

A Comprehensive Model of Reliability, Availability, and Maintainability (RAM) for Industrial Systems Evaluations

Mohamed F. Aly^a, Islam H. Afefy^a, Ragab K. Abdel-Magied^b, Eman K. Abd Elhalim^{c*}

^a Industrial Engineering Department, Faculty of Engineering, Fayoum University, Egypt.

^b Mechanical Engineering Department, Faculty of Engineering, Beni-Suef University, Egypt.

^c Industrial Engineering Department, Faculty of Engineering, Misr University for Science and Technology (MUST), Egypt.

Received 29 April 2018

Accepted 21 June 2018

Abstract

Reliability, availability, and maintainability are considered as a crucial metrics that are used to evaluate the performance of the industrial systems. In this work, an integrated reliability, availability, and maintainability (RAM) model of the 3-out-of-4 system was proposed to quantify the values of RAM indices and to identify the most critical equipment which mainly affects the system performance. The Markovian approach was adopted to model the system behavior. A transition diagram for the proposed model was constructed and differential equations of the proposed model were formulated to obtain the state probability. The availability at steady state, reliability at transient state and maintainability were analyzed and investigated. The proposed model was verified and validated. A real data of industrial system in Oil and Gas Egyptian Company was applied to validate the proposed model and the effect of failure and repair rates at different mission time was presented and discussed. The results of the applied proposed system revealed that the system availability at steady state is 99 %, the system reliability is 0.59%, and the system maintainability is 0.99%. On the other hand Turbine no. three was found the most critical item in the system and need more attention to improve the system performance. It could be said that the proposed model is considered an excellent tool for industrial systems performance evaluations.

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Keywords: Reliability; Availability; Maintainability; Markov approach.

1. Introduction

RAM is an important performance metrics in system analysis and considered a good starting point for system improvements [1]. The Oil and Gas industry has consistently increased its requirements, combined with the rise in technological systems, and increased competitiveness of service providers to implement adequate management strategies for these systems to improve their availability and productivity to comply with those most demanding standards. One important point in this regard is to have knowledge about the RAM of the main equipment in this industry [2]. Evidentially, a fault-based (Breakdown) maintenance system in the Oil and Gas industry is a costly and time-consuming process, resulting in a substantial and intangible loss to system operators.

Corvaro, et al. [3], assesses operational performance of reciprocating compressors used in Gas and Oil industry using RAM model. The study aimed to evaluate the effect of different factors related to RAM and devoted to collaborating with the private sector aiming continuous quality improvement. Aoudia, et al. [4], studied the economic impact of maintenance management ineffectiveness of one of the main industrial plants of a major Oil and Gas group. Sharma and Kumar, [5] built a

RAM model applied to a process industry using the Markovian approach in steady state. Parametric computations and indices of RAM to assess system performance in repairable industrial systems using Genetic Algorithm (GA) and the Markovian approach were presented [6, 7].

Evidently, the integration of reliability, availability and maintainability of investigation tends to good results. However, reliability, Availability, and maintainability were investigated individually or two of them in the other industries. Much effort has been made by the researchers providing performance modelling and availability analysis applied on different industrial systems as Paper Plant, Paint, and thermal power plant Industry [8-10].

Aggarwal, et. al. [11] presented a model using Markov birth-death process, the concept of fuzzy reliability and availability. A numerical method with the assumption that the failure and repair rates of each subsystem follow the exponential distribution has been developed. In which a mathematical modeling of the system is carried out using the mnemonic rule to derive Chapman-Kolmogorov differential equations and solved it by Runge-Kutta fourth-order method Which is considered one of the most common methods that are used in solving the differential equations as well as it is considered one of the oldest and the best method in numerical analysis moreover; it

* Corresponding author e-mail: emankhtab2005@gmail.com.

provides a popular way to solve the differential equations. When the system includes a large numbers of differential equations, MATLAB program could be used to solve these large equations. For this, the MATLAB software is considered one of the multi programs that could be used for numerical computations.

Lin, et al. [12] presented a reliability study using both classical and Bayesian semi-parametric frame-works, they illustrated how a wheel- set's degradation data can be modeled and analyzed to ease the calculation of system reliability during applying preventive maintenance. Singh, and Goyal, [13] developed methodology to study the transient behaviour of repairable mechanical biscuit shaping system on a biscuit manufacturing plant for determining the availability of the system based on Markov modelling. The differential equations have been solved using Laplace Transforms. Laplace Transform commonly used in the transient state to obtain the state probabilities, in which the differential equations are converted to algebraic equations to simplify the system solution.

The K-out-of-N system is the most important type for the repairable system according to reliability theory and, it is used in many applications such as petroleum industry. An investigation of a 2-out-of-3 system has been presented recently in published work in which the reliability and availability have been evaluated and analyzed for the system using Kolmogorov's equations and applied on some particular cases. In this analysis, mean time to system failure (MTSF), steady-state availability, busy period and profit function were derived to evaluate the system reliability and availability [14-15]. Preeti, [16] presented an analysis which considered as a powerful tool to analyze reliability of a linear consecutive 2-out-of-3-F system with common cause shock failure in which the transient equations of the reliability and steady equations of the availability have been investigated. Yusuf, [17] evaluate the system reliability indices of a repairable 3-out-of-4 system with preventive maintenance involving four types of failures using Kolmogorov equations.

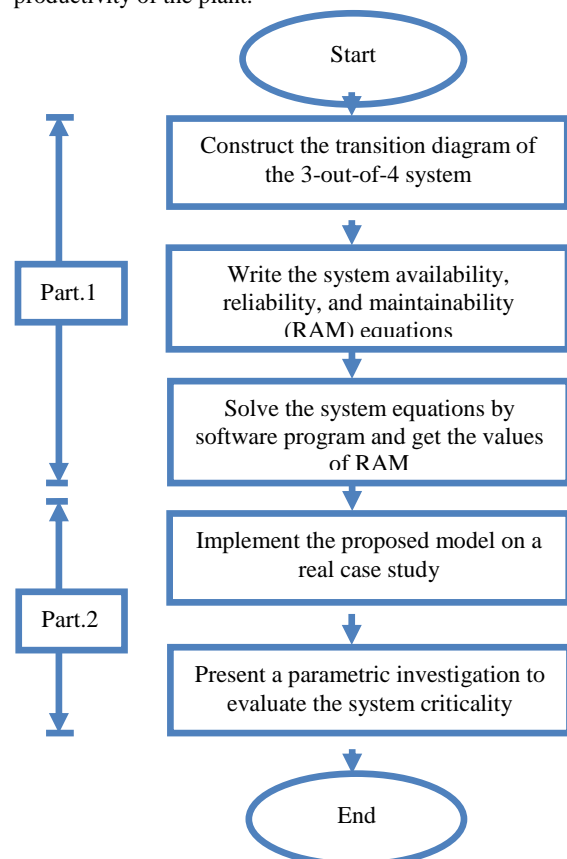
Apparently, the literature review -up to our reading-revealed the following points:

- The researchers concentrated, to more extent, on the investigation of availability and reliability of the industrial systems.
- Oil and Gas industry need more attention to improve and maintain its system performance.
- A little attention is paid to investigate the integration of RAM for different industrial application.
- No more work on studying of RAM analysis for a multi-component system such as 3-out-of-4 system.

Moreover, the applications of RAM analysis as an adopted approach for maintenance policies for Oil and Gas Industrial systems could be proposed and applied for increasing customer satisfaction, reduce the frequency of failures and maintenance costs. This is a motivation of the present work.

The aim of this work is to develop a comprehensive RAM model for industrial systems evaluation. This study has two main parts, as presented in Fig.1; the first is to develop 3-out-of-4 system RAM model based on the Markovian approach. Availability at steady state, reliability at transient state and maintainability are

analyzed and investigated. The differential equations are solved using Rung-Kutta method with aided of MATLAB software to get the system availability at steady state (A_{ss}) and solved by Laplace transform to get reliability at transient state. The second is to apply the proposed model for the performance measure of a real case of Oil and Gas industrial system. This model provides results for a complete reliability, availability, and maintainability (RAM) analysis utilizing data sets from a production system in an Oil and Gas plant. A parametric investigation of various values of system failure rates and repair rates on system reliability (R_s), availability, and maintainability and their effects on the system performance are presented. The results of that analysis help the designers/engineers and managers to quantify and measure the system performance; conversely, suitable maintenance policies/strategies can be selected to enhance the productivity of the plant.



Figuer1: Steps of the presented work.

2. System Description

In this section, the 3-out-of-4 system is described. The system consists of four units in which one unite is standby (sb) and the other three units must be in the operating state (o) for the system to work. The system failed (F) when two units failed and the other two units are in good state (g). Based on Markov assumption [18], differential equations that describe the proposed system are written to analyze the probability for each state. These equations are further solved for determining the RAM indices. The states of the system according to Markov are shown below in Table (1), and the transition diagram in Fig.2 depicts a model showing all the possible states of the system.

Table 1: System states

| State | Component state | | System condition |
|----------------|---|---------------------------------|------------------|
| | Available and standby | Failed | |
| S ₀ | T ₁ , T ₂ , T ₃ , T ₄ | - | Working |
| S ₁ | T ₂ , T ₃ , T ₄ | T ₁ | Working |
| S ₂ | T ₁ , T ₃ , T ₄ | T ₂ | Working |
| S ₃ | T ₁ , T ₂ , T ₄ | T ₃ | Working |
| S ₄ | T ₃ , T ₄ | T ₁ , T ₂ | Failed |
| S ₅ | T ₂ , T ₄ | T ₁ , T ₃ | Failed |
| S ₆ | T ₂ , T ₃ | T ₁ , T ₄ | Failed |
| S ₇ | T ₁ , T ₄ | T ₂ , T ₃ | Failed |
| S ₈ | T ₁ , T ₃ | T ₂ , T ₄ | Failed |
| S ₉ | T ₁ , T ₂ | T ₃ , T ₄ | Failed |

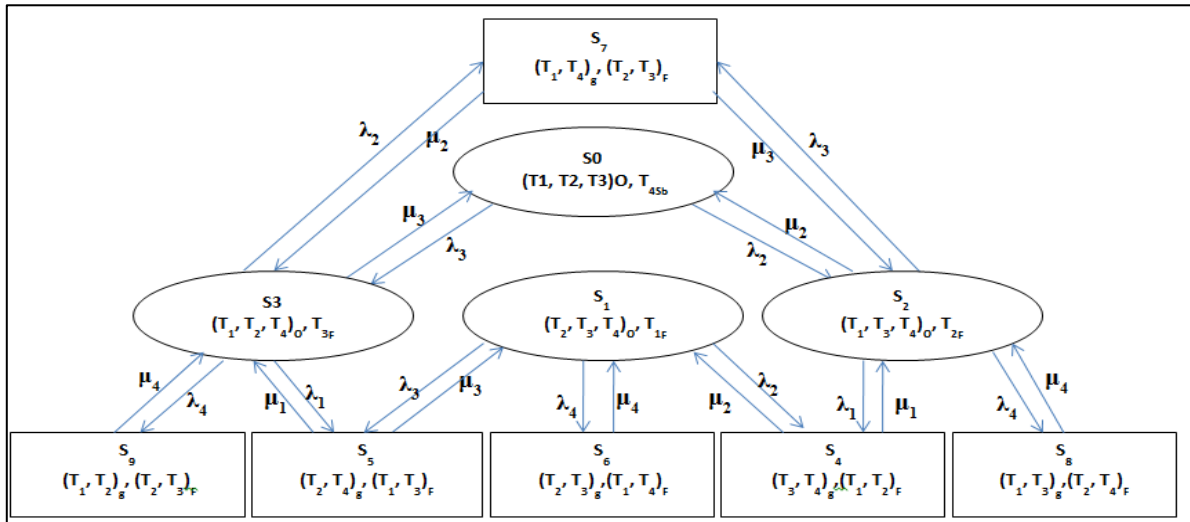


Figure 2: State transition diagram.

Where: : Unit working at full capacity : Failed state

3. Model Proposed

The differential equations associated with the transition diagram are derived on the basis of Markov birth-death process. Various probability considerations generate the following sets of differential equations:

$$[(d/dt) + \lambda_1 + \lambda_2 + \lambda_3] P_0(t) = \mu_1 P_1(t) + \mu_2 P_2(t) + \mu_3 P_3(t) \quad (1)$$

$$[(d/dt) + \mu_1 + \lambda_2 + \lambda_3 + \lambda_4] P_1(t) = \mu_2 P_4(t) + \mu_3 P_5(t) + \mu_4 P_6(t) + \lambda_1 P_0(t) \quad (2)$$

$$[(d/dt) + \lambda_1 + \lambda_3 + \lambda_4 + \mu_2] P_2(t) = \mu_1 P_4(t) + \mu_3 P_7(t) + \mu_4 P_8(t) + \lambda_2 P_0(t) \quad (3)$$

$$[(d/dt) + \lambda_1 + \lambda_2 + \lambda_4 + \mu_3] P_3(t) = \mu_1 P_5(t) + \mu_2 P_7(t) + \mu_4 P_9(t) + \lambda_3 P_0(t) \quad (4)$$

$$[(d/dt) + \mu_1 + \mu_2] P_4(t) = \lambda_1 P_2(t) + \lambda_2 P_1(t) \quad (5)$$

$$[(d/dt) + \mu_1 + \mu_3] P_5(t) = \lambda_1 P_3(t) + \lambda_3 P_1(t) \quad (6)$$

$$[(d/dt) + \mu_4] P_6(t) = \lambda_4 P_1(t) \quad (7)$$

$$[(d/dt) + \mu_2 + \mu_3] P_7(t) = \lambda_2 P_3(t) + \lambda_3 P_2(t) \quad (8)$$

$$[(d/dt) + \mu_4] P_8(t) = \lambda_4 P_2(t) \quad (9)$$

$$[(d/dt) + \mu_4] P_9(t) = \lambda_4 P_3(t) \quad (10)$$

Where, the initial conditions at time t = 0 are:

$$p_i(t) = \begin{cases} 1, & \text{if } i = 0 \\ 0, & \text{if } i \neq 0 \end{cases} \quad (11)$$

Where:

d/dt: derivative with respect to t, P₀(t): probability that the system is working at full capacity at time t, P_i(t): state probability that the system is in the ith state at time t, λ_i: failure rate for unit i, μ_i: repair rate for unit i.

3.1. Availability Equations

To get the steady state availability of the system (A_{SS}), (i.e., time independent performance behavior) which is mean d/dt=0 and t → ∞, the above equations (eq. 1 to 10) become:

$$(\lambda_1 + \lambda_2 + \lambda_3) P_0 = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 \quad (12)$$

$$(\mu_1 + \lambda_2 + \lambda_3 + \lambda_4) P_1 = \mu_2 P_4 + \mu_3 P_5 + \mu_4 P_6 + \lambda_1 P_0 \quad (13)$$

$$(\lambda_1 + \lambda_3 + \lambda_4 + \mu_2) P_2 = \mu_1 P_4 + \mu_3 P_7 + \mu_4 P_8 + \lambda_2 P_0 \quad (14)$$

$$(\lambda_1 + \lambda_2 + \lambda_4 + \mu_3) P_3 = \mu_1 P_5 + \mu_2 P_7 + \mu_4 P_9 + \lambda_3 P_0 \quad (15)$$

$$(\mu_1 + \mu_2) P_4 = \lambda_1 P_2 + \lambda_2 P_1 \quad (16)$$

$$(\mu_1 + \mu_3) P_5 = \lambda_1 P_3 + \lambda_3 P_1 \quad (17)$$

$$(\mu_4) P_6 = \lambda_4 P_1 \quad (18)$$

$$(\mu_2 + \mu_3) P_7 = \lambda_2 P_3 + \lambda_3 P_2 \quad (19)$$

$$(\mu_4) P_8(t) = \lambda_4 P_2 \quad (20)$$

$$(\mu_4) P_9 = \lambda_4 P_3 \quad (21)$$

These equations were solved using Rung-Kutta Forth order method and MATLAB, the values of steady state probabilities are as follows:

$$P_1 = (\lambda_1 / \mu_1) P_0, P_2 = (\lambda_2 / \mu_2) P_0, P_3 = (\lambda_3 / \mu_3) P_{0m},$$

$$P_4 = (\lambda_1 \lambda_2) / (\mu_1 \mu_2) P_0, P_5 = (\lambda_1 \lambda_3) / (\mu_1 \mu_3) P_0,$$

$$P_6 = (\lambda_1 \lambda_4) / (\mu_1 \mu_4) P_0, P_7 = (\lambda_2 \lambda_3) / (\mu_2 \mu_3) P_0,$$

$$P_8 = (\lambda_2 \lambda_4) / (\mu_2 \mu_4) P_0, P_9 = (\lambda_3 \lambda_4) / (\mu_3 \mu_4) P_0$$

The probability of full working capacity (P_0) is determined using normalizing conditions (i.e., $\sum_{i=0}^9 P_i = 1$) as follows:

$$P_0 = [(\mu_1 \mu_2 \mu_3 \mu_4) / (\lambda_1 \mu_2 \mu_3 \mu_4 + \lambda_2 \mu_1 \mu_3 \mu_4 + \lambda_3 \mu_1 \mu_2 \mu_4 + \lambda_1 \lambda_2 \mu_3 \mu_4 + \lambda_1 \lambda_3 \mu_2 \mu_4 + \lambda_1 \lambda_4 \mu_2 \mu_3 + \lambda_2 \lambda_3 \mu_1 \mu_4 + \lambda_2 \lambda_4 \mu_1 \mu_3 + \lambda_3 \lambda_4 \mu_1 \mu_2)] \quad (22)$$

Having the values of probabilities (P_0 - P_9) determined, A_{SS} is calculated as a summation of all working state probabilities as follows:

$$A_{SS} = P_0 + P_1 + P_2 + P_3 \quad (23)$$

3.2. Reliability Equations.

To get the reliability (R_S) of the system under consideration at any time, the equations (1 to 10) are solved taking Laplace transform and the probability transform are as follows:

$$[S + \lambda_1 + \lambda_2 + \lambda_3] P_0(S) = \mu_1 P_1(S) + \mu_2 P_2(S) + \mu_3 P_3(S) \quad (24)$$

$$[S + \mu_1 + \lambda_2 + \lambda_3 + \lambda_4] P_1(S) = \mu_2 P_4(S) + \mu_3 P_5(S) + \mu_4 P_6(S) + \lambda_1 P_0(S) \quad (25)$$

$$[S + \lambda_1 + \lambda_3 + \lambda_4 + \mu_2] P_2(S) = \mu_1 P_4(S) + \mu_3 P_7(S) + \mu_4 P_8(S) + \lambda_2 P_0(S) \quad (26)$$

$$[S + \lambda_1 + \lambda_2 + \lambda_4 + \mu_3] P_3(S) = \mu_1 P_5(S) + \mu_2 P_7(S) + \mu_4 P_9(S) + \lambda_3 P_0(S). \quad (27)$$

$$[S + \mu_1 + \mu_2] P_4(S) = \lambda_1 P_2(S) + \lambda_2 P_1(S) \quad (28)$$

$$[S + \mu_1 + \mu_3] P_5(S) = \lambda_1 P_3(S) + \lambda_3 P_1(S) \quad (29)$$

$$[S + \mu_4] P_6(S) = \lambda_4 P_1(S) \quad (30)$$

$$[S + \mu_2 + \mu_3] P_7(S) = \lambda_2 P_3(S) + \lambda_3 P_2(S) \quad (31)$$

$$[S + \mu_4] P_8(S) = \lambda_4 P_2(S) \quad (32)$$

$$[S + \mu_4] P_9(S) = \lambda_4 P_3(S) \quad (33)$$

Where S is the Laplace transform variable.

To determine R_S , a verification model is applied. It could be noted that the probabilities of the failed states haven't any effect on the system reliability, so the system reliability could be calculated considering only the working states (i.e., excluding the failed states). Based on this result, the probability of failed states could be neglected during solution of the complex systems and the probabilities of operating states are only considered.

The previous equations (24 to 33) are solved using MATLAB 2015 at the following initial conditions at time $t=0$ where,

$$P_i(t) = \begin{cases} 1, & \text{if } i = 0 \\ 0, & \text{if } i \neq 0 \end{cases} \quad (34)$$

After solving these equations, the probabilities of operating states for the system, under consideration, are calculated as follows:

$$P_0 = (C A B) / D \quad (35)$$

$$P_1 = (\lambda_1 A B) / D \quad (36)$$

$$P_2 = (\lambda_2 C B) / D \quad (37)$$

$$P_3 = (\lambda_3 C A) / D \quad (38)$$

Where:

$$A = \mu_2 + S + \lambda_1 + \lambda_3 + \lambda_4 \quad (39)$$

$$B = \mu_3 + S + \lambda_1 + \lambda_2 + \lambda_4 \quad (40)$$

$$C = \mu_1 + S + \lambda_2 + \lambda_3 + \lambda_4 \quad (41)$$

$$D = \text{see appendix A} \quad (42)$$

Having the values of probabilities of working states determined, R_S is calculated as follows:

$$R_S(S) = P_0(S) + P_1(S) + P_2(S) + P_3(S) \quad (43)$$

Taking the inverse of Laplace transforms then, P_0 , P_1 , P_2 , and P_3 , are calculated as $F(t)$ and the system reliability is calculated at time t as follows:

$$R(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t) \quad (44)$$

3.3. Maintainability Equations

For any system, the system maintainability (M_S) is calculated as follows:

$$M_S(t) = 1 - e^{-\mu t} \quad (45)$$

Where (μ) is the repair rate ($\mu = 1 / MTTR$), $MTTR$ is a mean time to repair of the system and is calculated as a function in mean time to repair ($MTTR$) and mean time between failure ($MTBF$) of system component i where:

$$MTTR_S = \frac{\sum_{i=1}^n MTTR_i / MTBF_i}{\sum_{i=1}^n 1 / MTBF_i} \quad (46)$$

4. Implementation of Proposed Model

Fig. 3 depicts the block diagram of the real industrial system in Egyptian Petrol Company. This system is consists of two unites of fuel supply connected in parallel with each other; in which the natural gas unit is active and the diesel unit is standby. The fuel supply units are connected in series with four turbines which are connected in parallel with each other (3-out-of-4); three of them are in operating state while the fourth is in the standby state and the system fails when two components (Turbines) fail.

The turbines are considered the most critical items in that system because it is used to operate four plants as shown in the block diagram. Whereas the actual output for each turbine is 3.8 MW and the total power required must be not lower than 11.4 MW; otherwise, the system will stop working which tends to big losses.

The transition diagram of this system is built based on the Markov, as explained previously (in section 2). Table (2) illustrates the number of failures, the repair time, and the operating time of this real system that were collected from historical data during year 2015. These data are applied to validate the proposed model. The RAM analysis and discussions are presented in this section.

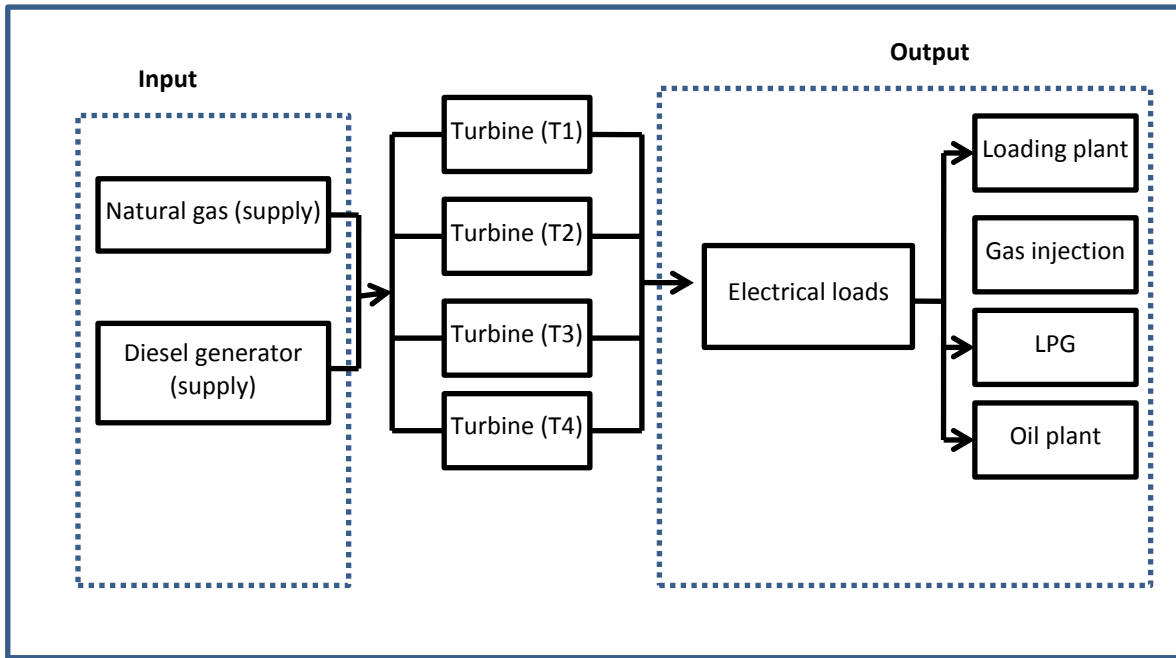


Figure 3: Block diagram of the industrial real system of Egyptian Petrol Company (Turbines of power generation).

The value of the failure rate and repair rate for each turbine is illustrated in Table (3) where:

$$\text{Failure rate} = \text{number of failures} / \text{operating time} \quad (47)$$

$$\text{Repair rate} = \text{number of failures} / \text{repair time} \quad (48)$$

As an example for T₁;

$$\text{Failure rate } (\lambda) = 11 / 6471 = 0.0017 \quad (49)$$

$$\text{Repair rate } (\mu) = 11 / 88 = 0.125 \quad (50)$$

Table 2: The collected historical data of the considered system for year 2015.

| | Number of failures | Repair time (hours) | Operating time (hours) |
|----------------|--------------------|---------------------|------------------------|
| T ₁ | 11 | 88 | 6471 |
| T ₂ | 8 | 53 | 6509 |
| T ₃ | 6 | 38 | 6526 |
| T ₄ | 9 | 105 | 6456 |

Table 3: Failure and repair rates of the considered system.

| | T ₁ | T ₂ | T ₃ | T ₄ |
|------------------|----------------|----------------|----------------|----------------|
| Failure rate (λ) | 0.0017 | 0.0012 | 0.001 | 0.0014 |
| Repair rate (μ) | 0.125 | 0.15 | 0.15 | 0.08 |

4.1. System Availability

By substituting values of failure and repair rates in system equations from (Eq.12 to 21) the state probabilities are calculated as a function in P₀ as follows:

$$P_1 = 0.0136 P_0, P_2 = 0.008 P_0, P_3 = 0.006 P_0, P_4 = 0.000011 P_0, P_5 = 0.00001 P_0, P_6 = 0.0002 P_0, P_7 = 0.000005 P_0, P_8 = 0.00014 P_0, P_9 = 0.00001 P_0$$

Where, P₀ and A_{SS} are calculated from equations (22) and (23) and equal 0.97 and 0.9995 respectively.

4.2. System Reliability

R_S is calculated as follows:

- Substitute about λ and μ from Table (3) in equations (39 to 42), the variables A, B, C and D could be calculated as follows:

$$A = 10000.0S + 1541.0 \quad (51)$$

$$B = 10000.0S + 1543.0 \quad (52)$$

$$C = 2500.0(5000.0S + 643.0) \quad (53)$$

$$D = (1.25e^{15}S^4 + 5.5112e^{14}S^3 + 8.075e^{13}S^2 + 3.9329e^{12}S + 4.1045e^8) \quad (54)$$

- Substitute about the variables A, B, C, and D in (eq., 35-38) to get the probability of the working states.
- Taking the inverse Laplace of system reliability (eq., 35 - 38) to get :

$$P_0(t) = 0.97e^{(-0.0001t)} + 0.015e^{(-0.15t)} + 0.00003e^{(-0.15t)} + 0.01e^{(-0.13t)} \quad (55)$$

$$P_1(t) = 0.01e^{(-0.0001t)} - 0.0001e^{(-0.15t)} - 2e^{-6}e^{(-0.15t)} - 0.012e^{(-0.13t)} \quad (56)$$

$$P_2(t) = 0.0075e^{(-0.0001t)} - 0.007e^{(-0.15t)} - 0.0003e^{(-0.15t)} + 0.0005e^{(-0.13t)} \quad (57)$$

$$P_3(t) = 0.006e^{(-0.0001t)} - 0.007e^{(-0.15t)} + 0.0003e^{(-0.15t)} + 0.0004e^{(-0.13t)} \quad (58)$$

- Substitute about P₀, P₁, P₂ and P₃ in equation (44) to get RS as follows:

$$R_s = 0.9935e^{(-0.0001t)} + 0.0009e^{(-0.15t)} + 0.000028e^{(-0.15t)} - 0.0011e^{(-0.13t)} \quad (59)$$

4.3. System maintainability

$MTTR_s$ is calculated from equation (46) based on the historical data of the system components which is illustrated in Table.2, (i.e., $\mu_s = 0.1196$, $MTTR_s = 8.36$ hr) where,

$$MTTR = \text{repair time} / \text{number of failure} \quad (60)$$

$$MTBF = 1 / \text{failure rate} \quad (61)$$

So, $MTTR_s = \sum_{i=1}^4 \frac{((88/11+53/8+38/6+105/9))/(1/0.0017+1/0.0012+1/0.001+1/0.0014))}{0.0017+0.0012+0.001+0.0014} = 8.36$ h

$$\sum_{i=1}^4 (0.0017+0.0012+0.001+0.0014) \quad (62)$$

Then the system maintainability at time t is as follows:

$$M(t) = 1 - e^{-(0.1196t)} \quad (63)$$

5. Results Discussion

An analysis of system performance has been carried out at different values of failure and repair rates of system components. The effects of these values on the system availability, reliability, and maintainability are discussed in the following sections.

5.1. Availability analysis

The A_{SS} was calculated at different values of failure rate and repair rate as follows:

Fig. (4) Shows the effect of failure rate for each turbine on the system availability at different values of failure rate (i.e., $\lambda = 0.001, 0.002, 0.003, 0.004, 0.005, 0.006$) without changing repair rate values mentioned in Table (3). It could be seen from this Figure that the system availability decreases, slightly, with increasing failure rate of the system component. Moreover, it is observed that turbine (T_4) has the lower availability than the other turbines.

On the other hand Fig. (5) depicts the effect of repair rate for each turbine on system availability at different values of repair rate (i.e., $\mu = 0.06, 0.09, 0.12, 0.15, 0.18, 0.21$) without changing failure rate values mentioned in Table (3). It is observed that the system availability increases, slightly, with increasing repair rate of turbines, and (T_4) has the higher availability than the other turbines this because it has the lower value of repair rate. This revealed that increasing the failure rate reduces the availability while increasing the repair rate leads to increasing the availability.

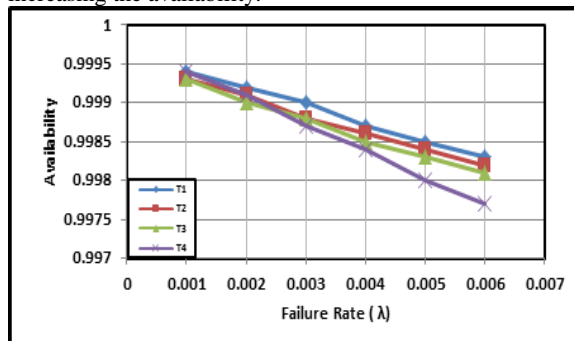


Figure 4: Effect of turbines failure rate (λ) on system availability at steady state.

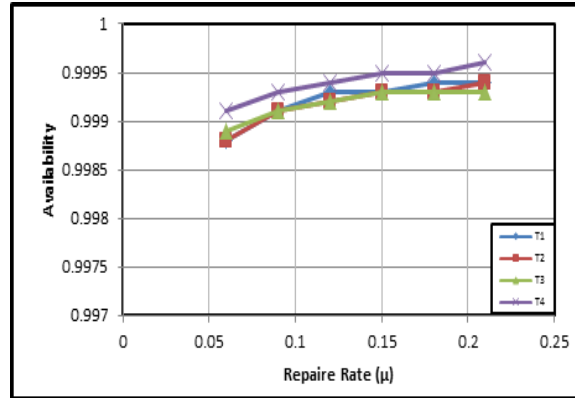


Figure 5: Effect of turbines repair rate (μ) on system availability at steady state.

5.2. Reliability Analysis

Fig. (6), depicts R_s of the system under consideration at real data of λ and μ mentioned previously in Table (3), along operating time and it is concluded that the system reliability decreases with time and the system reliability after 5000 running hours is 0.59%.

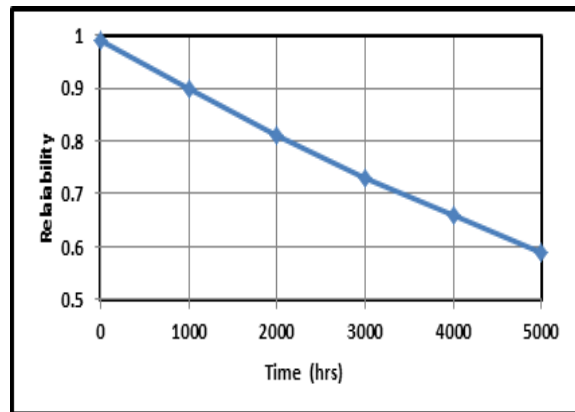


Figure 6: Effect of time on reliability of the system.

To investigate the effect of the failure rate of system components individually on R_s at different mission time, the failure rate of each turbine is changed within range (0.001 to 0.006 with incremental value 0.001) while, the values of rapier rates for all turbines and failure rates of the other turbines are the same real data.

Fig. (7), Illustrates the criticality of the system components, i.e., which turbine decreases R_s . It could be noted with comparing the cases of ($\lambda = 0.001$) for each turbine that T_3 has the higher effect on the system reliability and therefore it is the critical component as shown in Fig. (7-a), furthermore, with increasing λ to 0.006 for the same cases, it is also still the critical one as shown in Fig (7-b).

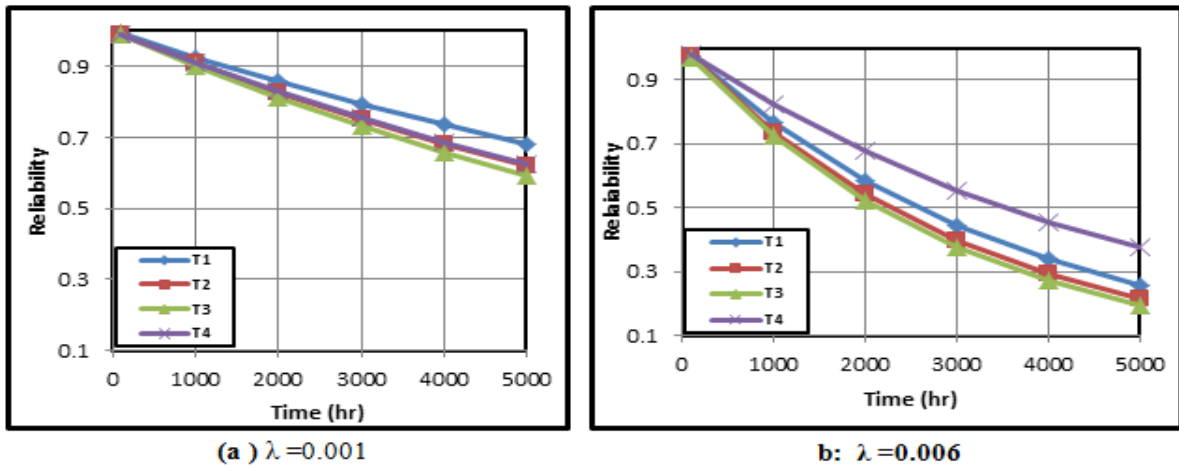


Figure 7:(a) and (b) Effect of failure rate (λ) on the system reliability at transient state.

5.3. Maintainability Analysis

Fig. (8) Illustrates the maintainability of each turbine as well as M_s of the overall system at real data along first 100 operating hours. It could be seen that the maintainability of T_4 is lower than the maintainability of the other components; this is due to the lower value of its repair rate than the others.

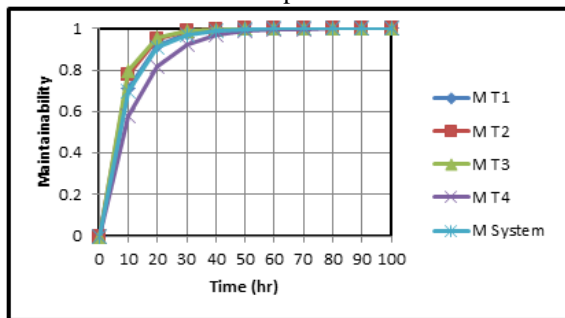


Figure 8: Maintainability of each turbine and the overall system versus time.

To investigate the effect of repair rate of overall system on M_s along the first 100 hrs of operating time, the repair rate of the system is assumed within range (0.0 to 0.21 with incremental value 0.03) as shown in Fig. (9). It could be seen that the increase in the system repair rate increases M_s .

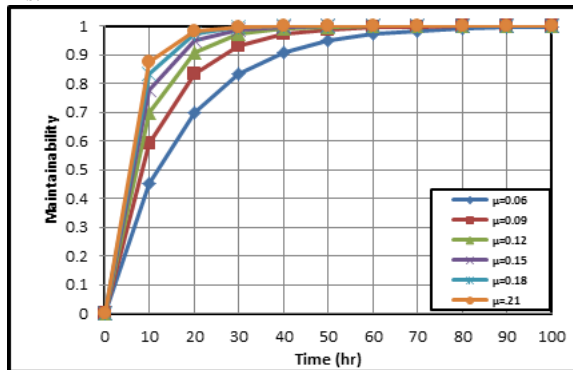


Figure 9: Effect of Repair Rate on Maintainability of the Overall System at Different Mission Time.

6. Conclusion

A RAM model of a 3-out-of-4 system has been proposed based on the Markovian approach. Availability at steady state, reliability at transient state and maintainability equations have been formulated. A real data of Oil Gas Egyptian Company was applied to obtain the system reliability, availability, and maintainability. A parametric investigation of various values of system failure and repair rates on system reliability, availability, and maintainability, as well as their effects on the system performance, are presented. The finding of this study could be concluded as follows:

- The proposed RAM model could be used as an integrated model, to investigate system reliability, availability, and maintainability of 3-out-of-4 system.
- It could be also used to determine the most critical component of the system.
- The proposed model helps maintenance engineers and designers to evaluate the system performance and carried out modification.
- The implementation of the proposed model revealed that the system availability at steady state is 99%, and the system maintainability is 0.99% but the system reliability after 5000 running hours is 0.59%. This means that, an enhancement required improving the system reliability and reducing the system down time. It is observed that T_3 is the most critical component in the system and need special attention with careful observation to reduce its down time and increase the system performance.

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Appendix (A)

$$\begin{aligned}
D = & (\mu_1 S^3 + \mu_2 S^3 + \mu_3 S^3 + S \lambda_1^3 + 3.0 S^3 \lambda_1 + S \lambda_2^3 + 3.0 S^3 \lambda_2 + S \lambda_3^3 + 3.0 S^3 \lambda_3 + S \lambda_4^3 + 3.0 S^3 \lambda_4 + \lambda_1 \lambda_2^3 + \lambda_1^3 \lambda_2 \\
& \lambda_2^3 + \lambda_1 \lambda_3^3 + \lambda_1^3 \lambda_3 + \lambda_1 \lambda_4^3 + \lambda_2 \lambda_3^3 + \lambda_1^3 \lambda_4 + \lambda_2^3 \lambda_3 + \lambda_2 \lambda_4^3 + h_2^3 \lambda_4 + \lambda_3 \lambda_4^3 + \lambda_3^3 \lambda_4 + S^4 + 3.0 S^2 \lambda_1^2 + 3.0 S^2 \lambda_2^2 \\
& + 3.0 S^2 \lambda_3^2 + 3.0 S^2 \lambda_4^2 + 2.0 \lambda_1^2 \lambda_2^2 + 2.0 \lambda_1^2 \lambda_3^2 + 2.0 \lambda_1^2 \lambda_4^2 + 2.0 \lambda_2^2 \lambda_3^2 + 2.0 \lambda_2^2 \lambda_4^2 + 2.0 \lambda_3^2 \lambda_4^2 + \mu_1 \mu_2 S^2 + \\
& \mu_1 \mu_3 S^2 + \mu_2 \mu_3 S^2 + \mu_1 S \lambda_1^2 + 2.0 \mu_1 S^2 \lambda_1 + \mu_1 S \lambda_2^2 + 2.0 \mu_1 S^2 \lambda_2 + \mu_2 S \lambda_1^2 + 2.0 \mu_2 S^2 \lambda_1 + \mu_1 S \lambda_3^2 + 2.0 \mu_1 S^2 \lambda_3 + \\
& \mu_2 S \lambda_2^2 + 2.0 \mu_2 S^2 \lambda_2 + \mu_3 S \lambda_1^2 + 2.0 \mu_3 S^2 \lambda_1 + \mu_1 S \lambda_4^2 + 2.0 \mu_1 S^2 \lambda_4 + \mu_2 S \lambda_3^2 + 2.0 \mu_2 S^2 \lambda_3 + \mu_3 S \lambda_2^2 + 2.0 \mu_3 S^2 \lambda_2 \\
& + \mu_2 S \lambda_4^2 + 2.0 \mu_2 S^2 \lambda_4 + \mu_3 S \lambda_3^2 + 2.0 \mu_3 S^2 \lambda_3 + \mu_3 S \lambda_4^2 + 2.0 \mu_3 S^2 \lambda_4 + \mu_1 \lambda_1 \lambda_2^2 + \mu_1 \lambda_1^2 \lambda_2 + \mu_1 \lambda_1 \lambda_3^2 + \mu_1 \lambda_1^2 \lambda_3 \\
& + \mu_2 \lambda_1 \lambda_2^2 + \mu_2 \lambda_1^2 \lambda_2 + \mu_1 \lambda_2 \lambda_3^2 + \mu_1 \lambda_2^2 \lambda_3 + \mu_2 \lambda_1 \lambda_3^2 + \mu_2 \lambda_1^2 \lambda_3 + \mu_3 \lambda_1 \lambda_2^2 + \mu_3 \lambda_1^2 \lambda_2 + \mu_1 \lambda_2 \lambda_4^2 + \mu_1 \lambda_2^2 \lambda_4 + \\
& \mu_2 \lambda_1 \lambda_4^2 + \mu_2 \lambda_2 \lambda_3^2 + \mu_2 \lambda_1^2 \lambda_4 + \mu_2 \lambda_2^2 \lambda_3 + \mu_3 \lambda_1 \lambda_3^2 + \mu_3 \lambda_1^2 \lambda_3 + \mu_1 \lambda_3 \lambda_4^2 + \mu_1 \lambda_3^2 \lambda_4 + \mu_3 \lambda_1 \lambda_4^2 + \mu_3 \lambda_2 \lambda_3^2 + \mu_3 \\
& \lambda_1^2 \lambda_4 + \mu_3 \lambda_2^2 \lambda_3 + \mu_2 \lambda_3 \lambda_4^2 + \mu_2 \lambda_3^2 \lambda_4 + \mu_3 \lambda_2 \lambda_4^2 + \mu_3 \lambda_2^2 \lambda_4 + 5.0 S \lambda_1 \lambda_2^2 + 5.0 S \lambda_1^2 \lambda_2 + 7.0 S^2 \lambda_1 \lambda_2 + 5.0 S \lambda_1 \lambda_3^2 \\
& + 5.0 S \lambda_1^2 \lambda_3 + 7.0 S^2 \lambda_1 \lambda_3 + 5.0 S \lambda_1 \lambda_4^2 + 5.0 S \lambda_2 \lambda_3^2 + 5.0 S \lambda_1^2 \lambda_4 + 5.0 S \lambda_2^2 \lambda_3 + 7.0 S^2 \lambda_1 \lambda_4 + 7.0 S^2 \lambda_2 \lambda_3 + 5.0 S \\
& \lambda_2 \lambda_4^2 + 5.0 S \lambda_2^2 \lambda_4 + 7.0 S^2 \lambda_2 \lambda_4 + 5.0 S \lambda_3 \lambda_4^2 + 5.0 S \lambda_3^2 \lambda_4 + 7.0 S^2 \lambda_3 \lambda_4 + 4.0 \lambda_1 \lambda_2 \lambda_3^2 + 4.0 \lambda_1 \lambda_2^2 \lambda_3 + 4.0 \lambda_1^2 \lambda_2 \\
& \lambda_3 + 4.0 \lambda_1 \lambda_2 \lambda_4^2 + 4.0 \lambda_1 \lambda_2^2 \lambda_4 + 4.0 \lambda_1^2 \lambda_2 \lambda_4 + 4.0 \lambda_1 \lambda_3 \lambda_4^2 + 4.0 \lambda_1 \lambda_3^2 \lambda_4 + 4.0 \lambda_1^2 \lambda_3 \lambda_4 + 4.0 \lambda_2 \lambda_3 \lambda_4^2 + 4.0 \lambda_2 \\
& \lambda_3^2 \lambda_4 + 4.0 \lambda_2^2 \lambda_3 \lambda_4 + \mu_1 \mu_2 \mu_3 S + \mu_1 \mu_2 S \lambda_1 + \mu_1 \mu_2 S \lambda_1 + \mu_1 \mu_3 S \lambda_1 + \mu_1 \mu_2 S \lambda_3 + \mu_1 \mu_3 S \lambda_2 + \mu_2 \mu_3 S \lambda_1 + \mu_1 \mu_2 S \lambda_4 + \\
& \mu_1 \mu_3 S \lambda_3 + \mu_2 \mu_3 S \lambda_2 + \mu_1 \mu_3 S \lambda_4 + \mu_2 \mu_3 S \lambda_3 + \mu_2 \mu_3 S \lambda_4 + \mu_1 \mu_2 \lambda_1 \lambda_3 + \mu_1 \mu_3 \lambda_1 \lambda_2 + \mu_1 \mu_2 \lambda_2 \lambda_3 + \mu_2 \mu_3 \lambda_1 \lambda_2 + \mu_1 \mu_3 \lambda_2 \\
& \lambda_3 + \mu_2 \mu_3 \lambda_1 \lambda_3 + \mu_1 \mu_2 \lambda_3 \lambda_4 + \mu_1 \mu_3 \lambda_2 \lambda_4 + \mu_2 \mu_3 \lambda_1 \lambda_4 + 3.0 \mu_1 S \lambda_1 \lambda_2 + 3.0 \mu_1 S \lambda_1 \lambda_3 + 3.0 \mu_2 S \lambda_1 \lambda_2 + 2.0 \mu_1 S \lambda_1 \\
& \lambda_4 + 3.0 \mu_1 S \lambda_2 \lambda_3 + 3.0 \mu_2 S \lambda_1 \lambda_3 + 3.0 \mu_3 S \lambda_1 \lambda_2 + 3.0 \mu_1 S \lambda_2 \lambda_4 + 3.0 \mu_2 S \lambda_1 \lambda_4 + 3.0 \mu_2 S \lambda_2 \lambda_3 + 3.0 \mu_3 S \lambda_1 \lambda_3 + 3.0 \\
& \mu_1 S \lambda_3 \lambda_4 + 2.0 \mu_2 S \lambda_2 \lambda_4 + 3.0 \mu_3 S \lambda_1 \lambda_4 + 3.0 \mu_3 S \lambda_2 \lambda_3 + 3.0 \mu_2 S \lambda_3 \lambda_4 + 3.0 \mu_3 S \lambda_2 \lambda_4 + 2.0 \mu_3 S \lambda_3 \lambda_4 + 2.0 \mu_1 \lambda_1 \lambda_2 \\
& \lambda_3 + 2.0 \mu_1 \lambda_1 \lambda_2 \lambda_4 + 2.0 \mu_2 \lambda_1 \lambda_2 \lambda_3 + 2.0 \mu_1 \lambda_1 \lambda_3 \lambda_4 + 2.0 \mu_2 \lambda_1 \lambda_2 \lambda_4 + 2.0 \mu_3 \lambda_1 \lambda_2 \lambda_3 + 2.0 \mu_1 \lambda_2 \lambda_3 \lambda_4 + 2.0 \mu_2 \lambda_1 \lambda_3 \\
& \lambda_4 + 2.0 \mu_3 \lambda_1 \lambda_2 \lambda_4 + 2.0 \mu_2 \lambda_2 \lambda_3 \lambda_4 + 2.0 \mu_3 \lambda_1 \lambda_3 \lambda_4 + 2.0 \mu_3 \lambda_2 \lambda_3 \lambda_4 + 11.0 S \lambda_1 \lambda_2 \lambda_3 + 11.0 S \lambda_1 \lambda_2 \lambda_4 + 11.0 S \lambda_1 \lambda_3 \\
& \lambda_4 + 11.0 S \lambda_2 \lambda_3 \lambda_4 + 9.0 \lambda_1 \lambda_2 \lambda_3 \lambda_4)
\end{aligned}$$