

# Simulation of Optimal Exergy Efficiency of Solar Flat Plate Collector

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Received 12 Sep 2014

Accepted 25 Sep 2015

## Abstract

Exergy analysis identifies potential factors responsible for thermodynamic losses and leads to efficiency improvements. In the present paper, exergy efficiency is expressed as a function of dimensionless mass flow rate and outlet fluid temperature. A computer program was developed for determining the optimal performance parameters for maximum exergy efficiency in a flat plate collector. The study was conducted for six collectors of different areas, having a different overall loss coefficient and a heat removal factor. It is observed that for given values of incident solar radiation, inlet fluid temperature and ambient temperature, the optimal mass flow rate varied from 0.0019 -0.0022 kg/s and exergy efficiency varied from 5.2- 8.2% for the collectors depending on its gross area, overall heat loss coefficient and heat removal factor.

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**Keywords:** Optimization, Exergy Efficiency, Outlet Fluid Temperature, Mass Flow Rate, Flat Plate Collector.

## Nomenclature

$A_c$	Gross area of the collector, m <sup>2</sup> .
$A_p$	Absorber plate area, m <sup>2</sup>
$c_p$	Specific heat of the heat transfer fluid, J/ kg°C.
$F_R$	Heat removal factor of the collector, dimensionless.
$F'$	Collector efficiency factor, dimensionless.
$H_a$	Absorbed solar radiation per unit area of the collector, W/m <sup>2</sup> .
$H_i$	Incident solar radiation per unit area, W/m <sup>2</sup> .
$\dot{m}$	Mass flow rate, kg/s.
$M$	Mass flow number, dimensionless.
$T_a$	Ambient temperature, °C.
$T_{f, in}$	Inlet fluid temperature, °C
$T_{f, out}$	Outlet fluid temperature, °C
$T_p$	Absorber plate temperature, °C
$T_{p, max}$	Stagnation temperature, °C
$T_s$	Apparent temperature of Sun, °C
$U_L$	Overall heat loss coefficient, W/m <sup>2</sup> °C.

## Greek Symbols

$\alpha$	Absorptance of the absorber plate, dimensionless.
$\tau$	Transmittance of the cover, dimensionless.
$\theta_{in}$	Dimensionless inlet fluid temperature.

$\theta_{max}$	Maximum collector temperature, dimensionless
$\theta_{out}$	Dimensionless outlet fluid temperature.
$\theta_{out}^*$	Dimensionless optimal outlet fluid temperature.
$\theta_s$	Dimensionless apparent temperature of Sun.
$\eta_I$	Energy efficiency of collector, dimensionless.
$\eta_{II}$	Exergy efficiency of collector, dimensionless.

## 1. Introduction

Solar flat plate collectors are devices used for low temperature applications. The heat absorbed by absorber is partly transferred from absorber plate to the fluid flowing in the tubes and the rest is lost to ambient. Heat transfer irreversibility decreases with the increase in fluid flow rate but this increases losses due to fluid friction. To optimize heat transfer to fluid from the absorber plate, an optimal mass flow rate of the fluid needs to be determined which takes care of both heat transfer irreversibility and losses due to fluid friction.

In recent past, various methods have been applied to optimize the design of a collector. Analysis of a solar collector was conducted by Howell and Bannerot [1] in order to determine the optimum outlet temperature for a given solar collector that would maximize the work output for various idealized heat engine cycles. The analysis demonstrated the effect of the radiative and convective heat losses from the collector. Second law analysis for the optimization of flat plate solar air heaters was performed

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by Altfeld *et al.* [2] where net exergy flow was maximized by minimizing exergy losses by absorption of radiation at absorber temperature level. Based on this analysis, optimal designs of the absorbers and flow ducts were determined. Having developed the optimal designs for air heaters, Altfeld *et al.* [3] conducted a sensitivity analysis to study the influence of varying operational conditions on optimal results. Hepbasli [4] comprehensively reviewed and evaluated the performance of a wide range of renewable energy resources and had defined exergy efficiency of solar flat plate collector. Luminosu *et al.* [5] conducted an exergy analysis of a flat plate collector with the assumption that the global solar radiation is equal to solar flux and inlet fluid temperature is equal to ambient temperature. Optimal operation mode of flat plate collector was determined by maximizing exergy efficiency of the collector with respect to various parameters. The global optimal operation mode of a flat plate collector was calculated considering exergy efficiency as a function of mass flow rate and collector area.

Exergy analysis was applied by various authors [6, 7, 8, 9] to judge a system and showed how exergy analysis provided illuminating and meaningful assessment of solar thermal processes and can assist in improving and optimizing designs. Kalogirou *et al.* [10] presented a review of exergy analysis of solar thermal systems. It includes exergy analysis of solar collectors like flat plate collectors, hybrid PV/T systems, parabolic trough collectors, parabolic dish collectors and reported various applications of solar thermal systems.

Though exergy analysis provides valuable information about the system but it is very complex to apply. Thus, the second law analyses which are simplified forms of exergy analysis are often employed. The Entropy Generation Minimization (EGM) technique was widely studied by Bejan [11] to optimize system performance in various heat transfer processes including solar thermal applications. In the past, many authors used the EGM method to judge and optimize processes [12, 13, 14]. Torres-Reyes *et al.* [15] established a procedure for the determination of optimal performance parameters for minimum entropy generation during the collection of solar energy. Doos *et al.* [16] presented Fuzzy ARTMAP neural network model to improve the process real-time performance of a power station in Al-Daura Refinery for the multi-agent process as a classifying system. Agent based fuzzy method has been employed by various authors in decision making problems to obtain the optimal solution [17, 18, 19, 20]. A numerical simulation was employed for the performance analysis of Stirling engine cycle by Tarawneh *et al.* [21].

F. Jafarkazemi *et al.* [22] conducted an energetic and exergetic evaluation of flat plate collector. The theoretical model was verified experimentally wherein flat plate collectors were tested in open loop with water as heat transfer fluid. The energy and exergy efficiency of flat plate collector were determined for constant mass flow rates of 0.03, 0.04 and 0.05 kg/s. The theoretical and experimental values were compared by computing the root mean square error. The effect of design parameters on the collector performance was also studied. Khademi *et al.* [23] studied the optimal exergy efficiency of flat plate collector by employing Sequential Quadratic Programming (SQP) and Genetic Algorithm (GA). Nonlinear constraint

optimization technique was adopted in the present paper wherein objective function  $(1 - \eta_{II})$  is minimized w.r.t two inequality constraints viz.  $[1 \leq A_p \leq 5$  and  $0.001 \leq \dot{m} \leq 0.1]$ . Khademi *et al.* [23] suggested that the rate of convergence of SQP [24] was much higher than that of GA, but GA provides results with a higher accuracy for exergy efficiency. They also suggested that the smaller collector could also have a similar and a better performance compared to the collector with a larger surface area. SQP algorithm has a high convergence rate. But the rate of convergence of SQP depends highly on the starting point, first and second order derivatives of the objective function and also it stops in local optimum points. These are the weaknesses of SQP. GA, on the other hand, requires an initial population for training and rate of convergence is also low. Mukhopadhyay *et al.* [25] optimized exergy efficiency of flat plate collector to obtain the optimal operational mode, i.e., mass flow rate and outlet fluid temperature for a given collector whereas Khademi *et al.* [23] optimized exergy efficiency to determine the optimal design parameters, i.e., area of collector and mass flow rate.

In the present paper, an analytical study [25] is conducted for six different collectors with different surface area  $A_c$ , heat removal factor  $F_R$  and overall heat loss coefficient  $U_L$  to determine the optimal mode of operation for which exergy and energy efficiency would be maximum for the fixed values of uncontrollable parameters such as solar radiation and ambient temperature. The present paper is an extension of our work [25], to present the simulation done to obtain the optimal operational mode of a flat plate collector. A computer program is written based on the proposed mathematical model to solve the nonlinear constrained optimization problem using Direct Substitution Method (DSM) to obtain the optimal results. DSM to solve nonlinear optimization problem is simple to implement compared to SQP and GA. The simulator thus developed requires initial input-specification data of collector ( $F'U_L$ ,  $F'\tau\alpha$ ,  $A_c$ ,  $F_rU_L$ ,  $F_R\tau\alpha$ ,  $\tau\alpha$ ), values of uncontrollable parameters ( $H_p$ ,  $T_w$ ,  $T_s$ ) and heat capacity of heat transfer fluid,  $c_p$  to compute the optimal solution. The simulator is capable of determining the initial starting solution and thereby minimizes human error in its prediction. The optimal solution is obtained in less than 20 iterations. The development of the computer program based on the proposed optimization technique eliminates the dependency of author to use software which has its own limitations and complexities.

## 2. Optimization of Exergy Efficiency

Exergy efficiency of any process, as defined by Öztürk [6], is the ratio of the exergy transfer rate associated with the output to the exergy transfer rate associated with the driving input. Instantaneous exergy efficiency of flat plate collector can be defined as the ratio of the increased exergy to the exergy of solar radiation.

The exergy transfer rate associated with output (fluid) at a given time is given by:

Exergy output associated with the fluid = Energy output  $- T_a \times$  (Entropy generation in fluid)

∴ Exergy<sub>output</sub> =

$$\dot{m}c_p \left[ (T_{f,out} - T_{f,in}) - T_a \ln \frac{T_{f,out}}{T_{f,in}} \right] \quad (1)$$

Exergy transfer rate associated with solar radiation at a given time [7] is:

$$Exergy_{input} = A_c H_t \left[ 1 + \frac{1}{3} \left( \frac{T_a}{T_s} \right)^4 - \frac{4}{3} \left( \frac{T_a}{T_s} \right) \right] \quad (2)$$

The instantaneous exergy efficiency of the collector is given by:

$$\eta_{II} = \frac{Exergy_{output}}{Exergy_{input}} = \frac{\dot{m}c_p \left[ (T_{f,out} - T_{f,in}) - T_a \ln \frac{T_{f,out}}{T_{f,in}} \right]}{A_c H_t \left[ 1 + \frac{1}{3} \left( \frac{T_a}{T_s} \right)^4 - \frac{4}{3} \left( \frac{T_a}{T_s} \right) \right]} \quad (3)$$

Exergy efficiency given in Eq. (3) is expressed in dimensionless form as:

$$\eta_{II} = \frac{M}{\eta_e} \left[ (\theta_{out} - \theta_{in}) - \ln \left( \frac{\theta_{out}}{\theta_{in}} \right) \right] \quad (4)$$

where mass flow number

$$M = \frac{\dot{m}c_p T_a}{A_c H_t};$$

exergy fraction of solar radiation,

$$\eta_e = \left[ 1 + \frac{1}{3} \left( \frac{T_a}{T_s} \right)^4 - \frac{4}{3} \left( \frac{T_a}{T_s} \right) \right] \quad (5)$$

Dimensionless outlet and inlet fluid temperature respectively are,

$$\theta_{out} = \frac{T_{f,out}}{T_a}; \theta_{in} = \frac{T_{f,in}}{T_a} \quad (6)$$

It is evident from Eq. (4) that the exergy efficiency of a given collector is a function of mass flow number, *M* and dimensionless outlet fluid temperature  $\theta_{out}$ , for given solar insolation and ambient temperature.

Outlet fluid temperature for a collector can be computed using the following relation [26]:

$$\frac{T_{f,out} - T_a - H_a / U_L}{T_{f,in} - T_a - H_a / U_L} = \exp \left( \frac{-A_c F' U_L}{\dot{m}c_p} \right) \quad (7)$$

In dimensionless form the outlet fluid temperature in a flat plate collector can be expressed as:

$$\theta_{out} = \theta_{max} + (\theta_{in} - \theta_{max}) \exp \left( \frac{-F'(\tau\alpha)}{M(\theta_{max} - 1)} \right) \quad (8)$$

where  $\theta_{max}$  is the maximum collector temperature in dimensionless form. The maximum temperature of the plate ( $T_{p,max}$ ) called the ‘stagnation temperature’, occurs when the entire solar heat transfer is lost to the ambient. That is, when the useful energy gains by the collector is zero. The maximum collector temperature is given in dimensionless form as [10]:

$$\theta_{max} = \frac{T_{p,max}}{T_a} = 1 + \frac{H_t}{U_L T_a} \quad (9)$$

Eliminating  $\theta_{out}$  from Eq. (4) using Eq. (8) and differentiating  $\eta_{II}$  with respect to mass flow number *M* assuming  $\theta_{in}$  and  $\eta_e$  constant and equating to zero we obtain:

$$\ln \left[ \theta_{max} + (1 - \theta_{max}) \exp \left( \frac{-\tau\alpha F' / (\theta_{max} - 1) M}{\theta_{max} - 1} \right) \right] + \frac{\left( \frac{-\tau\alpha F' / (\theta_{max} - 1)}{\theta_{max} - 1} \right) (1 - \theta_{max}) \exp \left( \frac{-\tau\alpha F' / (\theta_{max} - 1) M}{\theta_{max} - 1} \right)}{M \left[ \theta_{max} + (1 - \theta_{max}) \exp \left( \frac{-\tau\alpha F' / (\theta_{max} - 1) M}{\theta_{max} - 1} \right) \right]} - (\theta_{max} - 1) \left[ 1 - \exp \left( \frac{-\tau\alpha F' / (\theta_{max} - 1) M}{\theta_{max} - 1} \right) \right] \left( 1 + \left( \frac{-\tau\alpha F' / (\theta_{max} - 1) M}{\theta_{max} - 1} \right) \right) = 0 \quad (10)$$

For the computer program, the left hand side of Eq. (10) is denoted by dEffdM. Eq. (10) is solved using Bisection Method to find the optimal mass flow number *M\** for which exergy efficiency is maximum for a given collector. Substituting *M\** in Eq. (8) gives the optimal

outlet fluid temperature  $\theta_{out}^*$ . The optimal exergy efficiency  $\eta_{II}^*$  is obtained by substituting *M\** and  $\theta_{out}^*$  in Eq. (4).

### 3. Optimization Process

The mathematical model for optimizing exergy efficiency is based on the following nonlinear constrained optimization problem:

$$\text{Maximize } \eta_{II} = \frac{M}{\eta_e} \left[ (\theta_{out} - \theta_{in}) - \ln \left( \frac{\theta_{out}}{\theta_{in}} \right) \right] \text{ eq.(4)}$$

subject to the constraints

$$M = \frac{\dot{m} c_p T_a}{A_c H_t} \text{ eq.(5)}$$

$$\theta_{out} = \theta_{max} + (\theta_{in} - \theta_{max}) \exp \left( \frac{-F'(\tau\alpha)}{M(\theta_{max} - 1)} \right) \text{ eq.(8)}$$

$$M \geq 0, \theta_{out} \geq 0$$

The Direct Substitution Method was applied to solve the optimization problem as discussed in Section 2. The optimization process is described in the following schematic diagram, Figure 1.

Variation of the exergy efficiency was studied as a function of mass flow rate with the following assumptions:

- For a given collector assume that the parameters like area of the collector  $A_c$ ; heat removal factor,  $F_R$ ; collector efficiency factor  $F'$  and overall heat loss coefficient,  $U_L$ ; transmissivity of cover ( $\tau$ ) and absorptance of plate ( $\alpha$ ) are constant.
- Inlet fluid temperature is assumed to be equal to ambient temperature, i.e.,  $\theta_{in} = T_{f,in}/T_a = 1$ ; incident solar radiation on the collector,  $H_t = 800 \text{ W/m}^2$ ; ambient temperature,  $T_a = 30^\circ\text{C}$  and apparent sun temperature,  $T_s = 6000 \text{ K}$ .
- Mass flow rate is varied from 0.0001 to 0.0419 kg/s and the corresponding exergy efficiency and energy efficiency are calculated for a given collector.

The flow chart for the optimization process to compute optimal mass flow rate and outlet fluid temperature, which maximizes exergy efficiency of the collector, is shown in Figures 2-4. Figure 2 represents the process for computing energy and exergy efficiency for different values of mass flow rate. Flow chart for computing two initial values of mass flow rate such that dEffdM takes a positive value for one and a negative value for the other, shown in Figure 3. These two values are the initial input for starting bisection method to obtain the optimal mass flow rate. Figure 4 shows the flow chart for computing the optimal mass flow rate that optimizes exergy efficiency.

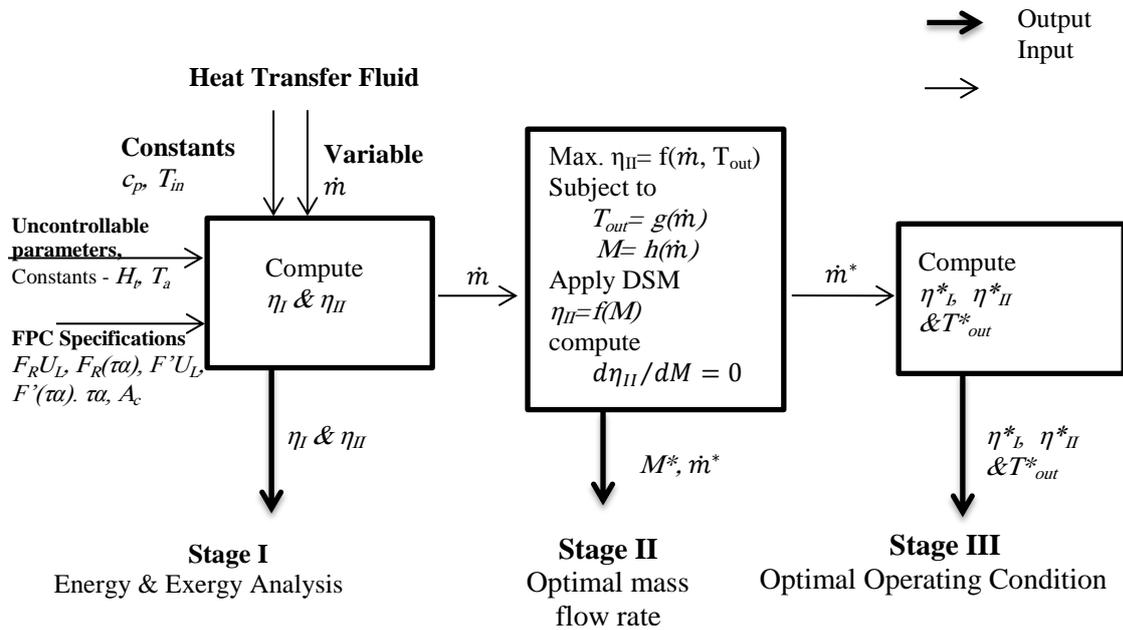
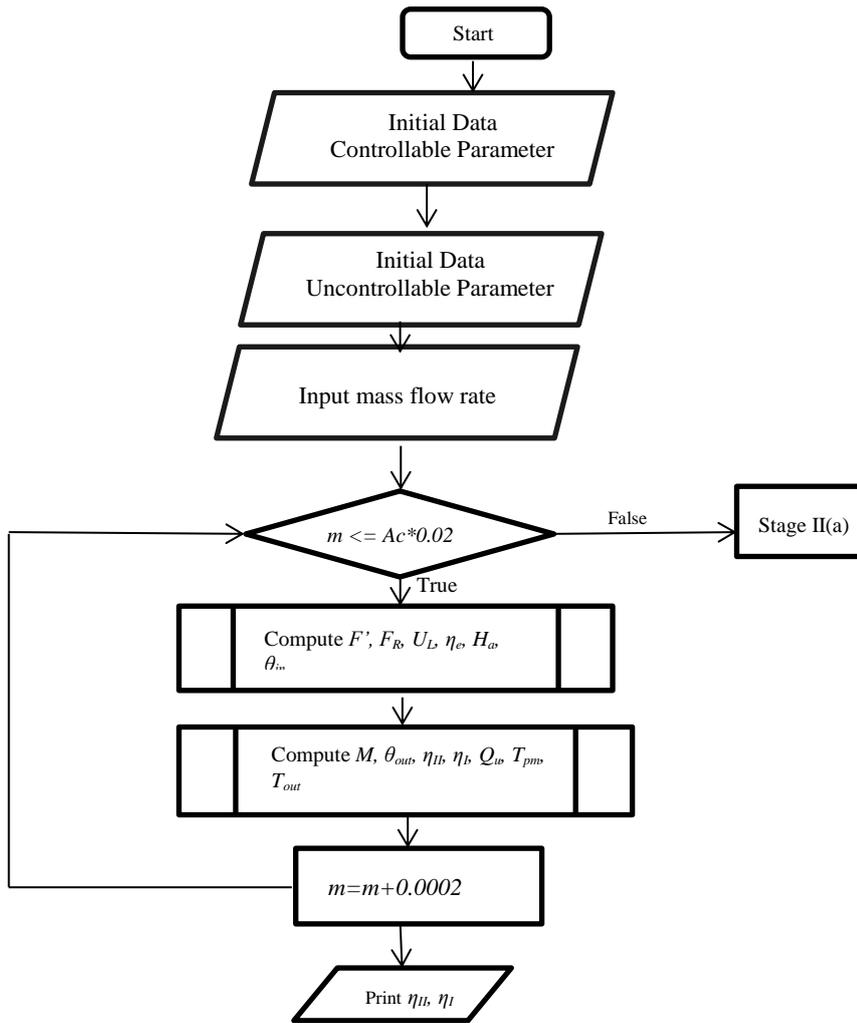
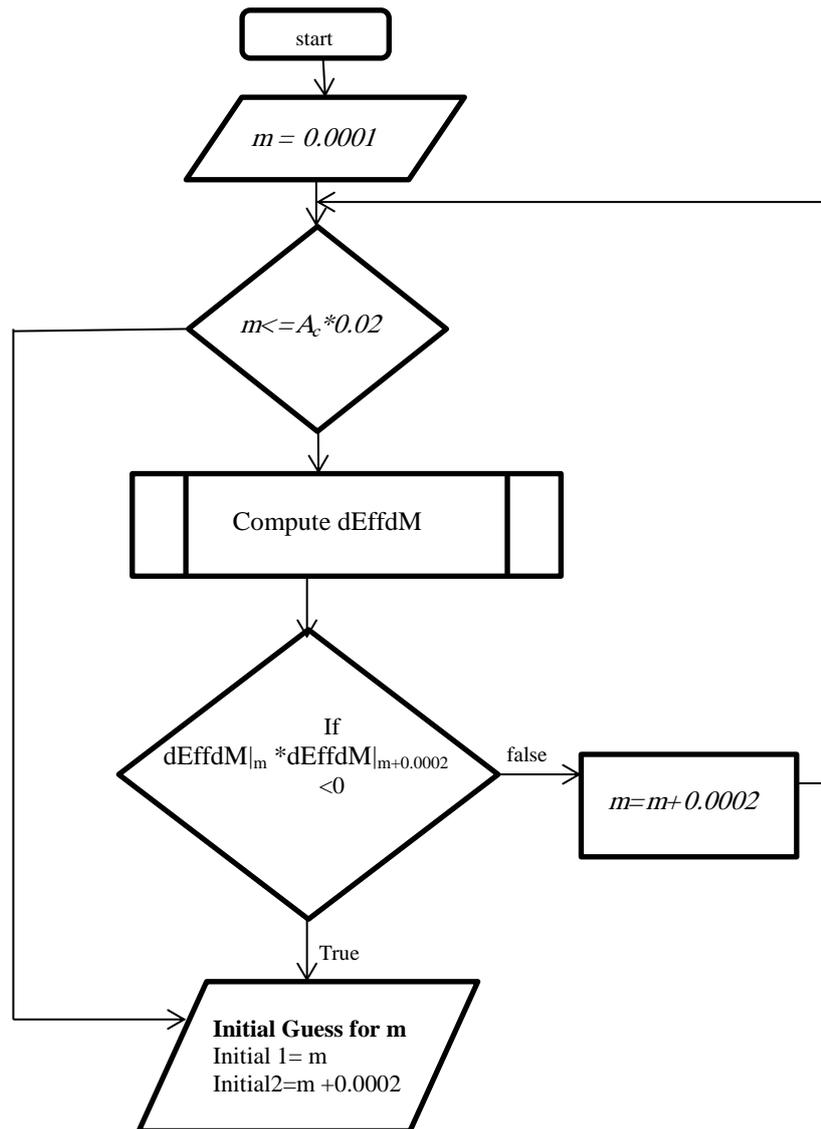


Figure 1. Schematic Diagram of Optimization Process



**Figure 2.** Flow Chart for Energy & Exergy Analysis



**Figure 3:** Stage II (a)- Flow chart for computing initial values of mass flow rate to start Bisection method

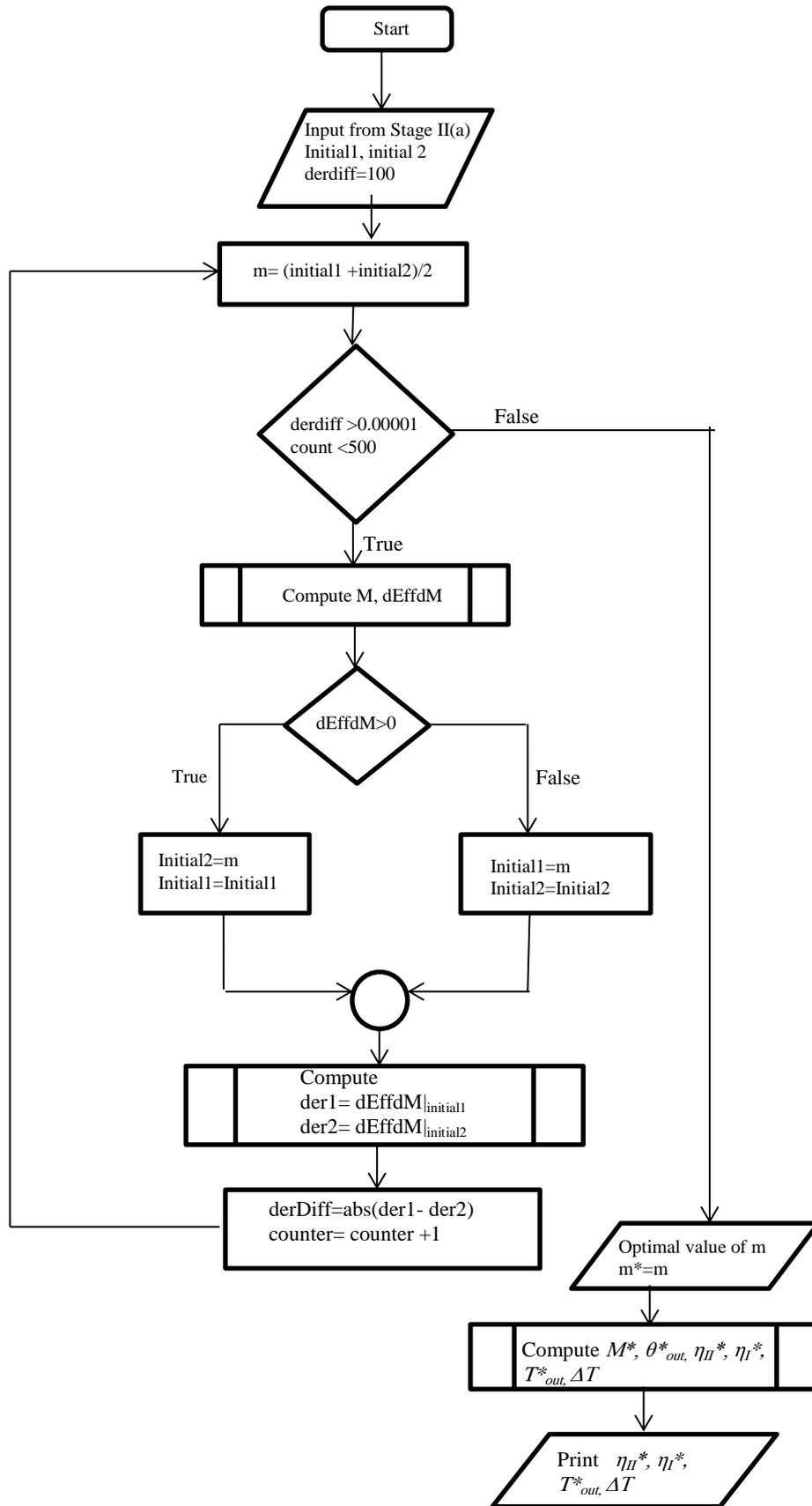


Figure 4. Flow chart for computing optimal mass flow rate and corresponding optimal energy and exergy efficiency, optimal outlet temperature

#### 4. Results and Discussion

A computer program in C language was developed based on the method discussed in Section 3 for computing the optimal mass flow rate that maximizes exergy efficiency of a flat plate collector. Figures 5 and 6, respectively, show the variation of energy and exergy efficiency of a flat plate collector with respect to mass flow rate. From Figure 5, it is observed that energy efficiency increases exponentially with the increase in mass flow rate then saturation begins, the growth slows and finally the growth stops and the energy efficiency remains unchanged with the increase in mass flow rate. Energy efficiency growth curve resembles the logistic curve in 1<sup>st</sup> quadrant. A similar pattern of the growth of energy efficiency with respect to mass flow rate is reported by Jafarkazemi [22].

Exergy efficiency increases rapidly with the increase in mass flow rate; it attains a maximum value and then decreases with the further increase in mass flow rate.

The optimal operating conditions, i.e., optimal mass flow rate  $\dot{m}^*$  and the corresponding optimal outlet fluid temperature, the exergy efficiency and  $\Delta T$  are determined for a given flat plate collector. The optimal operating conditions for six different collectors with a different collector area, a heat removal factor and an overall heat loss coefficient are computed assuming  $H_i = 800 \text{ W/m}^2$  and  $T_a = 30 \text{ }^\circ\text{C} = T_{f,in}$  and are tabulated in Table 1.

It is observed that for a given value of incident solar radiation, inlet fluid temperature and ambient temperature, optimal mass flow rate varied from 0.0019 - 0.0022 kg/s and exergy efficiency varied from 5.2 - 8.2% for the collectors depending on its gross area  $A_c$ , overall heat loss coefficient  $U_L$  and heat removal factor  $F_R$ .

Luminosu *et al.* [5] also determined the optimal operational mode of flat plate collector by exergetic analysis. They assumed that the exergy flow rate in global solar radiation is equal to the solar flux ( $HR$ ) and defined exergy efficiency for flat plate collector as:

$$\eta_{II, \text{Luminosu}} = \frac{\dot{m} c_p \left[ \frac{\Delta T - T_a \ln \left( \frac{T_{f,out}}{T_{f,in}} \right)}{A_c (HR)} \right]}{\quad} \quad (11)$$

The global maximum points suggested by Luminosu *et al.* [5] are:  $A_c = 3.3 \text{ m}^2$ , mass flow rate = 0.0031 kg/s,  $\eta_{II} = 3.9\%$  and  $\Delta T = 63.3 \text{ K}$ . The optimal operating conditions for collector with area  $A_c = 3.12 \text{ m}^2$  as obtained by author in this work are optimal mass flow rate = 0.0027 kg/s, exergy efficiency = 8.9%. For the same collector, Khademi *et al.* [23] obtained optimal mass flow rate = 0.0022 kg/s, exergy efficiency = 7.002%. The optimal mass flow rates obtained in all the three works above are almost equal. A significant difference in optimal values of exergy efficiency is noted in the work of Luminosu *et al.* which maybe because of the simplifying assumption for sun's exergy flow rate made by them.

Khademi *et al.* [23] applied SQP and Genetic Algorithm to obtain optimal design criteria for maximizing exergy efficiency. Optimal result obtained by Khademi *et al.* [22] applying SQP is:

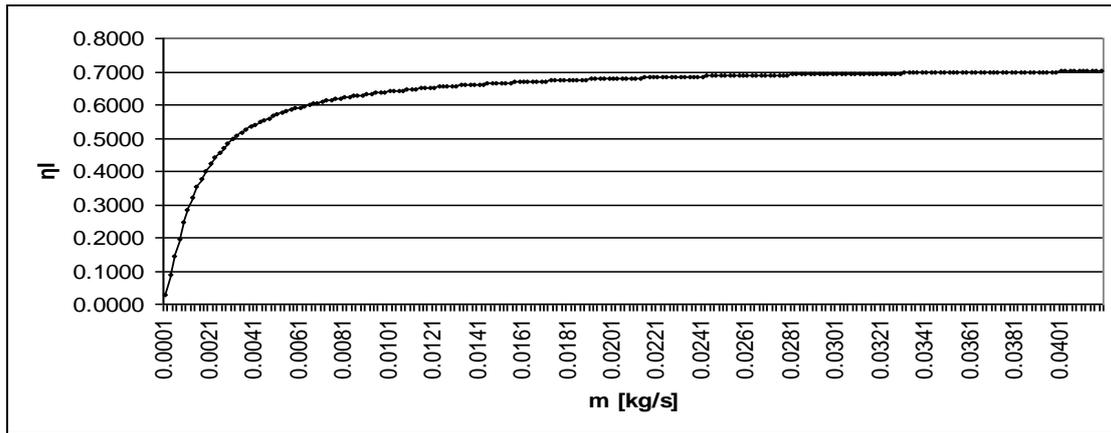
$$\dot{m} = 0.004365 \frac{\text{kg}}{\text{s}}, A_p = 5 \text{ m}^2, \eta_{II} = 6.1728\%, T_{f,out} = 388.997 \text{ K}, \eta_I = 46.4519\%.$$

The optimal results obtained by employing GA over a population of 500 and 150 generation are:

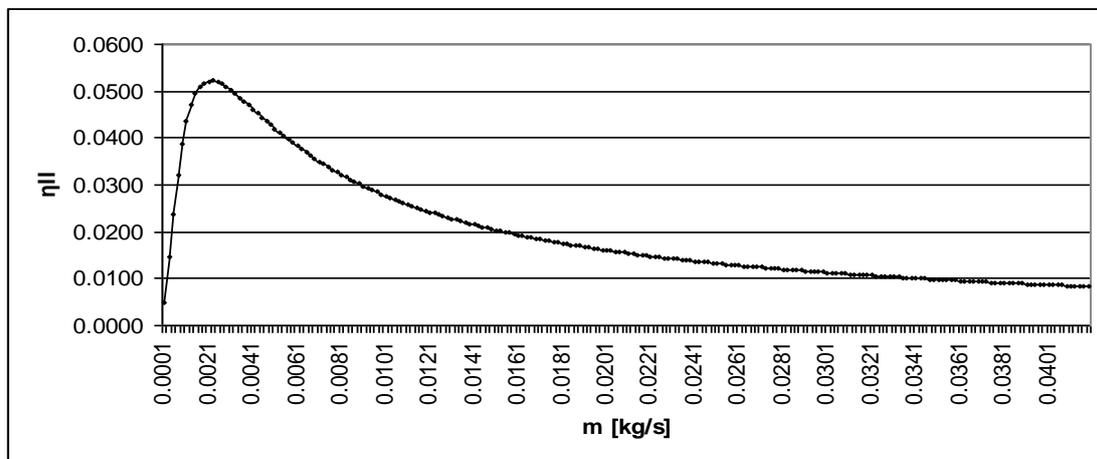
$$\dot{m} = 0.002178 \frac{\text{kg}}{\text{s}}, A_p = 3.12 \text{ m}^2, \eta_{II} = 7.0002\%, T_{f,out} = 407.684 \text{ K}, \eta_I = 44.9486\%.$$

**Table 1.** Optimal mass flow rate for different flat plate collectors and the corresponding optimal exergy efficiency, optimal outlet fluid temperature and  $\Delta T = T_{f,out}^* - T_{f,in}$  are tabulated where  $H_i = 800 \text{ W/m}^2$ ,  $T_a = 303 \text{ K} = T_{f,in}$  and  $c_p$  of water is considered at the inlet fluid temperature.

Collector	$A_c$	$F_R$	$U_L$	$\dot{m}^*$	$\eta_{II}^*$	$T_{f,out}^*$	$\Delta T$
	$\text{m}^2$		$\text{W/m}^2\text{K}$	$\text{kg/s}$	%	K	K
A	2.13	0.805	5.78	0.0023	5.2	380.01	77.01
B	2.26	0.76	3.68	0.0016	7.0	419.98	116.98
C	2.23	0.76	3.08	0.0019	8.2	419.99	116.99
D	1.55	0.78	3.56	0.0013	7.5	419.98	116.98
E	1.99	0.76	4.45	0.0017	6.1	405.01	102.01
F	2.125	0.805	5.8	0.0022	5.2	380.01	77.01



**Figure 5.** Variation of energy efficiency with respect to mass flow rate for a collector of area  $A_c = 2.13 \text{ m}^2$ ,  $FR = 0.805$ ,  $U_L = 5.76 \text{ W/m}^2\text{K}$ ,  $H_t = 800 \text{ W/m}^2$  and  $T_a = 30 \text{ }^\circ\text{C}$ .



**Figure 6.** Variation of exergy efficiency with respect to mass flow rate for a collector of area  $A_c = 2.13 \text{ m}^2$ ,  $F_R = 0.805$ ,  $U_L = 5.76 \text{ W/m}^2\text{K}$ ,  $H_t = 800 \text{ W/m}^2$  and  $T_a = 30 \text{ }^\circ\text{C} = T_{fin}$ .

**5. Validation of Computer Code**

To validate the computer code, optimal operation mode is determined for flat plate collector with design parameters considered by Khademi *et al.* [23] as input for the proposed model. The optimal results are compared with those reported by Khademi *et al.*. Tables 2 and 3 respectively presents a comparison of the optimal results obtained in the present work with those obtained by Khademi *et al.* using Genetic Algorithm and SQP. From

Tables 2 and 3, it is evident that the error in computing the optimal exergy efficiency, optimal mass flow rate and optimal outlet temperature using the proposed method is small in both cases. One of the sources of error may be the input solar radiation which is considered to be  $800 \text{ W/m}^2$  in the present work and is taken to be constant. Khademi *et al.* [23] did not report explicitly the value of input solar radiation used for computing the optimal design conditions.

**Table 2.** Comparison of optimal results obtained by using proposed model with that obtained by Khademi [23] using Genetic Algorithm; Number of iteration is 15 for proposed model.

Input					$\eta^*H$			$m^*$			$T^*out$		
$A_c$	$T_{in}$	$U_L$	$Q_u$	$\eta_I$	Applying GA	Present work	Error	Applying GA	Present work	Error	Applying GA	Present work	Error
$\text{m}^2$	K	$\text{W/m}^2\text{K}$	$\text{W/m}^2$								K		
3.12	300	3.29	1493.8	44.9	7.002	8.9	1.898	0.002178	0.002675	4.97E-04	407.684	414.514	6.83

**Table 3.** Comparison of optimal results obtained by using proposed model with that obtained by Khademi [23] using SQP; Number of iteration is 17 for proposed model and 9 for SQP.

Input					$\eta^*H$			$m^*$			$T^*out$		
$A_c$	$T_{in}$	$U_L$	$Q_u$	$\eta_I$	Applying SQP	Present work	Error	Applying SQP	Present work	Error	Applying SQP	Present work	Error
$\text{m}^2$	K	$\text{W/m}^2\text{K}$	$\text{W/m}^2$								K		
5	300	3.7978	1621.9	46.45	6.1728	5.28	0.893	0.004365	0.003063	1.30E-03	388.997	406.638	17.64

## 6. Conclusions

Exergetic optimization of flat plate collectors is carried out to evaluate the performance of a flat plate collector depending on mass flow rate and outlet fluid temperature. It is observed that decreasing the flow rate below optimal value increases the temperature of the fluid but a decrease in the exergy efficiency occurs. On the other hand, increasing the flow rate above the optimal value increases the energy efficiency but a decrease in the exergy efficiency and fluid temperature occurs. Thus, it can be concluded that the exergy analysis of the solar flat plate collectors allows the pre-determination of the optimal operational conditions for a collector for given values of controlled or uncontrolled parameters.

The simulator developed based on the proposed mathematical model can be used to determine the optimal operational mode of a flat plate collector for given environmental conditions. The rate of convergence of the proposed method is high and the accuracy of the result is also high. The results obtained are comparable to those reported by Khademi *et al.* [23] with an error of 0.8 for computing optimal exergy efficiency, error of the order of  $10^{-3}$  for computing the optimal mass flow rate and error of 17.64 for computing the optimal outlet fluid temperature when results were compared with those obtained by using SQP. When results were compared with results obtained by Genetic Algorithm, error in computing the optimal exergy is 1.9, error is of the order of  $10^{-4}$  for computing the optimal mass flow rate and an error of 6.83 for computing the optimal outlet fluid temperature is observed. It is observed that the error in computing the optimal operational mode using the proposed model is less when compared to the optimal solution obtained by Genetic Algorithm than SQP. Khademi *et al.* [23] reported that the results of GA represent more accuracy of algorithm. Hence, the simulator developed to solve the nonlinear constraint optimization problem based on direct substitution method gives an optimal result with a considerably good accuracy.

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## Appendix

### Computer Code Developed for Exergetic Optimization of Flat Plate Collectors

```

/*****
File Name      : Efficiency.cpp
PURPOSE       : Optimization of Second Law Efficiency w.r.t. mass flow rate
*****/
// **** HEADER FILES ****
#include<stdio.h>
#include<iostream.h>
#include<fstream.h>
#include<math.h>
void main()
{
    // **** DECLARATION AND INITIALIZATION ****
    double F_Ul = 4.86; // F'UL
    double F_TowAlpha = 0.722; // F'(Tow)(Alpha)
    double Ac = 2.13; // Gross Area of the Collector
    double Ts = 6000; // Aparent Temp of the Solar Radiation
    double FrUl = 4.66; // FRUL
    double FrTowAlpha = 0.688; // FR(Tow)(Alpha)
    double TowAlpha = 0.855; // (Tow)(Alpha)

    double Ht = 800; // Solar Insolation on the Collector Plane (W/m*m)
    double Ta = 303; // Ambient Temp. (in Kelvin)
    double cp = 4179; // Specific Heat Capacity of Water (J/kg-K)

    int i,j,counter;
    double tow,alpha,Fdash,Fr,Ul;
    double Tin,Tout,Ha,thetaIn, thetaMax,Qu;
    double etaE,M,md,eff2,dEffdM;
    double eta1,thetaOut,tpm,deltaT;
    double temp,mInitial1,mInitial2;
    double mdLower, mdUpper, mdMiddle, mdOptimum;
    double der1, der2, derDiff;
    ofstream result("Result.txt",ios::app);
    int slopSign1, slopSign2;
    double initial1 = -1000;
    double initial2 = -1000;
    int flagFound = 0;
    int flagStart = 1;

    // **** CALCULATION ****
    Fdash = F_TowAlpha/TowAlpha; // Fdash: Collector Efficiency Factor
    Fr = FrTowAlpha/TowAlpha; // Fr: Collector Heat removal factor

```

```

UI = FrUI/Fr; // UI: Overall Loss Coefficient (W/m*m-K)

Ha = TowAlpha*Ht; // Ha: Absorbed Solar Radiation (W/m*m)
Tin = Ta; // Tin: Inlet Fluid Temp. (K)
thetaIn = Tin/Ta; // thetaIn: Dimensionless Inlet fluid temp.
thetaMax = 1 + Ha/(UI*Ta); // thetaMax: Dimensionless max plate temp.
etaE = 1 + pow((Ta/Ts),4)/3 - 4*(Ta/Ts)/3;
// etaE: Exergy Fraction of Solar radiation

printf("Fdash:%f\n",Fdash);
printf("Fr:%f\n",Fr);
printf("UI:%f\n",UI);
printf("Ha:%f\n",Ha);
printf("Tin:%f\n",Tin);
printf("thetaIn:%f\n",thetaIn);
printf("thetaMax:%f\n",thetaMax);
printf("cp:%f\n",cp);
printf("Ta:%f\n",Ta);
printf("Ht:%f\n",Ht);
printf("Ac:%f\n",Ac);
printf("etaE:%f\n",etaE);

result<<"m-dot"<<" " <<"M"<<" " <<"Eff 2"<<" " <<"Derivation"<<" " <<"Eff 1"<<"
" <<"Tpm"<<" " <<"Delta-T"<<endl;

// ***CALCULATION OF ENERGY EFFICIENCY AND EXERGY EFFICIENCY OF COLLECTOR &
INITIAL GUESS FOR BISECTION METHOD*****

// md: Mass Flow rate (kg/s)
// M: Mass Flow Number
// eff2: Second Law Efficiency
// dEffdM: Derivative of Second Law Efficiency w.r.t M
// eta1: First Law Efficiency
// tpm: Mean Plate temp.(K)
// deltaT: Difference between Outlet and Inlet fluid temp (K)

mInitial1 = -1000;
mInitial2 = -1000;

for(md = 0.0001; md<= Ac*0.02 ; md=md+0.0002)
{
    M = md*cp*Ta/(Ac*Ht);

    eff2 = (M*(thetaMax - 1)/etaE)*(1 - exp(-F_TowAlpha/(M*(thetaMax-1)))) -
(M/etaE)*log(thetaMax + (1 - thetaMax)*exp(-F_TowAlpha/(M*(thetaMax-1))));

```

```

// Output results for Eta1, Tpm
thetaOut = thetaMax + (thetaIn - thetaMax)*exp(-F_TowAlpha/(M*(thetaMax-1)));
eta1 = M*(thetaOut - thetaIn);

Qu = Ac*Ht*eta1;
tpm = Tin + (Qu*(1-Fr))/(Ac*FrUl);

Tout = Ta*thetaOut;
deltaT = Tout - Tin;

// To Find the Initial Guess for Bisection Method
temp = F_TowAlpha/(thetaMax - 1);
dEffdM =log(thetaMax + (1-thetaMax)*exp(-temp/M))
        +(temp*(1-thetaMax)*exp(- temp/M))/(M*(thetaMax + (1-thetaMax)*exp(-
temp/M)))
        - (thetaMax-1)*(1-exp(-temp/M)*(1+(temp/M)));

if (flagStart==1)
{
if (dEffdM>0)
{
slopSign1 = 1;
}
else
{
slopSign1 = -1;
}
initial1 = md;
initial2 = md+ 0.0002;
flagStart = 0;
}
else
{
if (flagFound == 0)
{
if (dEffdM>0)
{
slopSign2 = 1;
der1= dEffdM;
}
else
{
slopSign2 = -1;
der2= dEffdM;
}
}
}
}

```

```

    if (slopSign1*slopSign2 < 0)
    {
        flagFound = 1;
    }
    else
    {
        initial1 = md;
        initial2 = md+ 0.02;
    }
}
} // End of Else

printf("m:%f      M:%f      Eff:%f      Derivative:%f      Eta1:%f Tpm:%f
      deltaT:%f\n",md,M,eff2,dEffdM,eta1,tpm,deltaT);
result<<md<<"      "<<M<<" "<<eff2<<"      "<<dEffdM<<"      "<<eta1<<"      "<<tpm<<"
"<<deltaT<<endl;

} // End of for (md)

printf("\nInitial 1: %f      Initial 2: %f\n",mInitial1,mInitial2);
/** CALCULATION OF OPTIMUM MASS FLOW RATE BY BISECTION METHOD **/
mdLower = mInitial1;
mdUpper = mInitial2;
mdMiddle = (mInitial1 + mInitial2)/2;

derDiff = 100;
counter = 0;
temp =      F_TowAlpha/(thetaMax - 1);
while(derDiff>0.000001 && counter < 500)
{
    M = mdMiddle*cp*Ta/(Ac*Ht);
    dEffdM =log(thetaMax + (1-thetaMax)*exp(-temp/M))
            +      (temp*(1-thetaMax)*exp(-temp/M))/(M*(thetaMax + (1-
thetaMax)*exp(-temp/M)))
            -      (thetaMax-1)*(1-exp(-temp/M)*(1+(temp/M)));

    if(dEffdM > 0)
        mdUpper = mdMiddle;
    else mdLower = mdMiddle;
    M = mdLower*cp*Ta/(Ac*Ht);
    der1 =  log(thetaMax + (1-thetaMax)*exp(-temp/M))
            +      (temp*(1-thetaMax)*exp(-temp/M))/(M*(thetaMax + (1-
thetaMax)*exp(-temp/M)))
            -      (thetaMax-1)*(1-exp(-temp/M)*(1+(temp/M)));

```

```

M = mdUpper*cp*Ta/(Ac*Ht);
der2 = log(thetaMax + (1-thetaMax)*exp(-temp/M))
      + (temp*(1-thetaMax)*exp(-temp/M))/(M*(thetaMax + (1-
thetaMax)*exp(-temp/M)))
      - (thetaMax-1)*(1-exp(-temp/M)*(1+(temp/M)));

mdMiddle = (mdUpper + mdLower)/2;
derDiff = fabs(der1 - der2);
counter++;
} // End of while()

// **** CALCULATE THE OPTIMUM VALUES ****
mdOptimum = mdMiddle;
M = mdOptimum*cp*Ta/(Ac*Ht);

eff2 = (M*(thetaMax - 1)/etaE)*(1 - exp(-F_TowAlpha/(M*(thetaMax-1)))) - (M/etaE)*log(thetaMax
+ (1 - thetaMax)*exp(-F_TowAlpha/(M*(thetaMax-1))));

// Output results for Eta1, Tpm

thetaOut = thetaMax + (thetaIn - thetaMax)*exp(- F_TowAlpha/(M*(thetaMax-1)));
eta1 = M*(thetaOut - thetaIn);

Qu = Ac*Ht*eta1;
tpm = Tin + (Qu*(1-Fr))/(Ac*FrUl);

Tout = Ta*thetaOut;
deltaT = Tout - Tin;

printf("Optimum m:%f      Eff2:%f  Eta1:%f      Tpm:%f
deltaT:%f\n",mdOptimum,eff2,eta1,tpm,deltaT);

result<<endl;
result<<"***** OPTIMUM VALUES *****"<<endl;
result<<"Optimum Mass Flow rate:"<<mdOptimum<<endl;
result<<"2nd law Efficiency:"<<eff2<<endl;
result<<"1st law Efficiency:"<<eta1<<endl;
result<<"Tpm:"<<tpm<<endl;
result<<"Delta T:"<<deltaT<<endl;
result.close();
} // End of main()

```