JJMIE

Volume 12, Number 4, December. 2018
ISSN 1995-6665
Pages 269 - 279

Jordan Journal of Mechanical and Industrial Engineering

Mathematical Modelling for Reliability Measures to Sugar Mill Plant Industry

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Received 1 Feb. 2018

Abstract

The present paper aims to investigate a sugar mill plant for obtaining its various performance measures. A sugar plant comprises many components, such as feeding system, evaporation system and crystallization system. These systems relate to each other in series configuration. Further, the feeding system consists of cutting, crushing, bagasse system, and heat generating system as well. After the raw material pass through the feeding system, output goes to be evaporated and then pass through the crystallization for the final output, which is sugar. A mathematical mode is devoloped, using Markov process, for evaluating the various reliability measures e.g. availability, reliability, MTTF and expected profit, of the sugar plant. Critical components is obtained through sensitivity analysis for the same. The information regarding failure rates and repair rate of various subsystems are taken from their past records. The results of this research show that the reliability of the sugar mill plant is equally sensitive with respect to all considered failures except bagesses carrying system. Also, the MTTF of the sugar mill are the most affected by the failiure of evaporation and crystallization process.

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Keywords: Performance measures; supplementary variable technique; Markov birth-death process; transition state probability; crystallization system; feeding system;

1. Introduction

As technology grows, the complications of the industrial systems increase rapidly. So, to maintain this devolvement, one must make the systems reliable (as much as possible). Therefore, the main task of the management of the system is to optimize the reliability for the same. A lot of research has been done in context of industrial system for improving the various performance measures of the systems [1, 2, and 3]. Sugar plant is one of the examples of such type of system. The role of reliability theory in complex industrial systems is widely studied in the literature of reliability. A sugar plant is such a complex system which is studied in this work. Many researchers, including Kakar et. al [4] studied a sulphate juice pump (SJP) system working in a sugar mill with the assumption that repair equipment may also fail during the repair and found the availability, MTSF and busy period of the system. Sachdeva et. al [5] presented a reliability study of the pulping system of paper industry using Petri nets technique and found the maintenance strategies to enhance the performance of the pulping system, and thus found ‘how the maintenance and operation cost reduce?’. Tiwari et. al [6] developed a mathematical model for a steam generating system of thermal power plant and analyzed the performance of various subsystems of them. Gupta et. al [7] found the various reliability measures of a coal handling unit of a thermal power plant using Markov process. Khanduja et. al [8] have discussed the performance evaluation for washing unit of a paper plant and analysed the digesting system for the same by using genetic algorithm. Besides, the effect of genetic algorithm, various system parameters, such as steady state availability is also calculated. Kumar and Ram [9] evaluated some important reliability characteristics of a coal handling unit of a thermal power plant. They also found the expected profit for the same. Ram et. al [10] analysed and evaluated the reliability measures for various engineering models under the concept of Gumbel-Hougaard family copula. Mariajayaprakash and Senthilvelan [11] discussed the failures of the fuel feeding system which is frequently occurring in the co-generation boiler, of a sugar mill, and gives the solution to overcome these failures. Gonzalez et. al [12] gave a practical view about the behaviour of an industrial (Bioethanol plant) system to access its reliability and availability and conclude that cost estimation is a key factor that should be considered. Kumar et. al [13] analysed the crushing system of a sugar mill with general repair distribution and constant failure rates and draw some important reliability measures. Kumar et. al [14] discussed refining system of a sugar mill which consist four subsystems and analysed the performance. Bakhshesh et. al [15] investigated the accumulation and deposition of solid particles in a pipeline with asymmetric branches by using Lagrangian method and find the effects of some

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Important parameters on it, and concluded that the dependency is higher at high flow velocities. Patel and Singh [16] studied a four stroke diesel engine using blended methyl ester (B50) in order to optimize nitrous oxide emission with addition of di-tert-butyl peroxide with cetane improver and found 1% di-tert-butyl peroxide would give the optimum results for nitrous oxide reduction in the diesel engine.

Here, in this paper the authors have developed a mathematical model of a sugar plant which consists of feeding system, evaporation and crystallization process. The considered system may work in three different states, which are good state, degraded state, and failed state, throughout the process of sugar making. The flow diagram and transition state diagram are shown in Fig. 1(a) and 1(b) respectively.

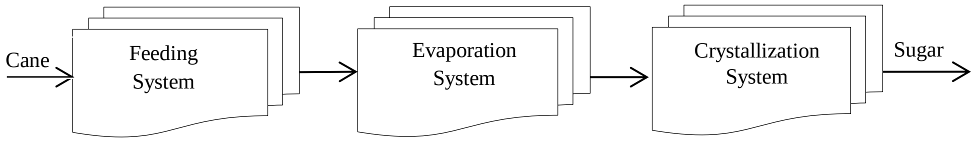

Figure 1(a). System configuration

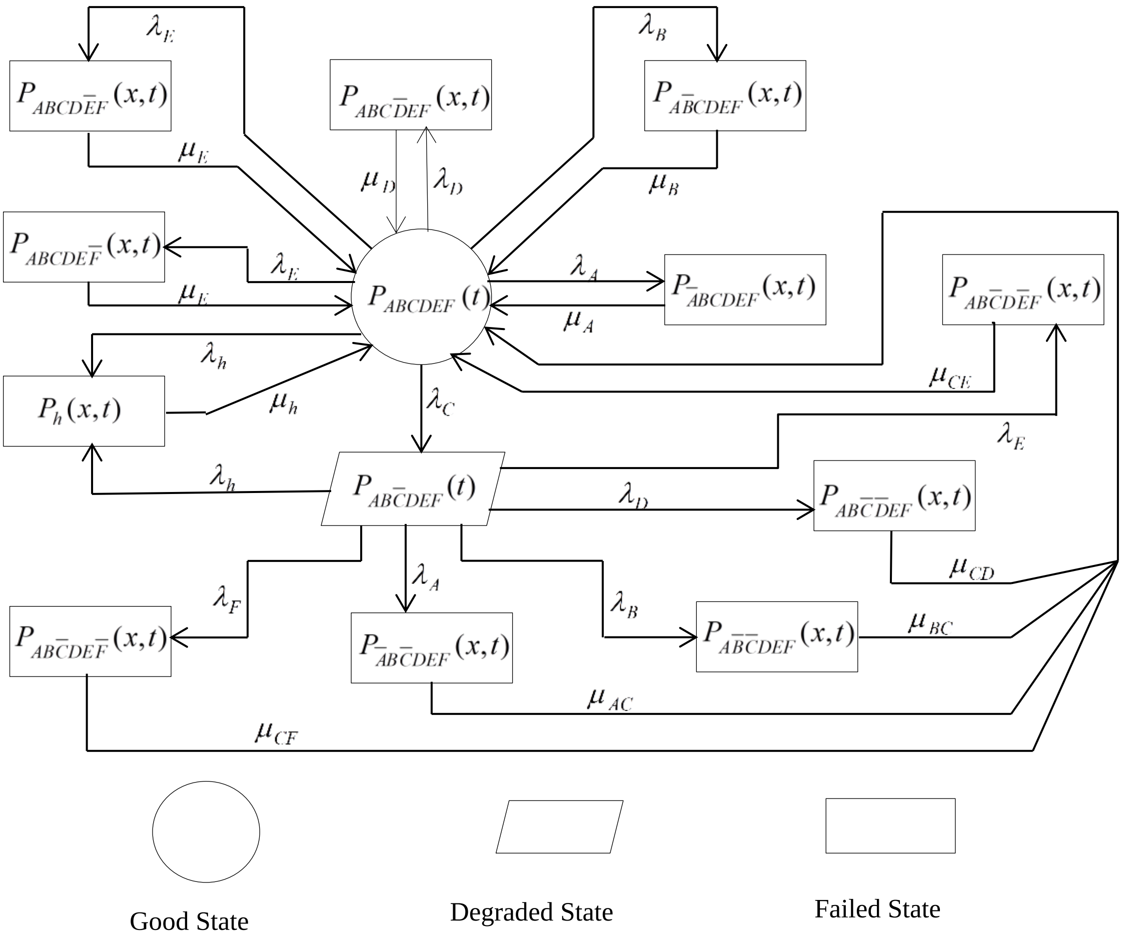

Figure 1(b). Transition State diagram

2. Assumptions and Notations

The following assumptions are used throughout the modelling

Initially all the components of the considered system are in good condition and hence it will work with full efficiency.

The system can also work in a reduced capacity (i.e. in degraded state).

At every instant repair, facilities are available.

All repair and failure rates are taken to be constant.

Raw material (i.e. cane) is always available to produce sugar.

The following notations have been used throughout the problem

$P_{ABCDEF}(t)$	The probability that at any instant *t* the plant is working with full efficiency.
$P_{AB\bar{C}DEF}(t)$	The probability that at any instant *t*, the plant is working in degraded state with failed bagasse carrying system.
$P_{\bar{A}BCDEF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of cutting process.
$P_{A\bar{B}CDEF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of crushing process.
$P_{ABC\bar{D}EF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of heat generating process.
$P_{ABCD\bar{E}F}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of evaporation process.
$P_{ABCDE\bar{F}}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of crystallization process.
$P_{\bar{A}B\bar{C}DEF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of cutting process and bagasse carrying process.
$P_{A\bar{B}\bar{C}DEF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of crushing process and bagasse carrying process.
$P_{AB\bar{C}\bar{D}EF}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of heat generating process and bagasse carrying process.
$P_{AB\bar{C}D\bar{E}F}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of evaporation process and bagasse carrying process.
$P_{AB\bar{C}DE\bar{F}}(x,t)$	The probability that at any instant *t*, the plant is failed by the failure of crystallization process and bagasse carrying process.
$P_h(x,t)$	The probability that at any instant *t*, the plant is failed by the human error.
$\lambda_A / \lambda_B / \lambda_C / \lambda_D / \lambda_E / \lambda_F / \lambda_h$	Failure rate of cutting process/crushing process/ bagasse carrying process/ heat generating process/evaporation process/crystallization process/human error.
$\mu_A / \mu_B / \mu_C / \mu_D / \mu_E / \mu_F / \mu_h$	Repair rate of cutting process/ crushing process/ bagasse carrying process/ heat generating process/ evaporation process/ crystallization process/ human failure.
$\mu_{AC} / \mu_{BC} / \mu_{CD} / \mu_{CE} / \mu_{CF}$	Simultaneous repair rate of cutting process and bagasse carrying process/ crushing process and bagasse carrying process/ heat generating process and bagasse carrying process/ evaporation process and bagasse carrying process/ crystallization process and bagasse carrying process.
K_1 / K_2	Revenue/service cost per unit time from the plant.
t/s	Time scale variable in years/ Laplace transforms variable.

3. Mathematical Formulation and Solution of the Considered Sugar Plant System

With the aid of Markov birth-death process the following set of intro-differential equation is developed

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$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h\right)P_{ABCDEF}(t)=\sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx \tag{1}$$

where $i = A, B, D, E, F, h, CF, AC, BC, CD, CE$; $j = \bar{A}BCDEF$, $A\bar{B}CDEF$, $ABC\bar{D}EF$, $ABCD\bar{E}F$, $ABCDE\bar{F}$, h, $AB\bar{C}DE\bar{F}$, $\bar{A}B\bar{C}DEF$, $A\bar{B}\bar{C}DEF$, $AB\bar{C}\bar{D}EF$, $AB\bar{C}D\bar{E}F$

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_D+\lambda_E+\lambda_F+\lambda_h\right)P_{AB\bar{C}DEF}(t)=\lambda_C P_{ABCDEF}(t) \tag{2}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_k\right)P_l(x,t)=0 \tag{3}$$

where $k = A, B, D, E, F, h, CF, AC, BC, CD, CE$;
$l = \bar{A}BCDEF$, $A\bar{B}CDEF$, $ABC\bar{D}EF$, $ABCD\bar{E}F$, $ABCDE\bar{F}$, h, $AB\bar{C}DE\bar{F}$, $\bar{A}B\bar{C}DEF$, $A\bar{B}\bar{C}DEF$, $AB\bar{C}\bar{D}EF$, $AB\bar{C}D\bar{E}F$

Boundary conditions

$$P_j(0,t)=\lambda_k P_l(t) \tag{4}$$

where $j = \bar{A}BCDEF$, $A\bar{B}CDEF$, $ABC\bar{D}EF$, $ABCD\bar{E}F$, $ABCDE\bar{F}$, h, $AB\bar{C}DE\bar{F}$, $\bar{A}B\bar{C}DEF$, $A\bar{B}\bar{C}DEF$, $AB\bar{C}\bar{D}EF$, $AB\bar{C}D\bar{E}F$;
$k = A, B, D, E, F, h, F, A, B, D, E$;
$l = ABCDEF$, $ABCDEF$, $ABCDEF$, $ABCDEF$, $ABCDEF$, $ABCDEF$, $AB\bar{C}DEF$, $AB\bar{C}DEF$, $AB\bar{C}DEF$, $AB\bar{C}DEF$, $AB\bar{C}DEF$

Initial condition

$$P_{ABCDEF}(t)=\begin{cases}1 & ,t=0\\ 0 & ,\text{othetwise}\end{cases} \tag{5}$$ and all other state probabilities are zero at t = 0

Taking Laplace transformation from equations (1) to (4), one gets

$$(s+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h)\bar{P}_{ABCDEF}(s)=1+\sum_{i,j}\int_0^\infty \mu_i \bar{P}_j(x,s)dx \tag{6}$$

$$(s+\lambda_A+\lambda_B+\lambda_D+\lambda_E+\lambda_F+\lambda_h)\bar{P}_{AB\bar{C}DEF}(s)=\lambda_C \bar{P}_{ABCDEF}(s) \tag{7}$$

$$\left(\frac{\partial}{\partial x}+s+\mu_k\right)\bar{P}_l(x,s)=0 \tag{8}$$

Boundary conditions

$$P_j(0,t)=\lambda_k P_l(t) \tag{9}$$

Solving equations from (6) to (8) with the help of boundary conditions, we get the transition state probabilities as

$$\bar{P}_{ABCDEF}(s)=\frac{1}{[H_1-H_3-H_4]} \tag{10}$$

$$\bar{P}_{AB\bar{C}DEF}(s)=\frac{\lambda_C}{H_2}\bar{P}_{ABCDEF}(s) \tag{11}$$

$$\bar{P}_{\bar{A}BCDEF}(s)=\frac{\lambda_A}{(s+\mu_A)}\bar{P}_{ABCDEF}(s) \tag{12}$$

$$\bar{P}_{A\bar{B}CDEF}(s)=\frac{\lambda_B}{(s+\mu_B)}\bar{P}_{ABCDEF}(s) \tag{13}$$

$$\bar{P}_{ABC\bar{D}EF}(s)=\frac{\lambda_D}{(s+\mu_D)}\bar{P}_{ABCDEF}(s) \tag{14}$$

$$\bar{P}_{ABCD\bar{E}F}(s)=\frac{\lambda_E}{(s+\mu_E)}\bar{P}_{ABCDEF}(s) \tag{15}$$

$$\bar{P}_{ABCDE\bar{F}}(s)=\frac{\lambda_F}{(s+\mu_F)}\bar{P}_{ABCDEF}(s) \tag{16}$$

$$\bar{P}_h(s)=\frac{\lambda_h}{(s+\mu_h)}\bar{P}_{ABCDEF}(s) \tag{17}$$

$$\bar{P}_{AB\bar{C}DE\bar{F}}(s)=\frac{\lambda_C\lambda_F}{H_2(s+\mu_{CF})}\bar{P}_{ABCDEF}(s) \tag{18}$$

$$\bar{P}_{\bar{A}B\bar{C}DEF}(s)=\frac{\lambda_A\lambda_C}{H_2(s+\mu_{AC})}\bar{P}_{ABCDEF}(s) \tag{19}$$

$$\overline{P}_{A\bar{B}\bar{C}DEF}(s)=\frac{\lambda_B\lambda_C}{H_2(s+\mu_{BC})}\overline{P}_{ABCDEF}(s) \tag{20}$$

$$\overline{P}_{AB\bar{C}\bar{D}EF}(s)=\frac{\lambda_C\lambda_D}{H_2(s+\mu_{CD})}\overline{P}_{ABCDEF}(s) \tag{21}$$

$$\overline{P}_{AB\bar{C}D\bar{E}F}(s)=\frac{\lambda_C\lambda_E}{H_2(s+\mu_{CE})}\overline{P}_{ABCDEF}(s) \tag{22}$$

Where

$$H_1=(s+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h),\ H_2=(s+\lambda_A+\lambda_B+\lambda_D+\lambda_E+\lambda_F+\lambda_h)$$

$$H_3=\frac{\lambda_A\mu_A}{(s+\mu_A)}-\frac{\lambda_B\mu_B}{(s+\mu_B)}-\frac{\lambda_D\mu_D}{(s+\mu_D)}-\frac{\lambda_E\mu_E}{(s+\mu_E)}-\frac{\lambda_F\mu_F}{(s+\mu_F)}-\frac{\lambda_h\mu_h}{(s+\mu_h)}$$

$$H_4=\frac{\lambda_C\lambda_F\mu_{CF}}{H_2(s+\mu_{CF})}-\frac{\lambda_A\lambda_C\mu_{AC}}{H_2(s+\mu_{AC})}-\frac{\lambda_B\lambda_C\mu_{BC}}{H_2(s+\mu_{BC})}-\frac{\lambda_C\lambda_D\mu_{CD}}{H_2(s+\mu_{CD})}-\frac{\lambda_C\lambda_E\mu_{CE}}{H_2(s+\mu_{CE})}$$

From the transition state diagram, the probability that the sugar plant system is in up and downstate is given by

$$\overline{P}_{up}(s)=\overline{P}_{ABCDEF}(s)+\overline{P}_{AB\bar{C}DEF}(s) \tag{23}$$

$$\overline{P}_{down}(s)=\sum_j \overline{P}_j(s) \tag{24}$$

where $j=\bar{A}BCDEF,\ A\bar{B}CDEF,\ ABC\bar{D}EF,\ ABCD\bar{E}F,\ ABCDE\bar{F},\ \bar{A}B\bar{C}DEF,$
$A\bar{B}\bar{C}DEF,\ AB\bar{C}\bar{D}EF,\ AB\bar{C}D\bar{E}F,\ AB\bar{C}DE\bar{F},\ h$

4. Particular Cases and Numerical Computations

4.1. Availability Assessment

The time dependent availability of the sugar mill plant can be obtained (as given below) by putting the value different failure rates as $\lambda_E=0.21,\ \lambda_F=0.21,$ $\lambda_h=0.11\ \lambda_A=0.04, \lambda_B=0.04,$ $\lambda_C=0.045, \lambda_D=0.09$ and all the repair are taken as one (i.e. 100% maintenance are taken into consideration) in (23) then taking inverse Laplace transform, we get the availability of sugar mill plant as

$$P_{up}(t)=0.4107799801\,e^{(-1.703033898t)}-0.0025783751\,0e^{(-0.7380277981t)}+0.5917983951e^{(-0.0039383035\ 45\ t)} \tag{25}$$

Varying time unit t in the equation (25), we get the following Table 1 and Fig. 2 for availability for the sugar plant system.

Table 1. Availability vs. Time

Time (t)	Availatbility
0	1
1	0.66305
2	0.60019
3	0.58704
4	0.58286
5	0.58027
6	0.57796
7	0.57569
8	0.57343
9	0.57118
10	0.56894

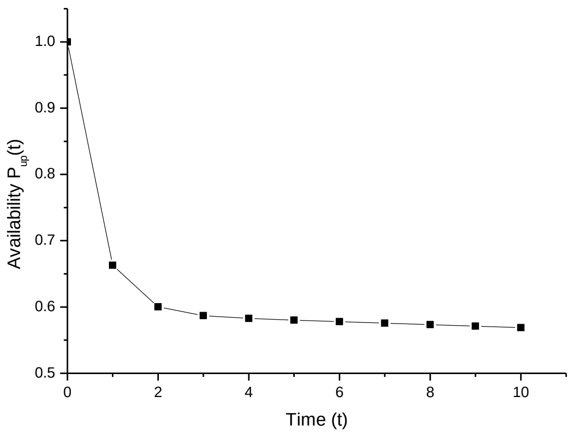

Figure 2. Availability vs. of Time

4.2. Reliability Assessment

Reliability investigation is one of the fundamental measures of ensuring safety in various industry/operations. However, the reliability assessment of these systems is too complex due to their multistate break down and multistate functionality. Reliability of a system is likelihood for performing its function for a given period of time under some specific/operating conditions. The reliability of the sugar mill plant will be obtained by taking its various failure rates as

$$\lambda_h=0.11, \lambda_A=0.04, \lambda_B=0.04, \lambda_C=0.045,$$
$$\lambda_D=0.09, \lambda_E=0.21, \lambda_F=0.21$$

and all repairs as zero (i.e. for calculating reliability of the considered system, the maintenance of its units are not taken into consideration) in (23) and taking the inverse Laplace transform, the reliability of the system is given as

$$R(t)=\mathrm{e}^{(-0.7450t)}+2\mathrm{e}^{(0.7225t)}\sinh(0.0225\ t) \tag{26}$$

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Now varying time unit t in the equation (26), one gets the Table 2 and Fig. 3 for reliability.

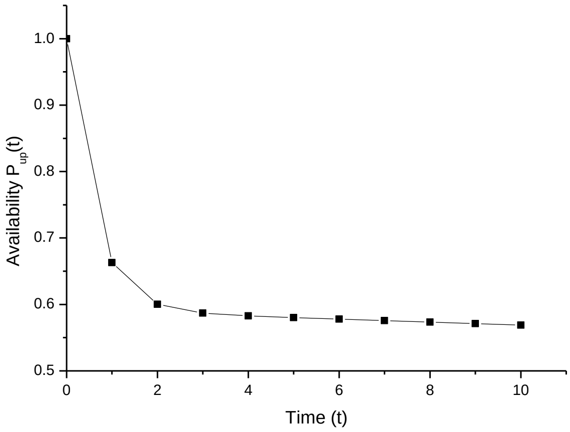

Figure 3. Reliability vs. Time

Table 2. Reliability vs. Time

Time (t)	Reliability $R(t)$
0	1.00000
1	0.49658
2	0.24659
3	0.12245
4	0.06081
5	0.03019
6	0.01499
7	0.00744
8	0.00369
9	0.00183
10	0.00091

4.3. Mean Time to Failure (MTTF) Assessment

Basically MTTF is the average failure time for an individual component. Mathematically it is calculated as

$$MTTF = \int_0^\infty t f(t)dt = \int_0^\infty R(t)dt = \lim_{s\to 0} \bar{R}(s)$$

Where $f(t)$ is the probability density function and $R(t)$ is system reliability

The MTTF of the considered system can be obtained by using (23) in the above expression of MTTF. Varying the failure rates one by one in the MTTF expression obtained in this step, Table 3 and corresponding Fig. 4 is obtained for MTTF of the considered system as:

4.4. Sensitivity Assessment

a) Sensitivity of Reliability

We carry out the sensitivity assessment of the reliability of sugar plant by differentiating the reliability expression with respect to various failure rates, and then setting

$$\lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09,$$
$$\lambda_E = 0.21, \lambda_F = 0.21, \lambda_h = 0.11$$

we get

$$\frac{\partial R(t)}{\partial \lambda_A}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_C}, \frac{\partial R(t)}{\partial \lambda_D}, \frac{\partial R(t)}{\partial \lambda_E}, \frac{\partial R(t)}{\partial \lambda_F}, \frac{\partial R(t)}{\partial \lambda_h}.$$

Now, setting t = 0 to 10 units of time in these partial derivatives, one can obtain the Table 4 and Fig. 5 respectively.

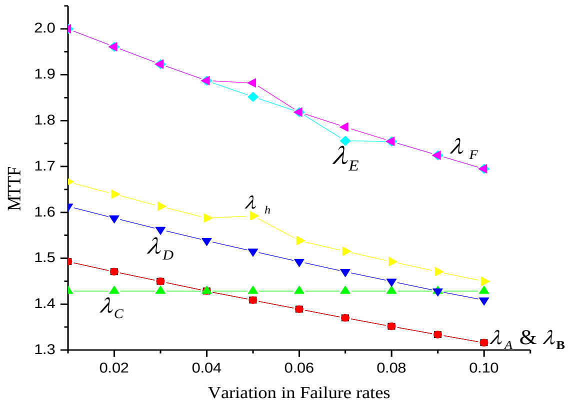

Figure 4. MTTF vs. Failure rates

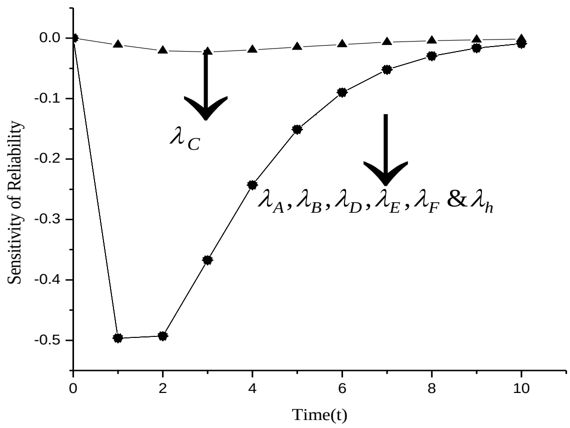

Figure 5. Sensitivity of Reliability vs. Time

Table 3. MTTF vs. Failure rates

Variations in $\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_E, \lambda_F, \lambda_h$	MTTF with respect to failure rates						
	λ_A	λ_B	λ_C	λ_D	λ_E	λ_F	λ_h
0.01	1.49253	1.49253	1.42857	1.61290	2.00000	2.00000	1.66666
0.02	1.47058	1.47058	1.42857	1.58730	1.96078	1.96078	1.63934
0.03	1.44927	1.44927	1.42857	1.56250	1.92307	1.92307	1.61290
0.04	1.42857	1.42857	1.42857	1.53846	1.88679	1.88679	1.58730
0.05	1.40845	1.40845	1.42857	1.51515	1.85185	1.88185	1.59250
0.06	1.38888	1.38888	1.42857	1.49253	1.81818	1.81818	1.53846
0.07	1.36986	1.36986	1.42857	1.47058	1.75571	1.78571	1.51515
0.08	1.35135	1.35135	1.42857	1.44927	1.75438	1.75438	1.49253
0.09	1.33333	1.33333	1.42857	1.42857	1.72413	1.72413	1.47058
0.10	1.31578	1.31578	1.42857	1.40845	1.69491	1.69491	1.44927

b) Sensitivity of MTTF Assessment

By differentiating MTTF expression with respect to failure rates and then putting various failure rates as $\lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09$, $\lambda_E = 0.21$, $\lambda_F = 0.21, \lambda_h = 0.11$ we get the values of $\frac{\partial(MTTF)}{\partial\lambda_A}, \frac{\partial(MTTF)}{\partial\lambda_B}, \frac{\partial(MTTF)}{\partial\lambda_C}, \frac{\partial(MTTF)}{\partial\lambda_D}, \frac{\partial(MTTF)}{\partial\lambda_E}, \frac{\partial(MTTF)}{\partial\lambda_F}, \frac{\partial(MTTF)}{\partial\lambda_h}$.

Varying the failure rates one by one as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in these partial derivatives, one can obtain the Table 5 and Fig. 6 respectively.

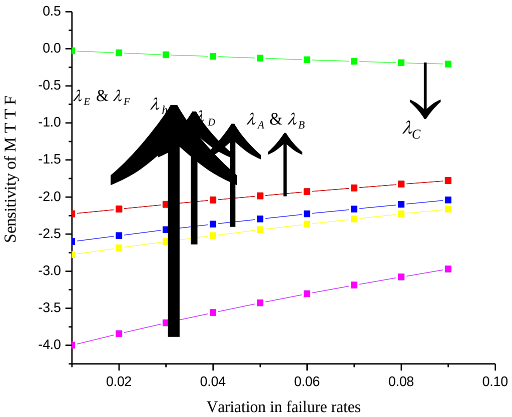

Figure 6. Sensitivity of MTTF vs. Failure rates

4.5. Expected Profit Assessment

The profit function [7] for the sugar plant during the time duration [0,t) is given as

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - tK_2 \tag{27}$$

Using Equation (25) in (27), profit function for the same set of parameters is given by

$$E_p(t) = \{K_1[-0.2412048172\ e^{(-1.703033\ 898\ t)} + 0.0034936016\ 05e^{(-0.7380277981\ )} - 150.2673393\ e^{(-0.003938303545\ t)} + 150.5050505] - K_2\ t\} \tag{28}$$

Now taking K_1= 1 and K_2 as 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 respectively than varying t in (28) one get the Table 6 and correspondingly Fig. 7.

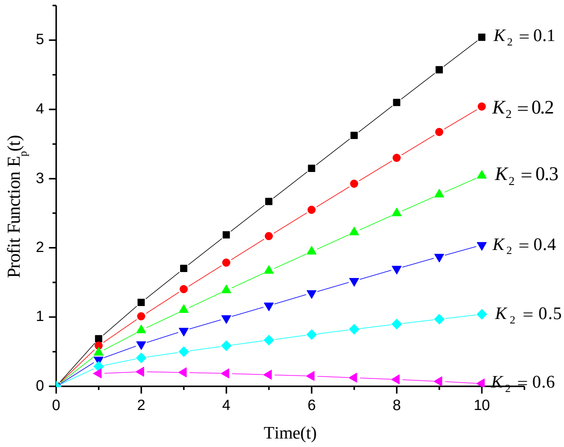

Figure 7. Expected profit vs. Failure rates

Table 4. Sensitivity of Reliability vs. Time

Time (t)	$\frac{\partial R(t)}{\partial\lambda_A}$	$\frac{\partial R(t)}{\partial\lambda_B}$	$\frac{\partial R(t)}{\partial\lambda_C}$	$\frac{\partial R(t)}{\partial\lambda_D}$	$\frac{\partial R(t)}{\partial\lambda_E}$	$\frac{\partial R(t)}{\partial\lambda_F}$	$\frac{\partial R(t)}{\partial\lambda_h}$
0	0	0	0	0	0	0	0
1	-0.49658	-0.49658	-0.01084	-0.49658	-0.49658	-0.49658	-0.49658
2	-0.49319	-0.49319	-0.02090	-0.49319	-0.49319	-0.49319	-0.49319
3	-0.36736	-0.36736	-0.02267	-0.36736	-0.36736	-0.36736	-0.36736
4	-0.24324	-0.24324	-0.01943	-0.24324	-0.24324	-0.24324	-0.24324
5	-0.15098	-0.15098	-0.01464	-0.15098	-0.15098	-0.15098	-0.15098
6	-0.08997	-0.08997	-0.01016	-0.08997	-0.08997	-0.08997	-0.08997
7	-0.05212	-0.05212	-0.00667	-0.05212	-0.05212	-0.05212	-0.05212
8	-0.02958	-0.02958	-0.00420	-0.02958	-0.02958	-0.02958	-0.02958
9	-0.01652	-0.01652	-0.00256	-0.01652	-0.01652	-0.01652	-0.01652
10	-0.00911	-0.00911	-0.00152	-0.00911	-0.00911	-0.00911	-0.00911

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Table 5. Sensitivity of MTTF vs. Failure rates

Variations in λ_A, λ_B, λ_h, λ_E, λ_{CCF}, λ_S	$\frac{\partial(MTTF)}{\partial\lambda_A}$	$\frac{\partial(MTTF)}{\partial\lambda_B}$	$\frac{\partial(MTTF)}{\partial\lambda_C}$	$\frac{\partial(MTTF)}{\partial\lambda_D}$	$\frac{\partial(MTTF)}{\partial\lambda_E}$	$\frac{\partial(MTTF)}{\partial\lambda_F}$	$\frac{\partial(MTTF)}{\partial\lambda_h}$
0.01	-2.22766	-2.22766	-0.02833	-2.60145	-4.00000	-4.00000	-2.77777
0.02	-2.16262	-2.16262	-0.05511	-2.51952	-3.84467	-3.84467	-2.68744
0.03	-2.10039	-2.10039	-0.08042	-2.44140	-3.69822	-3.69822	-2.60145
0.04	-2.04081	-2.04081	-0.10435	-2.36686	-3.55998	-3.55998	-2.51952
0.05	-1.98373	-1.98373	-0.12698	-2.29568	-3.42935	-3.42935	-2.44140
0.06	-1.92901	-1.92901	-0.14839	-2.22766	-3.30578	-3.30578	-2.36686
0.07	-1.87652	-1.87652	-0.16866	-2.16262	-3.18877	-3.18877	-2.29568
0.08	-1.82615	-1.82615	-0.18784	-2.10039	-3.07787	-3.07787	-2.22766
0.09	-1.77777	-1.77777	-0.20601	-2.04081	-2.97265	-2.97265	-2.16262

Table 6. Expected profit vs. Failure rates

Time(t)	Expected Profits					
	$K_2 = 0.1$	$K_2 = 0.2$	$K_2 = 0.3$	$K_2 = 0.4$	$K_2 = 0.5$	$K_2 = 0.6$
0	0	0	0	0	0	
1	0.68608	0.58608	0.48608	0.38608	0.28608	0.18608
2	1.20945	1.00945	0.80945	0.60945	0.40945	0.20945
3	1.70158	1.40158	1.10158	0.80158	0.50158	0.20158
4	2.18627	1.78627	1.38627	0.98627	0.58627	0.18627
5	2.66779	2.16779	1.66779	1.16779	0.66779	0.16779
6	2.14691	2.54691	1.94691	1.34691	0.74691	0.14691
7	3.62373	2.92373	2.22373	1.52373	0.82373	0.12373
8	4.09830	3.29830	2.49830	1.69830	0.89830	0.09830
9	4.57061	3.67061	2.77061	1.87061	0.97061	0.07061
10	5.04067	4.04067	3.04067	2.04067	1.04067	0.04067

5. Result Discussion

Keeping in mind the above figures we have:

- From Fig 2, it has been observed that the availability of the system is first decreasing rapidly and then swiftly as time passes.
- From Fig 3, the reliability of the system decreases smoothly as time passes.
- From Fig 4, it has been observed that the MTTF of the system is decreasing with respect to all type of failures except the failure rate of bagasse carrying unit. MTTF with respect failure rate of bagasse carrying system is approximate constant.
- Fig 5 shows the sensitivity assessment with respect to system reliability. From this one can see that the sensitivity of reliability is approximate constant with respect to failure rate of bagasse carrying system and for remaining failure rates of the system it first decrees and then increase.
- Fig 6 shows the sensitivity of MTTF. It reflects that the MTTF of sugar mill plant is equally sensitive with respect to the failure rate of cutting and crushing system, evaporation and crystallization and it is approximately constant with respect to failure rate of bagasse carrying system.
- From, Fig. 7, it is very clear that the profit decreases as the service cost increases with the passage of time unit.

6. Conclusion

In this work, the feeding system, evaporation system, and crystallization process of a sugar plant have been discussed. Based on the above calculation, we have concluded that the failure rate of bagasse carrying system has not so much impact on the production of the sugar mill plant and MTTF of the same is much sensitive with respect to the failure rate of sub parts of feeding system (cutting and crusher system). Also, the most surprising thing is that the reliability has same characteristics with respect to all types of failures Except bagasse carrying system. So, to make the sugar plant system more reliable, the managers and engineers must consider these points, and should try to reduce the failure rates for more production of sugar.

From this work, one could improve the sugar plant overall performance by restricting its failure rates and to make it less sensitive. Further, it asserts that the finding of this paper is highly advantageous to the management of the sugar plant industry.

References

[1] V. Kolhe, R. E. Shelke and S. S.Khandare, "Combustion Modeling with CFD in Direct Injection CI Engine Fuelled with Biodiesel". Jordan Journal of Mechanical and Industrial Engineering, Vol. 9 (2015) No. 1, 61-66.

[2] A. Goyal, S. Dhiman, S. Kumar and R. Sharma, "A Study of Experimental Temperature Measuring Techniques used in Metal Cutting". Jordan Journal of Mechanical and Industrial Engineering, Vol. 8 (2014) No. 2, 82-93.

[3] V. Kolhe, R. E. Shelke, S. S. Khandare, "Performance and Combustion Characteristics of a DI Diesel Engine Fueled With Jatropha Methyl Esters and Its Blends". Jordan Journal of Mechanical and Industrial Engineering, Vol. 8 (2014) No. 1, 7-12.

[4] M. Kakar, A. K. Chitkar, and J. Bhatti, "Probability analysis of a complex system working in a sugar mill with repair equipment failure and correlated life time". Mathematical Journal of Interdisciplinary Science, Vol. 1 (2012) No. 1, 57-66.

[5] A. Sachdeva, D. Kumar and P. Kumar, "Reliability Analysis of pulping system using Petri nets". International Journal of Quality and Reliability Management, Vol. 25 (2008) No. 8, 860-877.

[6] P. C. Tewari, S. Kajal and R. Khanduja, "Performance evaluation and availability analysis of steam generating system in a thermal power plant". In Proceedings of the World Congress on Engineering, Vol. 3 (2012), 4-6.

[7] S. Gupta, P. C. Tewari and A. K. Sharma, "A Markov model for performance evaluation of a coal handling unit of a thermal power plant". Journal of Industrial and System Engineering, Vol. 3 (2009) No. 2, 85-96.

[8] R. Khanduja, P. C. Tewari and D. Kumar, "Mathematical modelling and performance optimization for the digesting system of a paper plant". International Journal of Engineering, Vol. 23 (2010) No. 3&4, 215-225.

[9] A. Kumar, M. Ram, "Reliability measure improvement and sensitivity analysis of a coal handling unit for a thermal power plant". International Journal of Engineering, Vol. 26 (2013) No. 9, 1059-1066.

[10] M. Ram, S. B. Singh and V. V. Singh, "Stochastic analysis of a standby system with waiting repair strategy". IEEE Transactions on Systems, Man, and Cybernetics: Systems, Vol. 43 (2013) No. 3, 698-707.

[11] A. Mariajayaprakash, T. Senthilvelan, "Failure detection and optimization of sugar mill boiler using FMEA and Taguchi method". Engineering Failure Analysis, Vol. 30 (2013), 17-26.

[12] V. Gonzalez, J. F. Gomez, M. Lopez, A. Crespo and P. M. de Leon, "Availability and reliability assessment of industrial complex systems: A practical view applied on a bioethanol plant simulation". Safety, Reliability and Risk Analysis: Theory, Methods and Applications–Martorell et al.(eds); 2009, 687-695.

[13] D. Kumar, J. Singh and P. C. Pandey, "Design and cost analysis of a refining system in the sugar industry". Microelectronics Reliability, Vol. 30 (1990) No. 6, 1025-1028.

[14] D. Kumar, J. Singh, and I. P. Singh, "Availability of the feeding system in the sugar industry". Microelectronics Reliability, Vol. 28 (1988) No. 6, 867-871.

[15] M. Bakhshesh, E. G. Rad and M. Mehrvar, "Effect of Asymmetric Branches on Solid Particles Distribution in Central Gas Stations (CGSs)". Jordan Journal of Mechanical and Industrial Engineering, Vol. 8 (2014) No. 2, 56-65.

[16] N. K. Patel and R. N. Singh, "Optimization of NOx Emission from soya Biodiesel Fuelled Diesel Engine using cetane Improver". Jordan Journal of Mechanical and Industrial Engineering, Vol. 8 (2014) No. 4, 213-217.

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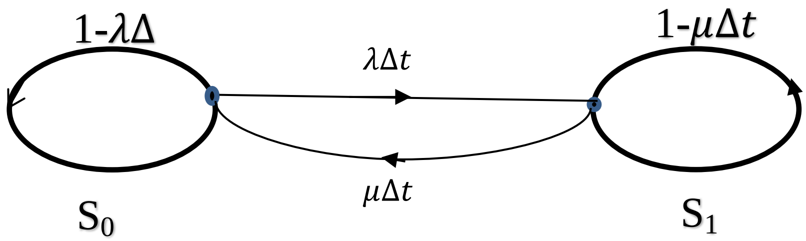
Appendix A

Markov Birth-death process

Markov process or Markov birth-death process is a very useful tool for analysing random events which are dependent on each other. It is the most powerful technique in the field of reliability, which helps us to evaluate the system's various performance measures. It is named after the Russian mathematician "Andrei Andreyevich Markov".

It is a process in which transition from one state to another state (future state) depends only on the present state of the system and does not depend on the past state or one can say that the transition does not depend on what happened in the past with the system or we can say that the future stage is dependent only on the present state. As the part of the process, initially based on system configuration a state transition diagram is created, then by the Markov process, a number of differential equations are generated (on the basis of input and output or repair and failure) and then by solving these equations with the help of Laplace transformation, we get the required system's transition state probabilities. Now with the help of these transition state probabilities, the various reliability measures are calculated.

Let us consider a system having only one element. The element can be in one of two states, s_0 or s_1 (i.e., functioning or non-functioning states as shown in following figure). Since the system considered is repairable, a transition is possible from state s_1 to state s_0.



The equation which represents the state S_0 is given as

$p_{s_0}(t+\Delta t) = (1-\lambda\Delta t)p_{s_0}(t) + p_{s_1}(t)\mu\Delta t$

Similarly, the equation for the state S_1 can be developed.

Appendix B

Formulation of the intro-differential equations for the various state of sugar mill plant

Using Markov birth–death process, we can find the probability of the system to be in initial state in the interval $(t, t+\Delta t)$ as

For the state good state $P_{ABCDEF}(t)$

$$P_{ABCDEF}(t+\Delta t) = (1-\lambda_A\Delta t)(1-\lambda_B\Delta t)(1-\lambda_C\Delta t)(1-\lambda_D\Delta t)(1-\lambda_E\Delta t)(1-\lambda_F\Delta t)$$

$$(1-\lambda_h\Delta t)P_{ABCDEF}(t) + \sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx$$

$$\frac{P_{ABCDEF}(t+\Delta t) - P_{ABCDEF}(t)}{\Delta t} + (\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h)P_{ABCDEF}(t)$$

$$= \sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx$$

Now taking $\lim_{\Delta t\to 0}$, we get

$$\lim_{\Delta t\to 0}\frac{P_{ABCDEF}(t+\Delta t) - P_{ABCDEF}(t)}{\Delta t} + (\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h)P_{ABCDEF}(t)$$

$$= \sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx$$

$$\left(\frac{\partial}{\partial t} + \lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E+\lambda_F+\lambda_h\right)P_{ABCDEF}(t) = \sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx \quad (1)$$

where $i = A, B, D, E, F, h, CF, AC, BC, CD, CE;\ j = \overline{A}BCDEF,$
$A\overline{B}CDEF, ABC\overline{D}EF, ABCD\overline{E}F, ABCDE\overline{F}, h, AB\overline{C}DE\overline{F},$
$\overline{A}B\overline{C}DEF, A\overline{B}\overline{C}DEF, AB\overline{C}\overline{D}EF, AB\overline{C}D\overline{E}F$

For degraded state (when the Sugar mill plant is working in degraded state due to failure of bagasse carrying process) $P_{AB\overline{C}DEF}(t)$

$$P_{AB\overline{C}DEF}(t+\Delta t) = (1-\lambda_A \Delta t)(1-\lambda_B \Delta t)(1-\lambda_D \Delta t)(1-\lambda_E \Delta t)(1-\lambda_F \Delta t)(1-\lambda_h \Delta t)P_{AB\overline{C}DEF}(t) + \lambda_C P_{ABCDEF}(t)$$

$$\frac{P_{AB\overline{C}DEF}(t+\Delta t) - P_{AB\overline{C}DEF}(t)}{\Delta t} + (\lambda_A + \lambda_B + \lambda_D + \lambda_E + \lambda_F + \lambda_h)P_{AB\overline{C}DEF}(t) = \lambda_C P_{ABCDEF}(t)$$

Now taking $\lim\limits_{\Delta t \to 0}$, we get

$$\lim_{\Delta t \to 0} \frac{P_{AB\overline{C}DEF}(t+\Delta t) - P_{AB\overline{C}DEF}(t)}{\Delta t} + (\lambda_A + \lambda_B + \lambda_D + \lambda_E + \lambda_F + \lambda_h)P_{AB\overline{C}DEF}(t) = \lambda_C P_{ABCDEF}(t)$$

$$\left(\frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_D + \lambda_E + \lambda_F + \lambda_h\right)P_{AB\overline{C}DEF}(t) = \lambda_C P_{ABCDEF}(t) \tag{2}$$

For Failed states (states which occurs due to complete failure of cutting process/crushing process/heat generating process/ evaporation process/ crystallization process/ human error) $P_i(x,t)$

$$P_i(x+\Delta x, t+\Delta t) = \{1-\mu_k \Delta t\} P_i(x,t)$$

$$\Rightarrow \frac{P_i(x+\Delta x, t+\Delta t) - P_i(x,t)}{\Delta t} + \mu_k P_i(x,t) = 0$$

Taking $\underset{\Delta x \to 0, \Delta t \to 0}{\text{limit}}$, we get

$$\Rightarrow \underset{\Delta x \to 0, \Delta t \to 0}{\text{limit}} \frac{P_i(x+\Delta x, t+\Delta t) - P_i(x,t)}{\Delta t} + \mu_k P_i(x,t) = 0$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_k\right)P_l(x,t) = 0 \tag{3}$$

where $k = A, B, D, E, F, h, CF, AC, BC, CD, CE;$
$i = \overline{A}BCDEF, A\overline{B}CDEF, ABC\overline{D}EF, ABCD\overline{E}F, ABCDE\overline{F}, h, AB\overline{C}DE\overline{F},$
$\overline{A}B\overline{C}DEF, A\overline{B}\overline{C}DEF, AB\overline{C}\overline{D}EF, AB\overline{C}D\overline{E}F$

Boundary conditions of the system are obtained corresponding to transitions between the states where transition from a state with and without elapsed repair time exists, with elapsed repair times x and 0. Hence we have the following boundary/initial conditions:

$$P_j(0,t) = \lambda_k P_l(t) \tag{4}$$

where $j = \overline{A}BCDEF, A\overline{B}CDEF, ABC\overline{D}EF, ABCD\overline{E}F, ABCDE\overline{F}, h,$
$AB\overline{C}DE\overline{F}, \overline{A}B\overline{C}DEF, A\overline{B}\overline{C}DEF, AB\overline{C}\overline{D}EF, AB\overline{C}D\overline{E}F;$
$k = A, B, D, E, F, h, F, A, B, D, E;$
$l = ABCDEF, ABCDEF, ABCDEF, ABCDEF, ABCDEF, ABCDEF,$
$AB\overline{C}DEF, AB\overline{C}DEF, AB\overline{C}DEF, AB\overline{C}DEF, AB\overline{C}DEF$

Initial condition

$$P_{ABCDEF}(t) = \begin{cases} 1 & ,t=0 \\ 0 & ,\text{othetwise} \end{cases} \quad \text{and all other state probabilities are zero at t = 0} \tag{5}$$