Optimal Batch Size Considering Partial Outsourcing Plan and Rework

Yuan-Shyi Peter Chiu a, Chen-Ju Liu b, Ming-Hon Hwang c*

aDepartment of Industrial Engineering & Management, Chaoyang University of Technology, Taichung 413, Taiwan
bDepartment of Business Administration, Chaoyang University of Technology, Taichung 413, Taiwan
cDepartment of Marketing & Logistics Management, Chaoyang University of Technology, Taichung 413, Taiwan

Received Oct 30, 2016 Accepted April 15, 2017

Abstract

Coping with the severe competition in the global markets and the limited in-house capacity, the management of present-day manufacturing firm always pursues possible alternatives to level production schedule by shortening production uptime, assure product quality, and reduce overall system cost in order to stay competitive. Inspired by this concept, the present study attempts to derive the optimal batch size for a fabrication system with outsourcing policy and rework. Mathematical modeling and optimization techniques are employed to explore the problem. As a result, a closed-form optimal batch size for the proposed system is determined. Besides, various critical system performance indicators, such as the break-even points of unit outsourcing cost, in-house defective rate, and unit reworking cost, etc., can now be revealed for assisting diverse managerial decision-makings.

Keywords: Production Batch Size, Outsourcing, Rework, Defective Items.

Appendix A

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>in-house production setup cost per cycle,</td>
</tr>
<tr>
<td>C</td>
<td>unit fabrication cost,</td>
</tr>
<tr>
<td>h</td>
<td>unit holding cost,</td>
</tr>
<tr>
<td>CR</td>
<td>unit reworking cost,</td>
</tr>
<tr>
<td>Kz</td>
<td>fixed outsourcing cost per cycle,</td>
</tr>
<tr>
<td>Cz</td>
<td>unit outsourcing cost,</td>
</tr>
<tr>
<td>β1</td>
<td>the linking variable between Kz and K, where Kz = [(1 + β1)K] and -1 &lt;= β1 &lt;= 0, where we assume that practically the in-house setup cost is relative higher than fixed outsourcing cost,</td>
</tr>
<tr>
<td>β2</td>
<td>the linking variable between Cr and C, where Cr = [(1 + β2)C] and β2 &gt;= 0,</td>
</tr>
<tr>
<td>Q</td>
<td>batch size per cycle – the decision variable of the proposed system,</td>
</tr>
<tr>
<td>Tr</td>
<td>the replenishment cycle time,</td>
</tr>
<tr>
<td>H1</td>
<td>the level of on-hand perfect quality inventory when in-house production finishes,</td>
</tr>
<tr>
<td>H2</td>
<td>the level of on-hand perfect quality inventory when rework process ends,</td>
</tr>
<tr>
<td>H</td>
<td>maximum level of on-hand perfect quality inventory when outsourcing items are received,</td>
</tr>
<tr>
<td>t1</td>
<td>production uptime if x = 0,</td>
</tr>
<tr>
<td>t2</td>
<td>reworking time if x = 0,</td>
</tr>
<tr>
<td>t3</td>
<td>production down time if x = 0,</td>
</tr>
<tr>
<td>T</td>
<td>cycle time if x = 0,</td>
</tr>
<tr>
<td>l(t)</td>
<td>= the level of on-hand perfect quality inventory at time t,</td>
</tr>
<tr>
<td>Id(t)</td>
<td>= the level of on-hand defective inventory at time t,</td>
</tr>
<tr>
<td>TC(Q)</td>
<td>= total operating cost per cycle,</td>
</tr>
<tr>
<td>E[TCU(Q)]</td>
<td>= the expected operating cost per unit time.</td>
</tr>
</tbody>
</table>

© 2017 Jordan Journal of Mechanical and Industrial Engineering. All rights reserved
system. Recursive algorithms were also developed for solving two- and three-stage systems. Studies related to various aspects of fabrication systems with reworking of nonconforming items can also be found elsewhere [8-13].

To cope with production capacity constraints, shortening production uptimes is often used by management as an alternative of leveling/smoothing production schedules/operations [14-18]. Coman and Ronen [14] proposed a linear programming model to study the outsourcing problem and identify relevant parameters and their relationships. They examined the model using both approaches of cost accounting and theory of constraints, and obtained various results to assist management in determining whether or not to outsource. With fewer computation requirements, they concluded that their model is simpler than other existing models. Amaral et al. [15] proposed various strategies for Original Equipment Manufacturers (OEMs) for deciding what activities to outsource and the ways to establish effective controls over the outsourcing procedure so that the business risks can be minimized. They pointed out that several renowned manufacturers had implemented these strategies. Kumar and Arbi [16] discussed how the implementation of right Information Technology (IT) systems and supply chain measures can help apparel manufacture reduce lead-times and total cost. Through a case study, they demonstrated that major components of customer lead-time (such as freight transportation time, order processing time and manufacturing time) can be minimized by an improved IT and logistics capabilities. Chiao et al. [17] developed a model to determine the outsourcing quantity decision for a single-manufacturer two-outsourcer system. Wherein, one outsourcer offers a lower unit outsourcing price, but whose supplies are of higher deteriorating rate; on the contrary, the other outsourcer's supplies are of lower deteriorating rate, but requires a higher outsourcing price. Through a case study by adopting the real production data from an ice cream company in Taiwan, they demonstrated that their model can assist manufacture in allocating the optimum quantity decision to each outsourcer in order to receive the highest profits. They also provided guidelines for allocation of outsourcing items to different outsourcers with different qualities and costs. Lee and Lan [18] examined an extended EPQ model with stochastic demand. To cope with uncertain demand rate and decrease the complexity of production planning, they assumed a fixed batch size policy along with an outsourcing of a secondary facility to supplement the fixed batch size policy. Shortage is permitted in their study, and the relevant cost parameters, including production setup cost, variable production cost, stock holding cost, backorder cost, and the outsourcing cost. Although global optimization on the batch size cannot be obtained, through extensive computational tests, they demonstrated that their model can significantly reduce the system cost as compared to that of classic lot-size policy. Studies related to various aspects of fabrication systems with outsourcing policy, manufacturing strategy and quality assurances can also be found elsewhere [19-29].

With the aim of shortening production uptime, assuring product quality, and reducing overall system cost, the present study proposes a decision model to determine the optimal batch size for a fabrication system with outsourcing policy and rework. Because little attention has been paid to the area of proposing a decision model to study joint effects of outsourcing policy and rework on the optimal production lot-size, the present paper is intended to bridge the gap.

2. The Proposed System and Formulations

Assumptions of the present proposed study include: (i) It is a deterministic fabrication system with a partial outsourcing policy, wherein both annual demand and production rates are steady, (ii) a random defective rate exists in the in-house fabrication system and all defective products are assumed to be repairable thru a rework process in the same cycle with extra cost, and (iii) a portion of the batch in each cycle is outsourced and the schedule of receiving outsourcing products is predetermined (i.e., in the end of rework process) and these products are promised to be of perfect quality.

The description of the proposed system with rework and a partial outsourcing plan is as follows. Consider a specific product can be made at $P$ units per year by a production system to meet a steady demand of $\lambda$ units per year. An $\alpha$ portion of repairable defective products may be randomly produced during the fabrication at a rate of $d$, thus $d = \alpha \lambda$. In each production cycle, the reworking of defective items starts right after the end of regular fabrication process, at a reworking rate $P_1$ units per year (see Figure 1). By not allowing shortage, we assume that $P$ must be larger than the sum of $\lambda$ and $d$, or $P - d - \lambda > 0$.

![Figure 1. The level of on-hand perfect quality inventory in the proposed system (in black) compared to that of the same system without outsourcing plan (in purple).](image)

Furthermore, a $\pi$ portion (where $0 < \pi < 1$) of the batch size $Q$ is outsourced in the proposed system, and all outsourcing items are of perfect quality (i.e., guaranteed by the contractor) and are scheduled to be received in the end of rework time, prior to delivery (Figure 1). It is noted when $\pi = 1$ the proposed system becomes a purchase system, and if $\pi = 0$ the proposed system comes to be an in-house production system. Other notation used in the proposed system is listed in Appendix A. The level of on-hand defective products in production and reworking times of the proposed system is depicted in Figure (2).
Figure 2. The level of on-hand defective products in production and reworking times of the proposed system

From Figures (1) and (2), the following formulas can be directly obtained:

\[ H_1 = (P - d - \lambda) t_{1\pi} \]  \hspace{1cm} (1)

\[ H_2 = H_1 + (P_1 - \lambda) t_{2\pi} \]  \hspace{1cm} (2)

\[ H = H_2 + \pi Q = \lambda t_{3\pi} \]  \hspace{1cm} (3)

\[ t_{1\pi} = \frac{H_1}{P - d - \lambda} = \frac{(1 - \pi) Q}{P} \]  \hspace{1cm} (4)

\[ t_{2\pi} = \frac{x[(1 - \pi)Q]}{P_1} \]  \hspace{1cm} (5)

\[ t_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \]  \hspace{1cm} (6)

\[ T_\pi = t_{1\pi} + t_{2\pi} + t_{3\pi} = \frac{Q}{\lambda} \]  \hspace{1cm} (7)

\[ dt_\pi = xP t_{1\pi} = x \left[ (1 - \pi) Q^2 \right] \]  \hspace{1cm} (8)

Total operating cost per cycle for the proposed system, \( TC(Q) \), consists of production setup cost, variable production cost, fixed outsourcing cost, variable outsourcing cost, reworking cost, holding cost for reworked items, holding cost for perfect quality and defective items in \( t_{1\pi}, t_{2\pi}, \) and \( t_{3\pi} \). Therefore, \( TC(Q) \) is as follows:

By substituting \( K\pi \) and \( C\pi \) in Equation (9), the operating cost per cycle for the proposed system \( TC(Q) \) becomes as follows:

We use the expected values of \( x \) in cost analysis to deal with randomness of defective rate \( x \). By substituting all related parameters from equations (1) to (8) into equation (10) and with further derivations, the expected operating cost per unit time for the proposed system \( E[TCU(Q)] \) can be obtained as follows:

\[ E[TCU(Q)] = \frac{E[TC(Q)]}{E[T]} = \frac{\lambda K}{Q} + \lambda (1 - \pi) C + \frac{\lambda (1 + \beta_1) K}{Q} + \lambda \pi \left[ (1 + \beta_2) C \right] + \lambda (1 - \pi) E[x] C_R \]

\[ + \frac{Q(h_1 - h)}{2} \left( \frac{\lambda E[x] (1 - \pi)^2}{P} \right) + \frac{hQ}{2} \left[ 1 - \lambda \left( \frac{1 - \pi^2}{P} \right) + \frac{\lambda E[x] (1 - \pi)}{P_1} (-2\pi) \right] \]  \hspace{1cm} (11)
3. Determining Optimal Batch Size

Upon obtaining the expected operating cost per unit time \( E[TCU(Q)] \), we apply the first and second derivatives of \( E[TCU(Q)] \) with respect to \( Q \) as follows:

\[
\frac{dE[TCU(Q)]}{dQ} = -\frac{\hat{\lambda} K}{Q} - \frac{\lambda}{Q^2} \left[ 1 + (1 - \pi) \beta_1 \right]
\]

\[
+ \frac{1}{2} \left( \frac{k - h}{P} \right) \hat{\lambda} \hat{E} \left[ \frac{1 - \pi}{1 - \pi^2} \right] + \frac{1}{2} \left( \frac{1 - \pi^2}{P} \right) \frac{\hat{\lambda}}{P} \hat{E} \left[ \frac{1}{1 - \pi} \right] (1 - 2\pi)
\]

\[
\frac{d^2E[TCU(Q)]}{dQ^2} = \frac{2\hat{\lambda} K}{Q^3} \left[ 1 + (1 + \beta) K \right]
\]

(12)

(13)

Since \( \lambda, Q(1 + \beta), \) and \( K \) are all positive, one confirms that equation (13) is positive. Hence, \( E[TCU(Q)] \) is a strictly convex function for all \( Q \) different from zero. It follows that to derive the optimal replenishment batch size \( Q \), one can set first derivative of \( E[TCU(Q)] \) equal to zero, and with further derivation the optimal batch size \( Q^* \) can be obtained as follows:

\[
Q^* = \left[ \frac{2\hat{\lambda} K (2 + \beta)}{\lambda h - \frac{\hat{\lambda}}{2} \left( \frac{k - h}{P} \right) \hat{E} \left[ \frac{1 - \pi}{1 - \pi^2} \right] + \frac{1}{2} \left( \frac{1 - \pi^2}{P} \right) \frac{\hat{\lambda}}{P} \hat{E} \left[ \frac{1}{1 - \pi} \right] (1 - 2\pi) \right]
\]

(14)

4. Numerical Example with Sensitivity Analysis

To show the applicability of the obtained result, a numerical example with sensitivity analysis is provided in this section for this purpose. Assume a fabrication system can produce a particular product at a rate of \( P = 20,000 \) units per year to meet its steady demand rate of \( \lambda = 4,000 \) units per year. A portion \( \pi = 0.4 \) of the replenishment batch size is outsourced in each cycle to cope with the limited production capacity. The production setup cost \( K = $5,000 \) and unit fabrication cost \( C = $100 \). For outsourcing fixed and variable costs, assuming the relating parameters \( \beta_1 = -0.7 \) and \( \beta_2 = 0.2 \), or \( K_0 = $1,500 \) and \( C_0 = $120 \). An \( x \) portion of repairable defective items may randomly produce, where \( x \) follows a uniform distribution over the range of \([0, 0.2]\). A rework process is adopted right after the end of regular production in each cycle, and it can repair the defective items at a rate of \( P_1 = 5,000 \) units per year with unit rework cost \( C_{R} = $60 \). Other system parameters also include holding cost \( h = $30 \) per unit per year and holding cost \( h_1 = $40 \) per reworked item per year.

Applying Equations (14) and (11), we obtain the optimal batch size \( Q^* = 1,477 \) and the optimal operating cost per unit time for the proposed system \( E[TCU(Q^*)] = $481,607 \). Applying Equation (11) with different values of \( \pi \), the behavior of \( E[TCU(Q)] \), with respect to different replenishment batch size \( Q \), can be obtained as displayed in Figure (3).

Similarly, applying Equation (11) with different values of \( \pi \), different cost components of \( E[TCU(Q)] \) with respect to \( Q \) are shown in Figure (4). It is noted that as the batch size \( Q \) increases, the total holding cost of perfect quality and reworked items increase significantly, and the rework cost rises slightly; however, the production setup cost declines significantly.

Figure 3. Behavior of \( E[TCU(Q)] \) with respect to \( Q \)

Figure 4. Different cost components of \( E[TCU(Q)] \) with respect to \( Q \)

Figure 5. Analysis of the breakeven point of defective rate in the proposed system
Applying Equation (11) with both $\pi = 0$ and $\pi = 1$, and different values of $C_R$, the analytical result of the breakeven point of ratio of rework cost $C_R/C$ for $\beta_2 = 0.1$ in the proposed system can be obtained, as shown in Figure (6). It is noted that a breakeven point of $C_R/C = 0.722$ is obtained for the 'make or buy' decision making. For example, if the ratio of rework cost $C_R/C$ is less than 0.722, then the decision of 'make' is in favor in terms of cost savings.

**Figure 6.** Analysis of the breakeven point of ratio of rework cost $C_R/C$ for $\beta_2 = 0.1$ in the proposed system

Applying Equation (11) with different values of $\pi$ and $\beta_2$, the joint effects of different $\pi$ and $\beta_2$ on the expected system cost $E[TCU(Q)]$ are found and displayed in Figure (7). It is noted that there is a breakeven point on $\beta_2$ (i.e., $C_R/C - 1$), when $\beta_2$ goes beyond the breakeven point, as $\pi$ increases, the expected system cost $E[TCU(Q)]$ rises significantly; and as $\beta_2$ increases, $E[TCU(Q)]$ also goes up significantly.

**Figure 7.** Joint effects of different ratios of $\pi$ and $\beta_2$ on the expected system cost $E[TCU(Q)]$

Applying Equation (11) with both $\pi = 0$ and $\pi = 1$, and different values $\beta_2$, the analytical result of the breakeven point of $\beta_2$ in the proposed system is obtained and depicted in Figure (8). It is noted that a breakeven point of $\beta_2 = 0.086$ is found for the 'make or buy' decision making. For instance, if $\beta_2$ (i.e., $C_R/C - 1$) is less than 0.086, the decision of 'buy' is in favor in terms of cost savings.

**Figure 8.** Analytical result of the breakeven point of $\beta_2$ in the proposed system

5. Conclusions

The present study developed an exact model for solving a replenishment batch size problem considering rework and a partial outsourcing plan. We not only successfully achieved the goal of deriving the optimal batch size that minimizes overall production and outsourcing costs, but also provided a decision-support-system type of tools to help managerial decision makings (refer to Figures (4) to (8)). For instances, various critical system performance indicators, such as the breakeven points of unit outsourcing cost (see Figure (8)), in-house defective rate (refer to Figure (5)), and unit reworking cost (see Figure (6)), etc., can now be revealed for assisting diverse decision-makings. One interesting direction for future studies will be incorporating a stochastic product demand rate into the present system and examining its effect on the optimal batch size.

Acknowledgements

Authors appreciate the Ministry of Science and Technology of Taiwan for sponsoring the present study under Grant No.: MOST 104-2410-H-324-008-MY2.

References


[23] R. Malhotra, G. Taneja, "Comparative study between a single unit system and a two-unit cold standby system with varying demand". SpringerPlus, Vol. 4, No. 1, 2015, article no. 705.


