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Domains of Magnetic Moments of Two onboard Coils with Stable Damping of Microsatellite Angular Velocities

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Abstract

The present paper considers the problem of damping of microsatellite angular velocities. Two identical magnetic coils oriented in parallel to the axes of inertia of microsatellite are used as actuators. The aim of the paper is finding domains of the magnetic moments of coils ensuring asymptotically stable deceleration of microsatellite angular velocities. Lyapunov's second method is used to study the asymptotic stability of the microsatellite angular velocities. By analyzing the asymptotic stability condition, we have determined the domains of the magnetic moments of two onboard coils, which lead to stable a damping of the angular velocities. The calculation algorithm of the magnetic moments of coils ensuring asymptotic stability of the angular velocities is formulated.

Keywords: *Microsatellite, Magnetic Coils, Stability Domain, Lyapunov's Method, Algorithm.*

1. Introduction

As is known, in the process of separating from a base spacecraft, microsatellites and nanosatellites receive a certain angular velocity. Besides, during the uncontrolled motion on the Earth orbit the satellites are affected by aerodynamic and gravitational moments, which can lead to the change of their angular velocity [1]. Note that satellite angular velocity can be changed at a high-speed collision with a particle of space debris [2]. Despite the presence of a significant amount of the perturbing factors, the solution of the flight target problem of modern satellites is based on the effective control of a satellite angular velocity. In particular, the problem of ensuring a given orientation of a satellite assumes a controlled reduction of its angular velocity to small quantities with subsequent stabilization of the small level of the angular velocity. In the recent years, significant information is accumulated about the controlled reduction of the satellite angular velocity by means of onboard magnetic coils [4-18]. However, the publications do not contain a solution to the problem of defining the magnetic moments domains of two onboard coils which ensure stable damping of the angular velocities. The aim of the presented work is to find the domains of magnetic moments of two coils that provide asymptotically stable reduction of the microsatellite angular velocity.

The research object in this paper is the controlled motion of a microsatellite around its center of mass. The microsatellite has the shape of a rectangular parallelepiped. This form is the most efficient in terms of arranging equipment in the satellite [3].

Suppose that two fixed on board and oriented in parallel to microsatellite's axes of inertia electromagnetic coils are used as actuators. Two electromagnetic coils are sufficient for three-axis control of the microsatellite. Let us assume that the controlling moments from the interaction between the coils magnetic fields and the geomagnetic field make the main contribution to motion of the microsatellite around the center of mass. We use Lyapunov's second method to find the stability domains of the microsatellite relative motion. It is known, that mechanical moments from magnetic forces, gravity and aerodynamics are commensurable. However, with installation of strong onboard magnets and choosing a high orbit it is possible to obtain the mechanical moment from magnetic coils, which significantly exceeds the magnitudes of the mechanical moments of gravitational and aerodynamic forces [1]. In this case, we neglect the action of other force factors, such as gravitational or aerodynamic moments. The paper includes the following sections: introduction, condition of the asymptotic stability of the relative motion, stability domains of the relative motion, algorithm of finding of the magnetic moments, conclusion and references.

2. The condition of asymptotic stability of relative motion

The motion of a parallelepiped-shaped microsatellite around its center of mass is described by Euler's dynamic equations which in a vector form are:

$$J \cdot \frac{d\vec{\omega}}{dt} = \vec{M} \tag{1}$$

where J is the inertia tensor, $J = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix}$, J_x, J_y, J_z are the moments of inertia of the microsatellite relative to principal central axes *Ox*, *Oy*, *Oz*; $\vec{\omega} = (\omega_x, \omega_y, \omega_z)$ is the vector of microsatellite angular velocity; $\omega_x, \omega_y, \omega_z$ are the projections of microsatellite angular velocity to the coordinate axes *Ox*, *Oy*, *Oz*; $\vec{M}$ is the vector of the controlling mechanical moment.

Assume that the microsatellite has the shape of a parallelepiped with the sides *a*, *b* and *c*, respectively, along the axes *x*, *y* and *z*. The principal central moments of inertia of such microsatellite in body-fixed coordinate system *OXYZ* are:

$$J_x = \frac{m}{12}\left(b^2 + c^2\right); \quad J_y = \frac{m}{12}\left(a^2 + c^2\right); \quad J_z = \frac{m}{12}\left(a^2 + b^2\right), \tag{2}$$

where m is the mass of the microsatellite.

The kinetic energy of rotational motion of the microsatellite is:

$$V = \frac{1}{2} \cdot \vec{\omega}^T \cdot J \cdot \vec{\omega}. \tag{3}$$

The expression (3) can be considered as Lyapunov's function. In this case, according to Lyapunov's second method, the condition of the asymptotic stability of a stationary point $\omega_x = \omega_y = \omega_z = 0$ is:

$$\frac{dV}{dt} < 0. \tag{4}$$

Let us calculate the value of Lyapunov's function in a scalar form:

$$V = \frac{m}{24} \cdot \left(\left(b^2 + c^2\right)\omega_x^2 + \left(a^2 + c^2\right)\omega_y^2 + \left(a^2 + b^2\right)\omega_z^2 \right). \tag{5}$$

Differentiating function (5) with respect to the time of motion, we find the derivative of Lyapunov's function:

$$\frac{dV}{dt} = \frac{m}{12} \cdot \left(\left(b^2 + c^2\right)\omega_x \omega_x' + \left(a^2 + c^2\right)\omega_y \omega_y' + \left(a^2 + b^2\right)\omega_z \omega' \right), \tag{6}$$

where $\omega_x' = \varepsilon_x$, $\omega_y' = \varepsilon_y$, $\omega_z' = \varepsilon_z$ are the projections of angular acceleration of the microsatellite on the coordinate axes *Ox*, *Oy*, *Oz*.

The mechanical moment $\vec{M}$ is calculated as follows [1]:

$$\vec{M} = \vec{L} \times \vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ L_x & L_y & L_z \\ H_x & H_y & H_z \end{vmatrix} \tag{7}$$

where $\vec{L}$ is the vector of magnetic moment; $\vec{H}$ is the geomagnetic field intensity vector; L_x, L_y, L_z are the magnetic moments of the coils oriented in the direction of corresponding principal axes; H_x, H_y, H_z are the projections of the geomagnetic field vector to coordinate axes *Ox*, *Oy*, *Oz*. From equality (7) we find the projections of the vector of the controlling mechanical moment:

$$M_x = L_y H_z - L_z H_y; \quad M_y = L_z H_x - L_x H_z; \quad M_z = L_x H_y - L_y H_x. \tag{8}$$

Substituting the magnitudes of the angular accelerations according to the equations (1) into the expression (6), we obtain:

$$\frac{dV}{dt} = \omega_x\left(L_y H_z - L_z H_y\right) + \omega_y\left(L_z H_x - L_x H_z\right) + \omega_z\left(L_x H_y - L_y H_x\right). \tag{9}$$

Ensuring asymptotic stability of the system requires fulfillment of inequality (4) what taking into account expression (9) is given by:

$$\omega_x\left(L_y H_z - L_z H_y\right) + \omega_y\left(L_z H_x - L_x H_z\right) + \omega_z\left(L_x H_y - L_y H_x\right) < 0. \tag{10}$$

Here the components of angular velocity can be determined by means of angular velocity sensors. The components of the intensity vector can be calculated in one of several known ways as described in the studies [1], [6], [9], [16]. Let, for example, the components of the intensity vector $\vec{H}$ be calculated in the way described in the study [16]. In deducing these expressions, it is supposed that the geomagnetic field intensity is described by the model of straight dipole [9]. Projecting the vector $\vec{H}$ to the coordinate axes *OXYZ*, we get:

$$H_X = H\cos\theta, \; H_Y = -H\sin\theta\cos\varphi, \; H_Z = H\sin\theta\sin\varphi, \tag{11}$$

where $H = \frac{\mu_e}{r^3}(1 + 3\sin^2 i \sin^2 u)^{1/2}$ is the module of intensity vector, $\mu_e = 7{,}87 \cdot 10^{15} (Tc \cdot м^3)$ is Earth's magnetic constant, *i* is the orbit inclination, *u* is the argument of a latitude, *r* is the distance from a microsatellite to the center of Earth, $r = R_e + h$; R_e is the Earth radius, θ is the nutation angle, φ is the precession angle.

The dynamical Euler's equations (1) are considered jointly with the kinematic equations that allow us to determine Euler's angles [16]:

$$d\theta / dt = \omega_z \cos\varphi + \omega_y \sin\varphi + \tilde{\omega}_{YM} \sin\psi - \tilde{\omega}_{ZM} \cos\psi,$$
$$d\psi / dt = (\omega_z \sin\varphi - \omega_y \cos\varphi) / \sin\theta -$$
$$\tilde{\omega}_{XM} + \tilde{\omega}_{YM} ctg\theta \cos\psi + \tilde{\omega}_{ZM} ctg\theta \sin\psi, \quad (12)$$
$$d\varphi / dt = \omega_x - d\psi / dt \cos\theta - \tilde{\omega}_{ZM} \sin\theta \sin\psi,$$

where $\tilde{\omega}_{XM}$, $\tilde{\omega}_{YM}$, $\tilde{\omega}_{ZM}$ are the angular velocities of rotation of a magnetic coordinate system $OX_M Y_M Z_M$ around the fixed geocentric coordinate system $OX_e Y_e Z_e$; $\tilde{\omega}_{XM} = (\dot{\Omega} + \dot{\alpha}_2)\cos\alpha_1$, $\tilde{\omega}_{YM} = -(\dot{\Omega} + \dot{\alpha}_2)\sin\alpha_1$, $\tilde{\omega}_{ZM} = \dot{\alpha}_1$. Herein the angular velocity $\dot{\Omega}$ determines the orbit precession. The angles α_2 and α_1 are determined from a solution of the differential equations:

$$\dot{\alpha}_1 = \frac{3\cos u \sin i (1 + \sin^2 i \sin^2 u)}{(1 - \sin^2 i \sin^2 u)^{1/2} (1 + 3\sin^2 i \sin^2 u)} \dot{u},$$
$$\dot{\alpha}_2 = \frac{\cos i}{1 - \sin^2 i \sin^2 u} \dot{u}.$$

The deduced inequality (10) allows to find a value of one of magnetic moments (for example L_x) if the values of two other moments are known (in our case L_y and L_z). Let us group the left side of inequality (10) relative to magnetic moments. As a result the condition of the asymptotic stability becomes:

$$L_x\left(\omega_z H_y - \omega_y H_z\right) + L_y\left(\omega_x H_z - \omega_z H_x\right) +$$
$$+ L_z\left(\omega_y H_x - \omega_x H_y\right) < 0. \quad (13)$$

At each instant of time inequality (13) describes a half-space of asymptotic stability of the system which schematically may be displayed, for example, in Figure 1 as the domain of space indicated by arrows. On the contrary, in front of the plane specified in Figure 1 there is a domain of instability. In the domain of system instability the following condition is fulfilled:

$$L_x\left(\omega_z H_y - \omega_y H_z\right) + L_y\left(\omega_x H_z - \omega_z H_x\right) +$$
$$+ L_z\left(\omega_y H_x - \omega_x H_y\right) > 0.$$

3. Stability domains of relative motion

Note that only two coils fixed parallel to the corresponding axes of inertia make it possible to achieve effective control in three channels of rotational motion. This conclusion follows from the system of equations (1). Indeed, zeroing of one of components of magnetic moment L_i (i=x, y, z) does not lead to zeroing of any of three mechanical controlling moments M_i in formula (8). Therefore, to save the electric energy of the microsatellite, we will use two instead of three on-board magnetic coils. In this case, one of the magnetic components (e.g. L_y) will be equal to zero.

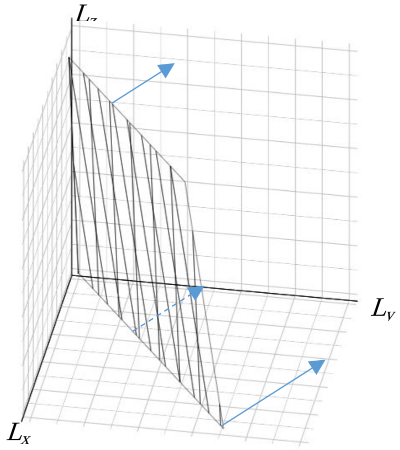

Figure 1. Domains of the asymptotic stability and instability of system with three magnetic coils in a satellite

Hence, the inequality (18) takes the form:

$$L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) < 0. \quad (14)$$

Inequality (14) allows to find possible values of one of magnetic moments (for example L_z) if the value of other moment is known (in our case L_x). In this case the domain of asymptotic stability is the half-plane. Indeed, the stability domain of the system lies under the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$, i.e. in the part of the plane, where the condition $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) < 0$ is fulfilled. Here the domain of system instability is above the straight line

$$L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0.$$

As the coils have own magnetic characteristics, let us set the limitations on the magnetic moments of coils:

$$\lambda_{1x} \le L_x \le \lambda_{2x},$$
$$\lambda_{1z} \le L_z \le \lambda_{2z}, \quad (15)$$

where $\lambda_{1x}, \lambda_{2x}, \lambda_{1z}, \lambda_{2z}$ are some constant positive values, which depend on physical characteristics of magnetic coils. At each instant of time an angular velocity sensor detects components of angular velocity of microsatellite. Let us assume that we know the value of the moment of the coil L_x and its axis is parallel to the axis *x*. It is necessary to find the limits in which the magnetic moment of the coil L_z must remain so that the system will be in stable equilibrium position. Without loss of generality, we separately consider two special cases.

In other words, we need to find out what maximal and minimal values of the magnetic moment L_x must be with limitative inequalities (14) and (15) being fulfilled. For example, it will graphically look like as shown in Figure 2. The domain of system stability lies under the boundary $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$ between the lines $L_x = \lambda_{1x}$, $L_x = \lambda_{1x}$, $L_z = \lambda_{1z}$, $L_z = \lambda_{2z}$ i.e. in the shaded part of the plan

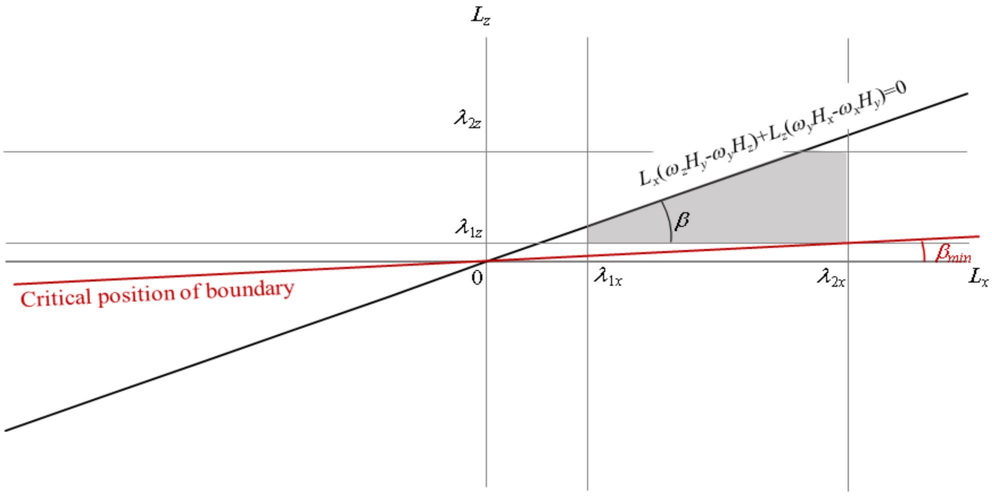

Figure 2. Domain of magnetic moments taking into account the limitations of system stability

When constructing stability domains the following question remains unresolved: at exactly which points the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ intersects the lines $L_x=\lambda_{1x}$, $L_x=\lambda_{2x}$, $L_z=\lambda_{1z}$, and $L_z=\lambda_{2z}$. Indeed, there are several possible cases of intersections of these lines.

In Figure 2 the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ on the boundary of stability domain intersects the straight lines $L_x=\lambda_{1x}$ and $L_z=\lambda_{2z}$.

Let us consider critical positions of the line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ in which at least at one point of the rectangle with the boundaries $L_x=\lambda_{1x}$, $L_x=\lambda_{2x}$, $L_z=\lambda_{1z}$, $L_z=\lambda_{2z}$ stability will remain. From inequality (14) we get

$$L_z\left(\omega_y H_x-\omega_x H_y\right)<-L_x\left(\omega_z H_y-\omega_y H_z\right). \quad (16)$$

If the following conditions are fulfilled

$$\omega_y H_x-\omega_x H_y>0, \quad (17)$$

$$\omega_z H_y-\omega_y H_z<0, \quad (18)$$

then from condition (16) we get

$$L_z<-L_x\cdot\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}, \quad (19)$$

that corresponds to the case presented in Figure 2.

If the following inequality is fulfilled

$$\omega_y H_x-\omega_x H_y<0, \quad (20)$$

then the domain of stability is above the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$.

Let us consider at first a case when inequalities (17) and (18) are fulfilled. Let us specify the angle β between the axis L_x and straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ as shown in Figure 2. For distinctness we will assume that all the values λ_{1x}, λ_{2x}, λ_{1z}, λ_{2z} are positive. With the given limitations the angle β may vary between 0 and $\frac{\pi}{2}$. Let us begin to decrease the angle β, simultaneously rotating the line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ around the origin of coordinates until the position where only one pair of possible values L_x and L_z is left with which the system remains in stable equilibrium position. The critical position of the boundary of the stability domain $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ is shown in Figure 3 with the red line. In critical position of the boundary $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ only one point of system stability is left.

Let us assume that a microsatellite has onboard magnetic coils with identical characteristics, then we obtain

$$\lambda_{1x}=\lambda_{1z}=\lambda_1, \lambda_{2x}=\lambda_{2z}=\lambda_2. \quad (21)$$

If the condition (21) and also the conditions specified earlier are fulfilled, then we obtain the graphic illustration of the problem represented in Figure 3.

At the moment when the boundary is in a critical position, the position of stable equilibrium of the system is determined only with one pair of magnetic moments of the coils, that is $L_x=\lambda_2$, $L_z=\lambda_1$. Let us begin to increase the angle β. While the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ is below the bisector of the first quadrant angle (see Figure. 3), the moment L_x may possess only a value from the interval $\left(\lambda;\lambda_2\right)$ where λ can be determined from the condition that the point $\left(\lambda;\lambda_1\right)$ belongs to the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$. In this case the equality

$L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ is fulfilled. Thus, while the angle $\beta<\frac{\pi}{4}$, then

$$-\lambda_1\cdot\frac{\omega_y H_x-\omega_x H_y}{\omega_z H_y-\omega_y H_z}<L_x<\lambda_2. \quad (22)$$

At the same time inequality (19) must be fulfilled. As with the specified limitations the inequality $-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}>0$ is fulfilled, from inequality (22) it follows that

$$\lambda_1\cdot\frac{\omega_y H_x-\omega_x H_y}{\omega_z H_y-\omega_y H_z}\cdot\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}<$$

$$<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_2.$$

Hence we get

$$\lambda_1<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_2.$$

Then owing to the inequality

$$\lambda_1<L_z<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_2 \quad (23)$$

is true. Here we defined the angle β as an angle between the axis L_x and the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$.

Hence it follows that the slope of the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ which is equal to $-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}$ coincides with the tangent of the angle β, i.e.

$$tg\beta=\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}. \quad (24)$$

Then equality (23) is true while $\operatorname{tg}\beta<\operatorname{tg}\frac{\pi}{4}=1$.

Hence it follows fulfillment of the condition

$$0<\omega_y H_z-\omega_z H_y<\omega_y H_x-\omega_x H_y. \quad (25)$$

And then jointly with the inequality (23) we obtain

$$\lambda_1<L_z<\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\cdot L_x<\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\cdot\lambda_2<\lambda_2. \quad (26)$$

If β becomes greater than $\frac{\pi}{4}$ (see Figure 4) then the moment L_x can possess all possible values from λ_1 to λ_2 :

$$\lambda_1<L_x<\lambda_2. \quad (27)$$

But at the same time inequality (19) must be fulfilled. As with the specified limitations the inequality $-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}>0$ is valid, then from condition (27) it follows:

$$\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\cdot\lambda_1<\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\cdot L_x<\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\cdot\lambda_2.$$

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If the condition (21) and also the conditions specified earlier are fulfilled, then we obtain the graphic illustration of the problem represented in Figure 3.

At the moment when the boundary is in a critical position, the position of stable equilibrium of the system is determined only with one pair of magnetic moments of the coils, that is $L_x = \lambda_2$, $L_z = \lambda_1$. Let us begin to increase the angle β. While the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$ is below the bisector of the first quadrant angle (see Figure. 3), the moment L_x may possess only a value from the interval $\left(\lambda;\lambda_2\right)$ where λ can be determined from the condition that the point $\left(\lambda;\lambda_1\right)$ belongs to the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$. In this case the equality $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$ is fulfilled. Thus, while the angle $\beta < \frac{\pi}{4}$, then

$$-\lambda_1 \cdot \frac{\omega_y H_x - \omega_x H_y}{\omega_z H_y - \omega_y H_z} < L_x < \lambda_2. \quad (22)$$

At the same time inequality (19) must be fulfilled. As with the specified limitations the inequality $-\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} > 0$ is fulfilled, from inequality (22) it follows that

$$\lambda_1 \cdot \frac{\omega_y H_x - \omega_x H_y}{\omega_z H_y - \omega_y H_z} \cdot \frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} <$$

$$< -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot L_x < -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot \lambda_2.$$

Hence we get

$$\lambda_1 < -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot L_x < -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot \lambda_2.$$

Then owing to the inequality

$$\lambda_1 < L_z < -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot L_x < -\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} \cdot \lambda_2 \quad (23)$$

is true. Here we defined the angle β as an angle between the axis L_x and the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$.

Hence it follows that the slope of the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$ which is equal to $-\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y}$ coincides with the tangent of the angle β, i.e.

$$tg\beta = \frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y}. \quad (24)$$

Then equality (23) is true while $\mathrm{tg}\,\beta < \mathrm{tg}\,\frac{\pi}{4} = 1$.

Hence it follows fulfillment of the condition

$$0 < \omega_y H_z - \omega_z H_y < \omega_y H_x - \omega_x H_y. \quad (25)$$

And then jointly with the inequality (23) we obtain

$$\lambda_1 < L_z < \frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y} \cdot L_x < \frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y} \cdot \lambda_2 < \lambda_2. \quad (26)$$

If β becomes greater than $\frac{\pi}{4}$ (see Figure 4) then the moment L_x can possess all possible values from λ_1 to λ_2 :

$$\lambda_1 < L_x < \lambda_2. \quad (27)$$

But at the same time inequality (19) must be fulfilled. As with the specified limitations the inequality $-\frac{\omega_z H_y - \omega_y H_z}{\omega_y H_x - \omega_x H_y} > 0$ is valid, then from condition (27) it follows:

$$\frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y} \cdot \lambda_1 < \frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y} \cdot L_x < \frac{\omega_y H_z - \omega_z H_y}{\omega_y H_x - \omega_x H_y} \cdot \lambda_2.$$

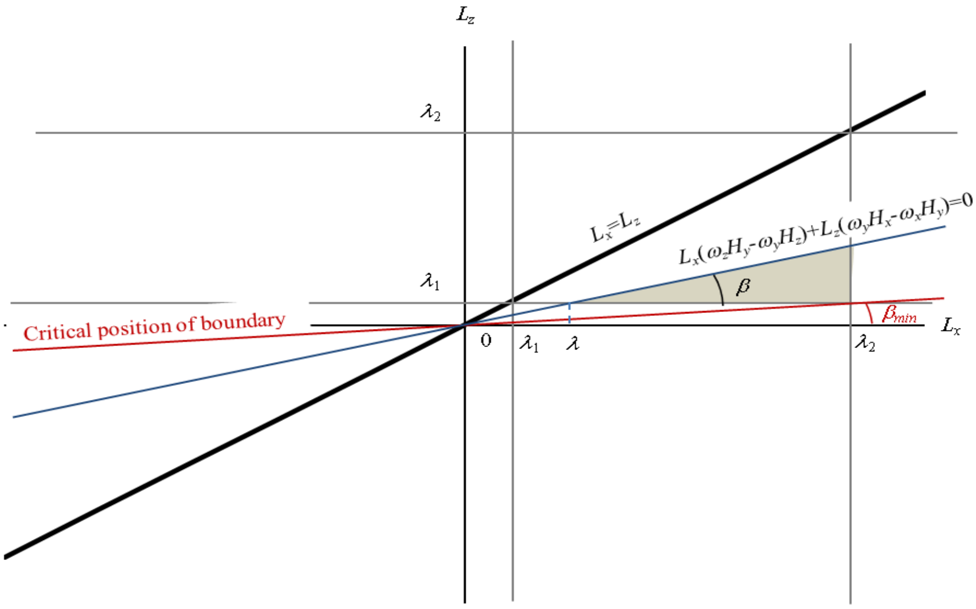

Figure 3. Stability domain with the angle $\beta < \frac{\pi}{4}$

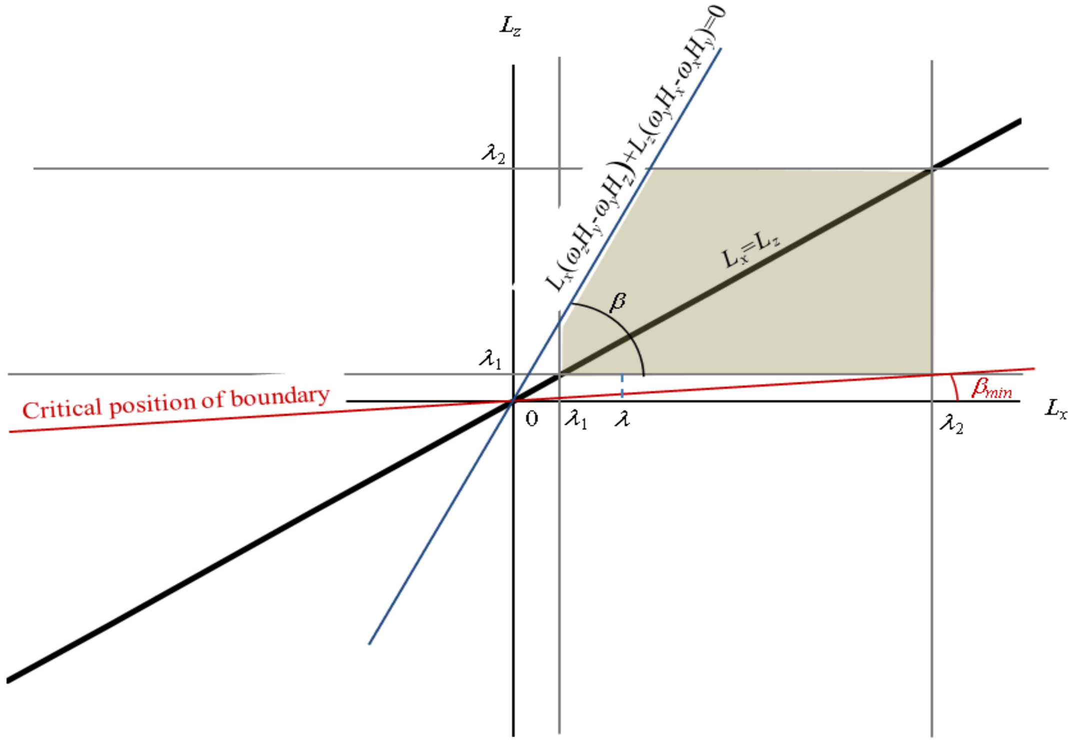

Figure 4. Stability domain in case of $\beta > \frac{\pi}{4}$

At the same time if the boundary of the domain is in a critical position, then the position of the stable equilibrium of the system is determined only by having one pair of values of magnetic moments on coils, that is $L_x = -\lambda_1$, $L_z = -\lambda_2$. Let us begin to decrease the angle β.

While the straight line $L_x\left(\omega_z H_y - \omega_y H_z\right) + L_z\left(\omega_y H_x - \omega_x H_y\right) = 0$ is below the bisector of the first quadrant shown in Figure 6 as thick straight line, the moment L_x can possess values from the interval $\left(-\lambda_x; -\lambda_1\right)$.

Here the value $-\lambda_x$ can be determined from the condition that the point $\left(-\lambda_x;-\lambda_2\right)$ belongs to the line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$. Then we get $\lambda_x=\lambda_2\cdot\dfrac{\omega_y H_x-\omega_x H_y}{\omega_z H_y-\omega_y H_z}$. Thus, while $\beta>\dfrac{\pi}{4}$ the following inequality is valid

$$\lambda_2\cdot\frac{\omega_y H_x-\omega_x H_y}{\omega_z H_y-\omega_y H_z}<L_x<-\lambda_1. \qquad (33)$$

At the same time the inequality (19) must be fulfilled. As in the case of the limitation $-\dfrac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}>0$, from inequality (33) it follows that

$$-\lambda_2\cdot\frac{\omega_y H_x-\omega_x H_y}{\omega_z H_y-\omega_y H_z}\cdot\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}<$$
$$<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_1.$$

Therefore, we get the inequality

$$-\lambda_2<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_1.$$

Then owing to inequality (19) the following inequality is valid

$$-\lambda_2<L_z<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot\lambda_1. \qquad (34)$$

The stability domain in this case is shown in Figure 6.

Thus, we defined the angle β as the angle between the axis L_x and the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$. As the slope of the straight line $L_x\left(\omega_z H_y-\omega_y H_z\right)+L_z\left(\omega_y H_x-\omega_x H_y\right)=0$ which is equal to $-\dfrac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}$ coincides with the tangent of the angle β (24), inequality (34) is valid while $\operatorname{tg}\beta>\operatorname{tg}\dfrac{\pi}{4}=1$. Hence it follows that

$$\omega_y H_z-\omega_z H_y>\omega_y H_x-\omega_x H_y. \qquad (35)$$

And then jointly with inequality (38) we get

$$-\lambda_2<L_z<-\frac{\omega_z H_y-\omega_y H_z}{\omega_y H_x-\omega_x H_y}\cdot L_x<-\lambda_1. \qquad (36)$$

As soon as β becomes less than $\dfrac{\pi}{4}$ (see Figure 7) L_x can possess all possible values from λ_1 to λ_1:

$$-\lambda_2<L_x<-\lambda_1. \qquad (37)$$

But at the same time the inequality (19) must be fulfilled. Moreover, jointly with inequality (37) owing to inequality (19) the following is true

$$-\lambda_2<L_z<\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}L_x. \qquad (38)$$

We note that inequality (38) is true while $\operatorname{tg}\beta\le\operatorname{tg}\dfrac{\pi}{4}=1$, therefore

$$\frac{\omega_y H_z-\omega_z H_y}{\omega_y H_x-\omega_x H_y}\le 1. \qquad (39)$$

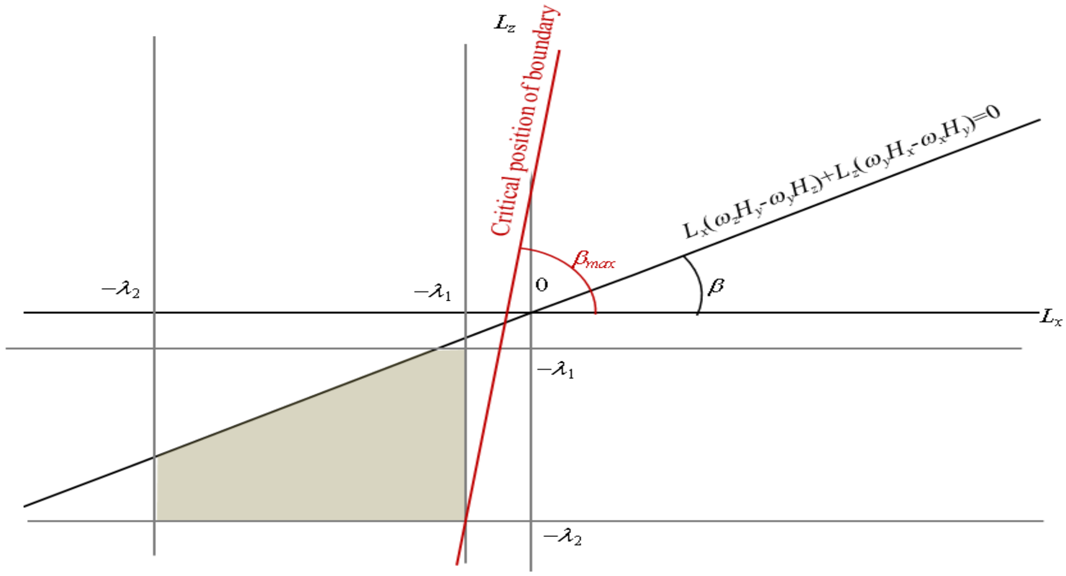

Figure 5. Graphical interpretation of the stability domain with using of two identical magnetic coils with negative values of magnetic moments

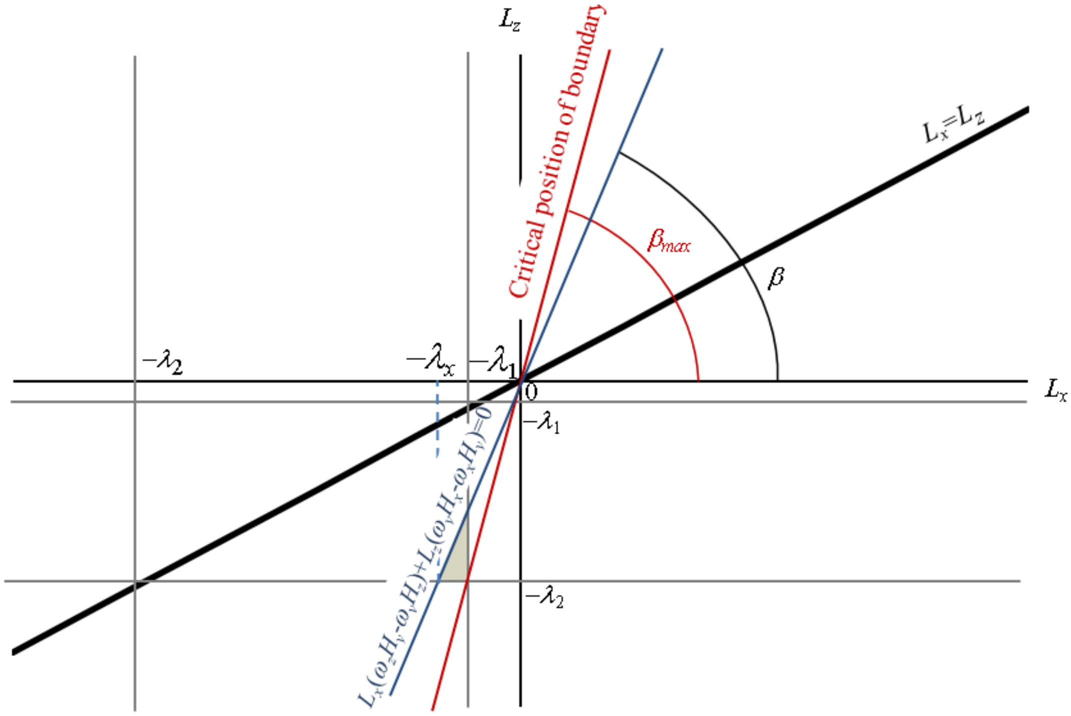

Figure 6. Stability domain in case of $\beta > \frac{\pi}{4}$ and negative values of magnetic moments

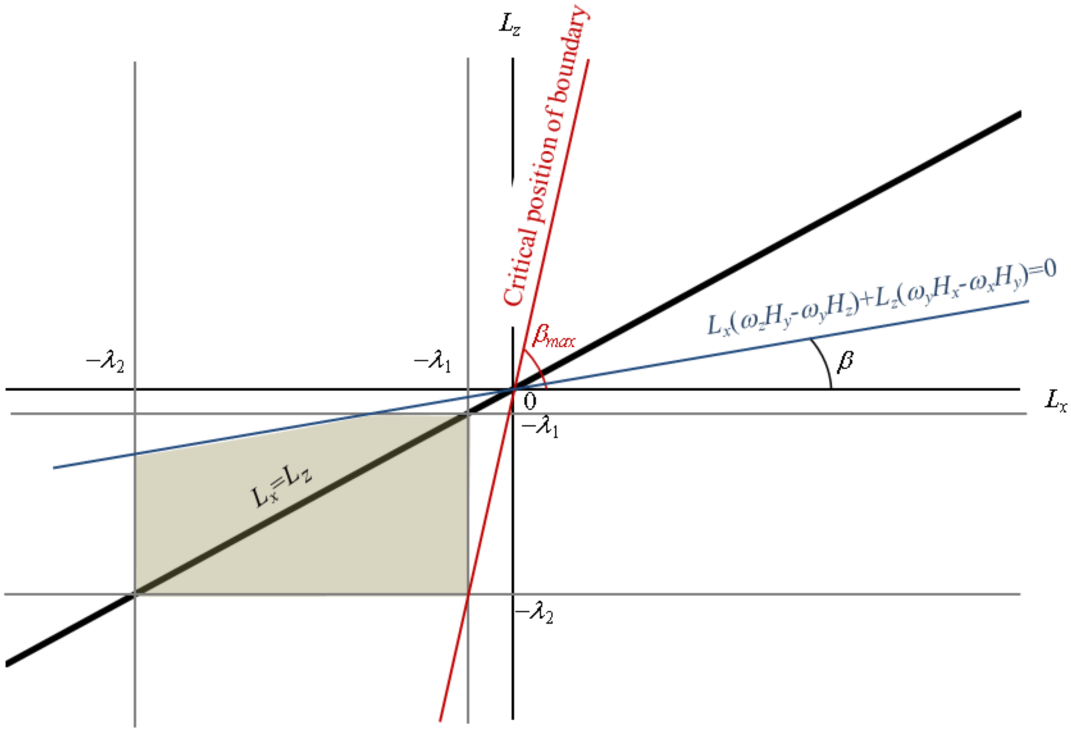

Figure 7. Stability domain in case of $\beta < \frac{\pi}{4}$ and negative values of moments of inertia

4. The algorithm of finding of the magnetic moments

The domains of magnetic moments of the microsatellite with the two onboard coils that ensure the asymptotic stability of the trivial solution $\omega_x = \omega_y = \omega_z = 0$ were determined. The study revealed the following characteristic cases of constructing the domains of magnetic moments.

1. When conditions (17) and (18) are fulfilled, in case inequality (25) is fulfilled and values of magnetic moments are positive, the asymptotic stability of the equilibrium position of the system requires the fulfillment of the inequality (26).

2. When conditions (17) and (18) are fulfilled, in case inequality (28) is fulfilled and values of magnetic moments are positive, the asymptotic stability of the equilibrium position of the system requires the fulfillment of the inequality (29).

3. When conditions (17) and (18) are fulfilled, in case the inequality (35) is fulfilled and values of magnetic moments are negative, the asymptotic stability of the equilibrium position of the system requires the fulfillment of inequality (36).

4. When conditions (17) and (18) are fulfilled, in case the inequality (39) is fulfilled and values of magnetic moments are negative, the asymptotic stability of the equilibrium position of the system requires joint the fulfillment of inequalities (37) and (38).

The found domains of stability of the system should be determined at each particular instant of the time of the microsatellite motion. Let us consider an algorithm of finding the magnetic moments what ensure damping angular velocities of a microsatellite with two onboard coils. The algorithm is presented in Figure 8. This algorithm contains the described above results of constructing of the asymptotic stability domains with numbers 1-4.

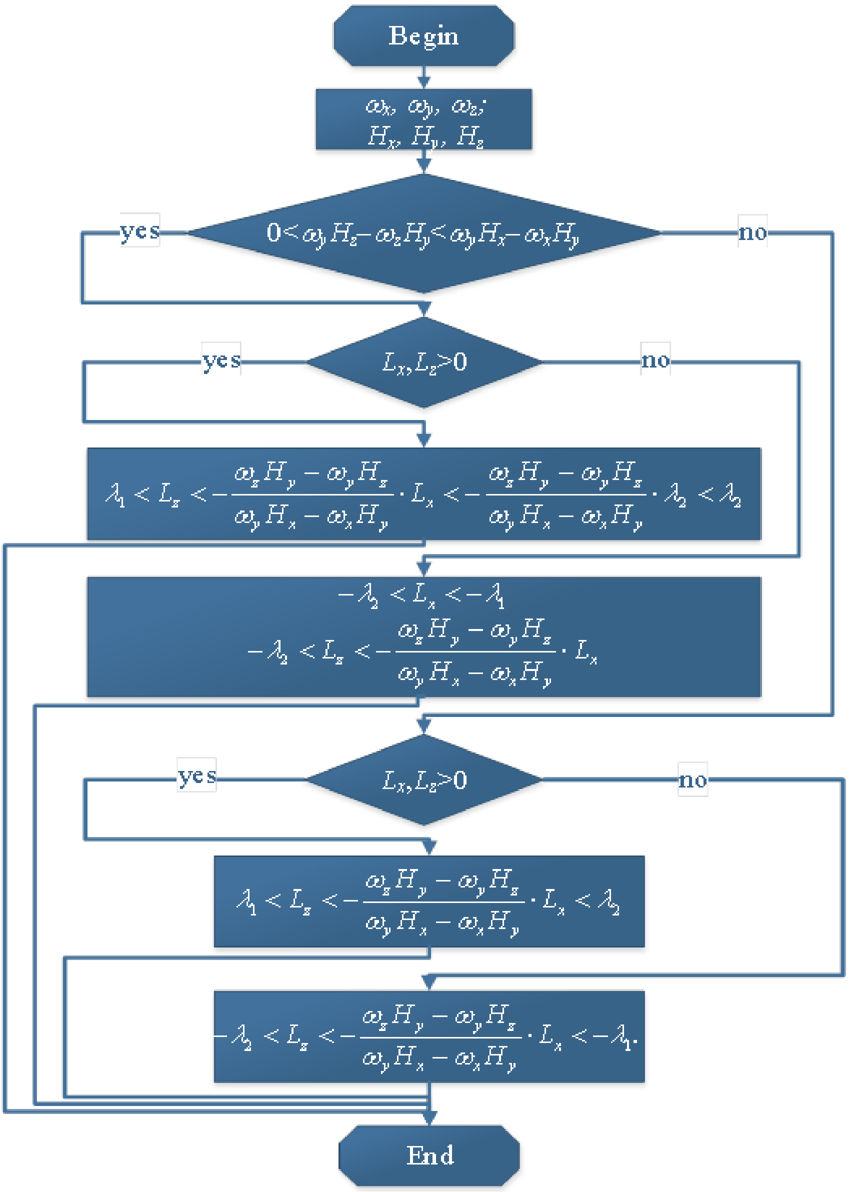

Figure 8. The algorithm of finding of the magnetic moments that ensure asymptotic stability of the system

5. Conclusion

The main problem considered in the paper is the calculation of domains of the magnetic moments of two onboard coils, which lead of stable damping of the microsatellite angular velocities. The paper is devoted to solving this problem. Application of Lyapunov's method allows us to obtain the conditions of asymptotic stability in the problem of controlled damping of angular velocity of the parallelepiped-shaped microsatellite with two onboard magnetic coils. The analysis of the stability conditions has determined the domains of the coils magnetic moments leading to the asymptotic stability of the system solution. This study considered a special case of the relation between the magnitudes of angular velocities and the magnitudes of the magnetic field intensities caused by fulfillment of the inequalities (17)-(18). Note that similar considerations are also possible to obtain the stability domains with other special limitations on the angular velocity and the intensity of the magnetic field. However, these results are beyond the scope of this paper, but can be considered in subsequent publications. It should be noted that the described angular velocity damping algorithm can be applied to modeling the rotational motion of new microsatellites or nanosatellites with the magnetic control system.

The domains of the asymptotic stability were obtained for a microsatellite in the form of a parallelepiped. However, similar stability domains can be defined for microsatellites of any other forms. It is necessary to recalculate the magnitudes of the principal central moments of inertia of the microsatellite by formulas (2). It should be noted that these results are valid for arbitrary initial angular velocities.

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