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# MHD Tangent Hyperbolic Nanofluid with Zero Normal Flux of Nanoparticles at the Stretching Surface with Thermal Radiation

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**Abstract**

This article presents the effect of thermal radiation and slip effects on MHD boundary layer flow of tangent hyperbolic nanofluid over a stretching sheet and condition of zero normal flux of nano particles introduced at the surface. Similarity transformations are used to transform the governing partial differential equations in terms of continuity, momentum, energy, and concentration which are reconstructed into ordinary differential form and then solved numerically using Runge-Kutta fourth order method with shooting technique. The behavior of the involved physical parameters Weissen berg number (We), thermal radiation (R), Lewis number (Le), velocity slip (λ) and power law index (n) are displayed graphically for velocity, temperature, and concentration profiles. Additionally, local skin friction, local Nusselt number, and local Sherwood number are computed and analyzed. Comparison of the present results with previously published literature is specified and found in good agreement. The numerical results show that the skin friction increases and the Nusselt number as well, whereas the Sherwood numbers are decreasing with the increase in velocity slip parameter.



*Keywords:* Tangent hyperbolic fluid, nanofluid, MHD, thermal radiation, partial slip effects;

## 1. Introduction

The steady flow over stretching sheet with heat transfer is a classical problem in fluid dynamics. In past few decades, boundary layer flows over stretching sheet got a lot of importance due to its occurrence in various engineering and mechanical processes, chemical and metallurgical processes like polymers pro-tuberating, glass blowing, paper and fiber glass production, metallic plate cooling, melt-spinning, hot rolling etc. Sakiadis [1] has discussed the study of the boundary layer flow over a continuous solid surface moving with constant speed. Crane [2] has extended the work for both linear and exponentially stretching sheet. Hayat *et al*. [3] examined the chemically reactive flow of third grade fluid by an exponentially convective stretching sheet. Vajravelu *et al*. [4] have studied the effects of thermo-physical property on unsteady flow and heat transfer in a Ostwald-de Waele liquid on a stretching sheet. Turkyilmazoglu [5] has derived the flow of a micropolar fluid due to a porous stretching sheet and heat transfer.

The study of MHD flow is very important as it has many industrial applications, such as magnetic materials processing, purification of crude oil, magneto hydrodynamic electrical power generation, glass manufacturing, geophysics, and paper production, and many other practical applications such as magnetic field effects on wound healing, or canter action causing hypothermia, surgical procedures and MRI (magnetic reason imaging) to diagnose disease. Furthermore, the influence of the magneto hydrodynamic (MHD) flow due to a stretching sheet has enormous applications in modern metallurgy and chemical industry e.g. fusing of metals and cooling of nuclear reactors etc. Rashidi *et al*.[6] have analyzed the entropy analysis of convective MHD flow of third grade non-Newtonian fluid over a stretching sheet. MHD flow and heat transfer of couple stress fluid over an oscillatory stretching sheet with heat source/sink in porous medium has been studied by Nasir Ali *et al*.[7]. Hayat *et al*.[8] derived MHD axisymmetric flow of third grade fluid between stretching sheets with heat transfer. Rana and Bhargava [9] have analyzed similar research for a nonlinear stretching sheet using finite element and finite difference methods. Mabood *et al*.[10] introduced the MHD flow over exponential radiating stretching sheet using homotopy analysis method. Forced convection heat transfer in a semi annulus under the influence of a variable magnetic field derived by Sheikholeslami *et al.[*11]. Rashidi *et al.[*12] investigated the entropy generation in MHD and slip flow over a rotating porous disk with variable properties.

The term nanofluid used for suspensions which comprise the nano size particles (silver, gold, aluminum,

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copper, diamond etc.) and conventional base fluid. Firstly, Choi [13] introduced the term nanofluid. Today nanofluids are sought for wide range of applications in medical application, Biomedical industry, detergency, power generation in nuclear reactors and more specifically in any heat removal involved industrial applications. Micropolar nanofluid flow with MHD and viscous dissipation effects towards a stretching sheet with multimedia feature studied by Kai-LongHsiao [14]. Ibrahim [15] analyzed the magnetohydrodynamic (MHD) boundary layer stagnation point flow and heat transfer of a nanofluid past a stretching sheet with melting. Mabood *et al*.[16] did a numerical study on MHD boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet. Hayat *et al*.[17] have studied the flow of nanofluid by nonlinear stretching velocity.

The fluids are mainly classified into two categories: Newtonian and Non-Newtonian. Moreover, the analysis of heat transfer of non-Newtonian fluids plays a very important role in various industrial applications, such as in petroleum production and metallurgical process. Numerous materials excluding dyes, blood at low shear rate, ketchup, certain paints, lubricants, mud and personal care products are non-Newtonian in nature. Flow of non-Newtonian fluids has a pivotal role in manufacturing polymer devolatisation, bubbles columns, fermentation, composite processing, boiling, plastic foam processing, and bubbles absorption etc. In the present research work, we will focus on the study of non-Newtonian fluid which is one of the important branches of non-Newtonian fluid i.e., hyperbolic tangent fluid which is useful for chemical engineering systems, and it has the capacity to describe shear thinning phenomena. Examples of tangent hyperbolic fluids are ketchup, whipped cream, blood, paint, and nail polish. Malik *et al*. [18] analyzed the MHD flow of tangent hyperbolic fluid over a stretching cylinder by using Keller box method. Prabhaker *et al*. [19] studied the impact of inclined Lorentz forces on tangent hyperbolic nanofluid flow with zero normal flux of nanoparticles at the stretching sheet. Hayat *et al*. [20] introduced magnetohydrodynamic (MHD) stretched flow of tangent hyperbolic nano liquid with variable thickness. Khan *et al*. [21] did a numerical investigation on boundary layer flow of MHD tangent hyperbolic nanofluid over a stretching sheet. Naseer *et al*. [22] derived the boundary layer flow of hyperbolic tangent fluid over a vertical exponentially stretching cylinder. Nadeem and Akram [23] studied the peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel.

Thermal radiation is important in some applications because of the way radiant emission depends on temperature and nanoparticle volume fraction. Thermal radiation is continuously emitted by all matter whose temperature is above absolute zero. Thermal radiation is also defined as the portion of the electromagnetic spectrum that extends from about 0.1 to 100μm, since the radiation emitted by bodies due to their temperature falls almost entirely into this wave length range. Hayat *et al*. [24, 25] investigated on radiative flow of a tangent hyperbolic fluid with convective conditions. Mahanthesh *et al*. [26] analyzed nonlinear convective and radiated flow of tangent hyperbolic liquid due to stretched surface with convective condition. Rehman *et al*. [27] introduced mutual effects of thermal radiations and thermal stratification on tangent hyperbolic fluid flow yields by both cylindrical and flat surfaces. Rashidi *et al*. [28] scrutinized the buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation. Thermal radiation and MHD effects on boundary layer flow of micropolar nanofluid past a stretching sheet with non-uniform heat source/sink has been derived by Pal and Mandal [29]. Hayat *et al*.[30] have analyzed the new thermodynamics of entropy generation minimization with nonlinear thermal radiation and nanomaterials.

Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation are studied by Swati Mukhopadhyay [31].Thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet are studied by Rizwan Ul Haq *et al*.[32]. MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions have been discussed by Ibrahim and Shankar [33]. Hayat *et al*.[34] studied radial MHD and mixed convection effects in peristalsis of non-Newtonian nanomaterial with zero mass flux conditions. Rehman *et al*.[35] analyzed entropy generation analysis for non-Newtonian nanofluid with zero normal flux of nanoparticles at the stretching surface. Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid is studied by Rashidi *et al*.[36].

From above literature, the aim of the present study is to investigate the condition of zero normal flux for MHD tangent hyperbolic fluid flow towards a stretching sheet with thermal radiation. Mathematical model is structured for non-Newtonian fluid in the presence of slip mechanism of nanofluid. To discard the gravitational settling at the surface of the sheet, we have considered the passive control of nanoparticles at the surface which are defined in the boundary condition. The governing nonlinear partial differential systems are transformed into ordinary nonlinear differential equations. Here the Runge-Kutta fourth order method with shooting technique is applied to construct the series solution for the nonlinear governing problems. The effects of governing parameters on fluid velocity, temperature and particle concentration were discussed and shown in graphs and tables as well.

## 2. Tangent hyperbolic fluid model:

For an incompressible fluid the balance of mass and momentum are given by [23]

$$div\, V = 0, \tag{1}$$

$$\rho \frac{dV}{dt} = divS + \rho f \tag{2}$$

Where $\rho$ is the density, $V$ is the velocity vector, $S$ is the Cauchy stress tensor, $f$ represents the specific body force and $d/_{dt}$ represents the material time derivative. The constitutive equation for hyperbolic tangent fluid is given by

$$S = -PI + \overline{\tau} \tag{3}$$

$$\overline{\tau} = -[\mu_\infty + (\mu_0 + \mu_\infty) \tanh(\Gamma\bar{\dot{\gamma}})^n]\bar{\dot{\gamma}}, \tag{4}$$

In which $-PI$ is the spherical part of the stress due to constraint of incompressibility, $\bar{\tau}$ is extra stress tensor, $\mu_\infty$ is the infinite shear rate viscosity, $\mu_0$ is the zero shear rate viscosity, $\Gamma$ is the time dependent material constant, n is the power law index, i.e., flow behavior index and $\bar{\dot{\gamma}}$ defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi}, \tag{5}$$

Where $\Pi = \frac{1}{2}\mathrm{tr}(\mathrm{grad}V + \mathrm{grad}V^{\mathrm{T}})^2$. Here $\Pi$ is the second invariant strain tensor. We consider Eq. (4), the case for which $\mu_\infty = 0$ because it is not possible to discuss the problem for the infinite shear rate viscosity and since we are considering tangent hyperbolic fluid that describing shear thinning effects so $\Gamma\bar{\dot{\gamma}} < 1$.Then Eq.(4) takes the form

$$\bar{\tau} = -\mu_0[(\Gamma\bar{\dot{\gamma}})^n]\bar{\dot{\gamma}} = -\mu_0[(1 + \Gamma\bar{\dot{\gamma}} - 1)^n]\bar{\dot{\gamma}} \tag{6}$$

$$\bar{\tau} = -\mu_0[1 + n(\Gamma\bar{\dot{\gamma}} - 1)]\bar{\dot{\gamma}}$$

## 3. Mathematical formulation

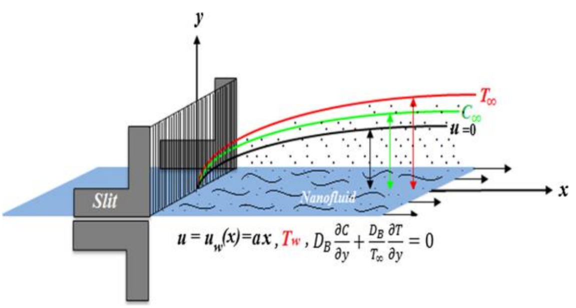


**Figure 1.** Geometry of the model.

Consider the steady, viscous, two-dimensional and incompressible boundary layer flow of tangent hyperbolic nano fluid over a stretching sheet with surface at y = 0 which is stretching linearly with velocity $u_w = ax$.The flow lies in the region y > 0 and magnetic field of strength $B_0^2$ is applied normal to the flow. It is assumed that at the stretching surface, the temperature and the nano particles fraction take constant values $T_W$ and $C_w$ whereas the ambient values of temperature $T_\infty$ and the nanoparticles fraction $C_\infty$ are attained as y tends to infinity. Taking all these assumptions into account, the governing equations under boundary layer assumptions are given below[23] :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon(1-n)\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\upsilon n\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma {B_0}^2(x)}{\rho_f}u, \tag{8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}, \tag{9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \tag{10}$$

here $u$ and $v$ are the velocity components along the $x$ and $y$ directions, respectively, $\rho_f$the density of the base fluid, $\alpha_m = {}^{k}/_{(\rho c)_f}$ the thermal diffusivity, $\upsilon$ the kinematic viscosity, $\Gamma$ is the time constant, $n$ is the power law index, T is the fluid temperature, $c_p$ is the specific heat, $D_B$the Brownian diffusion coefficient, $D_T$ the thermophoretic diffusion coefficient, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid and ρ the density of the particles.

Using Rosseland approximation for radiation we can write

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{11}$$

Where $\sigma^*$is the Stefan–Boltzman constant, $k^*$ is the absorption coefficient. Assuming that the temperature difference within the flow is such that $T^4$ may be expanded in a Taylor series and expanding $T^4$ about $T_\infty$, the free stream temperature and neglecting higher orders we get

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4.$$

We introduce subjective boundary conditions are $u = u_w(x) + \gamma_1\frac{\partial u}{\partial y},\ v = 0, T = T_w\ ,\ D_B\frac{\partial C}{\partial y} + \frac{D_B}{T_\infty}\frac{\partial T}{\partial y} = 0\quad as\ y = 0$ (Zero normal flux)

$$u = 0, T = T_\infty,\ C = C_\infty\ as\ y \to \infty \tag{12}$$

The similarity transformations for this problem can be written as

$$\eta = y\sqrt{\frac{a}{\upsilon}}, u = axf'(\eta), v = -\sqrt{a\upsilon}f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_\infty}. \tag{13}$$

After applying the above-defined transformations into governing equations, they take the following form

$$((1-n) + n\,Wef'')f''' - f'^2 + ff'' - M^2 f' = 0, \tag{14}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' + Nb\phi'\theta' + Nt\theta'^2\right) = 0, \tag{15}$$

$$\phi'' + PrLef\phi' + \frac{Nt}{Nb}\theta'' = 0. \tag{16}$$

Transformed boundary conditions

$$f(0) = 0, f'(0) = 1 + \lambda f''(0), \theta(0) = 1, Nb\phi'(0) + Nt\theta'(0) = 0, \tag{17}$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0.$$

where prime denotes differentiation with respect to η. The dimensionless parameters $Nb, Nt, Pr, Le, M, We, R, \lambda$ are the Brownian motion parameter, thermophoresis parameter, Prandtl, Lewis, the magnetic parameter, Weissenberg number, thermal radiation parameter, velocity slip parameter, respectively. These parameters are defined as follows:

$$Nb = \frac{(\rho c)_p D_B C_\infty}{(\rho c)_f \upsilon}, Nt = \frac{(\rho c)_p D_T (T_W - T_\infty)}{(\rho c)_f \upsilon T_\infty}, \Pr = \frac{\upsilon}{\alpha_m}, \mathrm{Le} = \frac{\alpha_m}{D_B}, M = \frac{\sigma B_0^2}{\rho_f a}.$$

$$We = \frac{\sqrt{2}a^{3/2}\Gamma x}{\sqrt{\upsilon}}, R = \frac{kk^*}{4\sigma^* T_\infty^3}, \lambda = \gamma_1\sqrt{\frac{a}{\upsilon}}\,. \tag{18}$$

The physical quantities of interest are the skin friction $C_f$ and the local Nusselt number $Nu_x$ which are defined as

$$C_f = \frac{\tau_w}{\rho {u_w}^2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{19}$$

Where the surface shear stress $\tau_w$ and the heat flux $q_w$ are given as

$$\tau_w = (1-n)\left(\frac{\partial u}{\partial y}\right)_{y=y_0} + \frac{n\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2_{y=y_0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} + (q_r)_w \tag{20}$$

Substituting Eq. (13) into Eqs. (19)- (20), we obtain

$$Re_x^{\frac{1}{2}}C_f = (1-n)f''(0) + \frac{nWe}{2}(f''(0))^2,$$

$$\frac{Nu_x}{Re_x^{\frac{1}{2}}} = -\left(1 + \frac{4}{3}R\right)\theta'(0) \tag{21}$$

where $Re_x = (ax^2/\upsilon)$ is the local Reynolds number.

## 4. Numerical Procedure

The system of coupled non-linear ordinary differential equations (14) - (16) along with the boundary conditions (17), which are solved numerically by Runge-Kutta fourth order method with shooting technique. The step size is taken as $\Delta\eta = 0.01$ and the convergence criteria were set to $10^{-5}$. Computational time is 1.569 s. The asymptotic boundary conditions defined in Eq.(17) are replaced by $f(\eta_{max}) = 1$ using a value similarity variable $\eta_{max} = 15$. The choice of $\eta_{max} = 15$ ensures that all numerical solutions approached the asymptotic values correctly.

## 5. Results and discussion

The coupled nonlinear ordinary differential Eqs.(14)-(16), subjected to the boundary conditions (17) have been calculated using Runge-Kutta fourth order method with shooting technique. Here the numerical solutions of tangent hyperbolic fluid for stretching sheet are presented. The dimensionless parameters including, magnetic parameter $(M)$, Prandtl number $(Pr)$, Brownian motion parameter $(Nb)$, thermophoresis parameter $(Nt)$, thermal radiation $R$, velocity slip parameters $\lambda$ are the main interests of the study. The numerical values of local skin friction are exhibited in Table1.The present results show a good agreement with the published data. From Table 2, the Nusselt number is a decreasing function of increasing R, and Sherwood number is found to be an increasing function of $R$.

**Table 1.** Comparison of local skin friction coefficient in the absence of nanoparticles, Weissenberg number $We = 0$.

| $M$ | Akbar et al.[37] | Malik et al.[18] | Hussain et al.[38] | Khan et al.[21] | Present results |
|---|---|---|---|---|---|
| 0 | -1 | -1 | -1 | -1 | -1.00017 |
| 1 | -1.41421 | -1.41419 | -1.4137 | -1.4139 | -1.41421 |
| 5 | -2.449449 | -2.44945 | -2.4495 | -2.4499 | -2.44949 |
| 10 | -3.31663 | -3.31657 | -3.3166 | -3.3170 | -3.31662 |
| 100 | -10.0498 | -10.04981 | -10.0500 | -10.0503 | -10.04988 |
| 500 | -22.38303 | -22.38294 | -22.3835 | -22.3839 | -22.38303 |
| 1000 | -31.63859 | -31.63851 | -31.6391 | -31.6399 | -31.63858 |

**Table 2.** Calculation of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ when $\mathrm{Pr} = 1.0,\ Le = 1.0,\ M = 0.5,\ Nb = 0.2, Nt = 0.3, We = n = 0.1$.

| $R$ | $\lambda$ | $Re_x^{1/2} C_f$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|---|
| 0.0 | | -1.10901 | 0.47841 | -0.71762 |
| 0.5 | 0.1 | -1.10901 | 0.33929 | -0.50894 |
| 1.0 | | -1.10901 | 0.27901 | -0.41852 |
| | 0.2 | -0.97330 | 0.32343 | -0.48514 |
| 0.5 | 0.3 | -0.86987 | 0.31053 | -0.46580 |
| | 0.5 | -0.72127 | 0.29055 | -0.43583 |

Figure.2 illustrates the effect of power law index on velocity, temperature and concentration profiles. It is noticeable that an increase in the values of power-law index $n$ correspond to reduction in velocity and boundary layer thickness, and it is also observed that the fluid nature changes from shear thinning to shear thickening. Hence velocity profile shows the decreasing behavior while temperature and concentration profile are increasing with increasing values of power law index. The influence of the Weissenberg number $We$ on the velocity, temperature and concentration profiles are presented in figure 3. An increase in the values of Weissenberg number $We$ correspond to the decrease in velocity of the flow and thinning of hydrodynamic boundary layer since $We$ is the ratio of the shear rate time to the relaxation time of the fluid. Hence with the increase of the Weissenberg number $We$ the relaxation time increases, which produces more resistance to the motion of the fluid and thus the velocity profile decreases. But the reverse behavior is obtained for temperature and concentration profiles. Figure 4, illustrates the effect of magnetic parameter on velocity, temperature and concentration profiles. It shows that velocity profile decreases with an increase in magnetic parameter. The existence of magnetic field produces a force called Lorentz force and it resists the fluid motion and therefore some useful energy is converted into heat. Therefore the flow velocity is decreased but temperature and concentration are enhanced with increasing values of $M$. Effect of Brownian motion parameter $Nb$ on nanoparticle concentration is observed in Figure 5, from the figure we can seen that nanoparticle concentration is found to decrease and the concentration boundary layer thickness decreases with an increase in the Brownian motion parameter $Nb$, due to an increase in Brownian motions causes irregular movement of nanoparticles, hence it ultimately deprecates the nanoparticle concentration.

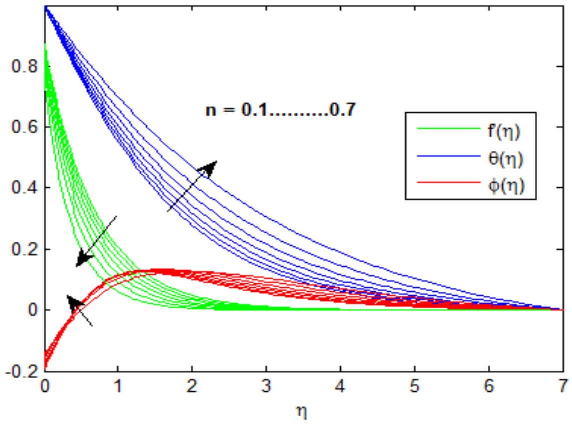


**Figure 2.** Effect of n on velocity, temperature and concentration profiles.

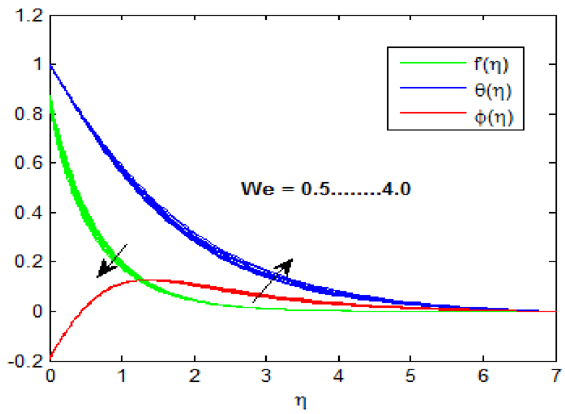


**Figure 3.** Effect of $We$ on velocity, temperature and concentration profiles.

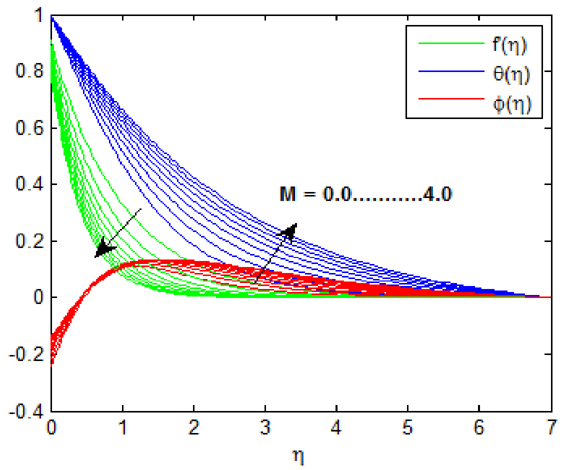


**Figure 4.** Effect of $M$ on velocity, temperature and concentration profiles.

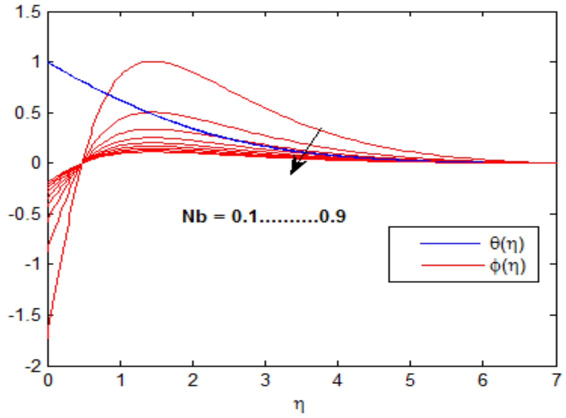


**Figure 5.** Effect of $Nb$ on temperature and concentration profiles.

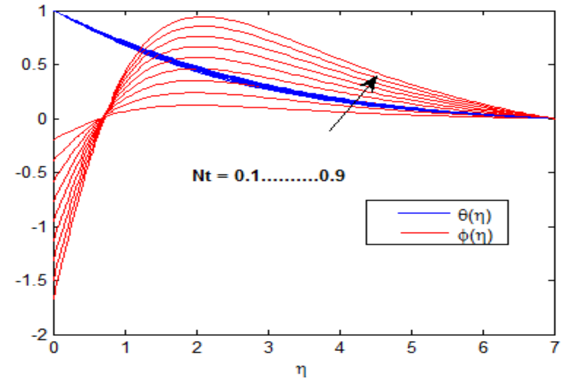


**Figure 6.** Effect of $Nt$ on temperature and concentration profiles.

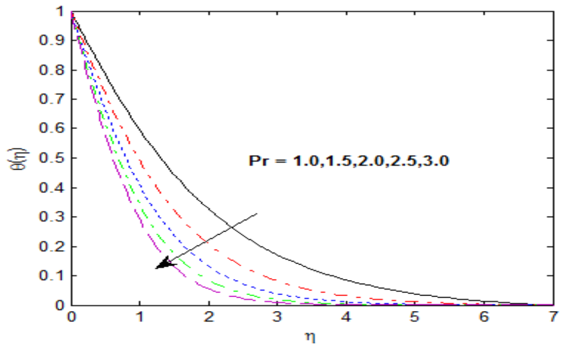


**Figure 7.** Effect of Pr on temperature profile.

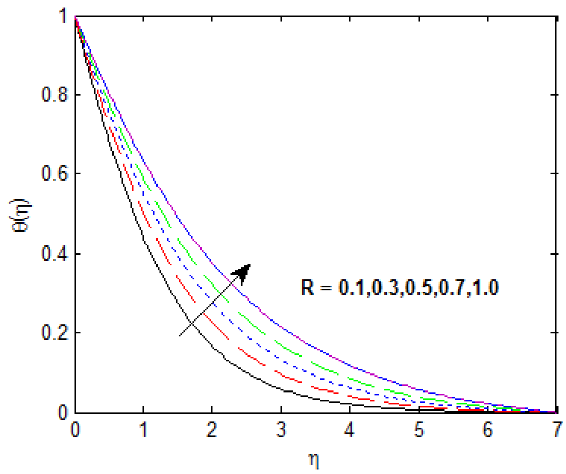


**Figure 8.** Effect of $R$ on temperature profile.

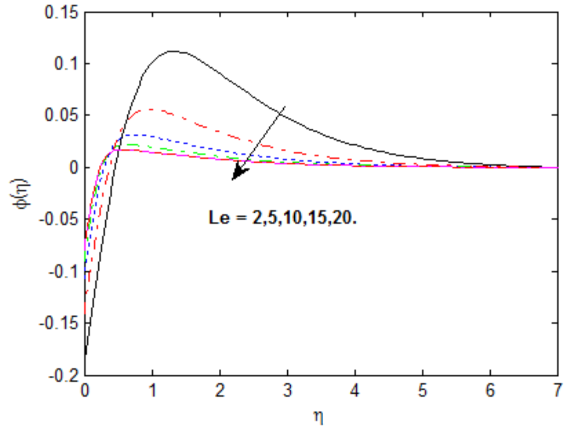


**Figure 9.** Effect of Le on concentration profile.

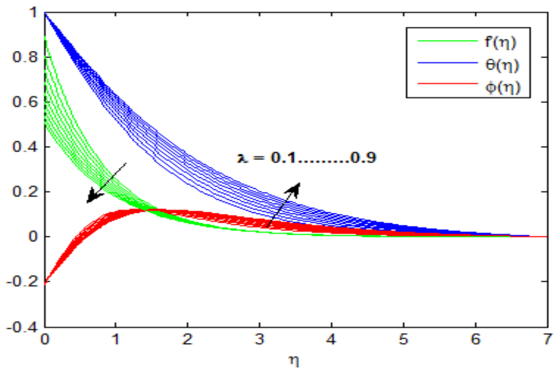


**Figure 10.** Effect of λ on velocity, temperature and concentration profiles.

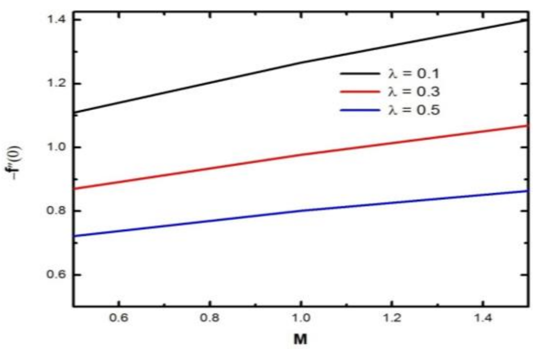


**Figure 11.** Variation of skin friction with various values of $M$ and $\lambda$

Figure 6 depicts the impact of thermophoresis parameter $Nt$ on temperature and nanoparticle concentration profiles. From the figure, large values of thermophoresis parameter $Nt$ temperature profile as well as nanoparticle concentration profile increases. It is observed that for large values of thermophoresis parameter $Nt$, nanoparticles migrated from the hot surface to cold ambient fluid, as a result the thermal boundary layer thickness enhances. Figure 7 shows the influence of Prandtl number $Pr$ on temperature profile. Here we observed that the temperature profile decreases by increasing $Pr$. In fact when $Pr$ increases then thermal diffusivity decreases. This indicates diminution in energy transfer ability and decrease of thermal boundary layer thickness. Influence of thermal radiation $R$ on temperature profile is shown in Figure 8. Here we observed that when the thermal radiation $R$ increases then the temperature profile enhanced. For large values of radiation parameter, generates a significant amount of heating to the nanofluid which increases the nanofluid temperature profile and thicker thermal boundary layer thickness. Influence of Lewis number $Le$ on concentration profile is shown in Figure 9. As Lewis number increases the concentration graph decreases and the concentration boundary layer thickness decreases. This is because for larger Lewis number $Le$, the Brownian diffusion is weaker, which leads to weak mass transfer as a result, the thinner concentration boundary layer at the surface.

Figure 10 illustrates the effect of velocity slip parameter on velocity, temperature and concentration profiles. An increase in the velocity slip parameter $\lambda$ corresponds to a decrease in the relative velocity of the fluid and the stretching sheet. Increasing $\lambda$ decreases the velocity because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid. The boundary layer thickness also decreases as the slip parameter $\lambda$ increases. In this figure, we also observed that as the value of velocity slip parameter increases the temperature profile as well as concentration profile increases. Results produced for skin friction are analyzed for magnetic parameter and velocity slip parameter $\lambda$ in Figure 11. It can be found that dcreasing the skin friction for increasing velocity slip paramter($\lambda$).

## 6. Conclusions

In general, the effect of thermal radiation, velocity slip, Weissenberg number, power-law index parameter and magnetic field on boundary layer flow and heat transfer of tangent hyperbolic fluid with zero normal flux of nanoparticles past a stretching sheet have been discussed. The boundary layer equations governing the flow problem are reduced into a couple of high order non-linear ordinary differential equations using the similarity transformation. The obtained differential equations are solved numerically using Runge-Kutta fourth order method with shooting technique from Matlab software. The effects of various governing parameters such as velocity slip parameters $\lambda$, magnetic parameter $M$, radiation parameter $R$, Prandtl number $Pr$, Brownian motion parameter $Nb$, thermophoresis parameter $Nt$ and Lewis number parameter $Le$ on momentum, energy and concentration equation are analyzed using figures and tables. The observations of the present study are as follows:

1. By increasing the values of velocity slip parameter λ, Weissenberg number $We$, power-law index $n$ and magnetic parameter $M$ on velocity, temperature and concentration profiles, there happens a reduction in velocity profile and increase in the temperature and concentration profiles.
2. Increase in Prandtl number $Pr$ leads to decrease in the temperature profile.
3. Nanoparticle concentration decreases with an increase in Brownian motion parameter $Nb$.
4. As the thermophoresis parameter $Nt$ enhances, both temperature and concentration profile increase.
5. Concentration profile is reduced when the Lewis number $Le$ increased.
6. Temperature profile is increased with the increasing values of thermal radiation parameter $R$.

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