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# A Hybrid Batch-Fabrication Decision for a Vendor–Buyer Integrated System with Multiple Deliveries, Rework, and Machine Failures

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**Abstract**

Transnational firms operate in turbulent and competitive marketplaces. They continually find ways to optimize their internal supply chains to guarantee that firms achieve operating goals of high quality, quick response time, smooth fabrication schedules, and timely deliveries under the reality of limited capacity and unreliable machines/processes. This study considers a vendor–buyer integrated batch-fabrication problem with outsourcing, rework, machine failures, and multiple deliveries to facilitate better decision making and assist enterprises in increasing their competitive advantages. We assume that a portion of a batch is outsourced in order to reduce manufacturing uptime, and in-house production experiences undesirable situations, such as machine failures and nonconforming stock making. Corrective action on failures and repair tasks of the nonconforming are undertaken in each cycle as they occur. The finished stocks are then shipped under the multiple-deliveries plan. We build a model to explicitly depict the problem and determine the problem's cost function through formulations and derivations. The convexity of cost function and the optimal uptime are obtained via differential calculus and a proposed specific algorithm. Lastly, we offer a numerical example to show our proposed model makes diverse crucial system information, such as the individual and joint impact of outsourcing, rework, random failures, as well as the frequency of delivery on different features and the optimal uptime of the problem, easily accessible, to assist enterprises in strategic planning, management, and decision making in their practical intra-supply-chain environments.



*Keywords:* Vendor-buyer integrated system, Hybrid batch fabrication, Machine failures, Rework, Multi-delivery;

## 1. Introduction

To facilitate better decision-making for transnational enterprises and assist them in increasing competitive advantages, we explore a vendor-buyer integrated batch-fabrication problem with outsourcing, rework, machine failures, and multiple deliveries. Many real-world production processes experience undesirable situations, such as machine failures and nonconforming stock making. Corrective actions on failures and repair tasks of the nonconforming are undertaken to avoid unwanted disruption/delay in production and poor/unacceptable product quality. Lee and Moinzadeh [1] considered a multi-echelon batch-ordering-based repairable system to analyze its operating characteristics. The authors evaluated the outstanding orders' distribution and subsequently backorders as two parameters in their proposed approximation scheme. Through tests of various scenarios of infinite and finite servers, they found that their proposed scheme could effectively determine the system stock levels that minimize the total inventory holding and backordering costs. The authors also compared certain outstanding orders' distribution to that in the existing literature. Groenevelt et al. [2] explored the batch-production problem considering an unreliable manufacturing facility and safety stocks. They assumed the machine failure rate is constant, but failure repair time is random. To meet the desired service level, safety stocks are used. Different bounds for service levels were set to explore the influence of various system parameters on these ranges. The authors introduced a policy for production control to show how safety stock works under the renewal process type of a specific single-server queue. They also demonstrated how their results could be fitted in a wider resource allocation managerial decision making. Moini and Murthy [3] considered the batch-sizing problem of an unreliable fabrication system. The authors built a model to clearly characterize their unreliable system and different repair strategies to determine the cost-minimization batch sizes under these separate strategies. Sha et al. [4] studied a photolithography scheduling problem for wafer fabrication considering an on-line rework policy. The authors incorporating on-line rework into the dispatching rule with the aim of reducing production procedure and machine workload, as well as increasing the output rate. Goerler and Voß [5] investigated

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the impacts of nonconforming stock and rework process on the batch-size problem with limited capacity. The mixed-integer programming technique was presented to deal with the problem. Diverse numerical experiments were carried out to analyze the influences of variations in defective occurrences on computer times needed to generate the batch size solution. Extra studies [6-14] explored the influence of various characteristics of the unreliable machine and rework strategy on manufacturing systems and operations management.

In the production planning/scheduling stage, the outsourcing alternative can effectively achieve these goals to reduce the manufacturing uptime and/or release some workloads from the machine. Gaimon [15] examined the strategic trade-offs problem regarding subcontracting versus capacity expansion, specifically, for the service-sector environments featuring excessive seasonal demand or quick technological improvement, where demand exceeds current inventory/capacity. The author explored the critical influence of subcontracting on the company's competitive pricing strategy and found out that the optimal price policy can be determined by the higher of either unit subcontracting cost or unit in-house fabrication cost. Jolayemi and Olorunniwo [16] examined a deterministic multi-plant production-transportation problem with multiple warehouses and subcontracting options. A profit-maximization model was developed to help decide the optimal plans for the (1) batch sizes for each factory, (2) shipping quantities from each factory to each warehouse, (3) on-hand inventories for each warehouse, and (4) subcontracting quantities for each warehouse. The authors gave numerical examples to demonstrate how their model works and discussed the limitations of their proposed model. Arias-Aranda et al. [17] considered the benefits and operations flexibility from implementing an outsourcing strategy in service-oriented enterprises. The authors carried out their study by the structural analysis to reveal that outsourcing benefits significantly increase as the operations' flexibility rises, especially in the operational areas of the firms' markets, personnel, expansion, and information systems. Rosar [18] studied the relationship between strategic subcontracting and the optimal purchase policy. The author first presented an outsourcing option that relies on non-cost-savings subcontracting mechanisms and then extended to cost-savings relevant rationale, discussing the incentives of sellers who are involved in nested subcontracting policies. Additional works [19-23] explored the influence of diverse outsourcing characteristics on the manufacturing sectors and business management.

Furthermore, in real-world supply-chain environments, goods' transportation is often arranged via multiple shipments at specific time intervals. Banerjee and Burton [24] examined the influence of coordinated versus independent stock replenishing disciplines on a single-vendor multi-buyers problem. The authors performed numerous simulation experiments to show that for discrete-lots procurements in a single-vendor multi-buyers problem, the classical batch-size models cannot satisfactorily solve them. The authors recommended that the coordinated common replenishing cycle model is a better approach. Swenseth and Godfrey [25] incorporated the existing transportation cost functions (in the literature) into the stock replenishing decision making without giving in the accuracy of the decision or increasing unnecessary complexity to its process. Toews et al. [26] examined the classical economic batch ordering/production quantity problems allowing partial backordering option, wherein the backlogging rate is a linear function of the time to delivery. Montarelo et al. [27] studied the four-echelon inventory management decision in the decentralized supply-chain environments using a Tabu searching metaheuristic. The authors assumed different service levels to cope with the market's stochastic demands. They proposed a global simulation approach to explore/optimize the four-echelon linear/nonlinear supply chains and revealed significant differences among echelons in critical inventory and cost parameters. Furthermore, the authors believed that their proposed approach could be generalized for border applications.

Recent studies explored the influence of various features of multiple deliveries on different fabrication-transportation and supply-chain systems. Focusing of these works, including on pricing and lot sizing in a two-echelon supply chain [28], optimization of a specific closed-loop green supply chain [29], price profit structuring for single-vendor and multi-buyer system [30], multiproduct multi-shipment system with expedited rate and rework [31], and stochastic supply chain featuring imperfect production and controllable defects [32]. Since few of the works mentioned above have examined the collective influence of machine failures, outsourcing, multiple deliveries, and rework on batch-fabrication decision making, this study aims to bridge the gap and explicitly build a decision support model to help today's transnational firms handle the challenge of vendor's unreliable fabrication process, meeting buyer's timely needs of quality multiproduct, and keeping the total operating expenses at a minimum.

## 2. The proposed hybrid batch fabrication vendor-buyer coordinated system

The description of the proposed vendor-buyer coordinated system with stochastic failure, rework, outsourcing, and multi-delivery is given below. Consider that a buyer's annual demand rate $\lambda$ units of a particular product are to be met by a vendor using a hybrid batch fabrication plan. In order to shorten the cycle length, a $\pi$ (where $0 < \pi < 1$) portion of the batch size $Q$ is subcontracted to an outside provider, and the remaining portion is made in-house with an annual fabrication rate $P_1$. Thus, a particular unit cost $C_\pi$ and setup cost $K_\pi$ are connected to the outsourcing plan, with $C_\pi = (1 + \beta_2)C$ and $K_\pi = (1 + \beta_1)K$, where $\beta_2$ and $\beta_1$ are the connecting parameters; and $C$ and $K$ denote the in-house unit and setup costs, respectively. We assume that the receipt time of outsourced items is scheduled before starting stock transportation time in each cycle. The outside provider promises the quality of outsourced products.

The fabrication machine is subject to a Poisson distributed failure, with the mean = $\beta$ per year. Thus, time to a machine failure $t$ obeys the Exponential distribution (i.e., density function $f(t) = \beta e^{-\beta t}$). An abort/resume discipline is used when a failure occurs. Under this discipline, the failure repair work starts immediately. A fixed repair time $t_r$ is assumed (in case that actual repair time is greater than $t_r$, a rental/spare machine will be put in use to avoid further fabrication schedule delay). Fabrication of the unfinished/interrupted lot continues immediately when the repair work completes. Further, a random $x$ portion of nonconforming items is fabricated at a rate of $d_1$ (where $d_1 = P_1x$). These nonconforming stocks can be repaired in each cycle via a rework process, at a rate $P_2$ and extra unit cost $C_R$, when the regular fabrication ends. Shortages are not permitted in the proposed model; hence, $(P_1 – d_1 – \lambda) > 0$ must hold.

Once the rework process is done and the outsourced products are received, $n$ fixed portions of the finished lot are distributed to the buyer at a fixed time interval $t'_{n\pi}$ during the product transportation $t'_{3\pi}$. The extra notation used by this study also includes the following:

$t_{1\pi}$ = uptime of the proposed system – the decision variable,
$t'_{2\pi}$ = rework time in failure occurrence case,
$T'_\pi$ = cycle length in failure occurrence case,
$Q$ = batch size,
$H_0$ = perfect stock level when a failure happens,
$H_1$ = perfect stock level when $t_{1\pi}$ ends,
$H_2$ = perfect stock level when the rework process ends,
$H$ = perfect stock level upon receipt of outsourced products,
$M$ = machine repair cost,
$h$ = unit holding cost for the perfect stock,
$h_1$ = unit holding cost for the reworked stock,
$h_2$ = unit holding cost for buyer stock,
$K_1$ = fixed distribution cost,
$C_T$ = unit distribution cost,
$C_1$ = unit cost for safety stock,
$h_3$ = unit holding cost for safety stock,
$g$ = $t_r$, fixed machine repair time,
$D$ = quantity per shipment,
$I$ = the leftover stock in $t'_{n\pi}$,
$I(t)$ = perfect stock level at time $t$,
$I_d(t)$ = nonconforming stock level at time $t$,
$I_F(t)$ = safety stock level at time $t$,
$I_c(t)$ = buyer stock level at time $t$,
$TC(t_{1\pi})_1$ = total system cost per cycle in failure occurrence case,
$E[TC(t_{1\pi})_1]$ = the expected total system cost per cycle in failure occurrence case,
$E[T'_\pi]$ = the expected cycle length in failure occurrence case,
$TC(t_{1\pi})_2$ = total system cost per cycle in no failure occurrence case,
$E[TC(t_{1\pi})_2]$ = the expected total system cost per cycle in no failure occurrence case,
$T_\pi$ = cycle length in no failure occurrence case,
$E[T_\pi]$ = the expected cycle length in no failure occurrence case,
$t_{2\pi}$ = rework time in no failure occurrence case,
$t_{3\pi}$ = stock distribution time in no failure occurrence case,
$t_{n\pi}$ = time interval between two consecutive shipments in no failure occurrence case,
$\boldsymbol{T_\pi}$ = replenishment cycle length for the proposed system with or without a failure occurrence,
$E[TCU(t_{1\pi})]$ = the expected system cost per unit time for the proposed system with or without a failure occurrence,
$T$ = cycle length for the proposed system without outsourcing, nor breakdown,
$t_1$ = uptime for the proposed system without outsourcing, nor breakdown,
$t_2$ = rework time for the proposed system without outsourcing, nor breakdown,
$t_3$ = delivery time for the proposed system without outsourcing, nor breakdown.

Two distinct situations regarding stochastic machine failure need to be separately explored as follows:

### *2.1. Situation 1: A stochastic failure happens during $t_{1\pi}$*

In situation one, we have time to failure occurrence $t < t_{1\pi}$. When a failure occurs, the level of perfect stock arrives at $H_0$, it continues to accumulate to $H_1$ when $t_{1\pi}$ ends, and the stock level reaches $H_2$ when $t'_{2\pi}$ completes. Meantime, the receipt of outsourced products further brings the finished stock level to $H$ at the beginning of distribution time $t'_{3\pi}$. Finally, the stock level declines to zero at the end of the fabrication cycle (see Figure 1).

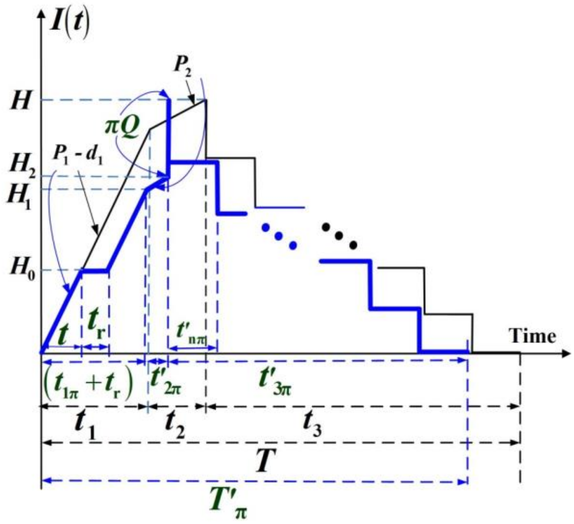


**Figure 1.** The level of perfect stock in the proposed hybrid batch fabrication vendor-buyer integrated system with multi-delivery, reworks, and machine failure (in blue) as compared to that of a system with rework and multi-delivery (in black)

The safety stock level in situation one is exhibited in Figure 2. It explicitly shows that at the beginning of $t'_{3\pi}$, the safety items along with the finished products are distributed to the buyer for meeting extra product demand during $t_r$. The nonconforming stock level in situation one is displayed in Figure 3. It illustrates the nonconforming stock levels at time $t$, $t_{1\pi}$, and $t'_{2\pi}$.

The following formulas can be directly obtained from the description of the proposed model and by observation of Figures 1 to 3:

 

$$T'_{\pi} = t_{1\pi} + t_{\mathrm{r}} + t'_{2\pi} + t'_{3\pi} \tag{1}$$

$$H_0 = (P_1 - d_1)t \tag{2}$$

$$t_{1\pi} = \frac{Q(1-\pi)}{P_1} \tag{3}$$

$$H_1 = (P_1 - d_1)t_{1\pi} \tag{4}$$

$$t'_{2\pi} = \frac{(1-\pi)Q(x)}{P_2} \tag{5}$$

$$H_2 = H_1 + P_2 t'_{2\pi} \tag{6}$$

$$t'_{3\pi} = T'_{\pi} - (t_{1\pi} + t_{\mathrm{r}} + t'_{2\pi}) \tag{7}$$

$$T'_{\pi} = \frac{Q + \lambda t_{\mathrm{r}}}{\lambda} \tag{8}$$

$$d_1 t_{1\pi} = x(P_1 t_{1\pi}) = (1-\pi)Q(x) \tag{9}$$

*2.1.1. Stock status in $t'_{3\pi}$ of situation 1*

At the beginning of product distribution time $t'_{3\pi}$, the outsourced and safety products are added to the finished lot to bring the shipping quantity up to $H$.

$$H = H_2 + \pi Q + \lambda t_r \tag{10}$$

The stock level during $t'_{3\pi}$ can be calculated by Eq. (11) [31].

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) Ht'_{3\pi} = \left(\frac{n-1}{2n}\right) Ht'_{3\pi} \tag{11}$$

*2.1.2. The buyer's stock holding status*

The buyer's stock holding status in $T'_{\pi}$ can be computed by Eq. (12) [31].

$$\frac{1}{2}\left[\frac{Ht'_{3\pi}}{n} + (H - \lambda t'_{3\pi})T'_{\pi}\right] \tag{12}$$

*2.1.3. Total cost per cycle of situation 1*

$TC(t_{1\pi})_1$, the total cost per cycle, consists of both the variable and setup costs for fabrication and outsourcing, safety products' relevant costs (refer to Fig. 2), repair cost for equipment failure, rework cost, both setup and variable distribution costs, and total holding costs (including buyer's stocks, reworked, finished, and nonconforming) during $T'_{\pi}$, as exposed in Eq. (13).

Substitute formulas (1) to (12) in Eq. (13), and apply the expected value to deal with the randomness of $x$, the following expected total system cost per cycle of situation one, $E[TC(t_{1\pi})_1]$ could be gained.

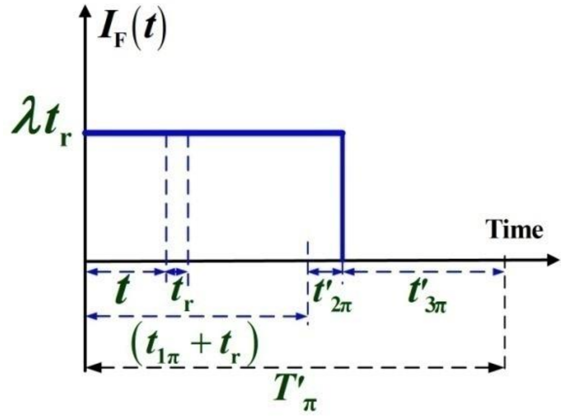


**Figure 2.** The safety stock level in situation 1 of the proposed system

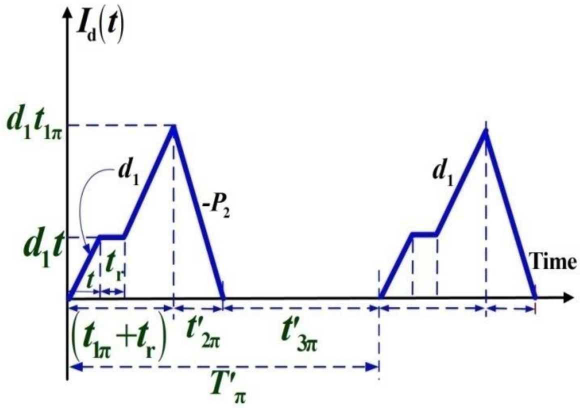


**Figure 3.** The nonconforming stock level in situation 1 of the proposed system

$$\begin{aligned} TC(t_{1\pi})_1 &= C[(1-\pi)Q] + K + C_{\pi}(\pi Q) + K_{\pi} + C_1(\lambda t_r) + h_3(\lambda t_r)(t_{1\pi} + t_r + t'_{2\pi}) + M \\ &+ C_R(1-\pi)Q(x) + nK_1 + C_T[Q + \lambda t_r] + \frac{h_2}{2}\left[\frac{Ht'_{3\pi}}{n} + (H - \lambda t'_{3\pi})T'_{\pi}\right] + h_1\frac{P_2 t'_{2\pi}}{2}(t'_{2\pi}) \\ &+ h\left[\frac{H_1 + d_1 t_{1\pi}}{2}(t_{1\pi}) + (H_0 t_r) + (d_1 t)t_r + \frac{H_1 + H_2}{2}(t'_{2\pi}) + \left(\frac{n-1}{2n}\right)Ht'_{3\pi}\right] \end{aligned} \tag{13}$$

$$\begin{aligned} E[TC(t_{1\pi})_1] &= C(t_{1\pi}P_1) + K + C_{\pi}(\pi t_{1\pi}P_1 y_1) + K_{\pi} + C_1\lambda g + h_3\left[\lambda g t_{1\pi} + \lambda g^2 + \frac{\lambda g E[x] t_{1\pi} P_1}{P_2}\right] \\ &+ M + C_R E[x] t_{1\pi} P_1 + nK_1 + C_T[t_{1\pi}P_1 y_1 + \lambda g] + h(P_1 t g) + \frac{t_{1\pi}^2 P_1^2 (h_2 - h)}{2n\lambda(1-\pi)}(y_1 - y_2) \\ &+ \frac{E[x]^2 t_{1\pi}^2 P_1^2 (h_1 - h)}{2P_2} + h_2\left[\frac{g t_{1\pi} P_1}{2}(y_1 + y_2)\right] + \frac{g t_{1\pi} P_1}{2n}(h_2 - h)(y_1 - y_2) + \frac{h_2 t_{1\pi}^2 P_1^2 y_1}{2}\left(\frac{y_2}{\lambda}\right) \\ &+ h\left[\frac{g t_{1\pi} P_1}{2}(y_1 - y_2)\right] + \frac{h t_{1\pi}^2 P_1^2 y_1}{2\lambda}\left[y_1 + \frac{\lambda(-\pi)}{P_1} + \frac{\lambda E[x](1-2\pi)}{P_2}\right] + h_2\left(\frac{\lambda g^2}{2}\right) \end{aligned} \tag{14}$$

where

$$y_1 = \frac{1}{(1-\pi)};\ y_2 = \lambda\left[\frac{1}{P_1} + \frac{E[x]}{P_2}\right].$$

The expected cycle length in situation one can be gained from Eq. (8) as follows:

$$E[T'_\pi] = \frac{Q+\lambda t_r}{\lambda} = \frac{t_{1\pi}P_1 y_1 + \lambda t_r}{\lambda} \tag{15}$$

### 2.2. Situation 2: No failure happening in $t_{1\pi}$

In situation two, we have time to failure occurrence $t \geq t_{1\pi}$. Because no failure is happening, the perfect stock level accumulates to $H_1$ when $t_{1\pi}$ ends, and it reaches $H_2$ when $t_{2\pi}$ completes. Meantime, the receipt of outsourced products further brings the finished stock level to $H$ at the beginning of distribution time $t_{3\pi}$. Finally, the stock level declines to zero at the end of the fabrication cycle (see Figure 4).

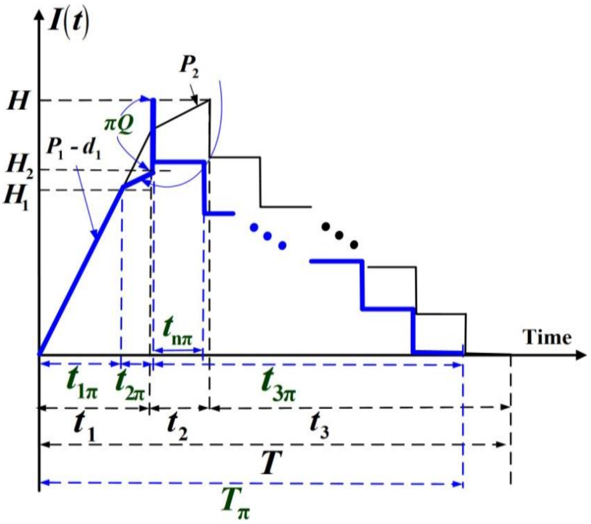


**Figure 4.** The level of perfect stock in the proposed hybrid batch fabrication vendor-buyer integrated system with multi-delivery and rework, but no failure happening (in blue) as compared to that of a system with rework and multi-delivery (in black)

The safety stock level in situation two remains unchanged throughout $T_\pi$. Likewise; the following formulas can be directly obtained from the description of the proposed model and by observation of Figure 4:

$$T_\pi = t_{1\pi} + t_{2\pi} + t_{3\pi} \tag{16}$$

$$H_1 = (P_1 - d_1)t_{1\pi} \tag{17}$$

$$H_2 = H_1 + P_2 t_{2\pi} \tag{18}$$

$$H = H_2 + \pi Q \tag{19}$$

$$t_{1\pi} = \frac{Q(1-\pi)}{P_1} \tag{20}$$

$$t_{2\pi} = \frac{(1-\pi)Q(x)}{P_2} \tag{21}$$

$$t_{3\pi} = T_\pi - (t_{1\pi} + t_{2\pi}) \tag{22}$$

Equation (9) remains valid in situation two. Total holding inventories in $t_{3\pi}$ and at the buyer's location (see Figs. 4 and 5) can be gained by applying Eqs. (23) and (24) [31].

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)H(t_{3\pi}) = \left(\frac{n-1}{2n}\right)H(t_{3\pi}) \tag{23}$$

$$\frac{1}{2}\left[\frac{H(t_{3\pi})}{n} + (H - \lambda t_{3\pi})T_\pi\right] \tag{24}$$

#### 2.2.1. Total cost per cycle of situation 2

$TC(t_{1\pi})_2$, the total cost per cycle, consists of both the variable and setup costs for fabrication and outsourcing, safety products' holding cost (see Figure 7), setup and variable distribution costs, rework cost, and total holding costs (including buyer's stocks, finished, nonconforming, and reworked items) during $T_\pi$, as exposed in Eq. (25).

Substitute equations (16) to (24) and (9) in Eq. (25), and use the expected value to deal with the randomness of $x$, the following expected total system cost per cycle for situation 2, $E[TC(t_{1\pi})_2]$ could be gained.

$$\begin{aligned} TC(t_{1\pi})_2 &= C[(1-\pi)Q] + K + C_\pi(\pi Q) + K_\pi + h_3(\lambda t_r)T_\pi + nK_1 \\ &+ C_T Q + C_R(1-\pi)Q(x) + \frac{h_2}{2}\left[\frac{Ht_{3\pi}}{n} + (H - \lambda t_{3\pi})T_\pi\right] + h_1\frac{P_2 t_{2\pi}}{2}(t_{2\pi}) \\ &+ h\left[\frac{H_1 + d_1 t_{1\pi}}{2}(t_{1\pi}) + \frac{H_1 + H_2}{2}(t_{2\pi}) + \left(\frac{n-1}{2n}\right)Ht_{3\pi}\right] \end{aligned} \tag{25}$$

$$\begin{aligned} E[TC(t_{1\pi})_2] &= C(t_{1\pi}P_1) + K + C_\pi(\pi t_{1\pi}P_1 y_1) + K_\pi + h_3 g(t_{1\pi}P_1 y_1) + nK_1 + C_T(t_{1\pi}P_1 y_1) \\ &+ C_R E[x]t_{1\pi}P_1 + \frac{E[x]^2 t_{1\pi}^2 P_1^2(h_1 - h)}{2P_2} + \frac{h_2 t_{1\pi}^2 P_1^2}{2(1-\pi)}\left(\frac{y_2}{\lambda}\right) + \frac{t_{1\pi}^2 P_1^2(h_2 - h)y_1}{2n\lambda}(y_1 - y_2) \\ &+ \frac{h t_{1\pi}^2 P_1^2 y_1}{2\lambda}\left[y_1 + \frac{\lambda(-\pi)}{P_1} + \frac{\lambda E[x](1-2\pi)}{P_2}\right] \end{aligned} \tag{26}$$

where

$$y_1 = \frac{1}{(1-\pi)};\ y_2 = \lambda\left[\frac{1}{P_1} + \frac{E[x]}{P_2}\right].$$

The expected cycle length in situation two can be gained as follows:

$$E[T_\pi] = \frac{Q}{\lambda} = \frac{t_{1\pi}P_1 y_1}{\lambda} \tag{27}$$

## 3. Solving the proposed system

As we assume that Poisson distributed failure rate $\beta$ per year, so, the time to failure $t$ follows the Exponential distribution, with $f(t) = \beta e^{-\beta t}$ and $F(t) = (1 - e^{-\beta t})$ (i.e., the density and cumulative density functions). We employ the renewal reward theorem to solve the $E[TCU(t_{1\pi})]$ as follows.

Substitute formulas (15) and (27) in formula (29), and then formulas (14), (26), and (29) in formula (28), along with additional efforts in derivations, $E[TCU(t_{1\pi})]$ can be gained as follows (see Appendix A for details).

The first- and second-derivatives of $E[TCU(t_{1\pi})]$ are shown in equations (B-1) and (B-2) in Appendix B. Since the first term on the right-hand side (RHS) of Eq. (B-2) is positive, it follows that the $E[TCU(t_{1\pi})]$ is convex if the second term on the RHS of Eq. (B-2) is also positive. That means if $\tau(t_{1\pi}) > t_{1\pi} > 0$ holds (see Eq. (B-3) for details).

Once Eq. (B-3) is verified to be true, we can solve the optimal $t_{1\pi}^*$ by setting the first-derivative of $E[TCU(t_{1\pi})] = 0$ (refer to Eq. (B-1)). Since the first term on the RHS of Eq. (B-1) is positive, we have the following.

$$E\left[TCU\left(t_{1\pi}\right)\right]=\frac{\left\{\int_0^{t_{1\pi}} E\left[TC\left(t_{1\pi}\right)_1\right]\cdot f\left(t\right)dt+\int_{t_{1\pi}}^{\infty} E\left[TC\left(t_{1\pi}\right)_2\right]\cdot f\left(t\right)dt\right\}}{E[\boldsymbol{T}_\pi]} \tag{28}$$

where

$$E[\boldsymbol{T_\pi}]=\int_0^{t_{1\pi}} E\left[T'_\pi\right]\cdot f\left(t\right)dt+\int_{t_{1\pi}}^{\infty} E\left[T_\pi\right]\cdot f\left(t\right)dt \tag{29}$$

$$E\left[TCU\left(t_{1\pi}\right)\right]=\left[\frac{\lambda}{y_1+\dfrac{\lambda g\left[1-e^{-\beta t_{1\pi}}\right]}{t_{1\pi}P_1}}\right]\left\{\begin{array}{l}\dfrac{u_0}{t_{1\pi}}+\dfrac{u_1}{t_{1\pi}}+u_2+t_{1\pi}v_3-hge^{-\beta t_{1\pi}}+\dfrac{u_3e^{-\beta t_{1\pi}}}{t_{1\pi}}\\+\left(v_0+v_1+v_2\right)-\left(v_0+v_1+v_2\right)e^{-\beta t_{1\pi}}\end{array}\right\} \tag{30}$$

$$\left\{\begin{array}{l}\left[\left(hg+v_0+v_1+v_2\right)P_1\left(y_1P_1\beta e^{-\beta t_{1\pi}}\right)+v_3P_1\left(y_1P_1-\lambda g\beta e^{-\beta t_{1\pi}}\right)\right]t_{1\pi}^{\ 2}\\+\left[u_3P_1\left(-y_1P_1\beta e^{-\beta t_{1\pi}}\right)+v_3P_1\left(2\lambda g-2\lambda ge^{-\beta t_{1\pi}}\right)+\left(hg-u_2\right)P_1\lambda g\left(\beta e^{-\beta t_{1\pi}}\right)\right]t_{1\pi}\\-\left(u_0+u_1\right)P_1\left(y_1P_1+\lambda g\beta e^{-\beta t_{1\pi}}\right)+u_3P_1\left(-\lambda g\beta e^{-\beta t_{1\pi}}-y_1P_1e^{-\beta t_{1\pi}}\right)\\-\left(hg+v_0+v_1+v_2\right)P_1\lambda g\left(-e^{-2\beta t_{1\pi}}+e^{-\beta t_{1\pi}}\right)-\left(u_2+v_0+v_1+v_2\right)P_1\lambda g\left(e^{-\beta t_{1\pi}}-1\right)\end{array}\right\}=0 \tag{31}$$

Let $w_0$, $w_1$, and $w_2$ stand for the following:

$$w_0=\left[\left(hg+v_0+v_1+v_2\right)P_1\left(y_1P_1\beta e^{-\beta t_{1\pi}}\right)+v_3P_1\left(y_1P_1-\lambda g\beta e^{-\beta t_{1\pi}}\right)\right]$$

$$w_1=\left[u_3P_1\left(-y_1P_1\beta e^{-\beta t_{1\pi}}\right)+v_3P_1\left(2\lambda g-2\lambda ge^{-\beta t_{1\pi}}\right)+\left(hg-u_2\right)P_1\lambda g\left(\beta e^{-\beta t_{1\pi}}\right)\right]$$

$$\begin{aligned}w_2=&-\left(u_0+u_1\right)P_1\left(y_1P_1+\lambda g\beta e^{-\beta t_{1\pi}}\right)+u_3P_1\left(-\lambda g\beta e^{-\beta t_{1\pi}}-y_1P_1e^{-\beta t_{1\pi}}\right)\\&-\left(hg+v_0+v_1+v_2\right)P_1\lambda g\left(-e^{-2\beta t_{1\pi}}+e^{-\beta t_{1\pi}}\right)-\left(u_2+v_0+v_1+v_2\right)P_1\lambda g\left(e^{-\beta t_{1\pi}}-1\right)\end{aligned}$$

Then, Eq. (31) can be rearranged as follows:

$$w_0\left(t_{1\pi}\right)^2+w_1\left(t_{1\pi}\right)+w_2=0 \tag{32}$$

Finally, $t_\pi^*$ can be gained by applying the following square roots solution:

$$t_{1\pi}^{\ *}=\frac{-w_1\pm\sqrt{w_1^2-4w_0w_2}}{2w_0} \tag{33}$$

Since $F(t_{1\pi}) = (1 - e^{-\beta t1\pi})$ and its complement $e^{-\beta t1\pi}$ are both over the interval [0.1], and Eq. (33) can be rearranged as follows:

$$e^{-\beta t_{1\pi}}=\frac{-v_3t_{1\pi}P_1\left(y_1P_1-2\lambda g\right)+\left(u_0+u_1\right)P_1^2y_1-\left(u_2+v_0+v_1+v_2\right)P_1\lambda g}{\left\{\begin{array}{l}\left(hg+v_0+v_1+v_2\right)P_1^2y_1\beta t_{1\pi}^{\ 2}+\left[-u_3P_1^2y_1\beta-v_3t_{1\pi}P_1\lambda g\beta+\left(hg-u_2\right)P_1\lambda g\beta\right]t_{1\pi}\\-\left[\left(u_2+v_0+v_1+v_2\right)P_1\lambda g\right]-\left[\left(hg+v_0+v_1+v_2\right)P_1\lambda g\left(1-e^{-\beta t_{1\pi}}\right)\right]-2v_3t_{1\pi}P_1\lambda g\\-\left[\left(u_0+u_1\right)P_1\left(\lambda g\beta\right)\right]+\left[u_3P_1\left(-\lambda g\beta-y_1P_1\right)\right]\end{array}\right\}} \tag{34}$$

To solve $t_{1\pi}$*, first, let $e^{-\beta t1\pi} = 0$ and $e^{-\beta t1\pi} = 1$, then calculate Eq. (33) to gain the bounds for $t_{1\pi}$ (i.e., $t_{1\pi U}$ and $t_{1\pi L}$). Next step, use the current $t_{1\pi U}$ and $t_{1\pi L}$ to calculate the update values of $e^{-\beta t1\pi U}$ and $e^{-\beta t1\pi L}$. Re-apply Eq. (33) with the current $e^{-\beta t1\pi U}$ and $e^{-\beta t1\pi L}$ to find a set of update bounds $t_{1\pi U}$ and $t_{1\pi L}$. If ($t_{1\pi U} = t_{1\pi L}$) is true, then $t_{1\pi}$* is found (i.e., $t_{1\pi U} = t_{1\pi L} = t_{1\pi}$*), otherwise, repeat the aforementioned steps, until $t_{1\pi U} = t_{1\pi L}$.

## 4. Numerical example

This section offers a numerical illustration of the proposed hybrid batch fabrication problem in a vendor-buyer integrated environment featuring multi-delivery, rework, and machine failure. The parameters' values are assumed as follows (see Table 1):

**Table 1.** The assumed parameters' values in this illustration

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $K_\pi$ | $C_\pi$ | $\lambda$ | $\beta$ | $\beta_2$ | $K_1$ | $P_2$ | $C_R$ | C | $C_1$ | K | $h_2$ |
| 60 | 2.8 | 4000 | 1 | 0.4 | 90 | 5000 | 1.0 | 2.0 | 2.0 | 200 | 1.6 |
| x | $\pi$ | $P_1$ | M | $\beta_1$ | $C_T$ | g | $h_1$ | n | $h_3$ | h | |
| 20% | 0.4 | 10000 | 2500 | -0.70 | 0.01 | 0.018 | 0.4 | 3 | 0.4 | 0.4 | |

We first verify the convexity of $E[TCU(t_{1\pi})]$ by using the aforementioned values of parameters (i.e., make sure that $\tau(t_{1\pi}) > t_{1\pi} > 0$ (see Eq. (B-3)).

Because $e^{-\beta t1\pi}$ falls within the range of [0, 1], we first set $e^{-\beta t1\pi} = 0$ and $e^{-\beta t1\pi} = 1$, and apply Eq. (33) to find $t_{1\pi U} = 0.2780$ and $t_{1\pi L} = 0.0886$. Then, use the present values of $t_{1\pi U}$ and $t_{1\pi L}$ to calculate $e^{-\beta t1\pi U}$ and $e^{-\beta t1\pi L}$. Lastly, apply Eq. (B-3) with the obtained values of $e^{-\beta t1\pi L}$, $e^{-\beta t1\pi U}$, $t_{1\pi L}$, and $t_{1\pi U}$ to confirm that $\tau(t_{1\pi U}) = 0.5205 > t_{1\pi U} = 0.2780 > 0$ and $\tau(t_{1\pi L}) = 0.3073 > t_{1\pi L} = 0.0886 > 0$, respectively. Hence, we confirm the convexity of $E[TCU(t_{1\pi})]$ for $\beta = 1.0$, thus, the optimal $t_{1\pi}$* exists. Furthermore, a wider range of $\beta$ values have been used for the convexity test to show the boarder applicability of our proposed system, for details please refer to Table 2.

**Table 2**: Convexity test on $E[TCU(t_{1\pi})]$ with a wider range of $\beta$ values

| $\beta$ | $\tau(t_{1\pi U})$ | $t_{1\pi U}$ | $\tau(t_{1\pi L})$ | $t_{1\pi L}$ |
|---|---|---|---|---|
| 10 | 0.7323 | 0.2750 | 0.0465 | 0.0215 |
| 8 | 0.5757 | 0.2751 | 0.0571 | 0.0261 |
| 6 | 0.4749 | 0.2752 | 0.0740 | 0.0334 |
| 4 | 0.4201 | 0.2755 | 0.1053 | 0.0456 |
| 3 | 0.4126 | 0.2758 | 0.1336 | 0.0553 |
| 2 | 0.4290 | 0.2763 | 0.1834 | 0.0690 |
| **1** | **0.5205** | **0.2780** | **0.3073** | **0.0886** |
| 0.5 | 0.7162 | 0.2813 | 0.5175 | 0.1013 |
| 0.01 | 6.0021 | 0.5099 | 5.5977 | 0.1160 |

Now, to find $t_{1\pi}$*, again we first set $e^{-\beta t1\pi} = 0$ and $e^{-\beta t1\pi} = 1$, then apply Eq. (33) to gain the initial bounds for $t_{1\pi}$ (i.e., $t_{1\pi U} = 0.2780$ and $t_{1\pi L} = 0.0886$). Next, we use the current $t_{1\pi U}$ and $t_{1\pi L}$ to compute and update the values of $e^{-\beta t1\pi U}$ and $e^{-\beta t1\pi L}$. Then, we apply Eq. (33) repeatedly using the current $e^{-\beta t1\pi U}$ and $e^{-\beta t1\pi L}$ to obtain a set of update bounds $t_{1\pi U}$ and $t_{1\pi L}$. If ($t_{1\pi U} = t_{1\pi L}$) is true, then $t_{1\pi}$* is found (i.e., $t_{1\pi U} = t_{1\pi L} = t_{1\pi}$*), otherwise, repeat the aforementioned steps, until $t_{1\pi U} = t_{1\pi L}$. Iterative results for finding $t_{1\pi}$* are exhibited in Table 3. Hence, the optimal uptime for this example $t_{1\pi}* = 0.1181$ and $E[TCU(t_{1\pi}*)]$ = $12,295.06.

### *4.1. Effect of main system feature on the problem*

The effect of uptime $t_{1\pi}$ on expected total system costs $E[TCU(t_{1\pi})]$ is illustrated in Figure 5. It shows the initial bounds for $t_{1\pi}$ and the result of the final solution along with the convexity of $E[TCU(t_{1\pi})]$.

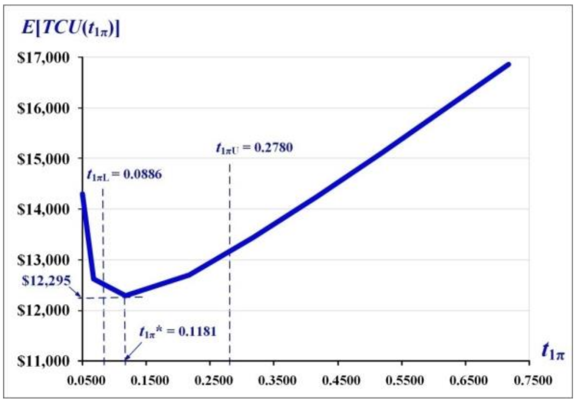


**Figure 5.** The effect of uptime $t_{1\pi}$ on expected total system costs $E[TCU(t_{1\pi})]$

**Table 3:** Iterative results for finding $t_{1\pi}$*

| Step # | $t_{1\pi U}$ | $e^{-\beta t1\pi U}$ | $t_{1\pi L}$ | $e^{-\beta t1\pi L}$ | $t_{1\pi U} - t_{1\pi L}$ | $E[TCU(t_{1\pi U})]$ | $E[TCU(t_{1\pi L})]$ |
|---|---|---|---|---|---|---|---|
| - | - | 0 | - | 1 | - | - | - |
| 1 | **0.2780** | 0.7573 | **0.0886** | 0.9152 | 0.1894 | $13,130.30 | $12,384.26 |
| 2 | 0.1480 | 0.8625 | 0.1115 | 0.8945 | 0.0365 | $12,349.70 | $12,298.62 |
| 3 | 0.1244 | 0.8830 | 0.1167 | 0.8899 | 0.0077 | $12,297.94 | $12,295.23 |
| 4 | 0.1195 | 0.8874 | 0.1178 | 0.8889 | 0.0017 | $12,295.20 | $12,295.07 |
| 5 | 0.1184 | 0.8883 | 0.1181 | 0.8886 | 0.0003 | $12,295.07 | $12,295.06 |
| 6 | 0.1182 | 0.8885 | 0.1181 | 0.8886 | 0.0001 | $12,295.06 | $12,295.06 |
| 7 | **0.1181** | 0.8886 | **0.1181** | 0.8886 | **0.0000** | **$12,295.06** | **$12,295.06** |

The influence of changes in rework-relevant ratio $C_R/C$ on $E[TCU(t_{1\pi}*)]$ is depicted in Figure 6. It indicates that as $C_R/C$ increases, $E[TCU(t_{1\pi}*)]$ rises accordingly; and it confirms that $E[TCU(t_{1\pi}*)]$ = \$12,295 when $C_R/C$ = 0.5 (as assumed in our example).

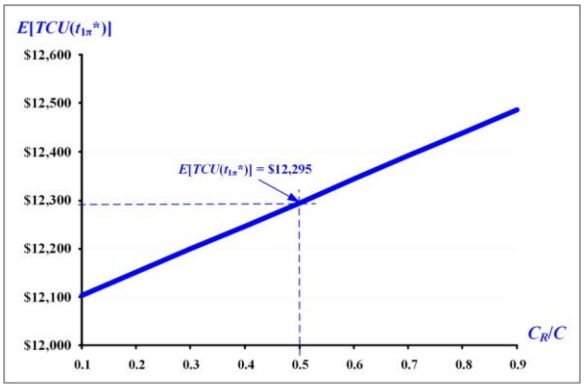


**Figure 6.** The influence of changes in rework relevant ratio $C_R/C$ on $E[TCU(t_{1\pi}*)]$

The influence of variations in the number of deliveries per cycle $n$ on relevant costs is illustrated in Figure 7. It specifies that as $n$ increases, the shipping cost rises drastically due to more frequent deliveries, and in-house stock holding goes up accordingly for the slow stock movement from the vendor to the buyer. When $n$ =1, it shows a significantly higher holding cost at the buyer end.

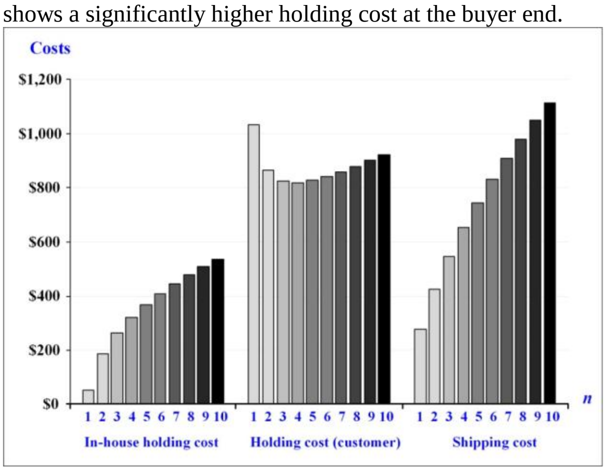


**Figure 7.** The impact of variations in the number of shipments per cycle $n$ on relevant costs

The influence of the number of deliveries $n$ on $E[TCU(t_{1\pi}*)]$ is displayed in Figure 8. It reconfirms at $n$ = 3, the solution of our example $t_{1\pi}*$ = 0.1181 and $E[TCU(t_{1\pi}*)]$ = \$12,295. Also, Figure 8 exposes that when $n$ = 2, we have the minimal $E[TCU(t_{1\pi}*)]$, and $E[TCU(t_{1\pi}*)]$ surges significantly, as $n$ rises.

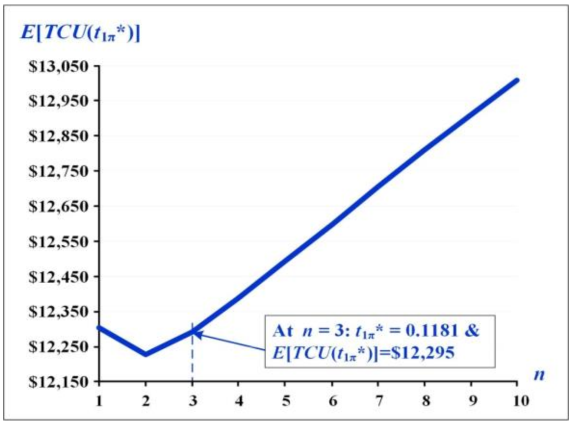


**Figure 8.** The influence on $E[TCU(t_{1\pi}*)]$ concerning $n$

Figure 9 exhibits the breakup of $E[TCU(t_{1\pi}*)]$. It shows that the setup and variable outsourcing costs add up to 37.2%. Total in-house fabrication relevant costs sum up to 51.3% (main contributors include variable cost 38.8% and breakdown-relevant cost 5.1%). The share of supply-chain integration-related costs is 11.5% (in which buyer holding cost contributes 6.7%).

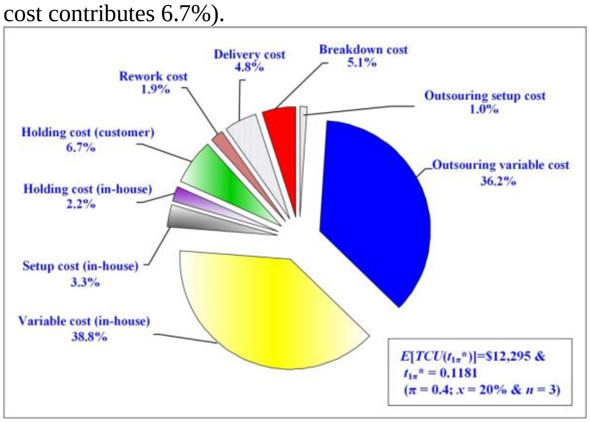


**Figure 9.** The breakup of $E[TCU(t_{1\pi}*)]$

The influences of variations in $\pi$ factor on utilization are shown in Figure 10. It indicates that when $\pi$ = 0.4 (as assumed in our example), utilization decreases from 47.68% to 28.68%, and in general, utilization declines significantly as $\pi$ increases.

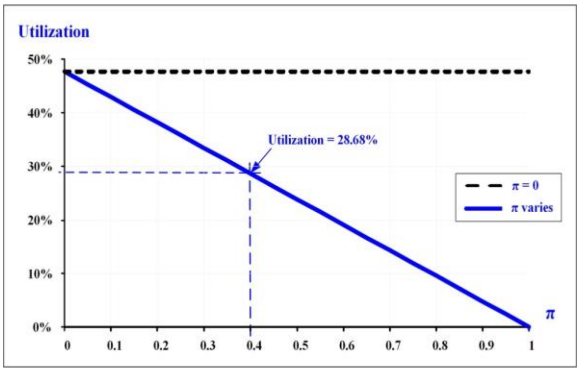


**Figure 10.** The effect of variations in outsourcing portion $\pi$ on utilization

Further analysis reveals that the critical $\pi$ value is 0.71, which can facilitate the "make-or-buy" decision making (see Figure 11). That is, once $\pi \geq 0.71$, the partial outsourcing policy will no longer be beneficial; the better decision is to use outside providers solely for meeting the product demand.

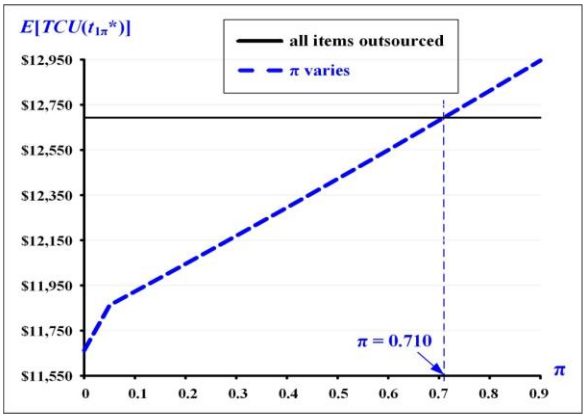


**Figure 11.** The critical $\pi$ value for make-or-buy decision making

The influence of differences in mean-time-to-failure $1/\beta$ on $E[TCU(t_{1\pi}*)]$ is exhibited in Figure 12. It confirms our optimal cost $E[TCU(t_{1\pi}*)]$ = \$12,295, and it also shows that $E[TCU(t_{1\pi}*)]$ declines significantly as $1/\beta$ increases (especially, when $1/\beta$ rises beyond 0.13). Furthermore, it specifies that as $1/\beta$ goes up to extremely large (for example, $1/\beta \geq 100$), $E[TCU(t_{1\pi}*)]$ = \$11,730, which is the same as the solution obtained from a problem without any failure occurrence.

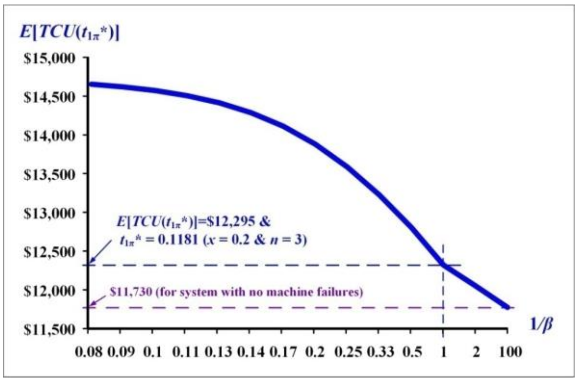


**Figure 12.** The influence of differences in $1/\beta$ on $E[TCU(t_{1\pi}*)]$

The effect of changes in the number of deliveries per cycle $n$ on the optimal decision variable $t_{1\pi}*$ is demonstrated in Figure 13. When $n = 3$, it confirms our result $t_{1\pi}* = 0.1811$ (years); and as $n$ increases, uptime $t_{1\pi}*$ rises significantly.

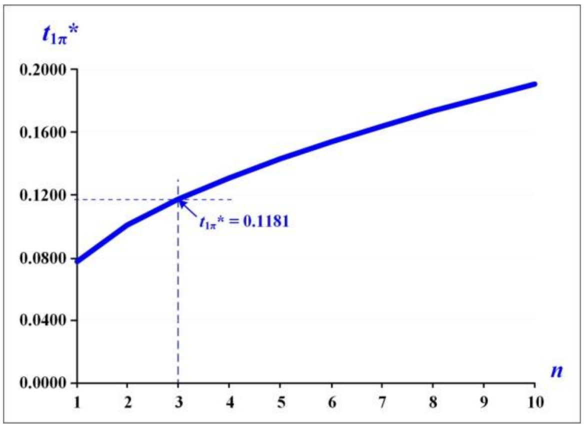


**Figure 13.** The influence of changes in $n$ on the delivery and stock-holding costs

*4.2. Combined effects of the main system feature on the problem*

Figure 14 exhibits analytical results of the combined effects of changes in rework relevant ratio $C_R/C$ and outsourcing factor $\pi$ on $E[TCU(t_{1\pi}*)]$. It reveals that when $\pi$ is small, as $C_R/C$ increases, $E[TCU(t_{1\pi}*)]$ rises considerably; and as $\pi$ increases, it seems to be irrelevant to $C_R/C$ ratios, $E[TCU(t_{1\pi}*)]$ upsurges drastically.

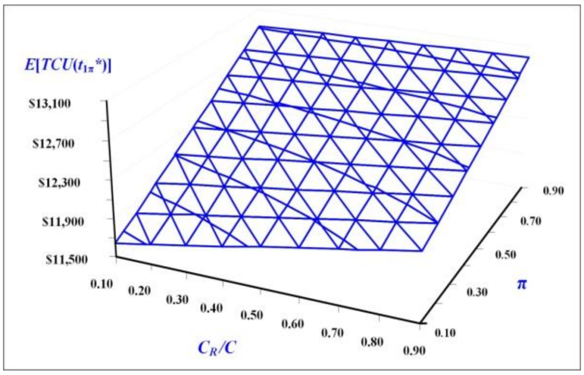


**Figure 14. C**ombined effects of changes in $C_R/C$ and $\pi$ on $E[TCU(t_{1\pi}*)]$

The joint influences of variations in random nonconforming rate $x$ and extra proportion $\beta_2$ of unit cost on the optimal uptime $t_{1\pi}*$ are illustrated in Figure 15. It reveals that $x$ has more influence on $t_{1\pi}*$ than $\beta_2$; for $t_{1\pi}*$ decreases significantly, as $x$ increases; and it is slightly changed, as $\beta_2$ increases.

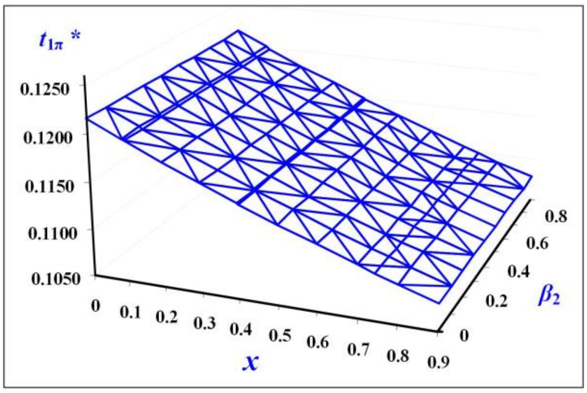


**Figure 15.** Joint influences of variations in $x$ and $\beta_2$ on $t_{1\pi}*$

*4.3. Discussion and limitation*

As this study assumed, the cases of only one or no machine failures take place in production uptime. The probabilities of various Poisson-distribution failure rates are exhibited in Table C-1 (see Appendix C). It indicates that the proposed model is appropriate for a good-condition machine (or with an average annual failure rate $\leq 1$). There is over 99.35% probability that the number of machine failures $\leq 1$ (refer to Table C-1). Further, for a fair-condition machine (or with an average annual failure rate $\leq 3$), our model is appropriate as there is over 94.11% chance of one or no failures occurring (see Table C-1). Our model's suitability falls below 94.11% as a machine having a mean failure rate higher than three per year. In such a case, the production planner needs to consider building a different model to explore the best fabrication policy for a system with such particular production equipment.

 

## 5. Conclusions

This study builds a mathematical model representing a vendor-buyer integrated system featuring machine failures, outsourcing, multiple deliveries, and rework. Through formulations and derivations, we determine the problem's cost function. The convexity of cost function and the optimal uptime are gained via differential calculus and a proposed specific algorithm. Lastly, we offer a numerical example to show our proposed model makes diverse crucial system information; such as the individual and joint impact of outsourcing, rework, random failures, as well as frequency of delivery on different features and the optimal uptime of the problem (see Figures 6 to 15), easily accessible, to assist enterprises in strategic planning, management, and decision making in their practical intra-supply-chain environments.

Managerial insights: upon completion of the proposed study, the production planners could apply this particular decision-support model to their hybrid batch fabrication vendor-buyer integrated systems to expose the following systems' characteristics: (1) the optimal fabrication runtime policy; (2) the total system expenses, relevant cost contributors, and machine utilization of their systems; (3) the individual/collective influence of their system's features on the optimal policy and other essential system performances. For future research, the incorporation of stochastic demand into the same context of the proposed model is an interesting direction.

## Acknowledgment

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**Appendix – A**

Derivations of Eq. (30) are provided as follows:

First, the integration outcomes of the numerator and denominator of Eq. (28) are given in Eqs. (A-1) and (A-2), respectively.

$$\left\{\int_0^{t_{1\pi}} E\left[TC\left(t_{1\pi}\right)_1\right]\cdot f(t)\,dt+\int_{t_{1\pi}}^{\infty} E\left[TC\left(t_{1\pi}\right)_2\right]\cdot f(t)\,dt\right\}$$

$$\begin{aligned}&=K_\pi+K+nK_1+t_{1\pi}\delta_1+t_{1\pi}^2\delta_2+M\left(1-e^{-\beta t_{1\pi}}\right)+C_T\lambda g\left(1-e^{-\beta t_{1\pi}}\right)+C_1\lambda g\left(1-e^{-\beta t_{1\pi}}\right)\\&+h\left(P_1 g\right)\left(-t_{1\pi}e^{-\beta t_{1\pi}}-\frac{1}{\beta}e^{-\beta t_{1\pi}}+\frac{1}{\beta}\right)+h_3\lambda g^2\left(1-e^{-\beta t_{1\pi}}\right)+\frac{1}{2}h_2\lambda g^2\left(1-e^{-\beta t_{1\pi}}\right)\\&+\frac{g}{2n}\left(h_2-h\right)\left(t_{1\pi}P_1\right)\left(y_1-y_2\right)\left(1-e^{-\beta t_{1\pi}}\right)+\frac{hg}{2}\left(t_{1\pi}P_1\right)\left(y_1-y_2\right)\left(1-e^{-\beta t_{1\pi}}\right)\\&+\frac{g}{2}\left[h_2+2h_3\right]\left(t_{1\pi}P_1\right)\left(y_1+y_2\right)\left(1-e^{-\beta t_{1\pi}}\right)\end{aligned}\tag{A-1}$$

$$E\left[\boldsymbol{T}_{\boldsymbol{\pi}}\right]=\frac{t_{1\pi}P_1y_1}{\lambda}+g\left[1-e^{-\beta t_{1\pi}}\right]\tag{A-2}$$

where

$$\delta_1=C_\pi\pi P_1y_1+CP_1+C_Ty_1P_1+C_RE[x]P_1\tag{A-3}$$

$$\begin{aligned}\delta_2&=\frac{E[x]^2P_1^2\left(h_1-h\right)}{2P_2}+\frac{P_1^2\left(h_2-h\right)y_1}{2n\lambda}\left(y_1-y_2\right)+\frac{h_2P_1^2y_1}{2}\left(\frac{y_2}{\lambda}\right)\\&+\frac{hP_1^2y_1}{2\lambda}\left[y_1+\frac{\lambda(-\pi)}{P_1}+\frac{\lambda E[x](1-2\pi)}{P_2}\right]\end{aligned}\tag{A-4}$$

With further derivation, one obtains $E[TCU(t_{1\pi})]$ as follows:

$$E\left[TCU\left(t_{1\pi}\right)\right]=\left[\frac{\lambda}{y_1+\dfrac{\lambda g\left[1-e^{-\beta t_{1\pi}}\right]}{t_{1\pi}P_1}}\right]\left\{\begin{array}{l}\dfrac{u_0}{t_{1\pi}}+\dfrac{u_1}{t_{1\pi}}+u_2+t_{1\pi}v_3-hge^{-\beta t_{1\pi}}+\dfrac{u_3e^{-\beta t_{1\pi}}}{t_{1\pi}}\\+\left(v_0+v_1+v_2\right)-\left(v_0+v_1+v_2\right)e^{-\beta t_{1\pi}}\end{array}\right\}\tag{30}$$

where

$$u_0=\frac{K_\pi}{P_1}+\frac{K}{P_1}+\frac{nK_1}{P_1}$$

$$u_1=\left[\frac{M}{P_1}+\frac{C_T\lambda g}{P_1}+\frac{C_1\lambda g}{P_1}+\frac{h_3\lambda g^2}{P_1}+\frac{1}{2}\frac{h_2\lambda g^2}{P_1}+\frac{hg}{\beta}\right]$$

$$u_2=\left[C_\pi\pi y_1+C+C_Ty_1+C_RE[x]\right]$$

$$u_3=\left[-\frac{M}{P_1}-\frac{C_T\lambda g}{P_1}-\frac{C_1\lambda g}{P_1}-\frac{h_3\lambda g^2}{P_1}-\frac{1}{2}\frac{h_2\lambda g^2}{P_1}-\frac{hg}{\beta}\right]$$

$$v_0=\frac{hg}{2}\left(y_1-y_2\right)$$

$$v_1=\frac{g}{2n}\left(h_2-h\right)\left(y_1-y_2\right)$$

$$v_2=\frac{g}{2}\left(h_2+2h_3\right)\left(y_1+y_2\right)$$

$$v_3=\left[\begin{array}{l}\dfrac{E[x]^2P_1\left(h_1-h\right)}{2P_2}+\dfrac{P_1\left(h_2-h\right)y_1}{2n\lambda}\left(y_1-y_2\right)+\left[\dfrac{h_2P_1y_2}{2\lambda(1-\pi)}\right]\\+\dfrac{hP_1y_1}{2\lambda}\left[y_1+\dfrac{\lambda(-\pi)}{P_1}+\dfrac{E[x]\lambda}{P_2}\left[1-2\pi\right]\right]\end{array}\right]$$

**Appendix – B**

The first- and second-derivatives of $E[TCU(t_{1\pi})]$ are shown in equations (B-1) and (B-2) below:

$$\frac{dE\left[TCU\left(t_{1\pi}\right)\right]}{d\left(t_{1\pi}\right)}=\frac{\lambda}{\left[y_1 t_{1\pi} P_1+\lambda g\left(1-e^{-\beta t_{1\pi}}\right)\right]^2}\left\{\begin{array}{l}
-\left(u_0+u_1\right)P_1\left(y_1 P_1+\lambda g\beta e^{-\beta t_{1\pi}}\right) \\
+u_3 P_1\left(-y_1 t_{1\pi} P_1\beta e^{-\beta t_{1\pi}}-\lambda g\beta e^{-\beta t_{1\pi}}-y_1 P_1 e^{-\beta t_{1\pi}}\right) \\
+v_3 t_{1\pi} P_1\left(y_1 t_{1\pi} P_1+2\lambda g-2\lambda g e^{-\beta t_{1\pi}}-t_{1\pi}\lambda g\beta e^{-\beta t_{1\pi}}\right) \\
-\left(hg+v_0+v_1+v_2\right)P_1\left(\begin{array}{l}-y_1 {t_{1\pi}}^2 P_1\beta e^{-\beta t_{1\pi}}-\lambda g e^{-2\beta t_{1\pi}} \\ -t_{1\pi}\lambda g\beta e^{-\beta t_{1\pi}}+\lambda g e^{-\beta t_{1\pi}}\end{array}\right) \\
-\left(u_2+v_0+v_1+v_2\right)P_1\lambda g\left(t_{1\pi}\beta e^{-\beta t_{1\pi}}+e^{-\beta t_{1\pi}}-1\right)
\end{array}\right\} \qquad \text{(B-1)}$$

and

$$\frac{d^2E\left[TCU\left(t_{1\pi}\right)\right]}{d\left(t_{1\pi}\right)^2}=\frac{\lambda}{\left(y_1 t_{1\pi} P_1+\lambda g\left(1-e^{-\beta t_{1\pi}}\right)\right)^3}\cdot$$

$$\left\{\begin{array}{l}
\left(u_0+u_1\right)P_1\left(2y_1^2P_1^2+y_1 t_{1\pi} P_1\lambda g\beta^2 e^{-\beta t_{1\pi}}+4y_1P_1\lambda g\beta e^{-\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-2\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-\beta t_{1\pi}}\right) \\
+u_3P_1e^{-\beta t_{1\pi}}\left(\begin{array}{l}y_1^2{t_{1\pi}}^2P_1^2\beta^2+2y_1^2P_1^2+2y_1^2t_{1\pi}P_1^2\beta+2y_1t_{1\pi}P_1\lambda g\beta^2+2y_1P_1\lambda g\beta \\ +y_1t_{1\pi}P_1\lambda g\beta^2e^{-\beta t_{1\pi}}+2y_1P_1\lambda g\beta e^{-\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-\beta t_{1\pi}}+\lambda^2g^2\beta^2\end{array}\right) \\
+v_3P_1\lambda g\left(\begin{array}{l}y_1{t_{1\pi}}^3P_1\beta^2e^{-\beta t_{1\pi}}+2\lambda g e^{-2\beta t_{1\pi}}+{t_{1\pi}}^2\lambda g\beta^2e^{-2\beta t_{1\pi}}-4\lambda g e^{-\beta t_{1\pi}} \\ +4t_{1\pi}\lambda g\beta e^{-2\beta t_{1\pi}}+{t_{1\pi}}^2\lambda g\beta^2e^{-\beta t_{1\pi}}-4t_{1\pi}\lambda g\beta e^{-\beta t_{1\pi}}+2\lambda g\end{array}\right) \\
-\left(hg+v_0+v_1+v_2\right)P_1e^{-2\beta t_{1\pi}}\left(\begin{array}{l}y_1^2{t_{1\pi}}^3P_1^2\beta^2e^{\beta t_{1\pi}}+2y_1{t_{1\pi}}^2P_1\lambda g\beta^2e^{\beta t_{1\pi}}-2y_1t_{1\pi}P_1\lambda g\beta e^{\beta t_{1\pi}} \\ +y_1{t_{1\pi}}^2P_1\lambda g\beta^2+2\lambda^2g^2\beta+4y_1t_{1\pi}P_1\lambda g\beta+t_{1\pi}\lambda^2g^2\beta^2e^{\beta t_{1\pi}} \\ +2y_1P_1\lambda g-2y_1P_1\lambda g e^{\beta t_{1\pi}}-2\lambda^2g^2\beta e^{\beta t_{1\pi}}+t_{1\pi}\lambda^2g^2\beta^2\end{array}\right) \\
+\left(u_2+v_0+v_1+v_2\right)P_1\lambda g\left(\begin{array}{l}t_{1\pi}\lambda g\beta^2e^{-2\beta t_{1\pi}}+t_{1\pi}\lambda g\beta^2e^{-\beta t_{1\pi}}+2\lambda g\beta e^{-2\beta t_{1\pi}}-2\lambda g\beta e^{-\beta t_{1\pi}} \\ +y_1{t_{1\pi}}^2P_1\beta^2e^{-\beta t_{1\pi}}+2y_1P_1e^{-\beta t_{1\pi}}+2y_1t_{1\pi}P_1\beta e^{-\beta t_{1\pi}}-2y_1P_1\end{array}\right)
\end{array}\right\} \qquad \text{(B-2)}$$

Since the first term on the right-hand side (RHS) of Eq. (B-2) is positive, it follows that the $E[TCU(t_{1\pi})]$ is convex if the second term on the RHS of Eq. (B-2) is also positive. That means if the following $\tau(t_{1\pi}) > t_{1\pi} > 0$ holds.

$$\tau\left(t_{1\pi}\right)=\frac{\begin{array}{l}
\left(u_0+u_1\right)\left(2y_1^2P_1^2+4y_1P_1\lambda g\beta e^{-\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-2\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-\beta t_{1\pi}}\right) \\
+u_3e^{-\beta t_{1\pi}}\left(\begin{array}{l}2y_1^2P_1^2+2y_1P_1\lambda g\beta+\lambda^2g^2\beta^2 \\ +2y_1P_1\lambda g\beta e^{-\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{-\beta t_{1\pi}}\end{array}\right)+v_3\lambda g\left(2\lambda g e^{-2\beta t_{1\pi}}-4\lambda g e^{-\beta t_{1\pi}}+2\lambda g\right) \\
-\left(hg+v_0+v_1+v_2\right)e^{-2\beta t_{1\pi}}\left(2y_1P_1\lambda g-2y_1P_1\lambda g e^{\beta t_{1\pi}}-2\lambda^2g^2\beta e^{\beta t_{1\pi}}+2\lambda^2g^2\beta\right) \\
+\left(u_2+v_0+v_1+v_2\right)\lambda g\left(2\lambda g\beta e^{-2\beta t_{1\pi}}-2\lambda g\beta e^{-\beta t_{1\pi}}+2y_1P_1e^{-\beta t_{1\pi}}-2y_1P_1\right)
\end{array}}{-\left\{\begin{array}{l}
\left(u_0+u_1\right)\left(y_1P_1\lambda g\beta^2e^{-\beta t_{1\pi}}\right)+u_3e^{-\beta t_{1\pi}}y_1\left(y_1t_{1\pi}P_1^2\beta^2+2y_1P_1^2\beta+2P_1\lambda g\beta^2+P_1\lambda g\beta^2e^{-\beta t_{1\pi}}\right) \\
+v_3\lambda g\left(y_1{t_{1\pi}}^2P_1\beta^2e^{-\beta t_{1\pi}}+t_{1\pi}\lambda g\beta^2e^{-2\beta t_{1\pi}}+4\lambda g\beta e^{-2\beta t_{1\pi}}+t_{1\pi}\lambda g\beta^2e^{-\beta t_{1\pi}}-4\lambda g\beta e^{-\beta t_{1\pi}}\right) \\
-\left(hg+v_0+v_1+v_2\right)e^{-2\beta t_{1\pi}}\left[\begin{array}{l}y_1^2{t_{1\pi}}^2P_1^2\beta^2e^{\beta t_{1\pi}}+4y_1P_1\lambda g\beta+y_1t_{1\pi}P_1\lambda g\beta^2+\lambda^2g^2\beta^2 \\ +2y_1t_{1\pi}P_1\lambda g\beta^2e^{\beta t_{1\pi}}-2y_1P_1\lambda g\beta e^{\beta t_{1\pi}}+\lambda^2g^2\beta^2e^{\beta t_{1\pi}}\end{array}\right] \\
+\left(u_2+v_0+v_1+v_2\right)\lambda g\left(\lambda g\beta^2e^{-2\beta t_{1\pi}}+\lambda g\beta^2e^{-\beta t_{1\pi}}+y_1t_{1\pi}P_1\beta^2e^{-\beta t_{1\pi}}+2y_1P_1\beta e^{-\beta t_{1\pi}}\right)
\end{array}\right\}}>t_{1\pi}>0 \qquad \text{(B-3)}$$

**Appendix C**

**Table C-1:** Probability of various Poisson-distribution failure rates

| $\beta$ | $t_{1\pi}$ * | $P(y=0)$ | $P(y=1)$ | $P(y \le 1)$ | $P(y>1)$ |
|---|---|---|---|---|---|
| 6.0 | 0.1736 | 35.30% | 36.76% | 72.06% | 27.94% |
| 5.0 | 0.1560 | 45.84% | 35.76% | 81.60% | 18.40% |
| 4.0 | 0.1412 | 56.85% | 32.11% | 88.96% | 11.04% |
| 3.0 | 0.1300 | 67.71% | 26.40% | 94.11% | 5.89% |
| 2.0 | 0.1224 | 78.29% | 19.16% | 97.45% | 2.55% |
| 1.0 | 0.1181 | 88.86% | 10.50% | 99.35% | 0.65% |
| 0.01 | 0.1172 | 99.88% | 0.12% | 100.00% | 0.00% |

$$\frac{e^{-\beta t_{1\pi}^{*}}\left(\beta t_{1\pi}^{*}\right)^{y}}{y!} \tag{C-1}$$

 

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