JJMIE

Volume 10 Number 1, March.2016
ISSN 1995-6665
Pages 75- 83

Jordan Journal of Mechanical and Industrial Engineering

Computational modeling of gaseous flow and heat transfer in a wavy microchannel

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Received 7 Nov 2015 *Accepted 28 Feb 2016*

Abstract

The present work is an investigation of the developing gas flow and heat transfer through a wavy micro-channel. Computational fluid dynamics (CFD) is used to solve the fluid flow and energy equations along with the slip flow and temperature jump boundary conditions. The simulations are carried out using three wavy amplitudes for Knudsen number 0.025 to 0.1 in the slip flow regime. It is shown that increasing the wavy amplitude results in augmentation of the heat transfer rate at the cost of increasing the frictional losses. However, the rarefication has a declining effect on both friction factor and Nusselt number. Correlations for the friction factor and Nusselt number as function of amplitude and Knudsen number are provided.

Keywords: *Wavy, microchannel, slip flow, temperature jump, Knudsen number*.

Nomenclature

A	amplitude of the wave (m)
C_f	skin friction coefficient
e_{Cf}	the ratio of wavy channel friction coefficient to that of planar channel
e_{Nu}	the ratio of wavy channel Nusselt number to that of planar channel
D_H	hydraulic diameter of the channel (m)
E	total energy
h	heat transfer coefficient (W/m^2 K)
H	the channel Height (m)
k	thermal conductivity of air (W/m.K)
k_B	the Boltzmann constant (J/K)
Kn	Knudsen number
L	length of single wave of the computational domain (m)
Ma	Mach number
n	unit vector normal to solid wall
Nu(x)	local Nusselt number
Nu	average Nusselt number
p	pressure (Pa)
$\bar{R}$	universal gas constant
Re	Reynolds number
s	wavy wall stream-wise local coordinate
t	unit vector tangent to solid wall
T	temperature, K
T_w	microchannel wall temperature, K
U_o	average velocity at inlet, m/s
V	velocity vector
WU	upper wall microchannel boundary
WL	lower wall microchannel boundary
x	axial coordinate, m
y	vertical coordinate, m

Greek Symbols

γ	ration of the specific heat (c_p/c_v)
λ	molecular mean free path (m)
λ_0	Length of the wave (m)
ν	kinematic viscosity
ρ	density of air, given by ideal gas equation (P/RT), Kg/m^3
σ	Lennard-Jones characteristic length (A^o)
σ_T	thermal accommodation coefficient
σ_v	momentum accommodation coefficient
τ_w	Wall shear stress (pa)

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1. Introduction

Design and fabrication of micro-electro mechanical systems (MEMS) have increased the need for understanding fluid flow and heat transfer due to wide spread use in micro-fluidic devices, micro heat exchangers, biological and medical applications. In gaseous flows the two-counter competing effects rarefaction and compressibility influence the physics of the flow in micro-scale systems. The slip effect is dominant when the characteristic length becomes comparable to the mean free path of the fluid molecules in the microchannel, or at low-pressure gas flows in ordinary channel.

To enhance the heat transfer characteristics the traditional way is the inclusion of finned surface as in the study by Yeom et al. [1] where a micro-pin fin array is micro-fabricated in a rectangle channel for thermal management in electronics. A maximum enhancement in the heat transfer of nearly 79% over the planar channel was found and it was attributed to the flow dynamics around the pin fins rather than the increase in heat transfer area since the array with larger pin diameter was superior to the array with smaller diameter, although the latter had greater overall surface area, since the porosity is lesser.

Departing from planar channel is typically associated with increased pressure drop penalty. A micro scale gas flow in a channel with three protruding steps carried out by Bakhshan et al. [2] showed the rarefication and inclusion of obstructing steps in the channel had a competing effect on pressure drop pently. Inserting a convergent divergent section in planar channel reduces the mixing length of two gas mixer was shown in a study by Darbandi and Sabouri [3] where Monte Carlo (DSMC) the solution method employed.

Kiwan and Al-Nimr [4] investigated convection over linearly stretched, isothermal microsurface via similarity solution of the boundary layer equations. They presented correlations for skin friction coefficient and Nusselt number in terms of velocity slip, and temperature slip parameters. Kiwan and Al-Nimr [5] also showed that complete similarity solution is possible for boundary layer flows only for a stagnation flow over isothermal microsurface. They found that skin friction coefficient is inversely proportional to both the slip velocity parameter and local Reynolds number. Shakir et al. [6] investigated flow and heat transfer in counter-flow rectangular microchannel heat exchanger for 3-D, laminar, incompressible, steady state, slip, airflow. It is found that the effectiveness decrease with increasing Re and increasing Kn number.

In an analysis of corrugation amplitude of two-dimensional steady laminar and incompressible liquid flow in micro-channel, Castellões et al. [7] found that increasing the amplitude corresponds to enhancement of the local heat transfer in comparison with the smooth parallel-plate channel. Zhang et al. [8] studied the effect of viscous heating on cooled and heated walls with other wall adiabatic in parallel flow microchannel. It was concluded that for a heated wall, viscous heating acts as volumetric heat source and raises the temperature of the cold fluid thus reducing the temperature difference between the wall, and fluid, which decreases the heat transfer and Nusselt.

Hooman and Ejali [9] presented closed form solutions for hydrodynamically and thermally fully developed, forced convection for both parallel plate and circular micro-channels using both no-slip and slip-flow forced convection of a liquid and a gas with temperature-dependent properties. It was shown that the Nusselt number decrease with increasing Kn number and slightly decreased with Brinkman number due to viscous dissipation. Ji et al. [10] investigated the roughness effects on friction factor using rectangular elements on two parallel plates with different spacing and heights. It is found that the effect of wall roughness reduces with increasing Knudsen number. Morini et al. [11] investigated the conditions for rarefaction effects on the pressure drop. It was demonstrated that for a fixed geometry of the microchannel cross-section, it is possible to determine the minimum value of the Knudsen number for which the rarefaction effects can be observed experimentally. Rached and Dahar [12] numerically investigated the effects of Knudsen number on the velocity and temperature fields for the microchannel flow.

The effects of rarefaction have been examined for simultaneously developing 2-D laminar, incompressible, constant-property flows in rectangular microchannels. Demsis et al. [13] conducted experiments using different gases in circular, and square cross-section channels with various wall roughnesses. They used a scaled-up version by employing low pressure in the test section. The ranges of Knudsen number and Reynolds numbers covered in this study were 0.0022– 0.024 and 0.54 –13.2 respectively. The results suggest that fRe (Poiseuille number) in the slip regime decreases monotonically with an increase in Knudsen number from continuum-laminar value of 64, for all the conditions studied. The results signify that the friction factor responds to changes in gas, tube material and geometry of cross-section. Further, fRe increases with an increase in wall roughness. Experiments conducted with copper and stainless steel as pipe material indicated that fRe is lower for the copper wall case than that of stainless steel pipe. The value of fRe for oxygen, nitrogen and argon in same hydraulic diameter, same roughness and same cross-section pipe is close.

ALsqirate et al. [14] provided heat transfer and pressure drop correlations for CO_2 flow in mini and micro tubes under superheated conditions; analytical and experimental results are found to be in good agreement. Demsis et al. [15-16] experimentally investigated flow channel with low pressure simulating slip flow in microchannel and reported unusually Nusselt number correlation, which is proportional to both Reynolds number and Brinkman number, and inversely proportional to Knudsen number. The reported Nusselt number values for the slip flow are two-five orders of magnitude less than the corresponding values in the continuum flow regime. Zade et al. [17] studied the effects of variable physical properties on the flow and heat transfer characteristics of simultaneously hydrodynamically and thermally developing slip-flow in rectangular microchannels with constant wall temperature; channel aspect ratios are studied at different Knudsen numbers. For instance, for a temperature difference of 50 K, the change in the Nusselt number in the entrance and fully developed regions can be as high as 20% and 15%, respectively.

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Mohammed et al. [18] numerically investigated the effect of various microchannel shapes such as zigzag, curvy, and step with rectangular cross-section on the overall thermal and hydraulic performance of water flow. Shojaeian et al. [19] employed three-dimensional numerical analysis for fully developed incompressible fluid flow and heat transfer through triangular microchannels over the slip flow regime. The influences of Knudsen, aspect ratio, and Reynolds number on the fluid flow and heat transfer characteristics are investigated. The results showed the rarefaction decreases the Poiseuille number, while its effect on the Nusselt number depends on the interaction between velocity slip and temperature jump. Gou and Li [20] reviewed and discussed the size effect impact on the flow and heat transfer characteristics in microchannel. Affirmed that the large surface to volume ratio in microchannel leads to higher friction factor and Nusselt numbers in comparison to the classical correlations. Sui et al. [21] Experimentally investigated friction and heat transfer in sinusoidal microchannels with rectangular cross sections using deionized water as the working fluid and the Reynolds numbers considered range from about 300 to 800. The results of Nusselt number and friction factor, for wavy microchannels are compared with those of baseline case of straight with the same cross section and footprint length. It is found that the heat transfer performance of the wavy microchannels is much better than that of straight baseline microchannels; at the same time the pressure drop penalty of the wavy microchannels can be much smaller than the heat transfer enhancement.

Shokouhmand and Bigham [22] investigated the developing fluid flow and heat transfer through a wavy microchannel numerically. The effects of creep flow and viscous dissipation are assumed. The results show that Knudsen number has declining effect on both the C_f Re and Nusselt number and rarefaction increased the temperature jump and slip velocity. In a CFD study of laminar flow and heat transfer in periodic trapezoidal channel with semi-circular cross-section, Geyer et al. [23] found that, for no-slip flow; the heat transfer enhancements of up to four times of that of fully-developed flow in a straight pipe at relatively small pressure loss penalty. Guzmán et al. [24] studied the enhancement characteristics of heat transfer, through a transition scenario of flow bifurcations, in asymmetric wavy wall channels by direct numerical simulations of the mass, momentum and energy equations, using the spectral element method. Hossain et al. [25] studied the case of unsteady sine-shaped wavy module with periodic boundary condition. It is found that the flow becomes unsteady with self-sustained oscillation at low Re on the order of 130 - 205 depending on the inlet spacing of the channel. It is found that the both the unsteady flow and increasing amplitude of the channel increases the Nusselt number and friction factor.

A comprehensive review on gas flow in microchannels is given by Agrawal [26]. It is noted that flow in planar channels of different cross-section has been extensively studied and analytical solution obtained by various approaches agree reasonably well amongst each other. Agrawal [26] affirms that studies focused on complex microchannels holds a great practical importance, since frequently the microchannels may not be straight.

In the present work, a steady, two-dimensional analysis of airflow through a wavy microchannel with uniform wall temperature is investigated. At micro level, the surface effects became more important than volume effects and presents and opportunity of enhancement of thermal transport. The flow is assumed steady, laminar and developing hydrodynamically and thermally. The continuum governing equations along with Maxwell slip [27] and Smoluchowski [28] temperature boundary conditions are solved numerically. The effect of the wave amplitude on both the friction coefficient and Nusselt number is investigated for range of Knudsen numbers in the slip regime. Knudsen is defined as the ratio of molecular mean free path length, λ, to the channel hydraulic diameter, D_H.

2. Problem statement and solution methodology

We consider the flow of air at low pressure through an asymmetric wavy microchannel. The physical domain of the channel, which consists of five wavy units, is shown in Fig. 1. The microchannel wavy walls geometry is given as

$$y = A\cos(2\pi x/\lambda_o) \tag{1}$$

Where A is amplitude of the wave and λ_o is length of wave. Here, three wave amplitudes are considered, the base case wavy channel amplitude is given by A1, whereas A2 and A3 are scaled from A1 by incrementing the amplitude by 25% and 50%, respectively, while maintain a constant wave foot-print. Hence, from a practical point of view if we are interested to cool a given surface with a specific designed geometry, we can adjust the degree of waviness of the channel to achieve the desired cooling rate.

The flow is assumed to be steady, two-dimensional and laminar. The fluid is assumed to be an ideal gas. Further, it is supposed that the flow is developing both hydrodynamically and thermally. The continuum governing equations are utilized in conjunction with the slip velocity and temperature jump boundary conditions. The continuity, momentum and energy equations governing the flow and heat transfer in the microchannel under consideration, are as follows

$$\nabla \cdot \rho\vec{V} = 0 \tag{2}$$

$$\left(\nabla . \rho\vec{V}\right) = -\nabla p + \mu\nabla^2\vec{V} \tag{3}$$

$$\nabla \cdot \left(\vec{V}\ (\rho E + p)\right) = k\nabla^2 T +\ \mu\Phi \tag{4}$$

Where $E = h - \frac{p}{\rho} + \frac{v^2}{2}$ and Φ is viscous dissipation.

The boundary conditions associated with the governing equations for the problem are:

Slip-condition and temperature jump at the channel walls; reported by Lokerby et al. [29] and Colins [30] as follows

$$u_s = \left(\frac{2-\sigma_v}{\sigma_v}\right)\lambda\ \left(\frac{\partial u_t}{\partial n} + \frac{\partial u_n}{\partial t}\right) \tag{5a}$$

$$T_s = \left(\frac{2-\sigma_T}{\sigma_T}\right)\frac{2\gamma}{\gamma+1}\frac{k}{\mu c_v}\lambda\left(\frac{\partial T}{\partial n}\right) \tag{5b}$$

The mean free path length is specified as follows:

$$\lambda = \frac{k_B T}{\sqrt{2}\pi\sigma^2 P} \tag{6}$$

Where $k_B = 1.38066 \times 10^{-23}\, JK^{-1}$ is the Boltzmann constant, T the temperature, P the pressure and σ the Lennard-Jones characteristic length of air.

The inlet gas temperature is specified to be T_{in} = 300K, while the wall temperature is maintained at 345 K. Relating the Knudsen number, to the Reynolds number and Mach number as follows specifies the mass flow rate at the channel inlet

$$Kn = k_2\sqrt{\gamma}\,\frac{Ma}{Re} \qquad (7)$$

Where, γ, is the specific heat ratio of the gas and the constant, $k_2 = \sqrt{\pi/2}$ is of the Maxwell model. This relationship shows the linking between rarefication and compressibility as explained in gas flows within a microchannel, detailed theoretical accounting of this is given by Colin [30]. It is noteworthy that at high *Re* the effect of *Ma* (compressibility) is significant, however, for low *Re* the *Kn* effect (rarefaction) is dominant. The governing equations along with the boundary conditions are solved in utilizing the Finite Volume Method (FVM). The computational domain consists of five-wave geometry as specified by Eq. (1). Progressions of meshes with increasing sizes, (200×110), (250×130), (300×150), and (350×170) are tested to select an optimal mesh size. The effect of mesh refinement on average Nusselt number and friction factor is shown in table 1. A non-uniform, quad mapped, mesh consisting of 45,000 elements is found to be adequate for all computation (For clarity the mesh of only the third wave is shown in Fig. 1). The steady governing equations were spatially discretized using the 2nd order upwind scheme. Pressure-velocity decoupling is handled using the SIMPLE algorithm [31]. The mean free path is calculated using the inlet condition temperature in K, and the operating pressure, in Pa, which is taken as one tenth of atmospheric pressure. Assuming totally thermally diffuse walls, the thermal accommodation coefficient, σ_T, is taken as unity for all the simulations. In a review of measurement techniques of the momentum accommodation factor, Agrawal and Prabhu [32] suggested a value of 0.926 for momentum accommodation factor, σ_v that is adopted in these in all the simulations.

Reynolds number of value two is considered in this study, which results in slight compressibility affects. Knudsen number was varied from 0.025 to 0.1, which is considered within the slip flow regime rarefaction is expected to dominate compressibility. The mass flow rate at the channel inlet is calculated using Eq. (7) above that relates the Mach number to the Knudsen number and Reynolds number.

The residuals for the convergence were monitored and solutions are deemed converged when the mass and velocity residuals were less than 10^{-7} and the energy residual are less than 10^{-9}. The local skin friction, $C_f(s) = \frac{\tau_w(s)}{\frac{1}{2}\rho\,\bar{U}^2}$ and local Nusslet number, $(s) = \frac{h\,D_H}{k}$, Where the heat transfer coefficient is computed using the local heat flux, wall temperature and a reference temperature, which is taken, as the film temperature. Whereas, the average Nusselt number is based on the total heat transfer rate and the inlet and outlet bulk temperatures.

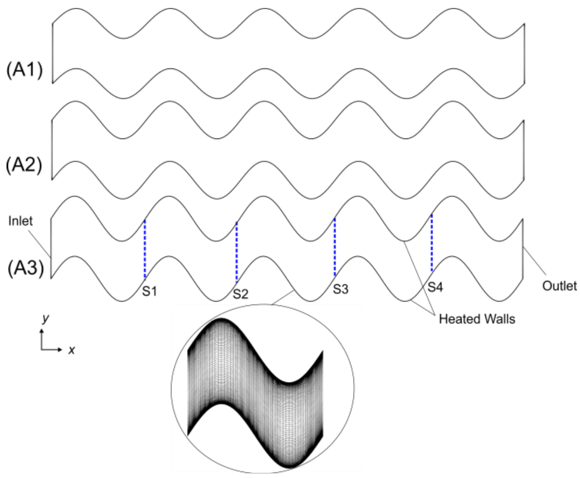

Figure 1 The geometry used for the computational domain, three amplitude and fixed

3. Results and Discussion

The velocity profiles are shown in Fig. (2a) for case of microchannel with amplitude A1. It can be observed that the slip velocity increases with increased rarefication, and is nearly 24%, 51%, and 67% of the average velocity, for Kn number 0.025, 0.075, and 0.1, respectively. At high Kn number the slip velocity on the lower channel wall is greater than on the upper wall, which corresponds to local acceleration of the flow. The increase in rarefication increases wall slip, which results in flattening of the velocity profiles. Also we note that the peak velocity does not coincide with centerline of the channel due to the centripetal acceleration of the flow caused by the local curvature and the peak velocity shifts toward the lower channel wall with increasing Kn number. The asymmetry in flow is evident by inspecting the local friction factor depicted in Fig. (2b), which indicates that the shear stress on the upper and lower walls is out of phase.

Table 1. Grid independent study

	cf	Nu	cf	Nu
Mesh size	Kn 0.025		Kn 0.1	
200, 100	2.5725E-02	2.1023E+01	3.6125E-03	1.0047E+01
250, 130	2.5536E-02	2.0730E+01	3.5609E-03	9.9484E+00
300, 150	2.5472E-02	2.0533E+01	3.5247E-03	9.8801E+00
% Error	0.739	1.417	1.449	0.993
% Error	0.253	0.958	1.027	0.691

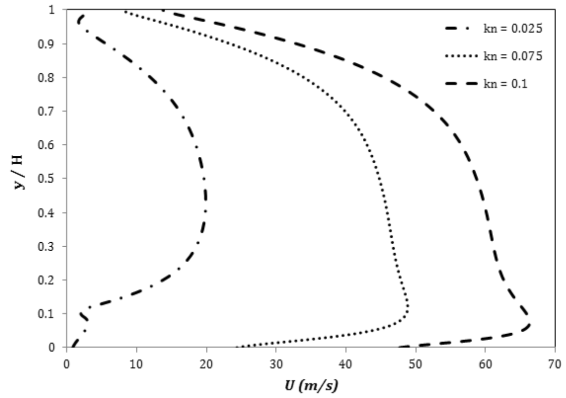

Figure 2a. Velocity profile at middle section of the wavy channel with amplitude, A1, shows the effect of Kn number on the velocity slip.

Figure 3 illustrates the effect of Kn numbers 0.025 and 0.1 on the flow field in terms of the velocity vectors and the corresponding stream function plots for case of microchannel with amplitude, A1. The vector plots shows maximum slip velocity in the vicinity of concave section of the wall and recirculating flow near the convex sections of the wall. The slip velocity significantly increase with increased rarefication effect as can be seen in velocity vector plots of Figure 3, Kn = 0.025 (left) in contrast to Kn = 0.1 (right). It is interesting to note the asymmetry in recirculating flow regions near the upper and lower walls as illustrated by stream function plots Fig. 3 (left) and Fig. 3 (right). Furthermore, the reduction in rarefication near the microchannel wall results in higher frictional losses as depicted in Fig. 4, for a given Knudsen number local friction coefficient varies along the channel dimensionless length. It can be observed that the frictional losses are extremely steep very near the channel entrance. However, there is a reduction of 50% in the frictional losses in the subsequent wave sections. The higher frictional losses at the inlet of the channel are related to uniform velocity boundary condition imposed there. However, spatial

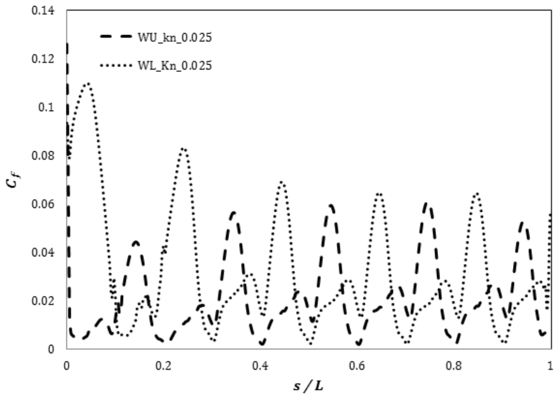

Figure 2b. Local friction coefficient vs. dimensionless stream-wise location, for the microchannel with amplitudes, A1, at the upper and lower channel walls, Kn = 0.025.

periodicity of the flow is almost established, after three waves, as illustrated by the similarity of the frictional-loss profiles of the fourth and fifth waves.

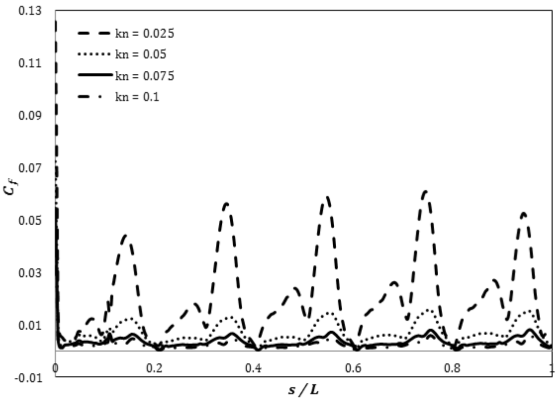

Figure 4. Friction coefficient vs. dimensionless stream-wise location, Kn = 0.025, 0.05, 0.075, and 0.1, wavy channel with amplitude A1.

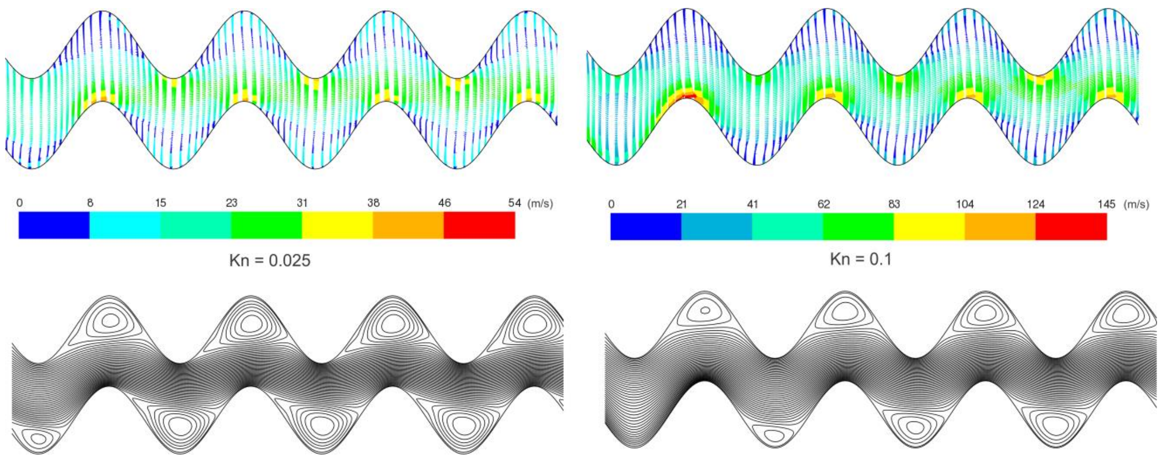

Figure 3. Velocity vectors and stream function, wavy channel with amplitude, A1, third wave in the center of the channel,

(a) Kn = 0.025, (b) Kn = 0.1

Figure 5 considers the simulation case of Knudsen number 0.5 for planar channel and three wavy channels with increasing amplitudes A1, A2, A3, respectively. For the hydrodynamically developing flow, in the case of planar channel, we observe the expected steep shear stress at channel entrance and tapering off with the development of the boundary layer. For the cases of wavy channel also we observed the spatially periodic behavior in the last three waves following a steep gradient at the inlet followed by developing flow in the first two-wave sections as mentioned earlier. It is interesting to note the effect of the wall shear stress as illustrated by the friction factor to increase with amplitude, due to increasingly tortuous path the flow must follow. An increase of the base amplitude by 25% on the average increases the local friction coefficient by nearly 50%.

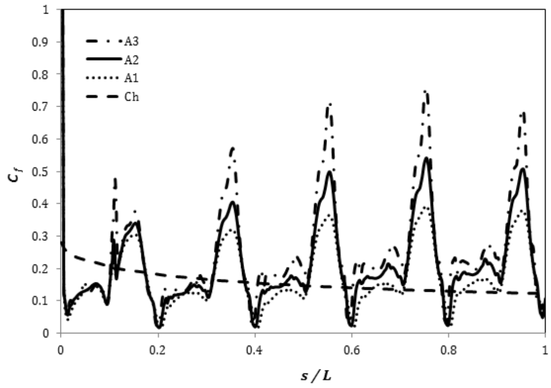

Figure 5. Friction coefficient vs. dimensionless stream-wise location, for the three wavy amplitudes and the planar channel, Kn = 0.05.

Figure 6 shows the temperature profiles at a cross-section in the middle of the microchannel. It can be noted that the fluid temperature near the wall diminishes with increased rarefication, or in other words as expected, increasing the Knudsen number results in increasing the temperature jump or an increase in rarefication affect result in flattening of the temperature profiles. The asymmetry in the temperature profiles is also inline with asymmetric in the velocity profiles, which was caused the varying local acceleration in the cross-stream direction. Presented in Fig. 7 is the effect of rarefication on the temperature field. Figure 7(b) shows the intensification of the temperature jump for case of Knudsen number 0.1, where the temperature contours near the wall are marginally higher than those near the microchannel centerline in contrast to the case of Kn number 0.025 illustrated in Fig 7(a), where we note significantly higher temperatures near the heated wall, particularly in the vicinity of the leeward portions of the channel. It can also be noted in Fig. 7(a) and Fig. 7(b); higher temperatures are found in the standing eddy regions where its expected that diffusion from these region to the core flow dominates convection. The variation in fluid density near the wavy boundaries and the core flow is caused by the compressibility effect, which increases with increasing Kn number as illustrated in Fig. 7(b).

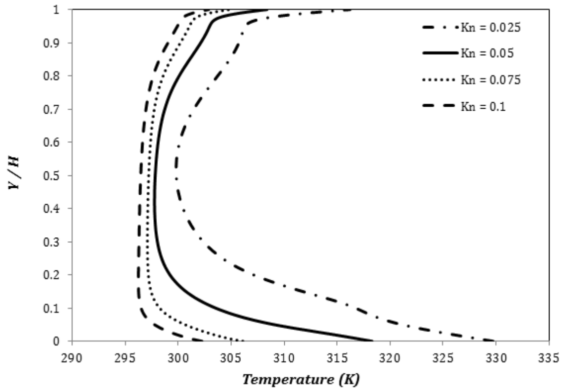

Figure 6. Temperature profile at S2, wavy channel with amplitude, A1, showing the effect of Kn number on the temperature jump.

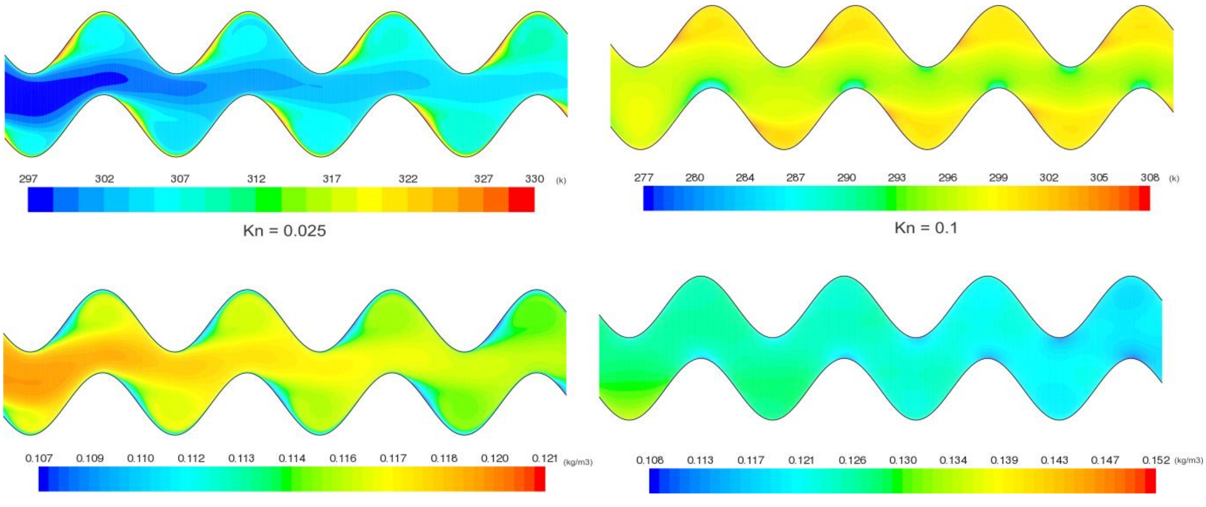

Figure 7. Temperature and density contours, wavy channel with amplitude, A1, third wave in the center of the channel, (a) Kn = 0.025, (b) Kn = 0.1

Figure 8 presents the variations of the local Nusselt number for Knudsen numbers within the slip flow regime. In the channel entrance, the heat transfer rates are steep due to high temperature gradients and conjunction with the thin thermal boundary layer, which presents little thermal resistance. After an adjustment from the uniform inlet temperature imposed on the entrance of the channel the Nusselt number exhibits a cyclic behavior in each subsequent wave, and noticeably the dimensionless heat transfer coefficient decrease with increased Knudsen number. Figure 9 shows the variations of the local Nusselt number with amplitude. It is clear that increasing the amplitude of the channel enhances the heat transfer rate.

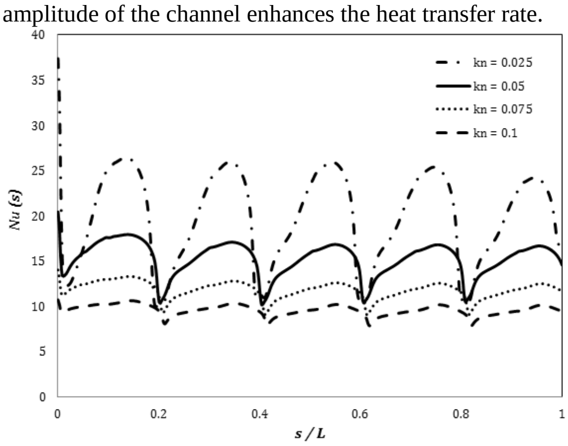

Figure 8. Variation of local Nusselt number vs. dimensionless stream-wise location, Kn = 0.025, 0.05, 0.075, and 0.1, for the wavy channel with amplitude, A1.

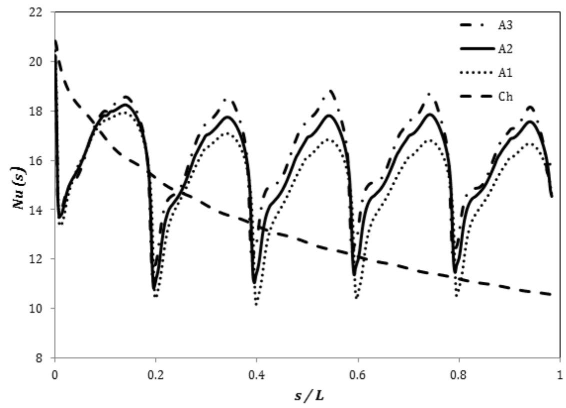

Figure 9. Variation of local Nusselt number along the microchannel, for the three amplitudes and the planar channel, Kn = 0.05.

Figure 10 and Figure 11 present the effect of rarefication and varying wave amplitude on the channel-averaged friction factor and Nusselt number. The frictional resistance decreases linearly with rarefication at two different rates. Whereas, the Nusselt number decreases linearly with increased rarefication at nearly fixed rate. The effect of the increase in the channel amplitude clearly enhances the heat transfer rate at the cost of increase in frictional losses. In addition, the slip velocity increases heat transfer rate since it increases advection in a region where diffusion is the dominate mode of transport. On the other hand, the temperature jump at the wall reduces the heat transfer rate by creating an effect similar to "contact resistance" such that the net effect is a reduction in heat transfer. In order to assess the effectiveness of waviness on the characteristic of flow and heat transfer in the microchannel, channel-averaged values of those of the planar channel normalize the friction factor and average Nusselt number of the wavy channel as follows

$$e_{C_f} = \frac{C_{f_{Wavy}}}{C_{f_{Planar\ Channel}}} \quad (8)$$

$$e_{Nu} = \frac{Nu_{Wavy}}{Nu_{Planar\ Channel}} \quad (9)$$

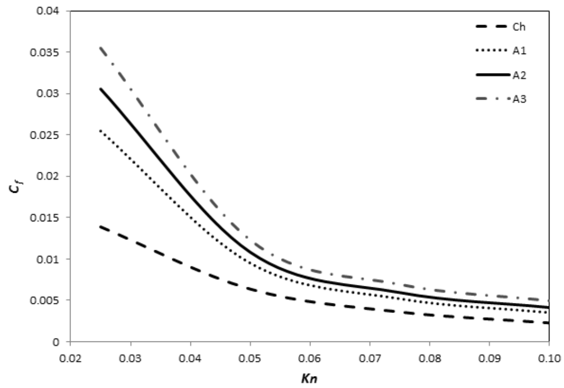

Figure 10. Effect of Kn number and wave amplitude on the average friction coefficient.

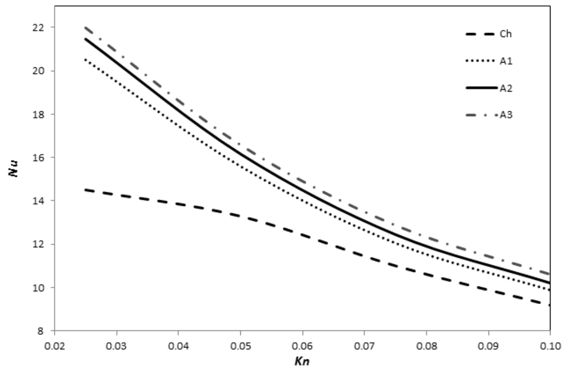

Figure 11. Effect of Kn number and wave amplitude on the average Nusselt number.

Figure 12 and 13 shows for a given Knudsen number the friction losses and augmentation in the heat transfer. The enhancement of heat transfer with increased waviness comes at the cost of increased frictional effects. On the average the pressure drop penalty is about 58%, 84%, and 115% for amplitudes A1, A2, and A3, respectively. However, nearly reaching an asymptotic value with increased rarefication effect. The corresponding heat transfer enhancement is on the average is 19%, 23%, and 27%. The average Nusselt number and friction factor are correlated to Kn number as follows:

$$Nu = c_1 \left(\frac{A}{\lambda_o}\right)^a Pr\ e^{(-b\ Kn)} \quad (10)$$

$$C_f = \left(\frac{A}{\lambda_o}\right)^d \left(c_2 + c_3\ e^{(-f\ Kn)}\right) \quad (11)$$

Where A and λ_0 are the amplitude and length of the wave, the constants are given as:

$a = 0.2$, $b = 10.16$, $d = 0.75$, $f = 52.5135$, $c_1 = 45.586$, $c_2 = 0.0077$and $c_3 = 0.1979$; the expected average error for either correlations is less than 2.5%.

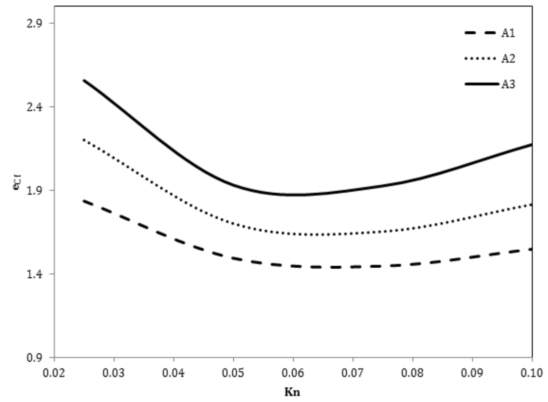

Figure 12. Effect of wave amplitude on the enhancement of heat transfer.

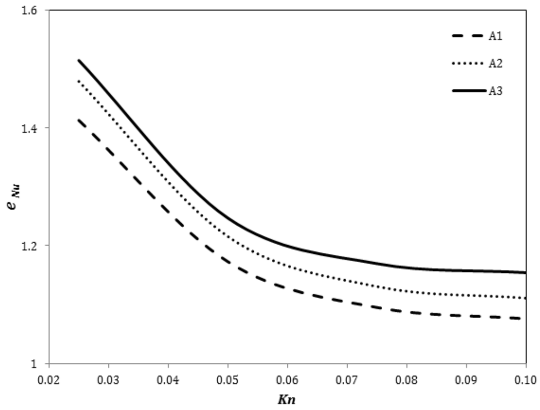

Figure 13. Effect of wave amplitude on the pressure drop penalty.

4. Conclusions

A steady two-dimensional analysis of developing flow through a wavy microchannel has been carried out. It is found that the slip velocity and temperature jump increase with increasing Kn number. Increased rarefication has a diminishing effect on both the heat transfer rate and frictional losses. However, the increase in channel waviness enhances the heat transfer at the cost of higher frictional losses. On the average the pressure drop penalty is about 58%, 84%, and 115%, for the three amplitudes considered which corresponds to average augmentation in heat transfer rate of 19%, 23%, and 27%, where a planar channel of same footprint as the three wavy channels is taken as the base-line reference. The friction factor and Nusselt number are correlated to Kn number.

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