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Availability Analysis of an Industrial System using Supplementary Variable Technique

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Abstract

The objective of this research is to develop a mathematical model for analyzing the availability of a butteroil production system. The industrial system consists of four subsystems; viz heater, clarifier, filling and granulation. From the state transition diagram of the system, using mnemonic rule and under the assumption of constant failure rates and variable repair rates, Chapman-Kolmogorov differential equations have been derived by applying supplementary variable techniques. These equations have been solved by Lagrange's method and availability of the system has been computed for various choices of failure and repair rates using Runge-Kutta fourth order method. Mean time between failure has been calculated numerically. Finally, criticality analysis has been done to get some ideas of the maintenance priority and assist the plant management in deciding maintenance priorities for optimum utilization of the resources.

Keywords: Availability; Supplementary Variable Technique; Lagrange's method; Runge-Kutta; MTBF;

NOMENCLATURE

H, C, F	Respective subsystems are working at full capacity
h, c, f	Respective subsystems are in failed state
C_s, F_s	One respective subsystem has failed
$P_0(t)$	Probability that at time't', all the units are working
$P_i(x,t)$	Probability that at time't', the system is in state i and having an elapsed repair time x
Δt	Time increment
$\frac{\partial}{\partial t}$	Derivative w.r.t. 't'

Greek Symbols

α_i $(i = 1\ to\ 3)$	Failure rates of subsystems H,C and F
β_i $(i = 1\ to\ 3)$	Repair rates of subsystems H,C and F

1. Introduction

It is important for an industrial system to run failure free for long duration of time without many interruptions. Availability of the system can be increased either by providing enough redundant parts or by increasing the reliability of its components. Although redundancy can be considered a best option but being very expensive, it is not always desirable. On the other hand, continuous monitoring of parts, provision of enough repair facilities and prompt response to any breakdown seems to be a better option. The purpose is to bring the failed system back to work in the shortest possible time. Study of butteroil production system in this paper has been undertaken under these considerations.

Reliability of various systems has been analyzed by various researchers using different techniques. [1] analyzed the availability of steam generation system of a thermal power plant taking constant failure and repair rates and derived expressions for steady state availability and Mean Time Between Failure (MTBF). [2] presented Markov models to derive the transient reliability and MTBF for repairable K-out-of-N: G systems subject to two failure modes. [3] developed a procedure based on graph theory and matrix approach for the reliability evaluation and selection of a rolling element bearing. [4] developed a multi-modal adaptive importance sampling method for reliability analysis of a vehicle body–door subsystem with respect to wind noise. [5] proposed an expression and an algorithm for computing reliability of K-out-of-N system. [6] studied the steady-state availability and the mean up-time of a series–parallel repairable system under the assumption of constant failure rate and arbitrary repair time, by using Supplementary Variable Technique (SVT) and vector Markov process theory. [7] introduced a fourth-order, implicit, low-dispersion, and low-dissipation Runge-Kutta scheme. [8] investigated the reliability analysis of a multi-state manufacturing system with different performance levels. [9] constructed a fifth-order explicit

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exponential Runge-Kutta method and proved its convergence for semi linear parabolic problems.

[10] presented a type of mixed Runge-Kutta methods by combining the underlying Runge-Kutta methods and the compound quadrature rules. [11] used Universal Generating Function (UGF) technique for reliability analysis of lithium-ion battery pack. [12] proposed a reliability analysis method of a multi-state system based on fuzzy Bayesian networks. [13] obtained some reliability measures of two cold standby units of a computer system using a semi-Markov process and a regenerative point technique. [14] worked on k-out-of-n standby subsystems with exponentially distributed component lifetimes and analyzed system reliability, mean time to failure, and steady-state availability as a function of the component failure rates. [15] developed performance model based on Markov birth-death process and calculated reliability, availability, maintainability, dependability, MTBF, Mean Time to Repair (MTTR) and dependability ratio for each subsystem of skim milk powder production system. [16] analyzed the availability of an engineering system by incorporating waiting time to repair and using supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula. [17] extended the matrix-based system reliability method to k out of N systems by modifying the formulations of event and probability vectors. [18] proposed a reliability, availability and maintainability (RAM) model to quantify the values of RAM indices and to identify the most critical equipment which mainly affects the system performance. [19] adopted six sigma and Gauss-Legendre quadrature formula to propose a generalized dynamic reliability model for calculating system reliability under complex load.

With a view to maximize availability and hence production; in this paper; reliability of the butteroil production system has been evaluated by considering constant failure rate and variable repair rate. System of differential equations has been developed using SVT and availability has been calculated using Runge-Kutta fourth order method. MTBF has been calculated using Simpson's 3/8 rule. In the conclusion part, criticality analysis of all the subsystems has been done to decide the maintenance priority of different subsystems.

The rest of this paper is organized as follows: Section 2 consists of brief description of the system and assumptions made in the analysis. Differential equations have been derived and solved in Section 3. In Section 4, transient state availability of the system has been computed by Runge-Kutta fourth order method using different combinations of failure and repair rates. MTBF has also been calculated in each case. Finally, conclusions have been drawn in Section 5.

2. Butteroil production system

Butteroil or ghee refers to the clarified butter fat obtained mainly from butter by means of removing all the water and SNF (solids-not-fat) contents. It is the richest source of milk fat and is prepared either by butter or cream. Butteroil production system consists of four subsystems i.e. heater, clarifier, filling and granulation. Out of these, except granulation subsystem, all the units are subject to random failures. Figure 1 gives us the flow chart of butteroil making process.

2.1. System description

2.1.1. Heater subsystem (H): It consists of a kettle in which temperature of butter is raised slowly with the help of steam. The final temperature is monitored to be not more than 107-109°C till its color is reddish brown. Ghee along with the residue can settle down for 25-30 minutes in the kettle before filtration. It consists of two units in series. Hence, if one unit fails, system fails.

2.1.2. Clarifier subsystem (C): Clarification is carried out at around 70°C in order to clarify all the residue particles from ghee. It consists of two units in parallel. Partial failure of this system reduces the capacity of the system. Major failure occurs only when both units fail.

2.1.3. Filling subsystem (F): In this section, ghee tins are filled, weighed and sealed simultaneously. The filling temperature is strictly watched to remain between 40-45°C. There are two filling units. Failure of any one unit reduces the working capacity while system completely fails when both units break down.

2.1.4. Granulation subsystem (G): This subsystem consists of a refrigerating unit where temperature is maintained between 15-20°C. This section rarely fails and hence has not been considered for analysis.

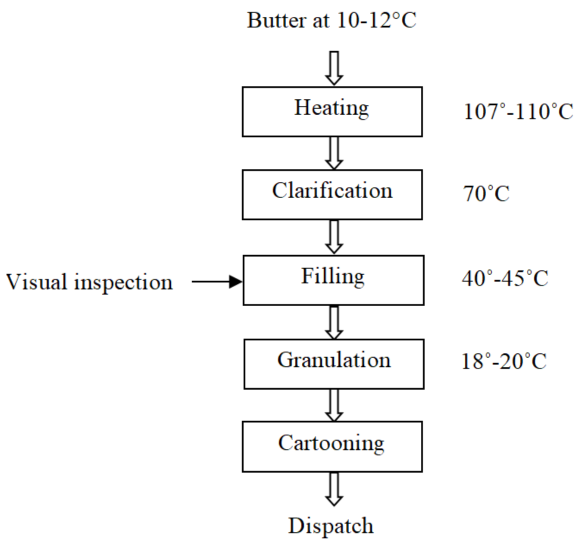

Figure 1. Schematic diagram of butteroil production system

2.2. Assumptions

Following assumptions have been made in the current analysis:

1. Failure and repair rates are constant and independent of each other and their unit is taken as per day.
2. In case of assessment of availability using SVT, repair rates are considered variable and failure rates as constant.
3. After repair, old unit is as good as new.
4. Enough repair/ maintenance facilities are available.
5. There are no simultaneous failures.
6. System may work at reduced capacity.

3. Mathematical Formulation of the System

To determine the reliability of the butteroil production system, Chapman-Kolmogorov differential equations have been developed by applying supplementary variable technique. A supplementary variable 'x' is added to change the non-Markovian event into Markovian event. Probability considerations, using mnemonic rule, give us the following set of differential equations associated with the transition diagram (Fig. 2) of the system at time (t+Δt):

$$P_0(t+\Delta t) = [1-\alpha_1\Delta t-\alpha_2\Delta t-\alpha_3\Delta t]P_0(t) + \int \beta_1(x)P_6(x,t)dx\Delta t + \int \beta_2(x)P_2(x,t)dx\Delta t + \int \beta_3(x)P_1(x,t)dx\Delta t$$

$$P_0(t+\Delta t) - P_0(t) = -[\alpha_1\Delta t+\alpha_2\Delta t+\alpha_3\Delta t]P_0(t) + \int \beta_1(x)P_6(x,t)dx\Delta t + \int \beta_2(x)P_2(x,t)dx\Delta t + \int \beta_3(x)P_1(x,t)dx\Delta t$$

Dividing both sides by Δt, we get

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -[\alpha_1+\alpha_2+\alpha_3]P_0(t) + \int \beta_1(x)P_6(x,t)dx + \int \beta_2(x)P_2(x,t)dx + \int \beta_3(x)P_1(x,t)dx$$

$$\left[\frac{\partial}{\partial t}+L_0\right]P_0(t) = M_0(t) \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_1(x)\right]P_1(x,t) = M_1(x,t) \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_2(x)\right]P_2(x,t) = M_2(x,t) \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_3(x)\right]P_3(x,t) = M_3(x,t) \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1(x)\right]P_j(x,t) = 0; \quad j = 4,6,7,11 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_2(x)\right]P_k(x,t) = 0; \quad k = 5,8 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_3(x)\right]P_l(x,t) = 0; \quad l = 9,10 \quad (7)$$

Where,

$$L_0 = \sum_{i=1}^{3}\alpha_i$$

$$L_1(x) = \sum_{i=1}^{3}\alpha_i + \beta_3(x)$$

$$L_2(x) = \sum_{i=1}^{3}\alpha_i + \beta_2(x)$$

$$L_3(x) = \sum_{i=1}^{3}\alpha_i + \beta_2(x) + \beta_3(x)$$

$$M_0(t) = \int P_1(x,t)\beta_3(x)dx + \int P_2(x,t)\beta_2(x)dx + \int P_6(x,t)\beta_1(x)dx$$

$$M_1(x,t) = \alpha_3P_0(t) + \int P_3(x,t)\beta_2(x)dx + \int P_{10}(x,t)\beta_3(x)dx + \int P_{11}(x,t)\beta_1(x)dx$$

$$M_2(x,t) = \alpha_2P_0(t) + \int P_3(x,t)\beta_3(x)dx + \int P_4(x,t)\beta_1(x)dx + \int P_5(x,t)\beta_2(x)dx$$

$$M_3(x,t) = \alpha_2P_1(t) + \alpha_3P_2(t) + \int P_7(x,t)\beta_1(x)dx + \int P_8(x,t)\beta_2(x)dx + \int P_9(x,t)\beta_3(x)dx$$

Initial Conditions

$$P_0(0) = 1$$

$$P_i(x,0) = 0 \qquad (i = 1,2,3 \ldots\ldots\ldots\ldots 11)$$

Boundary Conditions

$$P_1(0,t) = \alpha_3P_0(t)$$

$$P_2(0,t) = \alpha_2P_0(t)$$

$$P_3(0,t) = \int \alpha_2P_1(x,t)dx + \int \alpha_3P_2(x,t)dx$$

$$P_4(0,t) = \int \alpha_1P_2(x,t)dx$$

$$P_5(0,t) = \int \alpha_2P_2(x,t)dx$$

$$P_6(0,t) = \alpha_1P_0(t)$$

$$P_7(0,t) = \int \alpha_1P_3(x,t)dx$$

$$P_8(0,t) = \int \alpha_2P_3(x,t)dx$$

$$P_9(0,t) = \int \alpha_3P_3(x,t)dx$$

$$P_{10}(0,t) = \int \alpha_3P_1(x,t)dx$$

$$P_{11}(0,t) = \int \alpha_1P_1(x,t)dx$$

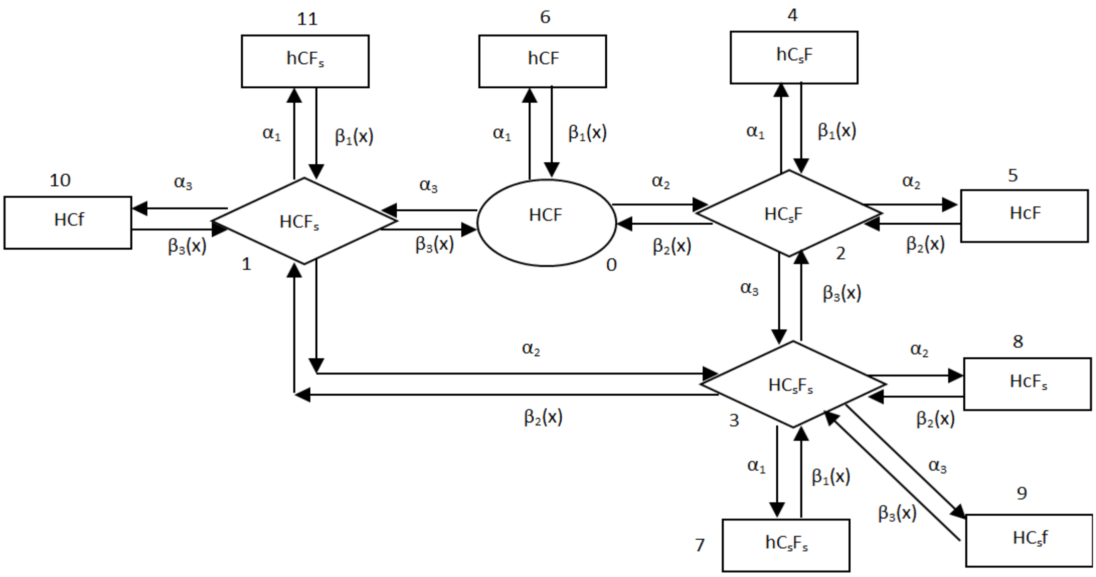

Figure 2. Transition diagram of butteroil production system

Set of differential equations from (1) to (7) along with initial conditions and boundary conditions is called Chapman-Kolmogorov differential difference equations Equation (1) is a linear differential equation of first order and eqs. (2) to (7) are linear partial differential equations of first order (Lagrange's type). All these equations have been solved using Lagrange's method. The probabilities of each state and expression of availability has been derived as follows:

$$P_0(t) = e^{-L_0 t}[1 + \int M_0(t) e^{L_0 t}\, dt]$$

$$P_1(x,t) = e^{-L_1(x)dx}\left[\int M_1(x,t) e^{\int L_1(x)dx}\, dx + \alpha_3 P_0(t-x)\right]$$

$$P_2(x,t) = e^{-L_2(x)dx}\left[\int M_2(x,t) e^{\int L_2(x)dx}\, dx + \alpha_2 P_0(t-x)\right]$$

$$P_3(x,t) = e^{-L_3(x)dx}\left[\int M_3(x,t) e^{\int L_3(x)dx}\, dx + \alpha_3 P_0(t-x)\right]$$

$$P_4(x,t) = e^{-\int \beta_1(x)dx} \int \alpha_1 P_2(x, t-x)dx$$

$$P_5(x,t) = e^{-\int \beta_2(x)dx} \int \alpha_2 P_2(x, t-x)dx$$

$$P_6(x,t) = e^{-\int \beta_1(x)dx} \alpha_1 P_0(t-x)$$

$$P_7(x,t) = e^{-\int \beta_1(x)dx} \int \alpha_1 P_3(x, t-x)dx$$

$$P_8(x,t) = e^{-\int \beta_2(x)dx} \int \alpha_2 P_3(x, t-x)dx$$

$$P_9(x,t) = e^{-\int \beta_3(x)dx} \int \alpha_3 P_3(x, t-x)dx$$

$$P_{10}(x,t) = e^{-\int \beta_3(x)dx} \int \alpha_3 P_1(x, t-x)dx$$

$$P_{11}(x,t) = e^{-\int \beta_1(x)dx} \int \alpha_1 P_1(x, t-x)dx$$

Finally, the expression of time dependent availability A(t) is obtained by summation of probabilities of all the working states and reduced capacity states, i.e.

$$A(t) = P_0(t) + \int \sum_{i=1}^{3} P_i\,(x,t)dx \tag{8}$$

Availability expression of the butteroil production system as given by equation (8) can be solved using constant failure rates and variable repair rates collected from the concerned plant.

3.1. *Performance modeling of the system*

As we have seen in the previous case, it is difficult to solve the problem analytically if either failure or repair rates are varied. Hence, in order to simplify the problem, failure and repair rates are considered constant. In this case, the system of equations (1) to (7) can be represented as follows:

$$P_0(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{3} \alpha_i\right] = P_6(t)\beta_1 + P_2(t)\beta_2 + P_1(t)\beta_3 \tag{9}$$

$$P_1(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{3} \alpha_i + \beta_3\right] = P_{11}(t)\beta_1 + P_3(t)\beta_2 + P_{10}(t)\beta_3 + P_0(t)\alpha_3 \tag{10}$$

$$P_2(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{3} \alpha_i + \beta_2\right] = P_4(t)\beta_1 + P_5(t)\beta_2 + P_3(t)\beta_3 + P_0(t)\alpha_2 \tag{11}$$

$$P_3(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{3} \alpha_i + \beta_2 + \beta_3\right] = P_7(t)\beta_1 + P_8(t)\beta_2 + P_9(t)\beta_3 + P_1(t)\alpha_2 + P_2(t)\alpha_3 \tag{12}$$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_1\right] = P_j(t)\alpha_1 \tag{13}$$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_2\right] = P_j(t)\alpha_2 \tag{14}$$

$for\ i = 5, j = 2; i = 8, j = 3$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_3\right] = P_j(t)\alpha_3 \tag{15}$$

$for\ i = 9, j = 3; i = 10, j = 1$

Initial Conditions

$P_i(t) = 1 \quad for\ i = 0$

$\quad\;\; = 0 \quad for\ i \neq 0$

To examine the effect of failure and repair rates on the availability in transient state, the system of differential equations (9) to (15) with initial conditions has been solved numerically using Runge-Kutta fourth order method. Analysis has been done for a period of 360 days divided over an interval of 30 days and the data has been tabulated in tables 1-6. These tables present the effect of failure and repair rates of various subsystems on the reliability of the system. MTBF, which has been computed using Simpson's 3/8 rule, with corresponding failure/repair rates, has been given in the last row of each table.

4. Results and Analysis

4.1. *Effect of failure rate of heater (α_1) on system availability*

By varying failure rate α_1 from 0.005 to 0.025 and keeping $\alpha_2 = 0.01$, $\alpha_3 = 0.002857$, $\beta_1 = 0.10$, $\beta_2 = 0.05$ and $\beta_3 = 0.04$, the availability of the system has been computed and compiled in Table 1, which shows that there is a decrease in availability up to 14.55 percent. Also, availability decreases by up to 1.98 percent as number of days increase from 30 to 360. MTBF shows a decline of around 50 days with the increase in failure rate from 0.005 to 0.025.

Table 1. Effect of failure rate of heater (α_1) on availability

Time ↓ (days) / α_1 →	0.005	0.01	0.015	0.02	0.025
30	0.9388	0.8980	0.8604	0.8255	0.7933
60	0.9258	0.8850	0.8476	0.8132	0.7815
90	0.9214	0.8809	0.8439	0.8098	0.7784
120	0.9199	0.8795	0.8425	0.8085	0.7771
150	0.9193	0.8790	0.8420	0.8080	0.7767
180	0.9191	0.8788	0.8418	0.8078	0.7765
210	0.9191	0.8787	0.8417	0.8077	0.7764
240	0.9191	0.8787	0.8417	0.8077	0.7764
270	0.9190	0.8787	0.8417	0.8077	0.7763
300	0.9190	0.8787	0.8417	0.8077	0.7763
330	0.9190	0.8787	0.8417	0.8077	0.7763
360	0.9190	0.8787	0.8417	0.8077	0.7763
MTBF	332.75	318.65	305.71	293.81	282.83

4.2. *Effect of failure rate of clarifier (α_2) on system availability*

As presented in Table 2, if failure rate α_2 increases from 0.01 to 0.033 and the values of α_1, α_3, β_1, β_2 and β_3 are kept at 0.005, 0.002857, 0.10, 0.05 and 0.04 respectively, availability goes down by 16 percent. However, availability decreases by up to 8.13 percent as time increases from 30 to 360 days. It is seen that MTBF also decreases by approximately 53 days as failure rate increases.

Table 2. Effect of failure rate of clarifier (α_2) on availability

Time (days) ↓ / α_2 →	0.01	0.01575	0.0215	0.02725	0.033
30	0.9388	0.9206	0.8972	0.8702	0.8405
60	0.9258	0.8970	0.8618	0.8232	0.7831
90	0.9214	0.8889	0.8502	0.8086	0.7663
120	0.9199	0.8862	0.8464	0.8040	0.7613
150	0.9193	0.8853	0.8452	0.8026	0.7599
180	0.9191	0.8850	0.8448	0.8021	0.7594
210	0.9191	0.8848	0.8446	0.8020	0.7593
240	0.9191	0.8848	0.8446	0.8019	0.7592
270	0.9190	0.8848	0.8446	0.8019	0.7592
300	0.9190	0.8848	0.8446	0.8019	0.7592
330	0.9190	0.8848	0.8446	0.8019	0.7592
360	0.9190	0.8848	0.8446	0.8019	0.7592
MTBF	332.75	321.60	308.37	294.19	279.83

4.3. Effect of failure rate of filling subsystem (α_3) on system availability

Next, the effect of failure rate of filling subsystem on the overall system availability has been analyzed. The results shown in Table 3 indicate that by varying failure rate α_3 = 0.002857, 0.004643, 0.006428, 0.008214 and 0.01 and taking $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\beta_1 = 0.10$, $\beta_2 = 0.05$ and $\beta_3 = 0.04$, the availability decreases by 3.67 percent. It is also observed that this decrease is 4.15 percent with the increase in time from 30 to 360 days. In this case, MTBF decreases by 11 days with the increase in failure rate.

Table 3. Effect of failure rate of filling subsystem (α_3) on availability

Time (days) ↓ / α_3 →	0.002857	0.004643	0.006428	0.008214	0.01
30	0.9388	0.9365	0.9331	0.9289	0.9239
60	0.9258	0.9214	0.9153	0.9077	0.8988
90	0.9214	0.9160	0.9086	0.8995	0.8888
120	0.9199	0.9141	0.9061	0.8963	0.8849
150	0.9193	0.9133	0.9051	0.8950	0.8834
180	0.9191	0.9131	0.9047	0.8945	0.8828
210	0.9191	0.9130	0.9046	0.8943	0.8825
240	0.9191	0.9129	0.9045	0.8943	0.8824
270	0.9190	0.9129	0.9045	0.8942	0.8824
300	0.9190	0.9129	0.9045	0.8942	0.8824
330	0.9190	0.9129	0.9045	0.8942	0.8824
360	0.9190	0.9129	0.9045	0.8942	0.8824
MTBF	332.75	330.84	328.20	324.96	321.44

4.4. Effect of repair rate of heater (β_1) on system availability

The results presented in Table 4 indicate the availability of the system when repair rate β_1 of the heater subsystem is varied from 0.10 to 0.40. Taking values of $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\alpha_3 = 0.002857$, $\beta_2 = 0.05$ and $\beta_3 = 0.04$, it is observed that availability improves up to 3.32 percent. Whereas, there is a decrease of 1.94-1.98 percent in availability as number of days increase from 30 to 360. MTBF increases by around 11 days with the increase in repair rate.

Table 4. Effect of repair rate of heater (β_1) on availability

Time (days) ↓ / β_1 →	0.10	0.175	0.25	0.325	0.40
30	0.9388	0.9563	0.9641	0.9685	0.9712
60	0.9258	0.9445	0.9522	0.9564	0.9590
90	0.9214	0.9399	0.9476	0.9517	0.9543
120	0.9199	0.9384	0.9460	0.9501	0.9527
150	0.9193	0.9378	0.9454	0.9495	0.9521
180	0.9191	0.9376	0.9452	0.9493	0.9519
210	0.9191	0.9375	0.9451	0.9493	0.9519
240	0.9191	0.9375	0.9451	0.9492	0.9519
270	0.9190	0.9375	0.9451	0.9492	0.9518
300	0.9190	0.9375	0.9451	0.9492	0.9518
330	0.9190	0.9375	0.9451	0.9492	0.9518
360	0.9190	0.9375	0.9451	0.9492	0.9518
MTBF	332.75	339.17	341.83	343.28	344.19

4.5. Effect of repair rate of clarifier (β_2) on system availability

Table 5 presents the effect of repair rate of clarifier (β_2) on the system availability. As β_2 is varied from 0.05 to 0.20 in five steps and the values of failure and repair rates of other subsystems i.e. α_1, α_2, α_3, β_1 and β_3 are taken as 0.005, 0.01, 0.002857, 0.10 and 0.04 respectively, it is observed that availability of the system decreases by 0.49-1.98 percent with the increase in time from 30 to 360 days. But, it increases by 2.69 percent as repair rate increases from 0.05 to 0.20. In this case, MTBF increases by around 9 days.

Table 5. Effect of repair rate of clarifier (β_2) on availability

Time (days) ↓ / β_2 →	0.05	0.0875	0.125	0.1625	0.20
30	0.9388	0.9450	0.9481	0.9498	0.9508
60	0.9258	0.9394	0.9442	0.9463	0.9473
90	0.9214	0.9383	0.9433	0.9454	0.9465
120	0.9199	0.9379	0.9430	0.9451	0.9462
150	0.9193	0.9378	0.9429	0.9450	0.9460
180	0.9191	0.9377	0.9428	0.9449	0.9460
210	0.9191	0.9377	0.9428	0.9449	0.9460
240	0.9191	0.9377	0.9428	0.9449	0.9460
270	0.9190	0.9377	0.9428	0.9449	0.9459
300	0.9190	0.9377	0.9428	0.9449	0.9459
330	0.9190	0.9377	0.9428	0.9449	0.9459
360	0.9190	0.9377	0.9428	0.9449	0.9459
MTBF	332.75	338.60	340.30	341.02	341.38

4.6. Effect of repair rate of filling subsystem (β_3) on system availability

Table 6 shows the effect of improvement of repair rate of filling subsystem (β_3) on the system availability. It is observed that as β_3 increases from 0.04 to 0.16 and the value of failure and repair rates of other subsystems are kept at $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\alpha_3 = 0.002857$, $\beta_1 = 0.10$ and $\beta_2 = 0.05$, availability increases by 0.38 percent. But as the number of days increase from 30 to 360, there is a decrease in availability of around 1.72-1.98 percent. MTBF increases by just 1 day with the increase in repair rate.

Table 6. Effect of repair rate of filling subsystem (β_3) on availability

Time (days) ↓ / β_3 →	0.04	0.07	0.10	0.13	0.16
30	0.9388	0.9394	0.9397	0.9399	0.9400
60	0.9258	0.9274	0.9280	0.9283	0.9284
90	0.9214	0.9236	0.9243	0.9246	0.9247
120	0.9199	0.9224	0.9230	0.9233	0.9234
150	0.9193	0.9219	0.9226	0.9229	0.9230
180	0.9191	0.9218	0.9225	0.9227	0.9229
210	0.9191	0.9217	0.9224	0.9227	0.9228
240	0.9191	0.9217	0.9224	0.9227	0.9228
270	0.9190	0.9217	0.9224	0.9227	0.9228
300	0.9190	0.9217	0.9224	0.9227	0.9228
330	0.9190	0.9217	0.9224	0.9227	0.9228
360	0.9190	0.9217	0.9224	0.9227	0.9228
MTBF	332.75	333.56	333.78	333.888	333.92

5. Conclusion

Tables 1-6 depict the effects of varying failure and repair rates of different subsystems on the availability of butteroil production system. On careful examination of these Tables, it reveals that failure rate of clarifier subsystem and repair rate of heater subsystems make maximum impact on availability of the system. This has also been demonstrated in Figures 3 and 4. However, in comparison to clarifier and heater subsystems, variation in failure/repair rate of filling subsystem makes lesser impact on system availability. Hence, we conclude that:

1. Utmost importance must be given to failure rate of Clarifier subsystem in order to improve system availability, since it decreases the system availability by 16 percent (more than any other subsystem)
2. Improvement in repair rate of Heater subsystem improves the system availability by 3.32 percent (more than any other subsystem). Hence, this subsystem should be repaired as soon as possible.
3. Based on failure rates, maintenance priority must be as per the following order:
   3.1. Clarifier subsystem
   3.2. Heater subsystem
   3.3. Filling subsystem
4. Similarly, based on repair rates, maintenance priority must be as per the following order:
   4.1. Heater subsystem
   4.2. Clarifier subsystem
   4.3. Filling subsystem

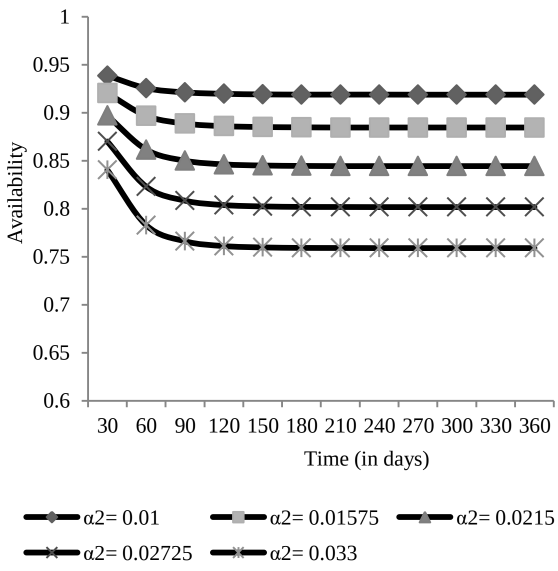

Figure 3. Effect of failure rate of Clarifier on system availability

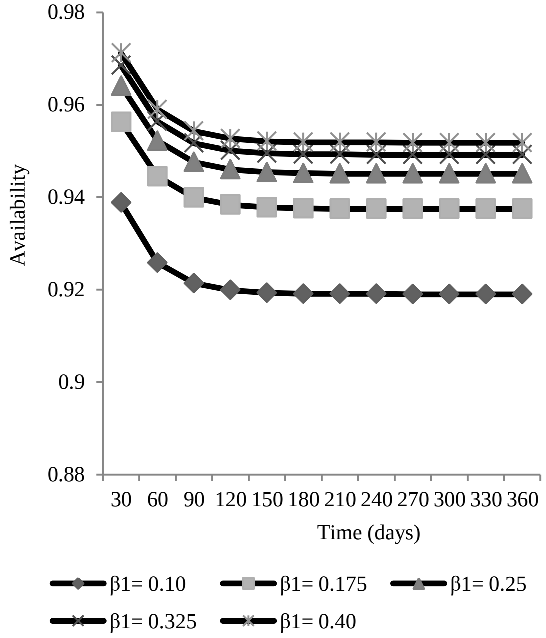

Figure 4. Effect of repair rate of heater on system availability

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