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Nonlinear Natural Frequencies and Frequency Veering of a Beam with an Arbitrary Initial Rise Supported by Flexible Ends and Resting on Elastic Foundation

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Abstract

The present study reports results for the effect of support flexibility on nonlinear natural frequencies and frequency veering phenomenon of an elastic Euler-Bernoulli beam resting on Winkler elastic foundation with an initial arbitrary rise. Also, it extends the analysis presented for the linear vibration analysis [1], to account for the effects of beam's ends flexibility and rise shape on; veering zones, and nonlinear natural frequencies.

The beam is supported by translational and rotational springs at each end. The effect of the induced force due to mid-plane stretching is accounted for due to its importance and significance on the nonlinear dynamic and vibrational behavior of the beam, as it was proved and presented in earlier investigations and studies.

The governing integro-partial differential equation is discretized using the assumed mode method "Galerkin's single mode approach" and the resulting nonlinear temporal equation was solved using the harmonic balance method to obtain results for the nonlinear natural frequencies. The results are presented for the nonlinear natural frequencies of the first three modes of vibration, for a selected range of physical parameters, like rotational and translational springs at both ends of the beam, elastic foundation stiffness and initial rise shape and level

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Keywords: *Non-Linear Frequency Veering, Rise Shape, Elastic Foundation, Flexible Supports*.

1. Introduction

Many civil structures, bridges, space structures and space frames can be modeled as beam like structures. In many cases and due to errors and defects in manufacturing, such beams may take the shape of a shallow arc which can be modeled as a beam with a half sine shape rise. Also, deformed pipe lines or pipes with geometrical imperfections can be modeled as a beam with a half sine / full sine rise resting on elastic foundation. Practically and in order to simulate the flexibility present in joints and fixing plates at the ends of the beam, the end supports of a given beam like structure are represented by translational and rotational springs.

The significance of the considered model lies in the varieties of engineering applications that it can model, such as soil-structure interaction problems, buried pipes, civil engineering structures and shafts in machinery, as well as the importance of studying and predicting the dynamic behavior of such systems needed for design, analysis, operation and evaluation. The model of an imperfect elastic beam element resting on elastic foundation, which can exhibit frequency curve veering, is used to study the dynamic behavior of a wide range of engineering systems found in, for example, foundation and structural engineering, fluid-structural interaction problems, micro-switches, and sensing devices in Micro-Electro-Mechanical Systems (MEMS).

The free and forced responses of such beam models, and other structures having similar frequency veering behavior, with various boundary conditions, vertical and axial loading conditions, types of elastic foundations, initial imperfections, and different assumptions about the effect of mid-plane stretching, have been the subject of numerous theoretical, numerical and experimental studies over the years, [2-9]. A review of the relevant literature related to beams with initial rise/imperfection can be found in [7-9].

The present work extends a previous study [1], which investigated the effect of supports flexibility on frequency veering in imperfect beams resting on elastic foundation based on linear analysis only. The extension includes the derivation of the mathematical model to account for the mid-plane stretching from which the nonlinearities are introduced to the governing integro-partial differential equation of motion. Also, the extension includes a thourough study of the effect of the beam flexible ends and the initial rise shape/level on the nonlinear natural frequencies and frequency veering phonmenon.

The deveriation of the mathematical model is similar to that followed in [7-9], but the nonlinear analysis for the natural frequencies and the initial rise shape are introduced in the present paper.

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2. System Description and Problem Formulation

A schematic of the two beam systems under consideration, initial ½ -sine rise and full sine rise, are shown in Figure (1). The beam is assumed to be uniform with length l and cross sectional area A. It has mass per unit length m, flexural rigidity EI, resting on a linear Winkler type elastic foundation of stiffness K_f. It is assumed that the beam is supported by two translational springs at the right and left ends (K_{TR} and K_{TL}) and two rotational springs at the right and left ends (K_{RR}, K_{RL}), respectively. The beam's initial rise shape "initial imperfection", regardless its form, is assumed to extended over the beam span. The moderately large vibrations $\hat{w}$ of the beam about the imperfect rest configuration $\hat{w}_0$, are governed in dimensional form, by the following non-linear integro-partial partial differential equation [1, 7-9]:

$$m\frac{\partial^2 \hat{w}}{\partial \hat{t}^2}+EI\frac{\partial^4 \hat{w}}{\partial \hat{x}^4}-\frac{EA}{l}\left[\int_0^l\left(\frac{1}{2}\left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2+\frac{\partial \hat{w}}{\partial \hat{x}}\frac{\partial \hat{w}_0}{\partial \hat{x}}\right)d\hat{x}\right]\left(\frac{\partial^2 \hat{w}}{\partial \hat{x}^2}+\frac{\partial^2 \hat{w}_0}{\partial \hat{x}^2}\right)+K_f\hat{w}=0 \tag{2.1}$$

Introducing the following non-dimensional parameters: $w=\hat{w}/r$, $\xi=\hat{x}/l$, $w_0=\hat{w}_0/r$ and $t=\hat{t}\sqrt{\frac{EI}{ml^4}}$, where r is the beam's radius of gyration $r=\sqrt{I/A}$, equation (2.1) can be re-written in the following form:

$$\frac{\partial^2 w}{\partial t^2}+\frac{\partial^4 w}{\partial \xi^4}-\left[\int_0^l\left(\frac{1}{2}\left(\frac{\partial w}{\partial \xi}\right)^2+\frac{\partial w}{\partial \xi}\frac{\partial w_0}{\partial \xi}\right)d\xi\right]\left(\frac{\partial^2 w}{\partial \xi^2}+\frac{\partial^2 w_0}{\partial \xi^2}\right)+K_f^* w=0 \tag{2.2}$$

where $K_f^*=K_f l^4/EI$. The boundary conditions in non-dimensional form for the considered beam with initial rise for instance, are:

$w'''+K_{TL}^* w=0$ at $\xi=0$, translational spring at the left end

$w''-K_{RL}^* w'=0$ at $\xi=0$, rotational spring at the left end

$w'''-K_{TR}^* w=0$ at $\xi=1$, translational spring at the right end

$w''+K_{RR}^* w'=0$ at $\xi=1$, rotational spring at the right end (2.3)

where $K_{TL}^*=K_{TL}l^3/EI$, $K_{TR}^*=K_{TR}l^3/EI$, $K_{RR}^*=K_{RR}l/EI$ and $K_{RL}^*=K_{RL}l/EI$. The initial rise or imperfection is assumed to take the form:

$$w_0=R\sin(n\pi\xi) \tag{2.4}$$

where R is a non-dimensional rise amplitude equals to the actual rise $\hat{R}$ divided by the radius of gyration, *i.e.* $R=\hat{R}/r$, n is the number of half -waves in the sinusoid. For a beam with a ½ -sine initial $n=1$ and $n=2$ for a beam with full sine rise.

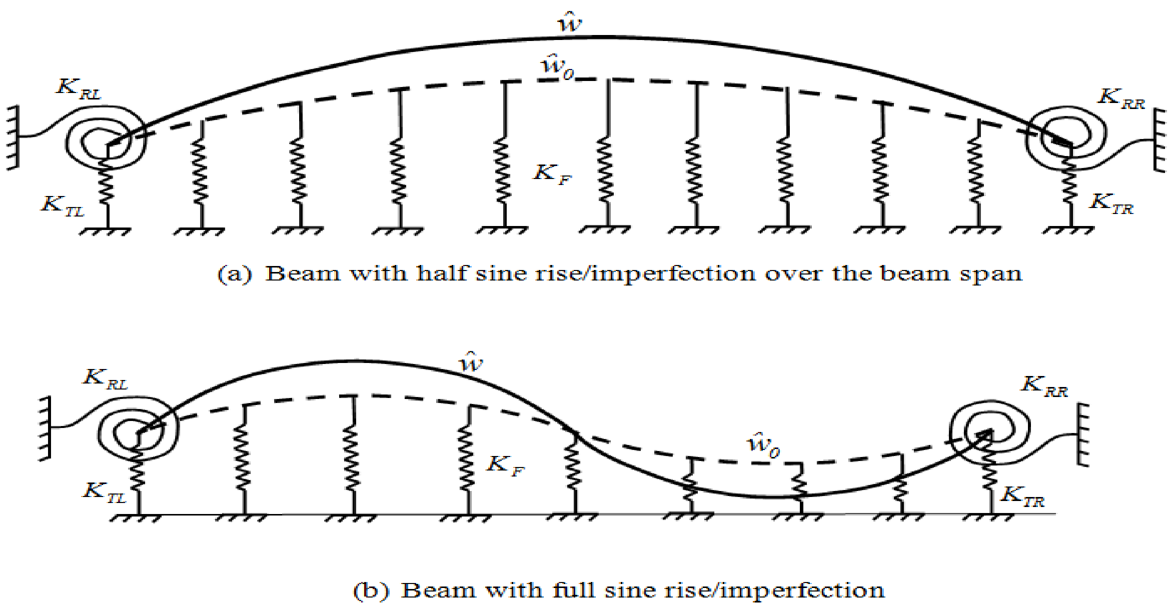

Figure 1. The schematic of beam resting on Winkler foundation with ½ and full sine rise

3. Analysis and Solutions

3.1. Non-Linear Temporal Model

The nonlinear integro-partial differential equation (2.2) can be discretized by assuming:

$$w = \sum_{i=1,2}^{\infty} \phi_i(\xi) q_i(t) \tag{3.1}$$

where $\phi_i(\xi)$ is the normalized, self-similar (i.e., independent of the motion amplitude) assumed mode shape of the beam and $q_i(t)$ is the generalized coordinates and it is the unknown time modulation of the assumed deflection mode $\phi_i(\xi)$. In the present work, Galerkin's method is used, whereby $\phi_i(\xi)$ is the eigenfunction of the *i-th* mode of the beam.

Using a simplified single mode approach, i.e., $w = \phi(\xi) q(t)$ and substituting w_0 from equation (2.4) into equation (2.2), multiplying by $\phi(\xi)$ integrating from *0* to *1*, and for convenience using the abbreviations $\phi(\xi) = \phi$, $q(t) = q$ one obtains the following reduced single-mode nonlinear temporal equation:

$$\begin{aligned} &\int_0^1 \phi\phi''''q d\xi + \\ &\int_0^1 \phi^2 \ddot{q} d\xi - \left\{ \int_0^1 \phi \left[\int_0^1 \frac{1}{2}\phi'^2 q^2 d\xi \right] (\phi'' q) d\xi \right\} - \\ &\left\{ \int_0^1 \phi \left[\int_{\xi_a}^{\xi_b} \phi' q \frac{dw_0}{d\xi} d\xi \right] \left(\frac{d^2 w_0}{d\xi^2} \right) d\xi \right\} + \\ &K_f \int_0^1 \phi^2 q d\xi = 0 \end{aligned} \tag{3.2}$$

Equation (3.2) can be re-arranged and written in the from:

$$\beta_0 \ddot{q} + \beta_1 q + \beta_2 q^2 + \beta_3 q^3 = 0 \quad , \tag{3.3}$$

Where

$$\beta_0 = \int_0^1 \phi^2 d\xi$$

$$\beta_1 = \int_0^1 \phi\phi'''' d\xi - \int_0^1 \left\{ \left[\int_{\xi_a}^{\xi_b} \phi \frac{dw_0}{d\xi} d\xi \right] \phi \frac{d^2 w_0}{d\xi^2} \right\} d\xi + K_f \int_0^1 \phi^2 d\xi$$

$$\beta_2 = -\int_0^1 \left(\int_0^1 \left(\frac{1}{2}\phi'^2 \right) d\xi \right) \phi \frac{d^2 w_0}{d\xi^2} d\xi - \int_0^1 \left(\int_0^1 \phi' \frac{dw_0}{d\xi} d\xi \right) \phi\phi'' d\xi$$

$$\beta_3 = -\int_0^1 \left(\int_0^1 \left(\frac{1}{2}\phi'^2 \right) d\xi \right) \phi\phi'' d\xi \tag{3.4}$$

3.2. Linear Model Mode Shapes and Natural Frequencies

The mode shapes and the associated natural frequencies can be obtained from the linearized version of the nonlinear integro-partial differential equation (2.2), which takes the form:

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial \xi^4} - \frac{\partial^2 w_0}{\partial \xi^2} \left\{ \int_0^l \frac{\partial w}{\partial \xi} \frac{\partial w_0}{\partial \xi} d\xi \right\} + K_f^* w = 0 \tag{3.5}$$

By substituting $w(\xi,t) = \phi A sin(\omega t)$, where ω is unknown natural frequency, into equation (3.5) and using the expression given by equation (2.4) for $w_0(\xi)$, equation (3.5) will have the following boundary value problem:

$$\begin{aligned} &\phi'''' - (\omega^2 - K_f)\phi = \\ &- R^2 (n\pi)^3 \sin(n\pi\xi) \int_0^1 \phi' \cos(n\pi\xi) d\xi \end{aligned} \tag{3.6}$$

The associated modes shape ϕ can be obtained by solving the non-homogeneous, boundary value ordinary differential equation (3.6), i.e., by combining the homogenous and particular solutions, such that $\phi = \phi_h + \phi_p$. The homogeneous solution, after substituting $\phi_h = e^{s\xi}$, is given by:

$$\begin{aligned} \phi_h &= A_1 \sin\alpha\xi + A_2 \cos\alpha\xi + \\ &A_3 \sinh\alpha\xi + A_4 \cosh\alpha\xi \end{aligned} \tag{3.7}$$

where $\alpha = \omega^2 - K_f^*$, A_1, A_2, A_3 and A_4 are arbitrary constants to be determined from the following boundary conditions:

$$\begin{aligned} &\phi'''(0) + K_{TL}^* \phi(0) = 0 \\ &\phi''(0) - K_{RL}^* \phi'(0) = 0 \\ &\phi'''(1) - K_{TR}^* \phi(1) = 0 \\ &\phi''(1) + K_{RR}^* \phi'(1) = 0 \end{aligned} \tag{3.8}$$

The particular ϕ_p to equation (3.6) takes the form:

$$\phi_p = D\sin(n\pi\xi) \tag{3.9}$$

Substituting equation (3.9) into equation (3.6), one obtains:

$$D = \frac{(n\pi)^3 R^2 \int_0^1 \phi' \cos(n\pi\xi) d\xi}{\alpha^4 - (n\pi)^4} \tag{3.10}$$

The total solution $\phi = \phi_h + \phi_p$ is, thus, given by:

$$\begin{aligned}\phi &= A_1 \sin\alpha\xi + A_2\cos\alpha\xi + \\ &A_3 \sinh\alpha\xi + A_4\cosh\alpha\xi + D\sin(n\pi\xi)\end{aligned} \tag{3.11}$$

Substituting the derivative of equation (3.11), i.e., the sum of the homogenous and particular solutions, into equation (3.10) one obtains:

$$D = \frac{(n\pi)^3 R^2}{\alpha^4 - (n\pi)^4} \left\{ \int_0^1 \begin{pmatrix} \alpha A_1 \cos\alpha\xi - \\ \alpha A_2 \sin\alpha\xi + \\ \alpha A_3 \cosh\alpha\xi + \\ \alpha A_4 \sinh\alpha\xi + \\ (n\pi) D\cos(n\pi\xi) \end{pmatrix} \cos(n\pi\xi) d\xi \right\} \tag{3.12}$$

Now the constant D can be obtained and calculated form the following expression, after some mathematical manipulations, which takes the form:

$$D = \left(\frac{a_5}{a_d} A_1 + \frac{a_6}{a_d} A_2 + \frac{a_7}{a_d} A_3 + \frac{a_8}{a_d} A_4 \right) \tag{3.13}$$

Where

$$a_5 = \left\{ \frac{R^2 (n\pi)^3}{\alpha^4 - (n\pi)^4} \alpha \right\} \int_0^1 \cos(\alpha\xi)\cos(n\pi\xi) d\xi$$

$$a_6 = -\left\{ \frac{R^2 (n\pi)^3}{\alpha^4 - (n\pi)^4} \alpha \right\} \int_0^1 \sin(\alpha\xi)\cos(n\pi\xi) d\xi$$

$$a_7 = \left\{ \frac{R^2 (n\pi)^3}{\alpha^4 - (n\pi)^4} \alpha \right\} \int_0^1 \cosh(\alpha\xi)\cos(n\pi\xi) d\xi$$

$$a_8 = \left\{ \frac{R^2 (n\pi)^3}{\alpha^4 - (n\pi)^4} \alpha \right\} \int_0^1 \sinh(\alpha\xi)\cos(n\pi\xi) d\xi$$

$$a_D = \left\{ \frac{R^2 (n\pi)^4}{\alpha^4 - (n\pi)^4} \alpha \right\} \int_0^1 (\cos(n\pi\xi))^2 d\xi \tag{3.14}$$

Applying the four boundary conditions given in (3.8) yields a system of equations for the four arbitrary constants A_i, $i = 1,2,3,4$, which is a homogeneous matrix equation and from its determinant the system natural frequency can be obtained and calculated by equating the determinant of the coefficient matrix to zero.

The procedure to obtain the natural frequency and the associated modshape is repeated here to show how the parameters β_i, $i = 1, 2, 3, 4$ of the nonlinear temporal equation, given in (3.3), can be calculated and evaluated. These parameters include all the system's physical parameters; K^*_f, K^*_{TL}, K^*_{TR}, K^*_{RR}, K^*_{RL} and initial rise shape R and n.

3.3. Non-Linear Natural Frequencies

The expressions of β_i, given in equation (3.3), were calculated numerically for a given value or combination of the physical system parameters. To simplify the analysis, and for convenience, equation (3.3) can be scaled and rewritten in the following non-dimensional form:

$$\ddot{q} + q + \varepsilon_2 q^2 + \varepsilon_3 q^3 = 0 \tag{3.15}$$

where a dot denotes a derivative with respect to $T = (\beta_1/\beta_0)^{1/2} t$, and ε_2 and ε_3 are dimensionless coefficients defined as $\varepsilon_2 = \beta_2/\beta_1$ and $\varepsilon_3 = \beta_3/\beta_1$. It is noted from the definitions of ε_2 and ε_3 that they are functions of all of beam system physical parameters " R, K^*_f, K^*_{TL}, K^*_{TR}, K^*_{RR}, K^*_{RL} and n".

In the present study, the nonlinear natural frequencies of the nonlinear equation of motion of the beam systems, shown in Figure (1) and given in equation (3.15) are obtained using the method of Harmonic Balance (HB). Since the oscillator includes asymmetric nonlinearity "quadratic term q^2", the assumed solution should contain a constant bias term, i.e., the approximate two terms- HB takes the form:

$$q(t) = A_0 + A_1 \cos(\omega t) \tag{3.16}$$

where ω is the non dimensional nonlinear natural frequency. The initial conditions are taken to be $q(0) = A_0 + A_1 = A$ and $\dot{q}(0) = 0$, where A is the amplitude of the motion.

Substituting equation (3.16) and its derivatives into equation (3.15) and balancing coefficients of different harmonics one obtains:

$$A_0\left(1 + \varepsilon_2 A_0 + \varepsilon_3 A_0^2 + \frac{3}{2} A_1^2\right) + \frac{\varepsilon_2}{2} A_1^2 = 0 \tag{3.17}$$

$$A_1\left(1 + 2\varepsilon_2 A_0 + 3\varepsilon_3 A_0^2 - \omega^2\right) + \frac{3\varepsilon_3}{4} A_1^3 = 0 \quad 3.18)$$

The above two coupled nonlinear algebraic equations were, for given physical parameters and amplitude of motion A, solved numerically to obtain results for the nonlinear natural frequency ω of a given mode of vibration. In addition to the above HB solution, results for the nonlinear natural frequencies can be obtained also using the method of multilple scales MMS, Nayfeh [10].

It was shown in previous studies [7, 8] that the HB and MMS methods can predict the dynamics of the beams with asignificant accuracy and both can capture the vibratory behavior of the nonlinear oscillator given in [10].

Here, in the present study, the HB method is used to obtain results for the nonlinear natural frequencies versus the amplitude, for given values of the system physical parameters and will be presented and discussed in the next section.

4. Results And Discussion

Dynamic behaviors of the beam systems, shown in Figure (1), were analyzed for some selected values of the system physical parameters: beam elastic foundation K_f^*, translational and torsional springs at the beam ends K_{TL}^*, K_{TR}^*, K_{RR}^*, K_{RL}^*, initial rise amplitude *R* and rise type parameter *n*.

Figures (2) – (4) display results for the variation of nondimensional linear natural frequencies ω versus the beam rise R for different physical parameters.

Figure (2) shows the results for a beam fixed at both ends, i.e., $K_{TL}^* = K_{TR}^* = K_{RR}^* = K_{RL}^* = \infty$, but for $K_f^* = 0$ and $n = 1$ "1/2 sine rise". As can be seen from Figure (2), two veering zones are present. The first veering occur between the 1st and 2nd natural frequencies at $R \approx 8.74$, and second veering occur between the 2nd and 3rd natural frequencies at a rise of $R \approx 16.7$. At these two zones, the two natural frequencies approach each other and then veer away. Here, it is worth mentioning that a drastic change in mode shapes occurred also, as it was presented by [1].

Other results are obtained and presented in Figures (3) – (4) for beams with some flexibility at the two ends. In Figure (3), the same trend is obtained for $K_{TL}^* = K_{TR}^* = 10^5$, $K_{RR}^* = K_{RL}^* = 50$, $K_f^* = 0$ and $n = 1$. From Figure (3), the qualitative behavior is the same, i.e., two veering zones between the first three natural frequencies but with some differences in natural frequencies as one may expect due to the flexibility of the beam supports. In Figure (4), the nondimensional linear natural frequencies ω versus the beam rise R are obtained for the same parameters of Figure (3), expect initial rise shape, i.e., $n = 2$. As it can be seen from Figure (4), due to the change in rise shape, an increase in natural frequencies can be noticed as well as a shift in the veering zones. For instance, the first veering zone between the 1st and 2nd natural frequencies at $R \approx 3.5$ compared to $R \approx 7.7$ for the case of $n = 1$, while the second veering occurs between the 2nd and 3rd natural frequencies at a rise of $R \approx 6.4$ for $n = 2$. From the results presented in Figures (3) and (4), it is demonstrated that the rise shape, whether it is ½ sine or full sine, has a siginificant role in the dynamical behavior as of the beam, as well as the other physical parameters, like elastic foundation stiffness K_f^*, translational and torsional springs at the beam ends K_{TL}^*, K_{TR}^*, K_{RR}^*, K_{RL}^*.

Results presented in Figures (2) – (4) are obtained from using the linearized version of the mathematical model. To have an idea about the nonlinear interaction between the modes of vibration and natural frequencies, the nonlinear derived mathematical model given in [6], will be analyzed and the nonlinear natural frequencies will be calculated, as mentioned in the previous section. This is due to the fact that the effect of vibration amplitude on natural frequency cannot be captured using the linear analysis. In addition, the frequency veering phenomenon is associated with drastic change in mode shapes and might have crossover instabilities between modes.

$K_r = 5$, $K_f = 0.5$

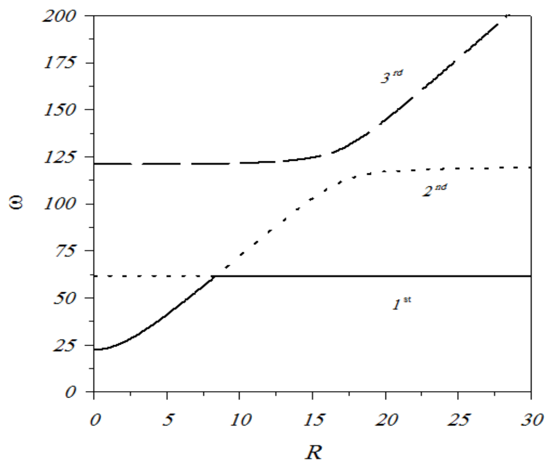

Figure 2. Variation of the non-dimensional linear natural frequencies ω with the non-dimensional rise R for $K_{TL}^* = K_{TR}^* = K_{RR}^* = K_{RL}^* = \infty$, $K_f^* = 0$ and $n = 1$ "1/2 sine rise"

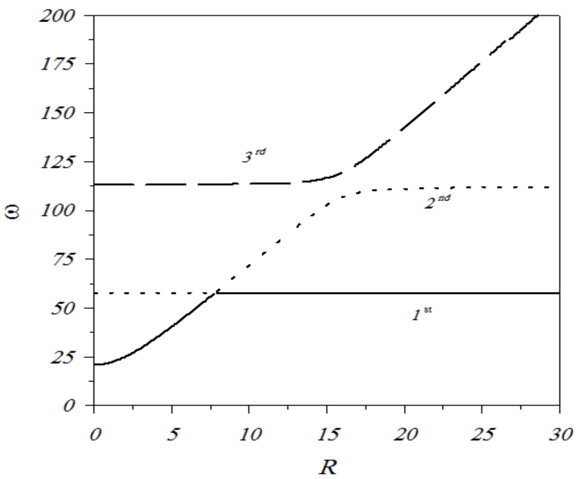

Figure 3. Variation of the non-dimensional linear natural frequencies ω with the non-dimensional rise R for $K_{TL}^* = K_{TR}^* = 10^5$, $K_{RR}^* = K_{RL}^* = 50$, $K_f^* = 0$ and $n = 1$

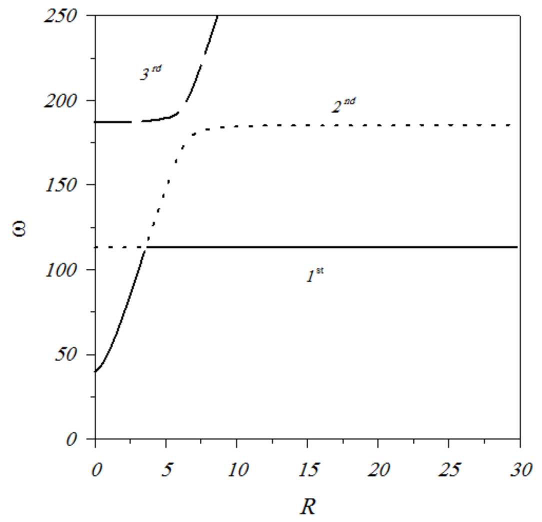

Figure 4. Variation of the non-dimensional linear natural frequencies ω with the non-dimensional rise R for $K^*_{TL} = K^*_{TR} = 10^5$, $K^*_{RR} = K^*_{RL} = 50$, $K^*_f = 0$ and $n = 2$

To have a clear picture about the interaction between modes and natural frequencies near or at the veering zones, Figures (5) – (8) display results for the nonlinear natural frequencies obtained for $K^*_{TL} = K^*_{TR} = 10^5$, $K^*_{RR} = K^*_{RL} = 100$, $K^*_f = 1$ and $n = 1$, for which the first veering zone between the 1st and 2nd natural frequencies occurs at $R \approx 8.1$.

Results in these Figures are presented as the variation of the nonlinear natural frequencies of the first three modes versus the amplitude of vibrational motion at a given beam rise R, near the veeering zone.

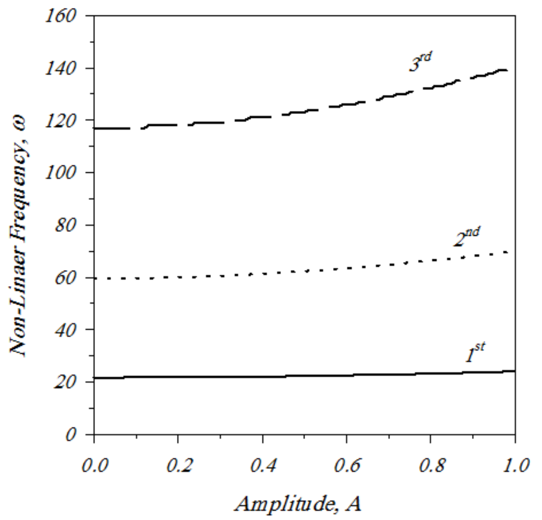

Figure 5. Variation of the non-linear natural frequencies ω with the Amplitude A for $K^*_{TL} = K^*_{TR} = 10^5$, $K^*_{RR} = K^*_{RL} = 100$, $K^*_f = 1$, $n = 1$ and $R = 0$

As it can be seen from Figure (5) that the three modes of vibration exhibit a hardening behavior, i.e., the natural frequency increases as the amplitude of motion increases, for the case of initial rise $R = 0$. On the other hand, as the values of the initial rise increase, the first natural frequency exhibits a softening behavior, i.e., the natural frequency decreases as the amplitude of motion increases, till it reaches a zero value which represents an unstable vibratory motion. This behavior, for the first natural frequency, is obvious in Figures (6) and (7), while the behavior of the second and third natural frequencies is of hardening type, regardless the beam rise. This behavior is due to the two competing nonlinearities of the beam system, i.e., ε_2 and ε_3 whether they are of softening or hardening type. As mentioned before, ε_2 and ε_3 contain all the system's physical parameters.

In addition, another phenomenon, like cross over, occurred between the second and third natural frequencies, as shown in Figure (8), for the case presented in Figure (2), near the second veering zone, between the second and third natural frequencies.

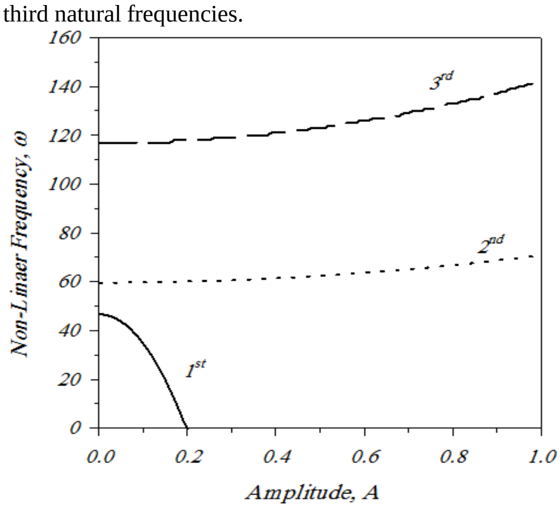

Figure 6. Variation of the non-linear natural frequencies ω with the Amplitude A for $K^*_{TL} = K^*_{TR} = 10^5$, $K^*_{RR} = K^*_{RL} = 100$, $K^*_f = 1$, $n = 1$ and $R = 6$

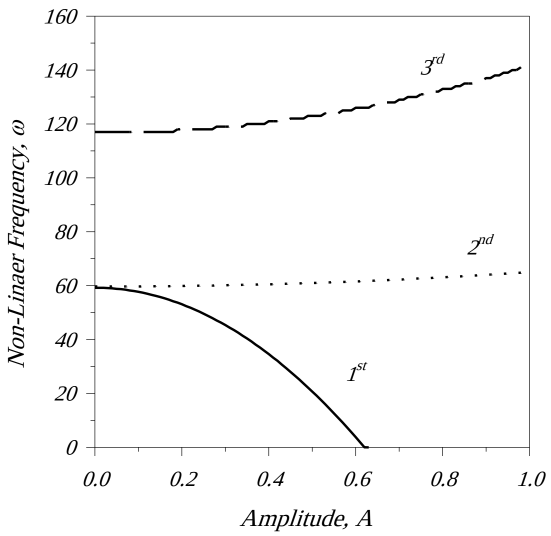

Figure 7. Variation of the non-linear natural frequencies ω with the Amplitude A for $K^*_{TL} = K^*_{TR} = 10^5$, $K^*_{RR} = K^*_{RL} = 100$, $K^*_f = 1$, $n = 1$ and $R = 8$

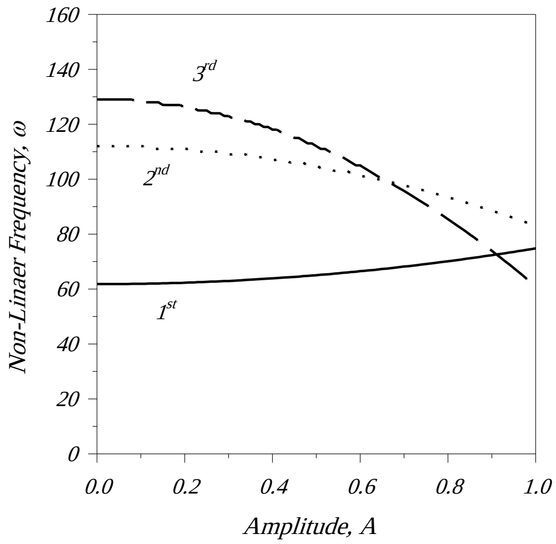

Figure 8. Variation of the non-linear natural frequencies ω with the Amplitude *A* for $K^*_{TL} = K^*_{TR} = K^*_{RR} = K^*_{RL} = \infty$, $K^*_f = 0$ $n = 1$ and $R = 17$

5. Conclusions

The present analyses of the nonlinear free vibration of a beam element, with flexible ends resting on elastic foundation with different rise shapes, show that the nonlinear natural frequency curves of the first three modes can exhibit a complex dynamic behavior which cannot be observed using the linear analysis. In addition to frequency veering phenomenon, these beam-like structures and, depending on system parameters, can exhibit crossover instabilities.

The obtained results indicate that the dynamic behavior is complicated enough and it needs a further and thorough analysis like the forced vibration and stability, which is beyond the scope of the present paper.

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