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Production and Distribution Decisions for a Multi-product System with Component Commonality, Postponement Strategy and Quality Assurance Using a Two-machine Scheme

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Abstract

Coping with present-day's more demanding customer requests on variety, fast delivery, and quality, a growing number of corporations constantly redesign their production scheme and reorganize their supply chains. Motivated by the benefits gained from postponement policy in multi-item manufacturing systems and response to customer's actual needs, this study adopts a two-machine fabrication scheme to explore the optimal fabrication-delivery policy for a two-stage multi-item system with common part, postponement policy and product quality reassurance (including product screening, scrap and rework). In the first stage, machine one is utilized to fabricate common parts that shared by all finished products. Then, in stage two, a separate machine produces the finished items under a rotation cycle time order. With the help of mathematical modeling, optimization methods, and a numerical illustration, our proposed fabrication system is capable of not only deciding the best fabrication-delivery policy, but also demonstrating its beneficial choice in cost saving and cycle length reduction in comparison with the results from a system using single-machine scheme.

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1. Introduction

Postponing product differentiation is an effective strategy which is often assessed by production managers when planning production of multiproduct that have a part in common, for it can reduce production cycle time and lower overall fabrication relevant costs. Zinn [1] presented four different heuristics to help identify the potentials of postponement in order to assess savings of safety stock from postponement. He provided a real example to applying heuristics to support his findings. Lee and Tang [2] developed a model to capture expenses and benefits relating to redesign of manufacturing strategy. Their model was applied to analyze some special cases of real examples. As a result, they identified and formalized three different ways of product/process redesigns. They are; the standardization, modular design, and process reforming, for delaying differentiation from these real examples. Some special theoretical cases were also studied to characterize the optimal point of product differentiation and derive managerial insights. Swaminathan and Tayur [3] observed a few leading producers in computer industry who adopted the delayed differentiation strategy in managing their product lines to lower cost while keeping customer service. Although this strategy is a challenging assignment, it is used to cope with stochastic demands by storing the half-way finished products to serve the fabrication needs for multiple end products. Accordingly, they modeled the problem as a two-stage integer program and employed the structural decomposition along with sub-gradient derivative methods in their solution process. A computational section is provided to express applicability of their solution procedure and offer insights on system characteristics, performances, and merits. Van Hoek et al. [4] provided a detailed investigation of the experiences from firms that handle process changes relating to adoption of postponement policy. They evaluated the benefits from implementing postponement strategy in each business environment, recognized the relevant managerial characteristics and potential bottlenecks, and suggested on how to successfully carry out the postponement strategy. Yang and Burns [5] explored the issues of decoupling spot, controlling, integrating, and planning capacity of the supply chain system from the viewpoint of postponement. Their objective was to expand the significance of postponement to the real-life supply-chain systems. Kumar and Wilson [6] investigated and explored the connection among inventory, delayed differentiation, and off-shoring. They

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pointed out the risks and reasons of off-shoring, the costs/benefits of delayed differentiation, and the relations and impacts of delayed differentiation and off-shoring on stock status. Then, the inventory impact of scenarios including diverse levels of delayed differentiation for an off-shore fabricated product is investigated using a set of real-world data. As a result, the best scenario is determined. Sensitivity analysis on the key variables was conducted and joint uncertainty owing to demand and cycle were identified as the significant input to every scenario. For any given strategy, they presented a fast approach to identify the dominated point within the uncertainty formula that affects stock status. Saghiri and Hill [7] explored the effect of supplier relationship on adopting delayed differentiation for a buying firm. Three separate postponement policies were proposed with 219 empirical data from manufacturing firms to test for the hypothetical connections between supplier relationships. The results indicated that buying firm’s ability on implementing postponement in product design phase is positively related to supplier’s level of commitment, anticipation of a long-lasting relationship, and joint actions with buyer. But another finding showed that buying firm's ability on implementing postponement in procurement phase is positively related to only the coordinated actions of supplier and buyer. Their findings provided greater insight into how different aspects of supplier relationship practically impact different types of postponement. Chiu et al. [8] derived optimal fabrication-distribution policy for a multiproduct system with quality reassurance and product differentiation. A single-machine two-stage manufacturing scheme is implemented. Consequently, they not only decided a closed-form optimal fabrication-distribution policy, but also showed a significant system cost savings and notable reduction in cycle time in comparison with the result from a single-stage scheme. Additional works [9-17] that also investigated diverse features of postponement issues in manufacturing systems.

In real-life production processes, owing due to diverse unpredictable factors fabrication of nonconforming items is inevitable. Product’s quality assurance has always been a challenging and critical task for production managers. Studies related to product’s quality assurance matters including inspection, scrap, and rework issues have been broadly performed in past years [18-42]. To cope with present-day’s more demanding customer needs in terms of variety, fast delivery, and quality, an increasing number of corporations constantly redesign their production scheme and restructure their supply chains. Motivated by the possible advantages of postponement policy in multiproduct manufacturing systems and response to actual customer’s needs, this study uses a dual-machine scheme to reconsider Chiu et al.’s problem [8]. The main difference between the present study and prior work [8] is that two separate machines are used in our proposed scheme, wherein in the first production stage machine one fabricates all common parts, and in the second stage machine two makes finished products under a rotation cycle time discipline, with the intention of further reducing cycle time. Past literature showed that combined impacts of postponement, quality reassurance, and a two-machine scheme to the multiproduct fabrication-distribution problem have not been explicitly explored. We aim to fill the gap.

2. Description, Modeling, and Formulation

Suppose that demands λ_i per year for L products must be met (where i = 1, 2, …, L) and these products share a common part which are produced in advance by a machine (i.e., in stage 1, see Figure 1). Then, right after that, in stage 2 a second machine manufactures L customized end products in sequence (see Figure 2) under a rotation cycle discipline. Such a two-machine scheme aims to reduce manufacturing cycle length and cut down total fabrication relevant cost. In stage 1, the manufacturing rate for common parts is $P_{1,0}$ per year, and the fabricating rate for finished items in stage 2 by a separate machine is $P_{1,i}$ per year.

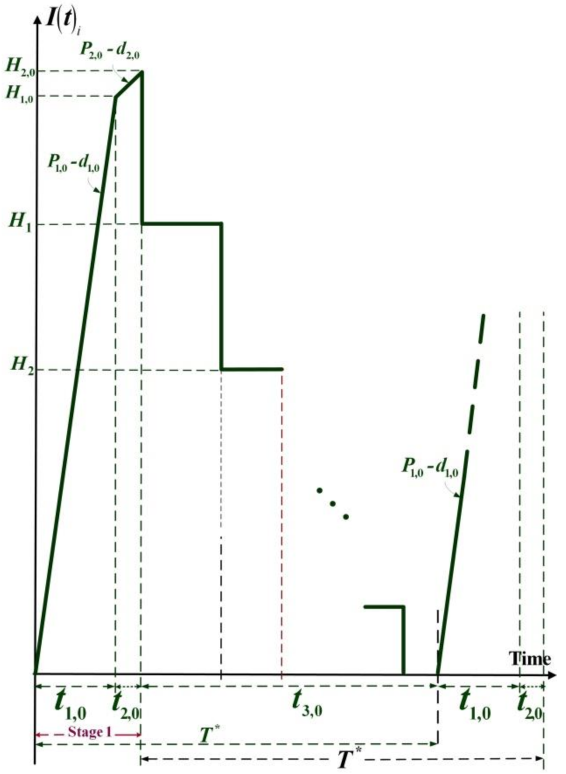

Figure 1. Level of on-hand common parts in stage 1 in the proposed model with a two-machine manufacturing scheme

All manufactured items are screened in both stages, cost of screening is a part of unit manufacturing cost C_i. Assuming x_i proportion of defective products may be manufactured randomly in both stages of the fabrication processes, at a rate of $d_{1,i}$ per year; so $d_{1,i} = P_{1,i}\, x_i$ (where i = 0, 1, 2, …, L; and i = 0 represents fabrication of common part in stage 1).

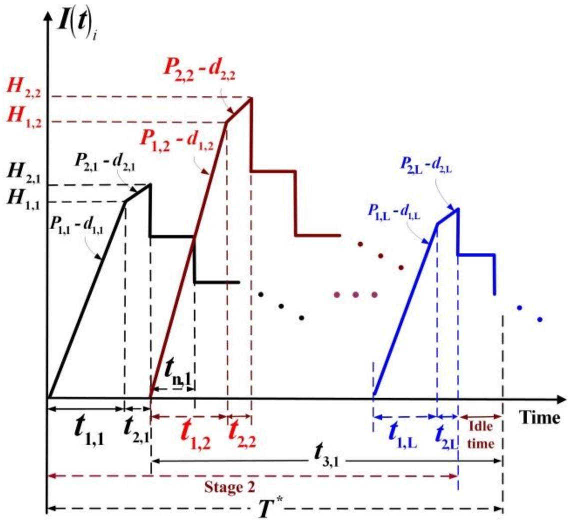

Figure 2. Level of on-hand finished items in stage 2 (machine 2) in proposed model using a two-machine production scheme

A portion $\theta_{1,i}$ of the nonconforming items are scraps (where $0 <= \theta_{1,i} <= 1$) and the rest are rework-able. The rework process immediately follows the regular fabrication, at $P_{2,i}$ items per year. During the rework, a $\theta_{2,i}$ portion fails (where $0 <= \theta_{2,i} <= 1$) that need to be scrapped (see Figures 3 and 4). Without permission of stock out, we further assume that $P_{1,i} - d_{1,i} - \lambda_i > 0$. In stage 2, when each rework process ends, fixed-quantity n installments of each finished batch are shipped to buyers at fixed time interval in $t_{3,i}$.

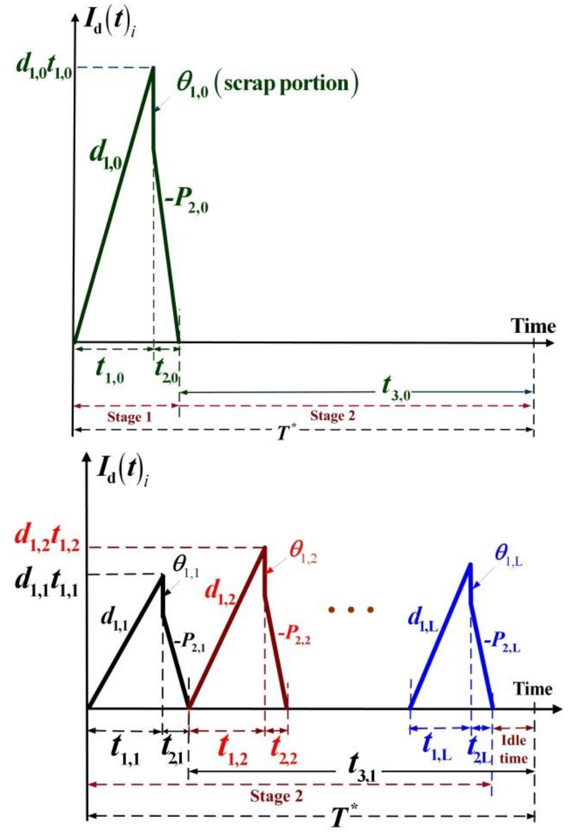

Figure 3. Inventory levels of on-hand defective common parts in stage 1 (left-hand side) and defective finished items in stage 2 (right-hand side) in a cycle

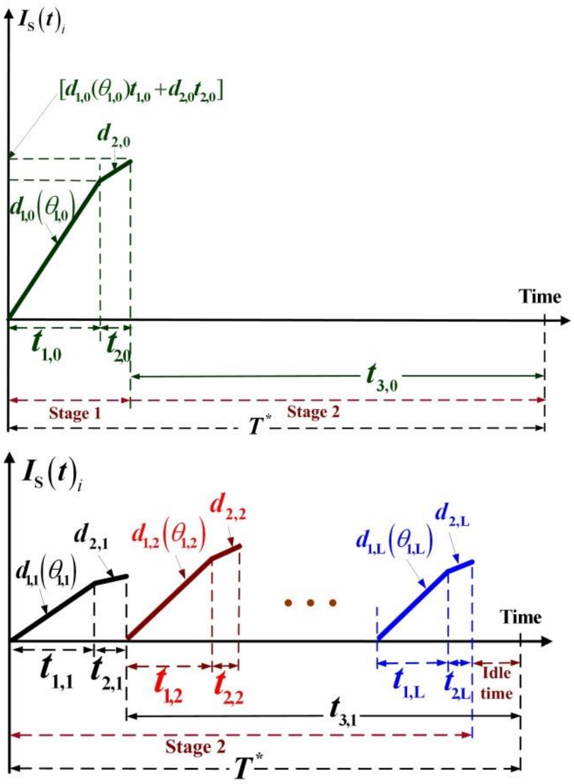

Figure 4. Inventory levels of scrapped common parts in stage 1 (left-hand side) and scrapped finished items in stage 2 (right-hand side) in a cycle

Inventory level of on-hand common parts awaiting the second stage's fabrication is exhibited in Figure 5. Nomenclature is exhibited in Appendix A.

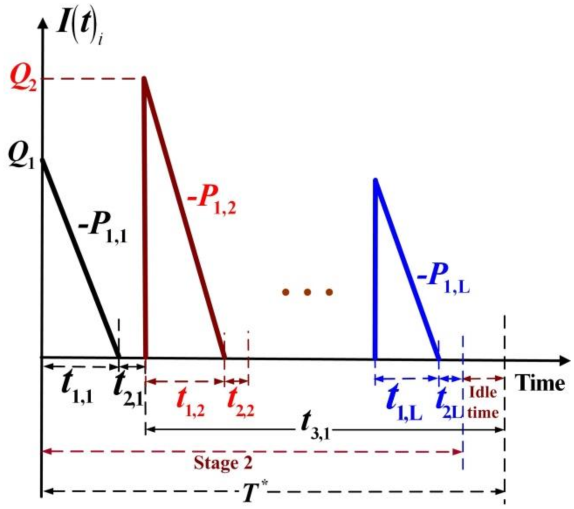

Figure 5. Inventory level of on-hand common parts awaiting the second stage's fabrication

2.1. Formulations and Mathematical Modeling

Since the proposed two-stage multi-product EPQ system using a two-machine production scheme, it releases the workload of fabricating the common intermediate parts from machine two. Therefore, an efficient end-item fabrication is expected stage two. The proposed solution procedure begins with deciding optimal rotation cycle

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length for stage 2. Then, uses the rotation cycle time for fabrication of common parts in stage one.

In order to satisfy demands, enough capacity must be ensured in stage 2 for fabricating L distinct products under the rotation cycle discipline. So, the prerequisite formulas (1) and (2) must hold:

$$\sum_{i=1}^{L}\left(t_{1,i}+t_{2,i}\right)<T \quad \text{or} \quad \sum_{i=1}^{L} Q_i\left[\frac{1}{P_{1,i}}+\frac{E[x_i]\left(1-\theta_{1,i}\right)}{P_{2,i}}\right]<T \tag{1}$$

or

$$\sum_{i=1}^{L}\left(\frac{\lambda_i}{\left[1-\varphi_i E[x_i]\right]}\right)\left[\frac{1}{P_{1,i}}+\frac{E[x_i]\left(1-\theta_{1,i}\right)}{P_{2,i}}\right]<1 \tag{2}$$

To meet product demands λ_i and by observing Figures 2 to 5, we obtain formulas for stage two as follows (for i = 1, 2, …, L):

$$Q_i=\frac{\lambda_i T}{1-\varphi_i E[x_i]} \tag{3}$$

$$T=t_{1,i}+t_{2,i}+t_{3,i}=\frac{Q_i\left(1-\varphi_i E[x_i]\right)}{\lambda_i} \tag{4}$$

$$\varphi_i=\theta_{1,i}+\theta_{2,i}(1-\theta_{1,i}) \tag{5}$$

$$Q_i=P_{1,i}\left(t_{1,i}\right) \tag{6}$$

$$H_{1,i}=\left(P_{1,i}-d_{1,i}\right)t_{1,i} \tag{7}$$

$$H_{2,i}=H_{1,i}+\left(P_{2,i}-d_{2,i}\right)t_{2,i} \tag{8}$$

$$t_{1,i}=\frac{Q_i}{P_{1,i}}=\frac{H_{1,i}}{P_{1,i}-d_{1,i}} \tag{9}$$

$$t_{2,i}=\frac{x_i Q_i\left(1-\theta_{1,i}\right)}{P_{2,i}}=\frac{d_{1,i}t_{1,i}\left(1-\theta_{1,i}\right)}{P_{2,i}}=\frac{H_{2,i}-H_{1,i}}{P_{2,i}-d_{2,i}} \tag{10}$$

$$t_{3,i}=nt_{n,i} \tag{11}$$

$$d_{1,i}\cdot t_{1,i}=P_{1,i}x_i\cdot t_{1,i} \tag{12}$$

At buyer's side, the inventory level in any given cycle is illustrated in Figure 6. From where, the following equations can be observed:

$$D_i=\frac{H_{2,i}}{n} \tag{13}$$

$$I_i=D_i-\lambda_i t_{n,i} \tag{14}$$

$$nI_i=\lambda_i\left(t_{1,i}+t_{2,i}\right) \tag{15}$$

From Figure 3, total holding costs during rework time $t_{2,i}$ are as follows:

$$\sum_{i=1}^{L}\left[h_{2,i}\left(\frac{P_{2,i}t_{2,i}}{2}\right)\left(t_{2,i}\right)\right] \tag{16}$$

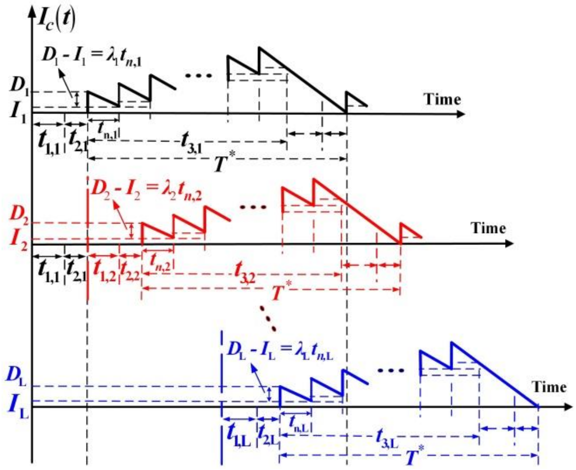

Figure 6. Inventory level of stocks at the buyer's in any given cycle

From Figure 5, total holding costs for *common parts* awaiting next stage of fabrication are as follows:

$$\sum_{i=1}^{L}\left\{h_{1,i}\left[\frac{Q_i}{2}\left(t_{1,i}\right)\right]\right\} \tag{17}$$

In delivering time t3,i (stage 2), total holding costs are as follows:

$$\sum_{i=1}^{L}\left\{h_{1,i}\left(\frac{n-1}{2n}\right)H_{2,i}t_{3,i}\right\} \tag{18}$$

In any given cycle, the fixed and variable shipping costs are

$$\sum_{i=1}^{L}\left\{nK_{1,i}+C_{T,i}\left[Q_i\left(1-\varphi_i x_i\right)\right]\right\} \tag{19}$$

At buyer's side (Figure 6), total holding costs in a given cycle are as follows [8]

$$\sum_{i=1}^{L}\left\{h_{3,i}\left[\frac{n\left(D_i-I_i\right)t_{n,i}}{2}+\frac{n(n+1)}{2}I_i t_{n,i}+\frac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right]\right\} \tag{20}$$

Therefore, overall system costs in a cycle for stage two, $TC_2(T, n)$ includes summation of setup, variable manufacturing, disposal, and rework costs, producer's inventory and safety stock holding costs, fixed and variable shipping costs, and buyer's holding costs. Hence, $TC_2(T, n)$ is

$$TC_2(T,n)=\sum_{i=1}^{L}\left\{\begin{array}{l} C_i Q_i+K_i+C_{R,i}\left[x_i\left(1-\theta_{1,i}\right)Q_i\right]+C_{S,i}\left[x_i\varphi_i Q_i\right]+nK_{1,i}+C_{T,i}\left[Q_i\left(1-\varphi_i x_i\right)\right] \\ +h_{1,i}\left[\frac{Q_i}{2}\left(t_{1,i}\right)+\frac{H_{1,i}t_{1,i}}{2}+\frac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right)+\left(\frac{n-1}{2n}\right)H_{2,i}t_{3,i}+\frac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)\right] \\ +h_{2,i}\left(\frac{P_{2,i}t_{2,i}}{2}\right)\left(t_{2,i}\right)+h_{3,i}\left[\frac{n\left(D_i-I_i\right)t_{n,i}}{2}+\frac{n(n+1)}{2}I_i t_{n,i}+\frac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right] \\ +h_{4,i}\left(x_i Q_i\right)T \end{array}\right\} \tag{21}$$

By substituting Eqs. (1) to (15) in Eq. (21), and using the expected values of x_i to cope with randomness of x_i, and after additional derivations, E[$TCU_2(T, n)$] becomes

$$E\left[TCU_2(T,\ n)\right]=E\left[TC_2(T,\ n)\right]/E[T]$$

$$=\sum_{i=1}^{L}\left\{\begin{array}{l}\left[C_i\lambda_i\pi_{0,i}+\dfrac{K_i}{T}+C_{R,i}\lambda_i\left(1-\theta_{1,i}\right)\pi_{1,i}+C_{S,i}\lambda_i\varphi_i\pi_{1,i}+\dfrac{nK_{1,i}}{T}+C_{T,i}\lambda_i\right]+\dfrac{h_{1,i}T\lambda_i^2}{2}\left[\pi_{3,i}+\pi_{4,i}-\dfrac{\pi_{5,i}}{n}\right]\\+\dfrac{h_{2,i}T\lambda_i^2\pi_{1,i}^2}{2}\left[\dfrac{\left(1-\theta_{1,i}\right)^2}{P_{2,i}}\right]+\dfrac{h_{3,i}T\lambda_i^2}{2}\left[\dfrac{2\pi_{0,i}}{P_{1,i}}+\dfrac{2\left(1-\theta_{1,i}\right)\pi_{1,i}}{P_{2,i}}-\dfrac{1}{\lambda_i}+\left(1+\dfrac{1}{n}\right)\pi_{5,i}\right]+Th_{4,i}\lambda_i\pi_{1,i}\end{array}\right\} \quad (22)$$

where

$$\pi_{0,i}=\frac{1}{\left(1-\varphi_iE[x_i]\right)};\ \pi_{1,i}=\frac{E[x_i]}{\left(1-\varphi_iE[x_i]\right)};\ \text{and}\ \pi_{2,i}=\left(1-\theta_{1,i}\right)\left(1-\theta_{2,i}\right)\quad for\ i=1,\ 2,\ \cdots,L;$$

$$\pi_{3,i}=\left[\frac{\pi_{0,i}^2}{P_{1,i}}+\frac{\left(1-E\left[x_i\right]\right)\left(1-\theta_{1i}\right)\pi_{0,i}\pi_{1,i}}{P_{2,i}}\right]\quad for\ i=1,\ 2,\ \cdots,L;$$

$$\pi_{4,i}=\left[\frac{1-E\left[x_i\right]}{\lambda_i}\pi_{0,i}+\frac{\pi_{2,i}\pi_{1,i}}{\lambda_i}+\frac{\pi_{0,i}\pi_{1,i}}{P_{1,i}}\left[1-\pi_{2,i}\right]\right]\quad for\ i=1,\ 2,\ \cdots,L; \quad (23)$$

$$\pi_{5,i}=\left[\pi_{4,i}-\pi_{3,i}-\frac{\left(1-\theta_{1,i}\right)\pi_{2,i}\pi_{1,i}^2}{P_{2,i}}\right]\quad for\ i=1,\ 2,\ \cdots,L$$

On the other hand, in stage one, in order to supply in time enough common parts (see Figure 1) to satisfy the needs of stage two's fabrications, machine 1 must fabricate common parts ($t_{1,0}$ + $t_{2,0}$) early. By observing Figures 1, 3, 4, and 5, we obtain the following formulations directly:

$$\sum_{i=1}^{L}Q_i=\lambda_0T \quad (24)$$

$$Q_0=\frac{\sum_{i=1}^{L}Q_i}{1-\varphi_0E\left[x_0\right]} \quad (25)$$

$$T=t_{1,0}+t_{2,0}+t_{3,0}=\frac{Q_0\left(1-\varphi_0E\left[x_0\right]\right)}{\lambda_0} \quad (26)$$

$$\varphi_0=\theta_{1,0}+\theta_{2,0}(1-\theta_{1,0}) \quad (27)$$

$$H_{1,0}=t_{1,0}\left(P_{1,0}-d_{1,0}\right) \quad (28)$$

$$t_{1,0}=\frac{Q_0}{P_{1,0}}=\frac{H_{1,0}}{P_{1,0}-d_{1,0}} \quad (29)$$

$$H_{2,0}=H_{1,0}+\left(P_{2,0}-d_{2,0}\right)t_{2,0} \quad (30)$$

$$t_{2,0}=\frac{x_0Q_0\left(1-\theta_{1,0}\right)}{P_{2,0}}=\frac{d_{1,0}t_{1,0}\left(1-\theta_{1,0}\right)}{P_{2,0}}=\frac{H_{2,0}-H_{1,0}}{P_{2,0}-d_{2,0}} \quad (31)$$

$$H_1=H_{2,0}-Q_1 \quad (32)$$

$$H_{2,0}=\sum_{i=1}^{L}Q_i \quad (33)$$

$$H_i=H_{(i-1)}-Q_i\quad for\ i=2,\ 3,\ ...,L \quad (34)$$

$$H_L=H_{(L-1)}-Q_L=0 \quad (35)$$

For stage 1, total cost in a cycle, $TC_1(T, n)$ comprises setup cost, variable manufacturing, disposal, and rework costs, inventory and safety stock holding costs. Hence, $TC_1(T, n)$ becomes:

$$TC_1(T,\ n)=$$

$$\left\{\begin{array}{l}C_0Q_0+K_0+C_{R,0}\left[x_0\left(1-\theta_{1,0}\right)Q_0\right]+C_{S,0}\left[x_0\varphi_0Q_0\right]+h_{2,0}\left[\dfrac{d_{1,0}t_{1,0}\left(1-\theta_{1,0}\right)}{2}\right]\left(t_{2,0}\right)\\+h_{1,0}\left[\dfrac{H_{1,0}t_{1,0}}{2}+\dfrac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right)+\dfrac{d_{1,0}t_{1,0}}{2}\left(t_{1,0}\right)+\sum_{i=1}^{L}H_i\left(t_{1,i}+t_{2,i}\right)\right]+h_{4,0}\left(x_0Q_0\right)T\end{array}\right\} \quad (36)$$

The prerequisite assumption for stage 1 is

$$\left(t_{1,0}+t_{2,0}\right)<T\ \text{ or }\left[Q_0\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)\right]<T \quad (37)$$

or

$$\left\{\left(\frac{\lambda_0}{\left[1-\varphi_0E\left[x_0\right]\right]}\right)\left[\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right]\right\}<1 \quad (38)$$

Substituting Eqs. (24) to (35) in Eq. (36) and using the expected values of x_i to cope with its randomness, and after additional derivations, E[$TCU_1(T, n)$] becomes

$$E\left[TCU_1\left(T,\ n\right)\right]=E\left[TC_1\left(T,\ n\right)\right]/E\left[T\right]$$
$$=\left[C_0\lambda_0\pi_{0,0}+\frac{K_0}{T}+C_{R,0}\lambda_0\left(1-\theta_{1,0}\right)\pi_{1,0}+C_{S,0}\lambda_0\varphi_0\pi_{1,0}+\gamma_0 T\right] \tag{39}$$

where

$$\pi_{0,0}=\frac{1}{\left(1-\varphi_0 E[x_0]\right)};\ \pi_{1,0}=\frac{E[x_0]}{\left(1-\varphi_0 E[x_0]\right)};\quad \pi_{0,j}=\frac{1}{\left(1-\varphi_j E[x_j]\right)}\qquad for\ \mathrm{j}=1,\cdots\cdots,i$$

$$\gamma_0=\begin{cases}\dfrac{h_{1,0}\lambda_0^2\left(\pi_{0,0}\right)^2}{2}\left[\dfrac{1}{P_{1,0}}+\dfrac{2E\left[x_0\right]\left(1-\theta_{1,0}\right)\left(1-E\left[x_0\right]\right)}{P_{2,0}}+\dfrac{E\left[x_0\right]^2\left(1-\theta_{1,0}\right)^2\left(1-\theta_{2,0}\right)}{P_{2,0}}\right]\\ +h_{1,0}\displaystyle\sum_{i=1}^{L}\left\{\left(\frac{\lambda_i\pi_{0,i}}{P_{1,i}}+\frac{\lambda_i\left(1-\theta_{1,i}\right)\pi_{1,i}}{P_{2,i}}\right)\left[\sum_{i=1}^{L}\left(\lambda_i\pi_{0,i}\right)-\sum_{j=1}^{i}\left(\lambda_j\pi_{0,j}\right)\right]\right\}\\ +h_{4,0}\lambda_0\pi_{1,0}+\dfrac{h_{2,0}\lambda_0^2\left(\pi_{1,0}\right)^2}{2}\left[\dfrac{\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right]\end{cases} \tag{40}$$

Therefore, E[$TCU(T, n)$] consists of the expected costs of both stages as follows:

$$E\left[TCU\left(T,\ n\right)\right]=E\left[TCU_1\left(T,\ n\right)\right]+E\left[TCU_2\left(T,\ n\right)\right]. \tag{41}$$

2.2. Deriving the Optimal Production and Shipment Policy

To decide the optimal fabrication-distribution policy, the convexity of E [$TCU_2(T, n)$] has to be proved first. That is Eq. (42) the Hessian matrix equations [43] must hold.

$$\left[T\quad n\right]\cdot\begin{pmatrix}\dfrac{\partial^2 E\left[TCU\left(T,n\right)\right]}{\partial T^2} & \dfrac{\partial^2 E\left[TCU\left(T,n\right)\right]}{\partial T\partial n}\\ \dfrac{\partial^2 E\left[TCU\left(T,n\right)\right]}{\partial T\partial n} & \dfrac{\partial^2 E\left[TCU\left(T,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T\\ n\end{bmatrix}>0 \tag{42}$$

From Eq. (22) we have

$$\frac{\partial E\left[TCU_2\left(T,n\right)\right]}{\partial T}=\sum_{i=1}^{L}\begin{Bmatrix}\left[\dfrac{-K_i}{T^2}-\dfrac{nK_{1,i}}{T^2}\right]+\dfrac{h_{1,i}\lambda_i^2}{2}\left\{\pi_{3,i}+\pi_{4,i}-\dfrac{\pi_{5,i}}{n}\right\}+\dfrac{h_{2,i}\lambda_i^2\pi_{1,i}^2}{2}\left[\dfrac{\left(1-\theta_{1,i}\right)^2}{P_{2,i}}\right]\\ +\dfrac{h_{3,i}\lambda_i^2}{2}\left[\dfrac{2\pi_{0,i}}{P_{1,i}}+\dfrac{2\left(1-\theta_{1,i}\right)\pi_{1,i}}{P_{2,i}}-\dfrac{1}{\lambda_i}+\left(1+\dfrac{1}{n}\right)\pi_{5,i}\right]+h_{4,i}\lambda_i\pi_{1,i}\end{Bmatrix} \tag{43}$$

$$\frac{\partial E\left[TCU_2\left(T,n\right)\right]}{\partial T^2}=\sum_{i=1}^{L}\left(\frac{2K_i}{T^3}+\frac{2nK_{1,i}}{T^3}\right) \tag{44}$$

$$\frac{\partial E\left[TCU_2\left(T,n\right)\right]}{\partial n^2}=\sum_{i=1}^{L}\left\{\frac{T\lambda_i^2}{n^3}\left[\left(h_{3,i}-h_{1,i}\right)\pi_{5,i}\right]\right\} \tag{46}$$

$$\frac{\partial E\left[TCU_2\left(T,n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left\{\frac{K_{1,i}}{T}+\frac{T\lambda_i^2}{2n^2}\left[\left(h_{1,i}-h_{3,i}\right)\pi_{5,i}\right]\right\} \tag{45}$$

$$\frac{\partial^2 E\left[TCU_2\left(T,n\right)\right]}{\partial T\partial n}=\sum_{i=1}^{L}\left\{-\frac{K_{1,i}}{T^2}+\frac{\lambda_i^2}{2n^2}\left[\left(h_{1,i}-h_{3,i}\right)\pi_{5,i}\right]\right\} \tag{47}$$

By substituting equations (44), (46), and (47) in Eq. (42) and after more derivations, Eq. (48) can be obtained.

$$\left[T\quad n\right]\cdot\begin{pmatrix}\dfrac{\partial^2 E\left[TCU_2\left(T,n\right)\right]}{\partial T^2} & \dfrac{\partial^2 E\left[TCU_2\left(T,n\right)\right]}{\partial T\partial n}\\ \dfrac{\partial^2 E\left[TCU_2\left(T,n\right)\right]}{\partial T\partial n} & \dfrac{\partial^2 E\left[TCU_2\left(T,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T\\ n\end{bmatrix}=\sum_{i=1}^{L}\frac{2K_i}{T}>0 \tag{48}$$

Since T and K_i are both positive, so Eq. (48) is positive. Therefore, for all n and T different from zero $E[TCU_2(T, n)]$ is strictly convex. Then, to simultaneously decide the fabrication and distribution policies, the linear system of the first derivatives (Eqs. (43) and (45)) of $E[TCU_2(T, n)]$ with respect to T and n, needs to be solved, respectively. Let these partial derivatives equal to zero and with extra derivations, the following can be gained:

$$T^{*}=\sqrt{\frac{\sum_{i=1}^{L}\left(K_{i}+nK_{1,i}\right)}{\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{h_{1,i}\lambda_{i}^{2}}{2}\left[\pi_{3,i}+\pi_{4,i}-\frac{\pi_{5,i}}{n}\right]+\frac{h_{2,i}\lambda_{i}^{2}\pi_{1,i}^{2}}{2}\left[\frac{\left(1-\theta_{1,i}\right)^{2}}{P_{2,i}}\right]+h_{4,i}\lambda_{i}\pi_{1,i}\\+\frac{h_{3,i}\lambda_{i}^{2}}{2}\left[\frac{2\pi_{0,i}}{P_{1,i}}+\frac{2\left(1-\theta_{1,i}\right)\pi_{1,i}}{P_{2,i}}-\frac{1}{\lambda_{i}}+\left(1+\frac{1}{n}\right)\pi_{5,i}\right]\end{array}\right\}}} \tag{49}$$

and

$$n^{*}=\sqrt{\frac{\left(\sum_{i=1}^{L}K_{i}\right)\sum_{i=1}^{L}\left\{\frac{\lambda_{i}^{2}}{2}\left[\left(h_{3,i}-h_{1,i}\right)\pi_{5,i}\right]\right\}}{\left(\sum_{i=1}^{L}K_{1i}\right)\left\{\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{h_{1,i}\lambda_{i}^{2}}{2}\left(\pi_{3,i}+\pi_{4,i}\right)+\frac{h_{2,i}\lambda_{i}^{2}\pi_{1,i}^{2}}{2}\left[\frac{\left(1-\theta_{1,i}\right)^{2}}{P_{2,i}}\right]+h_{4,i}\lambda_{i}\pi_{1,i}\\+\frac{h_{3,i}\lambda_{i}^{2}}{2}\left[\frac{2\pi_{0,i}}{P_{1,i}}+\frac{2\left(1-\theta_{1,i}\right)\pi_{1,i}}{P_{2,i}}-\frac{1}{\lambda_{i}}+\pi_{5,i}\right]\end{array}\right\}\right\}}} \tag{50}$$

3. Numerical Example with Sensitivity Analysis

In this section, we use a numerical example to explain the practical usage of our obtained results. Suppose five end items have annual demand rates λ_i: 3,800, 3,600, 3,400, 3,200, and 3,000 units; and $\alpha = 0.5$. We first assume the linear relationship between α and relevant system variables. Hence, in stage 1 we have $P_{1,0}$ = 120,000 (calculating by $P_{1,0} = (1/\alpha)$ * (the mean of $P_{1,i}$'s)) and $P_{2,0}$ = 96,000 (computing by $P_{2,0} = (1/\alpha)$ * (the mean of $P_{2,i}$'s)). The same linear relationship is also applied to other system variables and to relieve readers' comparison efforts, we adopt the same values of parameters as in [8] as follows:

K_0 = \$8,500,
C_0 = \$40,
$C_{R,0}$ = \$25,
$C_{S,0}$ = \$10,
$h_{1,0}$ = \$5,
$h_{2,0}$ = \$15,
$h_{4,0}$ = \$5,
x_0 = uniformly distributed over the interval of [0, 0.04],
$\theta_{1,0}$ = 0.2,
$\theta_{2,0}$ = 0.2,
$P_{1,i}$ = 128,276, 124,068, 120,000, 116,066, and 112,258 units, respectively; and $P_{1,i} = 1/(1/P_{1,i} - 1/P_{1,0})$,
$P_{2,i}$ = 102,621, 99,254, 96,000, 92,852, and 89,806 units, respectively; and $P_{2,i} = 1/(1/P_{2,i} - 1/P_{2,0})$,
K_i = \$10,500, \$10,000, \$95,000, \$90,000, and \$8,500, respectively,
C_i = \$80, \$70, \$60, \$50, and \$40,
$C_{R,I}$ = \$45, \$40, \$35, \$30, and \$25,
$C_{S,I}$ = \$30, \$25, \$20, \$15, and \$10,
$\theta_{1,i}$ = 0.30, 0.25, 0.20, 0.15, and 0.10,
$\theta_{2,i}$ = 0.30, 0.25, 0.20, 0.15, and 0.10,
x_i = uniformly distributed over the ranges of [0, 0.01], [0, 0.06], [0, 0.11], [0, 0.16], and [0, 0.21], respectively,
$h_{1,i}$ = \$30, \$25, \$20, \$15, and \$10,
$h_{2\ i}$ = \$50, \$45, \$40, \$35, and \$30,
$h_{4,0}$ = \$30, \$25, \$20, \$15, and \$10,
$K_{1,I}$ = \$2,200, \$2,100, \$2,000, \$1,900, and \$1,800,
C_{Ti}= \$0.5, \$0.4, \$0.3, \$0.2, and \$0.1,
$h_{3,i}$= \$90, \$85, \$80, \$75, and \$70, respectively.

Applying equations (49), (50), and (41), we found $n^* = 3$, $T^* = 0.4444$, and E[$TCU(T^*, n^*)$] = \$2,209,197. The impacts of different cycle times T to $E[TCU(T, n)]$ is depicted in Figure 7.

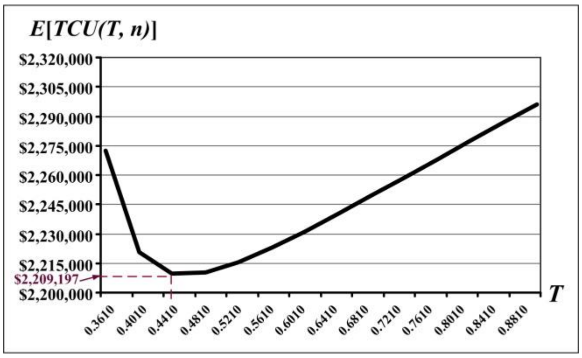

Figure 7. The impacts of different cycle times T to E[$TCU(T, n)$]

Figure 8 shows joint impacts of diverse expected values of x_i and φ_i on E[$TCU(T, n)$]. As both expected values of x_i and φ_i increase, E[$TCU(T, n)$] goes up significantly. These analytical results revealed the important/realistic information on production quality costs.

The behavior of E[$TCU(T, n)$] with respect to the completion rate α of common part is displayed in Figure 9. It indicates that as α increases, E[$TCU(T, n)$] decreases significantly, and a savings in cost 4.63% revealed at α = 0.5. That is the system costs decline to \$2,209,197 (from \$2,316,483) in comparison with the result from a prior work which utilized a fabrication scheme with a single stage.

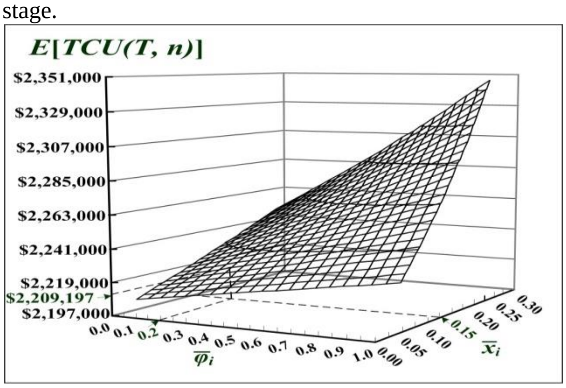

Figure 8. Joint impacts of diverse expected values of x_i and φ_i on E[$TCU(T, n)$]

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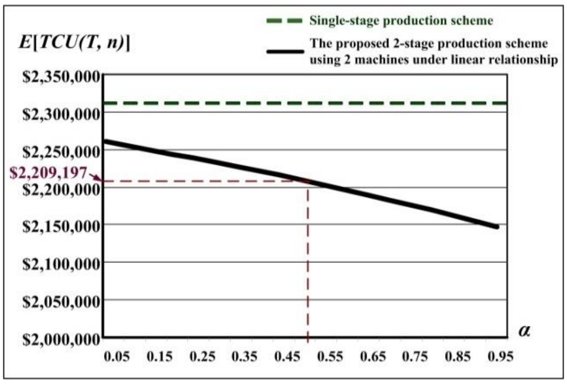

Figure 9. Behavior of E[$TCU(T, n)$] with respect to the completion rate α

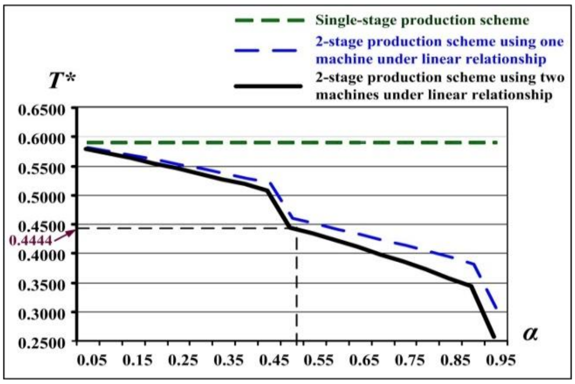

Figure 10. The impacts of different values of completion rate α to T^* for both the fabrication schemes with single-machine and two-machine

3.1. Exploration of Nonlinear Cost Relationship

This section examines the nonlinear relationship between α and its corresponding fabrication cost. It is assumed that 'α^(1/3)' is the relating factor. Therefore, we have C_0 = [α^(1/3)]C_1 = [(0.5)^(1/3)]($80) = $63, so its fabrication cost (or value) is higher than $40 as assumed linearly. Similarly, the following values of other parameters can also be obtained: $h_{1,0}$ = $8, K_0 = $13493, $h_{2,0}$ = $24, $C_{S,0}$ = $16, $h_{4,0}$ = $8, and $C_{R,0}$ = $40. For stage 2, we also have $C_{R,i}$ = $30, $25, $20, $15, and $10, respectively; $C_{,i}$ = $57, $47, $37, $27, and $17; K_i = $5,507, $5,007, $4,507, $4,007, and $3,507; and $C_{S,i}$ are $24, $19, $14, $9, and $4; and x_i are uniform distributed over the ranges [0, 0.01], [0, 0.06], [0, 0.11], [0, 0.16], and [0, 0.21], respectively. Other values of parameters remain the same as in earlier sub-section: x_0= [0, 0.04]; $\theta_{1,0}$ = 0.20; $\theta_{2,0}$ = 0.20; φ_0 = 0.36; scrap rates $\theta_{1,i}$ = 0.10, 0.15, 0.20, 0.25, and 0.30; $\theta_{2,i}$ = 0.10, 0.15, 0.20, 0.25, and 0.30, respectively; $P_{2,0}$ = 96,000; and $P_{1,0}$ = 120,000.

Applying equations (49), (50), and (22), we found T^* = 0.3659, n^* = 3, and E[$TCU(T^*, n^*)$] = $2,164,111. The behavior of E[$TCU(T, n)$] with respect to different α values under both nonlinear and linear relationships is displayed in Figure 11. It shows that as α increases, E[$TCU(T, n)$] declines. Specifically, E[$TCU(T, n)$] is reduced by 2.04% at α = 0.5. That is a savings of $45,086 (for it decreases from $2,209,197) in comparison with the result from the earlier linear case. This analytical outcome reveals as common part's value is higher, E[$TCU(T, n)$] decreases further in comparison with the earlier linear case.

For comparison purpose, Figure 10 exhibits impacts of different values of completion rate α to T^* for both the fabrication schemes with single-machine and two-machine. It reveals that as α increases, T^* decreases significantly, and T^* has reduced by 24.75% at α = 0.5 (i.e., it declines from 0.5906 to 0.4444) as compared to the result from a prior work which employed a fabrication scheme with single stage [30]. Additional analysis also indicates that T^* has reduced 3.39% further (at α = 0.5) comparing to the result from a two-stage fabrication scheme using one machine [8]. Clearly, the fabrication cycle length (or response time) is notably reduced in the proposed fabrication scheme in comparison with either the single-stage [30] or the two-stage single-machine production models [8].

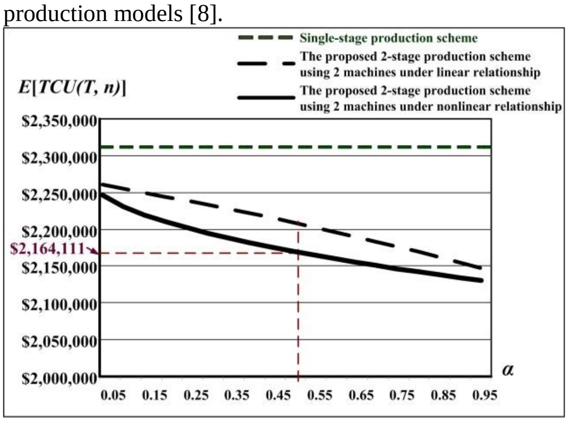

Figure 11. The behavior of E[$TCU(T, n)$] with respect to different α values under both nonlinear and linear relationships

Figure 12 illustrates the impacts of different values of α to T^* for both two-machine and single-machine manufacturing schemes under both nonlinear and linear relationships. It specifies that as α increases, T^* declines notably. Moreover, at α = 0.5, T^* is shortened by 17.66% (i.e., it decreases from 0.4444 to 0.3659) in comparison with earlier linear case. Additional analysis points out that at α = 0.5, T^* is cut down by 8.32% (i.e., T^* declines from 0.3991 to 0.3659) in comparison with the result in a prior work on a multiproduct two-stage single-machine manufacturing system with postponement and under the same nonlinear relationship [8]. Finally, if we judge the obtained results of nonlinear relationship case against that of a single-stage multi-item production system [30], we realize a significant 38.05% reduction in production cycle time at α = 0.5 (i.e., T^* declines from 0.5906 to 0.3659).

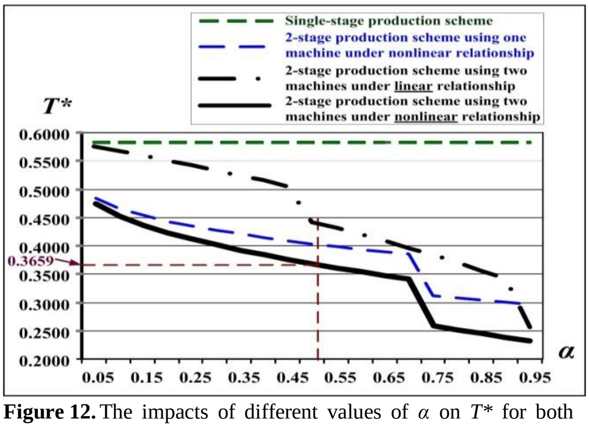

Figure 12. The impacts of different values of α on T^* for both two-machine and single-machine manufacturing schemes under both nonlinear and linear relationships

4. Conclusions

A two-machine fabrication scheme is proposed to re-explore a multiproduct manufacturing system featuring commonality of part, postponement strategy, and quality assurance (which was investigated previously [8] using a single-machine scheme). The objective is to further shorten the cycle time. Distinctively, in the first fabrication stage, machine one exclusively fabricates all common parts that needed by production of end products, and in stage two, a separate machine fabricates the end-product under a rotation cycle time discipline. With the help of mathematical modeling, optimization methods, and a numerical illustration, the proposed system is capable of not only deriving the best fabrication-delivery policy (see Figure 7), but also demonstrating that the proposed fabrication scheme is a beneficial choice in saving cost and shortening fabrication cycle length (Figures 9–12) in comparison with that obtained from a single-machine scheme.

The obtained analytical results exclusively expose the following valuable managerial information: (a) the behavior of E[$TCU(T, n)$] with respect to α; (b) the impacts of different α values to T^* for both the fabrication schemes with single-machine and two-machine, and under both nonlinear and linear relationship of component's value; (c) the behavior of E[$TCU(T, n)$] with respect to α for both nonlinear and linear relationships.

In summary, without an in-depth exploration on such a particular multi-item system utilizing a two-machine manufacturing scheme, the aforementioned valuable information remains inaccessible to managerial decision makings. For future research, examining effect of machine failure on the operating decisions is a practical direction.

Acknowledgements

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References

[1] W. Zinn, "Developing heuristics to estimate the impact of postponement on safety stock". International Journal of Logistics Management, Vol. 1 (1990) 11-16.

[2] H.L. Lee, C.S. Tang, "Modelling the costs and benefits of delayed product differentiation". Management Science, Vol. 43 (1997) 40-53.

[3] J.M. Swaminathan, S.R. Tayur, "Managing broader product lines through delayed differentiation using vanilla boxes". Management Science, Vol. 44 (1998) S161-S172.

[4] R.I. Van Hoek, B. Vos, H.R. "Commandeur, Restructuring European Supply Chains by Implementing Postponement Strategies". Long Range Planning, Vol. 32 (1999) 505-518.

[5] B. Yang, N. Burns, "Implications of postponement for the supply chain". International Journal of Production Research, Vol. 41 (2003) 2075-2090.

[6] S. Kumar, J. Wilson, "A manufacturing decision framework for minimizing inventory costs of a configurable off-shored product using postponement". International Journal of Production Research, Vol. 47 (2009) 143-162.

[7] S.S. Saghiri, A. Hill, "Supplier relationship impacts on postponement strategies". ". International Journal of Production Research, Vol. 52 (2014) 2134-2153.

[8] S.W. Chiu, J-S. Kuo, V. Chiu, Y-S.P. Chiu, "Cost minimization for a multi-product fabrication-distribution problem with commonality, postponement, and quality assurance". Mathematical and Computational Applications, Vol. 21(3) (2016) Art. No. 38, 1-14.

[9] D.A. Collier, "Aggregate safety stock levels and component part commonality". Management Science, Vol. 28 (1982) 1296-1303.

[10] Y. Gerchak, M. Henig, "An inventory model with component commonality". Operations Research Letters, Vol. 5(3) (1986) 157-160.

[11] A. Garg, C.S. Tang, "On postponement strategies for product multiple points of differentiation". IIE Transactions, Vol. 29 (1997) 641-650.

[12] R. Ernst, B. Kamrad, "Evaluation of supply chain structures through modularization and postponement". European Journal of Operational Research, Vol. 124 (2000) 495-510.

[13] R.S. Tibben-Lembke, Y. Bassok, "An inventory model for delayed customization: A hybrid approach". European Journal of Operational Research, Vol. 165 (2005) 748-764.

[14] G.A. Graman, "A partial-postponement decision cost model". European Journal of Operational Research, Vol. 201 (2010) 34-44.

[15] F. Bernstein, G.A. DeCroix, Y. Wang, "The impact of demand aggregation through delayed component allocation in an assemble-to-order system". Management Science, Vol. 57(6) (2011) 1154-1171.

[16] S. Qrunfleh, M. Tarafdar, "Lean and agile supply chain strategies and supply chain responsiveness: The role of strategic supplier partnership and postponement". Supply Chain Management, Vol. 18 (2013) 571-582.

[17] Y-S.P. Chiu, J-S. Kuo, S.W. Chiu, Y-T. Hsieh, "Effect of delayed differentiation on a multi–product vendor–buyer integrated inventory system with rework". Advances in Production Engineering & Management, Vol.11(4) (2016) 333-344.

[18] M.J. Rosenblatt, H.L. Lee, "Economic production cycles with imperfect production processes". IIE Transaction, Vol.18 (1986) 48-55.

[19] M. Henig, Y. Gerchak, "Structure of periodic review policies in the presence of random yield". Operations Research, Vol. 38 (1990) 634-643.

[20] A.M. Zargar, "Effect of rework strategies on cycle time". Computers & Industrial Engineering, Vol. 29 (1995) 239-243.

[21] H-M. Wee, Y-S. Shum, "Model development for deteriorating inventory in material requirement planning systems". Computers & Industrial Engineering, Vol. 36 (1999) 219-225.

[22] R.H. Teunter, S.D.P. Flapper, "Lot-sizing for a single-stage single-product production system with rework of perishable production defectives". OR Spectrum, Vol.25 (2003) 85-96.

[23] D. Ojha, B.R. Sarker, P. Biswas, "An optimal batch size for an imperfect production system with quality assurance and rework". International Journal of Production Research, Vol. 45 (2007) 3191-3214.

[24] W-N. Ma, D-C. Gong, G.C. Lin, "An optimal common production cycle time for imperfect production processes with scrap". Mathematical and Computational Modelling, Vol. 52 (2010) 724-737.

[25] T. Chakraborty, S.S. Chauhan, B.C. Giri, "Joint effect of stock threshold level and production policy on an unreliable production environment". Applied Mathematical Modelling, Vol. 37 (2013) 6593-6608.

[26] P. Jawla, S. R. Singh, "Multi-item economic production quantity model for imperfect items with multiple production setups and rework under the effect of preservation technology and learning environment". International Journal

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of Industrial Engineering Computations, Vol. 7(4) (2016) 703-716.

[27] Y. Hu, Z. Li, J. Zhao, L. Ren, D. Wang, "Optimization of hydraulic fracture-network parameters based on production simulation in shale gas reservoirs." Journal of Engineering Research, Vol. 4(4) (2016) 214-235.

[28] J. Gómez, I. Salazar, P. Vargas, "Sources of information as determinants of product and process innovation". PLoS ONE, Vol. 11(4) (2016) Article No. e0152743.

[29] S.W. Chiu, S-W. Chen, C-K. Chang, Y-S.P. Chiu, "Optimization of a multi–product intra-supply chain system with failure in rework." PloS ONE, Vol. 11(12) (2016) Article No. e0167511.

[30] A. Romero-Jabalquinto, A. Velasco-Téllez, P. Zambrano-Robledo, B. Bermúdez-Reyes, "Feasibility of manufacturing combustion chambers for aeronautical use in Mexico". Journal of Applied Research and Technology, Vol. 14(3) (2016) 167-172.

[31] B. Mičieta, J. Herčko, M. Botka, N. Zrnić, "Concept of intelligent logistic for automotive industry". Journal of Applied Engineering Science, Vol.14(2) (2016) 233-238.

[32] K.S. Dinesh, A.R. Ashwanth, S. Karthikeyan, G.K. Nithyanandam, "Investigation to improve the new product development process of mono block pump using additive manufacturing". International Journal of Mechanical and Production Engineering Research and Development, Vol. 6(2) (2016) 77-88.

[33] N. Abilash, M. Sivapragash, "Optimizing the delamination failure in bamboo fiber reinforced polyester composite". Journal of King Saud University - Engineering Sciences, Vol. 28(1) (2016) 92-102.

[34] M. Rakyta, M. Fusko, J. Herčko, L. Závodská, N. Zrnić, "Proactive approach to smart maintenance and logistics as a auxiliary and service processes in a company". Journal of Applied Engineering Science, Vol. 14(4) (2016) 433-442.

[35] P. Jindal, A. Solanki, "Integrated supply chain inventory model with quality improvement involving controllable lead time and backorder price discount". International Journal of Industrial Engineering Computations, Vol. 7(3) (2016) 463-480.

[36] A. Ajbar, E. Ali, "Study of advanced control of ethanol production through continuous fermentation". Journal of King Saud University – Engineering Science, Vol. 29(1) (2017) 1-11.

[37] S. Kovalevskyy, O. Kovalevska, P. Dašić, D. Ješić, P. Kovač, "Engineering consulting technology in production engineering intelligent mobile machines". International Journal of Industrial Engineering and Management, Vol. 8(4) (2017) 203-208.

[38] L. Oblak, M.K. Kuzman, P. Grošelj, "A fuzzy logic-based model for analysis and evaluation of services in a manufacturing company". Journal of Applied Engineering Science, Vol. 15(3) (2007) 258-271.

[39] I.D. Wangsa, "Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions". International Journal of Industrial Engineering Computations, Vol. 8(4) (2017) 453-480.

[40] M. Abubakar, U. Basheer, N. Ahmad, "Mesoporosity, thermochemical and probabilistic failure analysis of fired locally sourced kaolinitic clay". Journal of the Association of Arab Universities for Basic and Applied Sciences, Vol. 24 (2017) 81-88.

[41] H. Kaylani, A. Almuhtady, A. M. Atieh, "Novel approach to enhance the performance of production systems using lean tools". Jordan Journal of Mechanical and Industrial Engineering, Vol. 10(3) (2016) 215-229.

[42] Y-S.P. Chiu, C-J. Liu, M-H. Hwang, "Optimal Batch Size Considering Partial Outsourcing Plan and Rework". Jordan Journal of Mechanical and Industrial Engineering, Vol. 11(3) (2017) 195-200.

[43] R.L. Rardin, Optimization in Operations Research, Int. Ed., Prentice-Hall, New Jersey, 1998.

Appendix A

Nomenclature

For i = 0, 1, 2, …, L

Q_i = fabrication batch size,

φ_i = scrap rate of product i, where $0 <= \varphi_i <= 1$,

$t_{1,i}$ = uptime of product i,

$t_{2,i}$ = rework time of product i,

$t_{3,i}$ = shipping time of product i,

$H_{1,i}$ = level of perfect quality product i when regular production ends,

$H_{2,i}$ = level of perfect quality product i when rework process finishes,

K_i = setup cost,

C_i = unit manufacturing cost,

$C_{R,i}$ = unit rework cost,

$C_{S,i}$ = unit disposal cost,

$h_{1,i}$ = holding cost per product,

$h_{2,i}$ = holding cost per reworked item,

$h_{4,i}$ = holding cost per safety stock,

For *i* = 1, 2, …, *L*

$K_{1,i}$ = fixed cost per delivery of product i,

$C_{T,i}$ = unit shipping cost of product i,

$t_{n,i}$ = fixed time interval between two succeeding deliveries,

H_i = level of common part in the beginning of fabrication of end item i,

$I_c(t)_i$ = customer's stock level of product i at time t,

$h_{3,i}$ = holding cost per product for customer's stock,

I_i = the left-over quantities of product i in each $t_{n,i}$, at customer's side,

D_i = quantities of finished items for product i per delivery.

Other notation:

T = rotation cycle length – a decision variable,

n = number of deliveries per cycle – a decision variable,

α = common part's completion rate in comparison with finished product,

$E[T]$ = the expected fabrication cycle length,

$TC_1(T, n)$ = stage one's total cost per cycle,

$E[TC_1(T, n)]$ = stage one's expected cost per cycle,

$E[TCU_1(T, n)]$ = stage one's expected cost per unit time,

$TC_2(T, n)$ = stage two's total cost per cycle,

$E[TC_2(T, n)]$ = stage two's expected total cost per cycle,

$E[TCU_2(T, n)]$ = stage two's expected total cost per unit time,

$E[TCU(T, n)]$ = the expected system cost per unit time.