of the model was established as a cooperative agreement between both parties, in which the buyer shares the sales and inventory information of a set of products with the vendor. Subsequently, the vendor places a suggested order based on the information received. This will be a consolidated order, which means that it will group together the supply needs of all stores. In simple terms, in the traditional inventory management, a buyer makes their own decisions regarding the order size while in VMI, a buyer shares their sale and inventory data with a vendor such that the vendor can determine the order size for both. This policy can prevent stocking undesired inventories and hence provides greater visibility to inventory replenishment and supply planning. The inclusion of a safety stock and stock out costs were also considered. Table 2 presents the mathematical notation.

Table 2: Notations of parameters and variables

Index:	
$i = 1,2,..m$	Index for products.
$j = 1,2..w$	Index for stores.
Parameters:	
D_i	Forecasted demand.
P_i	Replenishment rate.
C_{oi}	Cost per item.
S	Fixed ordering cost (buyer).
C_{pi}	Variable ordering cost (buyer).
H	Order cost (vendor): order preparation.
C_{Pi}	Order cost (vendor): shipment.
G_i	Order cost (vendor): packaging.
C_{si}	Cost per item short.
$E[x_i]$	Expected number of shortages per order.
T	Time horizon.
C_{Hi}	Vendor's holding cost.
C_{hi}	Buyer's holding cost.
C_{Vi}	Pertain to the cost of holding stock in the store display shelf: cost of shelf space.
V_{Ai}	Pertain to the cost of holding stock in the store display shelf: cost per volume.
Decision variables:	
n	Common ordering frequency: number of orders per unit time.
n^*	Optimal ordering frequency: optimal number of orders per unit time
Q_i	Order size (items/order).
Q_i^*	Optimal order size.
L	Lead Time.
I_{buyer}	Average inventory (buyer).
I_{vendor}	Average inventory (vendor).
s_i	Reorder point.
t_1	Replenishment cycle.
SS_i	Safety stock.
TC_V	Total Cost (Vendor).
TC_B	Total Cost(Buyer).
TC_{SC}	Total Cost(Supply Chain).
TRC_{SC}	Total Relevant Cost.

Also, in this model the following assumptions were defined:

- *Single vendor and single buyer with m products (multiproduct systems). The buyer can have w stores.* VMI agreements are mostly implemented in a two-level supply chain. Furthermore, a collaborative agreement between two companies contains key parameters related to specific policies. This model then involves a single vendor serving a set of buyer's stores. This one-to-many model is not only ubiquitous, but it also describes the distribution activities of many companies while keeping the analytical complexity at a tractable level.
- *The information of the buyer's replenishment decision parameters is available to the vendor***.** Sharing sales and inventory information is an indispensable assumption in a VMI agreement.
- *Planning horizon of one period*. Considering an uncertain demand, it is necessary to contemplate a planning horizon of one period, which allows to reach a higher level of accuracy in the results.
- *The flow of information between the levels of the supply chain is automatic and in real time*. This is a condition that must be met to ensure that shared information is consistent, real and error-free.
- *The vendor has demand visibility of their buyer*. A major assumption of VMI is the transfer of information between the vendor and the buyer. The retailing industry in particular is sensitive to the vicissitudes of consumer demand. The uncertain nature of the demand is one of the motivations to consider aggregating multiple products in a single order. A forecasted demand by the vendor is considered.
- *Shortage cost is the loss of sales revenue from not meeting the demand*. This assumption allows us to consider certain real inventory replenishment policies that involve goals related to the level of service.
- *Quantity discounts are not permitted.* Quantity discounts is not an important aspect in the formulation of the model because the model seeks to optimize the replenishment cycle of the products.
- *The holding cost is same for all stores.* This assumption allows an adequate modeling of the holding cost in the stores, and at the same time, an appropriate simplification of reality is achieved.
- *The model assumes constant lead time*.
- *Safety Stock is required*. A safety stock is necessary to reduce the probability of stock-outs.
- *The model assumes that demand is uncertain and follows a normal distribution*: Although empirical probabilities can be used in this study, they are inconvenient for many reasons. Firstly, they require maintaining a record of the demand history for every product. This can be costly and unwieldy. Secondly, the distribution must be expressed as different probabilities for each of the past values, and products may have an even wider range of past values. Finally, it is more difficult and impractical to compute optimal inventory policies with empirical distributions. For these reasons, in practice, it is common and most popular for inventory application, to assume that demand follows a normal distribution. One reason is the frequency with which it seems to accurately model demand fluctuations. Another is its convenience and the fact that, according to the central limit theory, for sufficiently large samples (n ≥ 30), the sample means will be distributed around the population mean approximately in a normal distribution.

4.4.1. Buyer's total cost

When analyzing the different processes in a VMI model, it is clear that the purchase of products by the buyer is one of the main features. The purchase cost depends on each product $i$and the quantities ordered of that product. Each product $i$is assigned to a SKU (Stock-keeping unit) and has a unit purchase cost(C_{oi}), which is assumed by the buyer. At the beginning of each cycle, there is a forecasted demand for each product (D_i), which must be met in n_i orders. Then, the order size (Q_i) to meet this demand is defined as:

$$Q_i = \frac{D_i}{n_i} \tag{1}$$

In this way, the purchase cost will be the multiplication of the order size (Q_i), the purchase unit cost (C_{oi}), and the number of orders (n_i) in the cycle, for each of the m products. That is:

$$\text{Purchasing cost} = \sum_{i=1}^{m} n_i C_{oi} Q_i = \sum_{i=1}^{m} n_i C_{oi} \left(\frac{D_i}{n_i}\right) = \sum_{i=1}^{m} C_{oi} D_i \tag{2}$$

Also, an order cost is generated that depends on the characteristics of the product i. This cost has two components: a fixed cost (S) and a variable unit order cost (C_{pi}).The fixed cost (S)corresponds to the costs related to the administration and reception of the order.On the other hand, it was considered that for each type of product i, there is a variable unit cost of ordering (C_{pi}), which is associated with the follow-up costs that depend on the special requirements of these products. Hence, the cost of ordering is the sum of the fixed (S) and variable cost (C_{pi}) incurred in placing an order, multiplied by the number of orders $n_{i,}$:

$$\text{Ordering cost} = \sum_{i=1}^{m} n_i (S + C_{pi}) \tag{3}$$

Inventory management seeks to minimize stock out costs that occur when demand is greater than anticipated and cannot be met immediately. The process of estimating this cost involves an overview of non-quantitative variables including, but not limited to, customer perception, the long-term reliability of these perceptions, and the loss of consumer loyalty[48]. Subsequently, the stock-out costs consists of three elements. The first is $n_i$which is the ordering frequency, or the total number of orders placed over the whole length of time. The second term is C_{si}, which is the cost incurred due to the shortage of each product i. And then the last term is the Expected Unit Short $E[x_i]$, which is equal toσ_{DL} times $G_u(k)$, where σ_{DL} is the standard deviation of the demand over lead time and $G_u(k)$ is the unit normal loss function. The mathematical procedure for calculating the Expected Unit Shortis well-known, hence it can be entirely omitted.

$$Stock\ out\ costs = \sum_{i=1}^{m} n_i C_{si} E[x_i] = \sum_{i=1}^{m} n_i C_{si} \sigma_{DL} G_u(k) \tag{4}$$

In addition, it is important to estimate the buyer's holding cost which depends on inventory levels over time. The inventory level is represented in Figure 3, where Q is the order size, and SS is the safety stock.

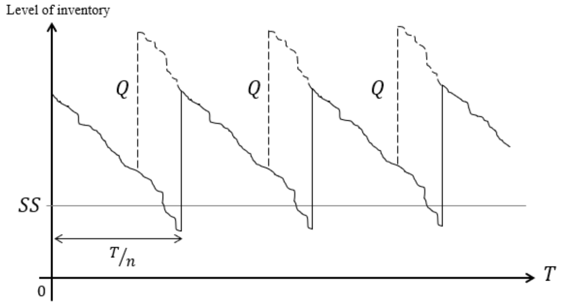

Figure 2: Estimating the buyer's holding cost.

According to the Figure 2, the inventory level can be approached to a linear function of the form $y = mx + b$, where m is equal to $-{Qn}/{T}$, and $b = Q$. Therefore, the average inventory (I_{buyer})can be readily obtained as follows (according to the linearity assumption adopted),

$$I_{buyer} = \frac{QTn}{2nT} + SS = \frac{Q}{2} + SS \tag{5}$$

Thus, the quantity ${T(Q + SS)}/{2n}$ is the area of the triangle with height $Q + SS$ and base ${T}/{n}$, which is divided by the cycle length ${T}/{n}$ to calculate the average inventory over the cycle. The cost of holding stock in the store j for a specific product is the sum of theaverage inventory I_{buyer}, multiplied by the cost of holding that product in that facility j (C_{hj}). Therefore, since each store has an order quantity Q_j, it is possible to estimate the cost of holding stock per unit of a certain product in the store j as:

$$C_{hj}\left(\frac{Q_j}{2} + SS_j\right) \tag{6}$$

Next, under the assumption that the cost of holding stock of a certain product in the store$j$ is the same for all stores and that there are w stores, this condition translates to:

$$C_h\left(\sum_{j=1}^{w} \frac{Q_j}{2} + \sum_{j=1}^{w} SS_j\right) \tag{7}$$

In addition, since it is possible to express the consolidated units of a product for all stores in terms of two variables Q and SS, That is,

$$\sum_{j=1}^{w} Q_j = Q \quad ; \sum_{j=1}^{w} SS_j = SS \tag{8}$$

Then, according to Eq. (7) and Eq. (8), after rearranging terms, the cost of holding stock in all buyer's stores for a particular product would be:

$$C_h\left(\frac{Q}{2} + SS\right) \tag{9}$$

Substituting into Eq. (9) the value Q according to Eq. (1), this condition translates to:

$$C_h\left(\frac{D}{2n} + SS\right) \tag{10}$$

Finally, using the fact that the VMI agreement included several products, Eq. (10) can be generalized to the case in which there are m products, as follows:

$$\sum_{i=1}^{m} C_{hi}\left(\frac{D_i}{2n_i} + SS_i\right) \tag{11}$$

Therefore, according to Eq.(2), Eq. (3), Eq. (4), and Eq. (11), the buyer's total cost(TC_B)would be:

$$TC_B = \sum_{i=1}^{m} C_{oi}D_i + \sum_{i=1}^{m} n_i(S + C_{pi}) + \sum_{i=1}^{m} n_i C_{si} E[x_i] + \sum_{i=1}^{m} C_{hi}\left(\frac{D_i}{2n_i} + SS_i\right) \tag{12}$$

4.4.2. Vendor's total cost

The vendor's order cost has three components. First, a fixed component (H) that is determined by the cost of preparing n_i orders. It typically includes fees for placing the order, and all kinds of clerical costs related to invoice processing, accounting, or communication. Second, a shipping cost ($C_{\boldsymbol{P}i}$) that depends on the type of product being shipped. Thus, it is multiplied by the number of orders. Finally, a packaging cost (G_i) associated with pre-consolidation or package formation includes weighting, labeling and packaging. This cost is dependent on the number of units to be shipped (D_i/n_i). Thus, the vendor's order cost is defined as:

$$\text{Vendor's ordering cost} = \sum_{i=1}^{m} n_i\left(H + C_{\boldsymbol{P}i} + \frac{D_i}{n_i}G_i\right) = \sum_{i=1}^{m} n_i(H + C_{\boldsymbol{P}i}) + \sum_{i=1}^{m} D_iG_i \tag{13}$$

Furthermore, in this vendor-managed inventory (VMI) model for impulse purchase products, the vendor's holding cost has two components. The first of these two components is the cost of holding stock in the storage facilities that is incurred before serving a set of buyer's stores. The second component is the cost of holding stock in the store display shelf that is incurred by allowing the vendor to display their product inside the buyer's store. Each of these components is discussed below:

Assuming a replenishment or production rate (P), the vendor's holding cost incurred in the storage activities is equal to the multiplication of the average inventory (I_{vendor}) and the cost of holding stock in their facility (C_H).The average inventory was calculated according to Figure3, where T is the total time horizon, n is the number of orders, and t_1 is the time period required for the vendor to replenish (or produce) an entire batch quantity$Q$ at a rate P.

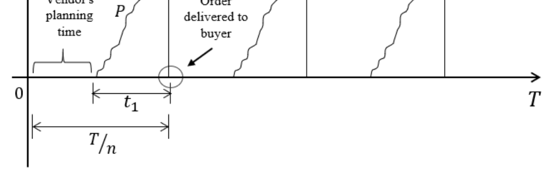

Figure 3. Estimating the vendor's holding cost.

According to the Figure 2, the inventory level can be approximated to a linear function of the form$y = mx + b$,where$m$is equal tothe replenishment or production rate $\left({}^{Q}/_{t_1}\right)$,and$b = y - mx = Q - {}^{Q}/_{t_1}\,({}^{T}/_{n})$, since the inventory level is Q, after ${}^{T}/_{n}$ units of time.The vendor's average inventory (I_{vendor}) can be readily obtained as follows (according to the linearity assumption adopted). The quantity ${}^{Qt_1}/_{2}$ is the area of the triangle with height Q and base t_1, which is divided by the cycle length ${}^{T}/_{n}$ to calculate the average inventory over the cycle.

$$I_{vendor} = \frac{nQt_1}{2T} \tag{14}$$

Thus, the vendor's holding cost stock incurred in the storage activities can be expressed as:

$$C_H I_{DC} = C_H\left(\frac{nQt_1}{2T}\right) \tag{15}$$

Using the fact that $t_1 = Q/P$ and $Q = D/n$, which are substituted into Eq. (15), the cost of holding stock in the vendor's distribution center would be:

$$C_H\left(\frac{D^2}{2nPT}\right) \tag{16}$$

Eq. (16) can be generalized to the case in which there are m products, as follows:

$$\sum_{i=1}^{m} C_{\boldsymbol{H}i}\left(\frac{D_{\boldsymbol{i}}^{2}}{2n_iP_iT}\right) \tag{17}$$

Secondly, the cost of holding stock in the store display shelf depends on the shelf space to display the product in the store and it is defined as the multiplication of the product volume (V_{Ai}) and the shelf space cost (C_{Vi}). Using the fact that there are m products on display, this condition translates into:

$$\sum_{i=1}^{m} V_{Ai}C_{Vi} \tag{18}$$

Therefore, the vendor's total cost is the sum of Eq. (13),Eq.(17), and Eq.(18):

$$TC_V = \sum_{i=1}^{m} n_i(H + C_{\boldsymbol{P}i}) + \sum_{i=1}^{m} D_iG_i + \sum_{i=1}^{m} C_{\boldsymbol{H}i}\left(\frac{D_{\boldsymbol{i}}^{2}}{2n_iP_iT}\right) + \sum_{i=1}^{m} V_{Ai}C_{Vi} \tag{19}$$

4.4.3.Supply Chain

Because we are looking for a general optimization with the VMI agreement, the total cost of the supply chain (CT_{SC}) will be equal to the sum of the total costs of each party (see Eq.(12) and Eq.(19)):

$$
\begin{aligned}
TC_{SC} = TC_B + TC_V = & \left[\sum_{i=1}^{m} C_{oi}D_i + \sum_{i=1}^{m} n_i(S + C_{pi}) \right. \\
& + \sum_{i=1}^{m} n_i C_{si} E[x_i] \\
& \left. + \sum_{i=1}^{m} C_{hi}\left(\frac{D_i}{2n_i} + SS_i\right)\right] \\
& + \left[\sum_{i=1}^{m} n_i(H + C_{\boldsymbol{Pi}}) \right. \\
& + \sum_{i=1}^{m} D_i G_i \\
& + \sum_{i=1}^{m} C_{\boldsymbol{Hi}}\left(\frac{D_i^{\,2}}{2n_i P_i T}\right) \\
& \left. + \sum_{i=1}^{m} V_{Ai} C_{Vi}\right]
\end{aligned} \tag{20}
$$

To optimize supply chain costs, it was proposed to exploit fixed costs by aggregating multiple products in a single order. In other words, the same amount of orders is placed for all products purchased from the same vendor. In this way, the objective function was defined as follows:

$$
\begin{aligned}
TC_{SC} = & \left[\sum_{i=1}^{m} C_{oi}D_i + n\sum_{i=1}^{m}(S + C_{pi}) \right. \\
& + n\sum_{i=1}^{m} C_{si}E[x_i] \\
& + \frac{1}{2n}\sum_{i=1}^{m} C_{hi}D_i \\
& \left. + \sum_{i=1}^{m} C_{hi}SS_i\right] \\
& + \left[n\sum_{i=1}^{m}(H + C_{\boldsymbol{Pi}}) \right. \\
& + \sum_{i=1}^{m} D_i G_i \\
& + \frac{1}{2nT}\sum_{i=1}^{m} C_{\boldsymbol{Hi}}\left(\frac{D_i^{\,2}}{P_i}\right) \\
& \left. + \sum_{i=1}^{m} V_{Ai}C_{Vi}\right]
\end{aligned} \tag{21}
$$

Note that in Eq.(21) the sub-index for variable n has been removed. In this way, the decision variable becomes the number of orders to be placed. Therefore, by eliminating those terms that do not depend on n (since they are not relevant for optimization), the supply chain cost function is simple function of n. This function was called Total Relevant Costs(CTR_{SC}), as shown below:

$$
\begin{aligned}
TRC_{SC} = & \left[n\left(\sum_{i=1}^{m} C_{pi} + S\right) + n\sum_{i=1}^{m} C_{si}E[x_i] \right. \\
& \left. + \frac{1}{2n}\sum_{i=1}^{m} C_{hi}D_i\right] \\
& + \left[n\left(\sum_{i=1}^{m} C_{\boldsymbol{Pi}} + H\right) \right. \\
& \left. + \frac{1}{2nT}\sum_{i=1}^{m} C_{\boldsymbol{Hi}}\left(\frac{D_i^{\,2}}{P_i}\right)\right]
\end{aligned} \tag{22}
$$

From the fact that classical optimization was selected to formulate the model, the purpose was to minimize the total relevant cost function by finding an extreme point solution. Using calculus, the derivative of the total relevant cost function (Eq.(22)) was taken and set equal to zero and solve for n. The total relevant costs curve was convex i.e. curvature is upward then a minimizer was obtained. In addition, taking into account that all function values were greater than 0, which indicated that $f''(n) > 0$, it was concluded that the function has a point where it takes a minimum value. By setting the first derivative equal to 0 and solving for n, the optimum number of orders is given by:

$$
n^* = \sqrt{\frac{\sum_{i=1}^{m} C_{hi}(D_i) + \frac{1}{T}\sum_{i=1}^{m} C_{\boldsymbol{Hi}}\left(\frac{D_i^{\,2}}{P_i}\right)}{2\left(\sum_{i=1}^{m} C_{pi} + \sum_{i=1}^{m} C_{si}E[x_i] + \sum_{i=1}^{m} C_{\boldsymbol{Pi}} + S + H\right)}} \tag{23}
$$

Similarly, based on Eq.(1) and the n^* definition, it is possible to represent the optimal order lot size as follows:

$$
Q_i^{\,*} = \frac{D_i}{n^*} \tag{24}
$$

In this way, benefits could be expected by optimizing the total inventory costs of the supply chain by exploiting the fixed costs present in the VMI agreement operations. The reduced fixed cost of receiving makes it optimal to reduce the lot size ordered for each product, thus reducing cycle inventory. Aggregation of orders for different products also brings advantages associated with administrative and implementation activities. In addition, the inventory policy has another parameter that needs to be estimated, the reorder point, s_i. This variable is important when trying to find what amount of inventory is sufficient to handle all the expected demand over the lead time for each product i. As previously mentioned, adding a safety stock is necessary because the demand is variable, and how much safety stock to add is determined by how badly the buyer would like to avoid stockout. Thus, the policy will be order $Q_i^{\,*}$ when inventory position is less than or equal to s_i, the reorder point for a product i. Therefore, it can be assumed that $s_i = \mu_{DL} + SS_i = \mu_{DL} + k\sigma_{DL}$, where the reorder point is equal to the forecast mean demand over lead time μ_{DL} (or the expected demand over the lead time period), plus the safety stock SS_i. And SS_i is simply, k, the safety stock factor, times σ_{DL}, the standard deviation of the error of the forecast over the lead time, or the root mean square error.

Now, considering a cost-minimization approach, as this approach is most commonly used in large and more sophisticated organizations, it is possible to analyze the value of k. Using the total cost equation and taking the first-order condition to minimize the total cost, the only relevant costs are the safety stock and the stock out cost because they are the only ones with the variable k(Eq. 4 and Eq. 11). By taking a first-order condition and setting it equal to 0, it ends with this condition: k is optimal for the minimum total relevant cost if the probability of the demand x greater than$k$ is equal to $Q_i^{\,*}$ times the cost of holding C_{hi}, divided by the demandD_i, times the cost of shortage C_{si}: $P[x \geq k] = Q_i^{\,*}C_{hi}/D_iC_{si}$. Notice that it makes sense that the ratio of C_{hi} over C_{si} would help determine how much a party would want to stock, because it is just a tradeoff between having too much and having too little. Then, the decision rule--when assuming cost of shortage--first has to make sure that the following expression is less than or equal to 1:$Q_i^{\,*}C_{hi}/D_iC_{si}$. And if it is, then it can be determined that the probability of stockout is equal to $P[x \geq k]$. Otherwise, k should be set as low as management allows.

In this way, it is valuable to summarize both why and how the management of impulse purchase products differs from other types of products as well as how this was reflected in the proposed model. Firstly, it is necessary to start by recognizing that VMI agreements are common among companies marketing impulse purchase products; that is, the companies that have a greater implementation of these types of collaborative models are big-box and department store retailers that offer a wide breadth of products. Second, the model assumed a planning horizon of one period because considering that impulse purchase products are characterized by an uncertain demand, it is necessary to contemplate a planning horizon that allows for the reaching higher levels of accuracy in the results. In relation to the above mentioned, in practice, shortage cost is the loss of sales revenue from not meeting the demand in the market place for impulse purchase products, and inventory management should minimize stock out costs that occur when demand is greater than anticipated and cannot be met immediately. All these aspects were considered in the model, even including some important variables by modelling the respective cost of inventory and stock-outs, like ordering frequency, cost incurred due to the shortage of each product, expected unit short, etc. Also, the safety stock was added to reduce the probability of stock-out because many buying decisions for these products are based on impulse and at the point of purchase.

It should also be noted that impulse buying products are characterized as products that cost little and are quickly consumed. Additionally, in general terms, an assumption related to constant lead time, it is an appropriate simplification of reality. Usually, a short lead time is a relatively common occurrence in the impulse purchase products market, as information distortion is magnified if replenishment lead times between stages are volatile or long. Then, by decreasing the replenishment lead time, companies and managers can minimize the uncertainty of demand during the lead time. Another critical aspect of these products is the way to calculate holding costs in practice. Commonly, managers consider the cost of holding stock in the storage facilities that are incurred before serving a set of buyer's stores, and the cost of holding stock in the display shelf defined as the multiplication of the product volume and the shelf space cost (remember that generally, these items are strategically displayed in hot spots), which depends on the shelf space to display the product in the store. Each of these cost components was included in the model. Last but not least, firms in this sector may order in large lots because the presence of fixed costs associated with ordering, quantity discounts in product pricing, and short-term promotions, encourages different stages of a supply chain to exploit economies of scale and order in large lots, not mention that another main advantage of ordering all products jointly is that a VMI agreement would be easy to administer and implement.

4.5. Validation and sensitivity analysis

To validate the model, a case study was conducted with two major companies that market impulse purchase products, where the cost reduction of the model was compared against a conventional form of inventory replenishment policy in a traditional supply chain. The vendor was a sugar confectionery producer company, and the buyer was a national retail company that is currently working on collaborative agreements with this impulse purchase products vendor. A copy of the data used in this study is available on request for any interested researcher. Please contact the corresponding author. The current scenario (traditional supply chain) was compared with two scenarios based on the proposed VMI model. Beyond having as it is purpose to execute the model in a real-life scenario, the main objective was to learn the economic impact as a key indicator of its implementation. It is also important to mention that the model was executed with eight products that are part of the VMI agreement, and historical data from the two previous years (24 months) were used to make key inferences from the model. The model measured performance over a five-month period (months 25, 26, 27, 28, and 29).

- *Scenario 1: VMI model with economies of scale to exploit fixed costs.* The VMI model was analytically executed with actual input data from both companies over a five-month period, to calculate and compare the total costs. In this scenario, the vendor and buyer have a VMI agreement for eight impulse purchase products, and the optimal number of orders$(n^*)$was calculated to reduce fixed costs in supply chain operation (Eq.(23)). In other words, products were ordered jointly (aggregate orders).
- *Scenario 2: VMI model without economies of scale to exploit fixed costs.* The VMI model was analytically executed with actual input data from both companies over a five-month period, to calculate and compare the total costs. In this scenario, the vendor and buyer have a VMI agreement for eight impulse purchase products, However, unlike scenario 1, the optimization proposal associated with the exploitation of fixed costs in the agreement, was not applied here. This means that products were ordered independently.
- *Scenario 3: traditional supply chain.* The total costs of both companies were calculated in a traditional scenario over a five-month period. In thistraditional supply chain, the buyer is responsible for tracking inventory levels at theirlocations and create a purchase order to make appropriate inventory replenishmentdecisions. The vendor has no information aboutfuture demand or inventory levels at theirbuyer’s facilities and so has no prior knowledge about the quantity and time of thepurchase order.

The results were compared for each of the five individual periods (months 25, 26, 27, 28, and 29) and for the entire period (from month 25 to month 29). These results can be found in the Table 3.

In the vendor’s case, it was noted that the greatest economic benefit is achieved in Scenario 1, with a savings of 35.52%, and 47.55% compared to Scenarios 2 and 3 respectively.As for the buyer’s total relevant cost, scenario 1 showed beneficial results as well, with a savings of 32.41%, and 42.78% compared to Scenarios 2 and 3 respectively. Comparing the total relevant cost for the supply chain, the results showed a reduction in cost of 45.13% in the proposed model compared to the traditional scheme of both companies (Scenario 3), and savings of 33.91% compared to Scenario 2. This demonstrated that the model proposed in this case study obtained the best results in each of the scenarios for each of the parties to the agreement, as well as for the supply chain in general. In addition, for this case study, the exploitation of fixed costs was beneficial in the agreement.

Although it is true that the model works with an aggregate demand (the sum of the demand of all the products), it was relevant to analyze the impact of variations in demand on the results. Therefore, knowing how well the model adapts to variations in demand and how these variations affect the optimal order size were important questions. By replacing in Eq. (24), the value of n^*established by Eq. (23), the following equation was obtained:

$$Q_i^* = \frac{D_i}{\sqrt{\frac{\sum_{i=1}^{m} C_{hi}(D_i) + \frac{1}{T}\sum_{i=1}^{m} C_{\boldsymbol{H}i}\left(\frac{D_i^2}{P_i}\right)}{2\left(\sum_{i=1}^{m} C_{pi} + \sum_{i=1}^{m} C_{si}E[x_i] + \sum_{i=1}^{m} C_{\boldsymbol{P}i} + S + H\right)}}} \quad (25)$$

Then, assuming a percentage change denoted as α in demand of a product i, the variation in optimal lot size (Q_i^*)is given by:

$$Q_i^* = \frac{(1+\alpha) * D_i}{\sqrt{\frac{(1+\alpha) * \sum_{i=1}^{m} C_{hi}(D_i) + \frac{(1+\alpha)^2}{T} * \sum_{i=1}^{m} C_{\boldsymbol{H}i}\left(\frac{D_i^2}{P_i}\right)}{2\left(\sum_{i=1}^{m} C_{pi} + \sum_{i=1}^{m} C_{si}E[x_i] + \sum_{i=1}^{m} C_{\boldsymbol{P}i} + S + H\right)}}} \quad (26)$$

From Eq. (26), it can be inferred that once an optimal order size has been calculated, a new optimal order size can be calculated according to a variation in demand of a product i. Also, if all products have the same variation in demand, the percentage variation will also be the same. Then, the increase or decrease in demand will be directly proportional to the size of the order.

To prove this, an analysis of the demand variation was carried out for all products. The results showed that the percentage change in order size was the same for each product. Variations were applied in an interval between ± 30%, the order size showed a variation between -9.48% and 6.52%, which indicates that the proposed model was adapted to fluctuations in demand without greatly varying the results in the period analyzed (months 25, 26, 27, 28, and 29), as shown in Figure 4.

Similarly, the total relevant costs showed a decrease at each level of the supply chain, which translates into a reduction in overall total relevant costs. Applying the same demand variation criterion used previously (variations in the order of ±30%), it was noticeable that the increase in costs at each level was proportional to the percentage increase in demand. This is illustrated in Figure 5. The results indicated that, although the total relevant cost of the supply chain was largely dependent on demand, the exploitation of fixed costs helped mitigate the impact due to large variations.

It is possible to point out that the validation carried out demonstrated positive benefits in relation to the economic impact that the implementation of the proposed model could bring about, provided that the set of assumptions and guidelines set forth throughout its formulation are complied with. Also, the exploitation of fixed costs and their inherent advantages were correct in regard to the financial terms within the agreement. Finally, with the development of the sensitivity analysis, it was noted that the research may lead us to inferences related to the generation of strategies in environments of uncertain demand. The model seems to adjust itself correctly to fluctuations in demand as a result of the variations in the optimal lot size, which would be a function of the percentage increase in demand and not of the associated costs.

Table 3: TotalRelevant cost savings (%): A comparison analysis

Savings percentage	Month	Scenario 1 vs. Scenario 2	Scenario 1 vs. Scenario 3
Total Relevant Cost: Vendor	25	34.79%	44.96%
	26	36.24%	52.83%
	27	35.40%	50.06%
	28	35.31%	43.14%
	29	36.04%	47.67%
	Entire period (5 months):	35.52%	47.55%
Total Relevant Cost: Buyer	25	31.86%	41.30%
	26	32.80%	45.37%
	27	32.25%	43.97%
	28	32.64%	40.79%
	29	32.51%	42.71%
	Entire period (5 months):	32.41%	42.78%
Total Relevant Cost: Supply Chain	25	33.29%	43.11%
	26	34.38%	48.98%
	27	33.73%	46.94%
	28	33.98%	41.97%
	29	34.21%	45.14%
	Entire period (5 months):	33.91%	45.13%

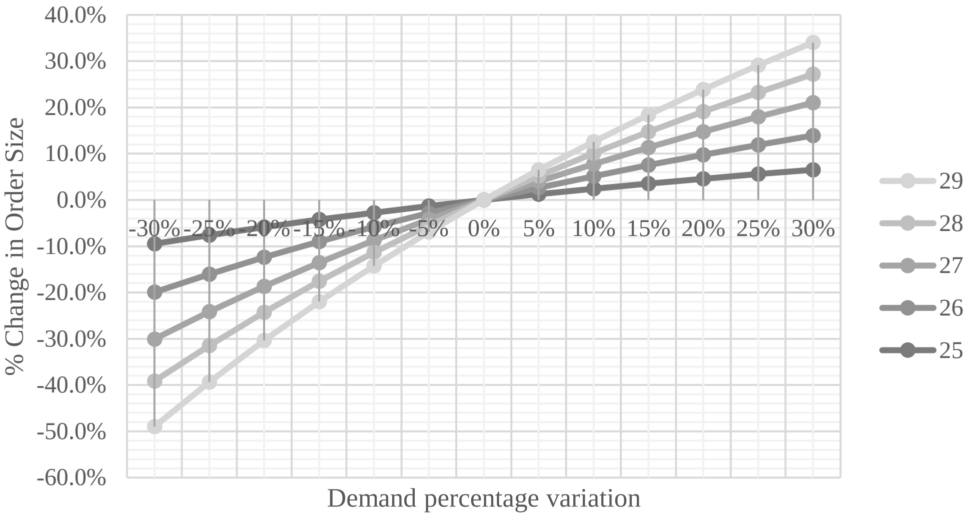

Figure 4. % Change in Order Size.

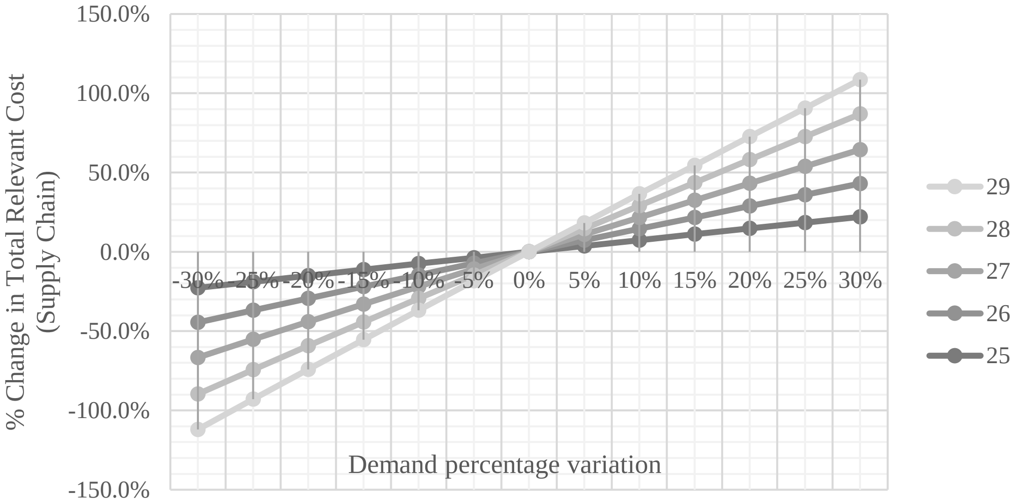

Figure 5. % Change in Total Relevant Cost (Supply Chain).

5. Conclusion

The proposed model mathematically represents a scheme that achieves a coordinated solution in a two-tier supply chain for impulse purchase products. Also, managing economies of scale in the model was formulated by exploiting fixed costs present in the supply chain. Hence, products are ordered and delivered jointly, and the model will distribute the ordering cost over a number of items. In this way, the model brings economic and administrative benefits related to the management of impulse purchase products for both parties, minimizing total costs and optimizing logistics operations. A lower cycle inventory is better because the average flow time is lower, inventory holding costs are lower, and working capital requirements are lower. Via contracts, the vendor and buyer are willing to operate with the same order sizes.

In the same way, the validation stage provides evidence that managing economies of scale that exploit fixed costs brings advantages in financial terms by reducing the ordering costs of the vendor and the buyer. Satisfactory results were obtained by comparing an executed scenario with the proposed model (scenario 1) against a scenario that did not consider the management of economies of scale (scenario 2) over a period of five months, obtaining savings of 35.52%, 32.41% and 33.91%, for the vendor, buyer, and supply chain respectively. Similarly, by comparing the economic impact of model execution (scenario 1) against a scenario based on a traditional supply chain without any non-cooperative agreement (scenario 3) for five months, the proposed model obtained the best measurement results in terms of costs, with an improvement of approximately 47.55%, 42.78%, and 45.13% for the vendor, buyer and supply chain respectively.

Finally, the validation phase of the model can be complemented with additional real cases. This is an aspect that would provide an enormous benefit to ensure greater credibility. Through the execution of case studies that contain different sets of variables, which may alter the behavior for demand, such as, the product portfolio or the agreement guidelines. In addition, it is possible to use this research as a starting point to move towards much more elaborate models, which reflect greater complexity of the real system, and better representation of operational characteristics along the supply chain. As far as future research is concerned, this model presents several aspects to take into account, mainly its implementation, since it would be interesting to see its development, impact, and response with respect to other business environments.

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