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Optimal Batch Size Considering Partial Outsourcing Plan and Rework

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Abstract

Coping with the severe competition in the global markets and the limited in-house capacity, the management of present-day manufacturing firm always pursues possible alternatives to level production schedule by shortening production uptime, assure product quality, and reduce overall system cost in order to stay competitive. Inspired by this concept, the present study attempts to derive the optimal batch size for a fabrication system with outsourcing policy and rework. Mathematical modeling and optimization techniques are employed to explore the problem. As a result, a closed-form optimal batch size for the proposed system is determined. Besides, various critical system performance indicators, such as the breakeven points of unit outsourcing cost, in- house defective rate, and unit reworking cost, etc., can now be revealed for assisting diverse managerial decision-makings.

Keywords: *Production Batch Size, Outsourcing, Rework, Defective Items*.

Appendix A

K = in-house production setup cost per cycle,
C = unit fabrication cost,
h = unit holding cost,
CR = unit reworking cost,
$K\pi$ = fixed outsourcing cost per cycle,
$C\pi$ = unit outsourcing cost,
$\beta 1$ = the linking variable between $K\pi$ and K, where $K\pi = [(1 + \beta 1)K]$ and $-1 <= \beta 1 <= 0$, where we assume that practically the in-house setup cost is relative higher than fixed outsourcing cost,
$\beta 2$ = the linking variable between $C\pi$ and C, where $C\pi = [(1 + \beta 2)C]$ and $\beta 2 >= 0$,
Q = batch size per cycle – the decision variable of the proposed system,
$T\pi$ = the replenishment cycle time,
H1 = the level of on-hand perfect quality inventory when in-house production finishes,
H2 = the level of on-hand perfect quality inventory when rework process ends,
H = maximum level of on-hand perfect quality inventory when outsourcing items are received,
t1 = production uptime if $\pi = 0$,
t2 = reworking time if $\pi = 0$,
t3 = production down time if $\pi = 0$,
T = cycle time if $\pi = 0$,
I(t) = the level of on-hand perfect quality inventory at time t,
Id(t) = the level of on-hand defective inventory at time t,
TC(Q) = total operating cost per cycle,
E[TCU(Q)] = the expected operating cost per unit time.

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1. Introduction

The present study determines the optimal batch size for a fabrication system with outsourcing policy and rework, with the aim of leveling production schedule by shortening production uptime, assuring product quality, and reducing overall system cost. The classic economic production lot-size problem [1] assumed a perfect production process and solved by using mathematical model which balanced the setup and stock holding costs in order to derive the optimal lot-size that minimizes the long-run total system costs. In most real manufacturing environments, however, caused by various controllable/uncontrollable factors, production of defective products is inevitable [2-5]. Reworking of the imperfect quality items could be an alternative to lower overall production cost [6-7]. Chern and Yang [6] assumed that a work center produces good, rework-able, and scrap items, and it adopts a threshold control policy to handle regular process and rework job. Guidelines were established for the machine in work center to switch between regular and rework processes. Their objective was to simultaneously find the optimal threshold and lot-size that maximize the expected long-term profit. Grosfeld-Nir and Gerchak [7] studied multistage fabrication systems where defective products can be repeatedly reworked at every stage. Random yield in each stage is assumed; hence, multiple production runs may be needed to meet the desired demand of the product. They explored and discussed how the studied multistage fabrication systems can be turned into a single-bottleneck system. Recursive algorithms were also developed for solving two- and three-stage systems. Studies related to various aspects of fabrication systems with reworking of nonconforming items can also be found elsewhere [8-13].

To cope with production capacity constraints, shortening production uptimes is often used by management as an alternative of leveling/smoothing production schedules/operations [14-18]. Coman and Ronen [14] proposed a linear programming model to study the outsourcing problem and identify relevant parameters and their relationships. They examined the model using both approaches of cost accounting and theory of constraints, and obtained various results to assist management in determining whether or not to outsource. With fewer computation requirements, they concluded that their model is simpler than other existing models. Amaral *et al*. [15] proposed various strategies for Original Equipment Manufacturers (OEMs) for deciding what activities to outsource and the ways to establish effective controls over the outsourcing procedure so that the business risks can be minimized. They pointed out that several renowned manufacturers had implemented these strategies. Kumar and Arbi [16] discussed how the implementation of right Information Technology (IT) systems and supply chain measures can help apparel manufacture reduce lead-times and total cost. Through a case study, they demonstrated that major components of customer lead-time (such as freight transportation time, order processing time and manufacturing time) can be minimized by an improved IT and logistics capabilities. Chiao *et al*. [17] developed a model to determine the outsource quantity decision for a single-manufacturer two-outsourcer system. Wherein, one outsourcer offers a lower unit outsourcing price, but whose supplies are of higher deteriorating rate; on the contrary, the other outsourcer's supplies are of lower deteriorating rate, but requires a higher outsourcing price. Through a case study by adopting the real production data from an ice cream company in Taiwan, they demonstrated that their model can assist manufacture in allocating the optimal quantity decision to each outsourcer in order to receive the highest profits. They also provided guidelines for allocation of outsourcing items to different outsourcers with different qualities and costs. Lee and Lan [18] examined an extended EPQ model with stochastic demand. To cope with uncertain demand rate and decrease the complexity of production planning, they assumed a fixed batch size policy along with an outsourcing of a secondary facility to supplement the fixed batch size policy. Shortage is permitted in their study, and the relevant cost parameters, including production setup cost, variable production cost, stock holding cost, backorder cost, and the outsourcing cost. Although global optimization on the batch size cannot be obtained, through extensive computational tests, they demonstrated that their model can significantly reduce the system cost as compared to that of classic lot-size policy. Studies related to various aspects of fabrication systems with outsourcing policy, manufacturing strategy and quality assurances can also be found elsewhere [19-29].

With the aim of shortening production uptime, assuring product quality, and reducing overall system cost, the present study proposes a decision model to determine the optimal batch size for a fabrication system with outsourcing policy and rework. Because little attention has been paid to the area of proposing a decision model to study joint effects of outsourcing policy and rework on the optimal production lot-size, the present paper is intended to bridge the gap.

2. The Proposed System and Formulations

Assumptions of the present proposed study include: (i) It is a deterministic fabrication system with a partial outsourcing policy, wherein both annual demand and production rates are steady, (ii) a random defective rate exists in the in-house fabrication system and all defective products are assumed to be repairable thru a rework process in the same cycle with extra cost, and (iii) a portion of the batch in each cycle is outsourced and the schedule of receiving outsource products is predetermined (i.e., in the end of rework process) and these products are promised to be of perfect quality.

The description of the proposed system with rework and a partial outsourcing plan is as follows. Consider a specific product can be made at P units per year by a production system to meet a steady demand of λ units per year. An x portion of repairable defective products may be randomly produced during the fabrication at a rate of d, thus $d = Px$. In each production cycle, the reworking of defective items starts right after the end of regular fabrication process, at a reworking rate P_1 units per year (see Figure 1). By not allowing shortage, we assume that P must be larger than the sum of λ and d, or $P - d - \lambda > 0$.

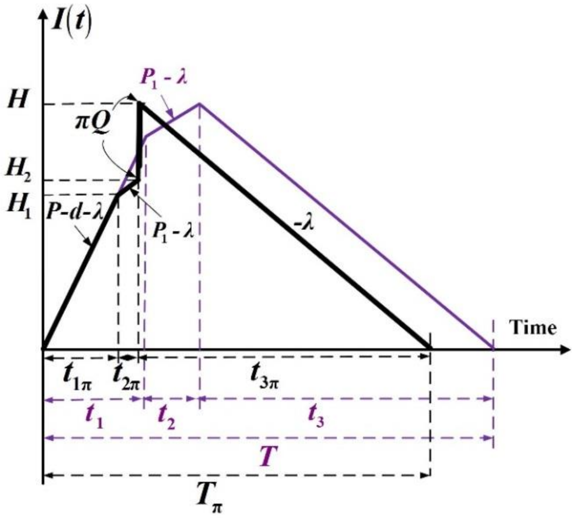

Figure 1. The level of on-hand perfect quality inventory in the proposed system (in black) compared to that of the same system without outsourcing plan (in purple)

Furthermore, a π portion (where $0 < \pi < 1$) of the batch size Q is outsourced in the proposed system, and all outsourcing items are of perfect quality (i.e., guaranteed by the contractor) and are scheduled to be received in the end of rework time, prior to delivery (Figure 1). It is noted when $\pi = 1$ the proposed system becomes a purchase system, and if $\pi = 0$ the proposed system comes to be an in-house production system. Other notation used in the proposed system is listed in Appendix A. The level of on-hand defective products in production and reworking times of the proposed system is depicted in Figure (2).

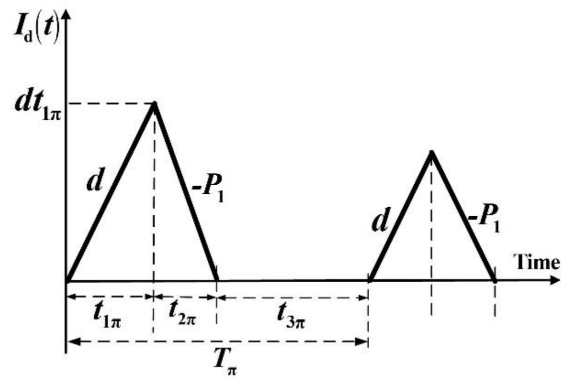

Figure 2. The level of on-hand defective products in production and reworking times of the proposed system

From Figures (1) and (2), the following formulas can be directly obtained:

$$H_1 = \left(P - d - \lambda\right) t_{1\pi} \tag{1}$$

$$H_2 = H_1 + \left(P_1 - \lambda\right) t_{2\pi} \tag{2}$$

$$H = H_2 + \pi Q = \lambda t_{3\pi} \tag{3}$$

$$t_{1\pi} = \frac{H_1}{P - d - \lambda} = \frac{\left(1-\pi\right)Q}{P} \tag{4}$$

$$t_{2\pi} = \frac{x\left[\left(1-\pi\right)Q\right]}{P_1} \tag{5}$$

$$t_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \tag{6}$$

$$T_\pi = t_{1\pi} + t_{2\pi} + t_{3\pi} = \frac{Q}{\lambda} \tag{7}$$

$$dt_{1\pi} = xPt_{1\pi} = x\left[\left(1-\pi\right)Q\right] \tag{8}$$

Total operating cost per cycle for the proposed system, $TC(Q)$, consists of production setup cost, variable production cost, fixed outsourcing cost, variable outsourcing cost, reworking cost, holding cost for reworked items, holding cost for perfect quality and defective items in $t_{1\pi}$, $t_{2\pi}$, and $t_{3\pi}$. Therefore, $TC(Q)$ is as follows:

By substituting Kπ and Cπ in Equation (9), the operating cost per cycle for the proposed system TC(Q) becomes as follows:

We use the expected values of x in cost analysis to deal with randomness of defective rate x. By substituting all related parameters from equations (1) to (8) into equation (10) and with further derivations, the expected operating cost per unit time for the proposed system E[TCU(Q)]can be obtained as follows:

$$\begin{aligned} TC\left(Q\right) &= K + C\left[\left(1-\pi\right)Q\right] + K_\pi + C_\pi\left(\pi Q\right) + C_R x\left[\left(1-\pi\right)Q\right] + h_1\frac{dt_{1\pi}}{2}\left(t_{2\pi}\right) \\ &\quad + h\left[\frac{H_1 + dt_{1\pi}}{2}\left(t_{1\pi}\right) + \frac{H_1 + H_2}{2}\left(t_{2\pi}\right) + \frac{H}{2}\left(t_{3\pi}\right)\right] \end{aligned} \tag{9}$$

$$\begin{aligned} TC\left(Q\right) &= K + C\left[\left(1-\pi\right)Q\right] + \left(1+\beta_1\right)K + \left(1+\beta_2\right)C\left(\pi Q\right) + C_R x\left[\left(1-\pi\right)Q\right] \\ &\quad + h_1\frac{dt_{1\pi}}{2}\left(t_{2\pi}\right) + h\left[\frac{H_1 + dt_{1\pi}}{2}\left(t_{1\pi}\right) + \frac{H_1 + H_2}{2}\left(t_{2\pi}\right) + \frac{H}{2}\left(t_{3\pi}\right)\right] \end{aligned} \tag{10}$$

$$\begin{aligned} E\left[TCU\left(Q\right)\right] &= \frac{E\left[TC\left(Q\right)\right]}{E\left[T\right]} = \frac{\lambda K}{Q} + \lambda\left(1-\pi\right)C + \frac{\lambda\left[\left(1+\beta_1\right)K\right]}{Q} + \lambda\pi\left[\left(1+\beta_2\right)C\right] + \lambda\left(1-\pi\right)E\left[x\right]C_R \\ &\quad + \frac{Q\left(h_1 - h\right)}{2}\left(\frac{\lambda E\left[x\right]^2\left(1-\pi\right)^2}{P_1}\right) + \frac{hQ}{2}\left[1 - \lambda\left(\frac{1-\pi^2}{P}\right) + \frac{\lambda E\left[x\right]\left(1-\pi\right)}{P_1}\left(-2\pi\right)\right] \end{aligned} \tag{11}$$

3. Determining Optimal Batch Size

Upon obtaining the expected operating cost per unit time E[*TCU*(*Q*)], we apply the first and second derivatives of *E[TCU(Q)]* with respect to *Q* as follows:

$$\frac{dE\left[TCU(Q)\right]}{dQ}=-\frac{\lambda}{Q^2}K-\frac{\lambda}{Q^2}\left[(1+\beta_1)K\right]+\frac{(h_1-h)}{2}\left[\frac{\lambda E[x]^2(1-\pi)^2}{P_1}\right]+\frac{h}{2}\left[1-\lambda\left(\frac{1-\pi^2}{P}\right)+\frac{\lambda E[x](1-\pi)}{P_1}(-2\pi)\right] \tag{12}$$

$$\frac{d^2E\left[TCU(Q)\right]}{dQ^2}=\frac{2\lambda}{Q^3}\left[K+(1+\beta_1)K\right] \tag{13}$$

Since λ, Q,$(1+\beta_1)$, and K are all positive, one confirms that equation (13) is positive. Hence, *E[TCU(Q)]* is a strictly convex function for all *Q* different from zero. It follows that to derive the optimal replenishment batch size *Q*, one can set first derivative of *E[TCU(Q)]*equal to zero, and with further derivation the optimal batch size *Q** can be obtained as follows:

$$Q^*=\sqrt{\frac{2\lambda K(2+\beta_1)}{(h_1-h)\left[\frac{E[x]^2(1-\pi)^2\lambda}{P_1}\right]+h\left[1-\lambda\left(\frac{1-\pi^2}{P}\right)+\frac{\lambda E[x](1-\pi)}{P_1}(-2\pi)\right]}} \tag{14}$$

4. Numerical Example with Sensitivity Analysis

To show the applicability of the obtained result, a numerical example with sensitivity analysis is provided in this section for this purpose. Assume a fabrication system can produce a particular product at a rate of P = 20,000 units per year to meet its steady demand rate of λ= 4,000units per year. A portion π = 0.4 of the replenishment batch size is outsourced in each cycle to cope with the limited production capacity. The production setup cost K =\$5,000 and unit fabrication cost C = \$100. For outsourcing fixed and variable costs, assuming the relating parameters β_1 = –0.7 and β_2 =0.2, or K_π = \$1,500 and C_π = \$120. An x portion of repairable defective items may randomly produce, where x follows a uniform distribution over the range of [0, 0.2]. A rework process is adopted right after the end of regular production in each cycle, and it can repair the defective items at a rate of P_1= 5,000 units per year with unit rework cost C_R = \$60.Other system parameters also include holding cost h= \$30per unit per year and holding cost h_1 = \$40 per reworked item per year.

Applying Equations (14) and (11), we obtain the optimal batch size Q^*=1,477 and the optimal operating cost per unit time for the proposed system $E[TCU(Q^*)]$ = \$481,607. Applying Equation (11) with different values of Q, the behavior of $E[TCU(Q)]$, with respect to different replenishment batch size Q, can be obtained as displayed in Figure (3).

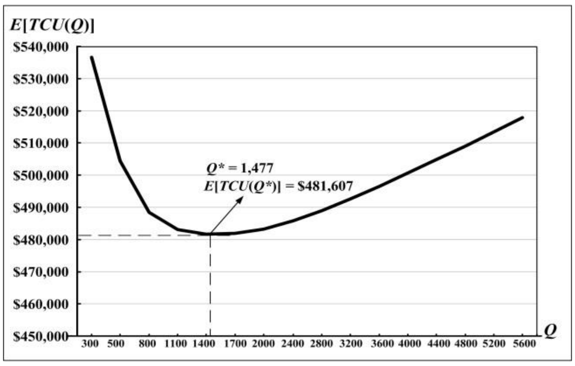

Figure 3. Behavior of $E[TCU(Q)]$ with respect to Q

Similarly, applying Equation (11) with different values of Q, different cost components of $E[TCU(Q)]$ with respect to Q are shown in Figure (4). It is noted that as the batch size Q increases, the total holding cost of perfect quality and reworked items increase significantly, and the rework cost rises slightly; however, the production setup cost declines significantly.

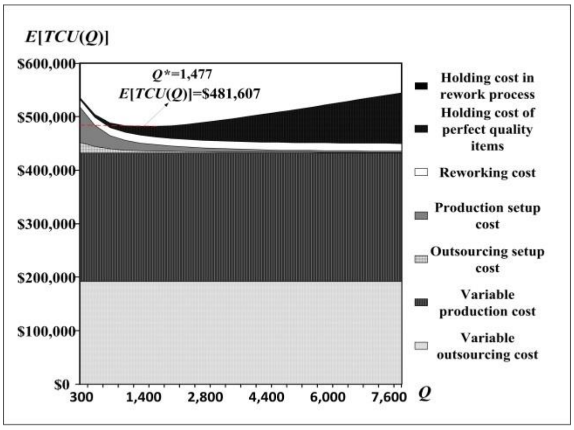

Figure 4. Different cost components of $E[TCU(Q)]$ with respect to Q

Applying Equation (11) with both π = 0 and π = 1, and different values of x, the analytical result of the breakeven point of defective rate x in the proposed system can be found as depicted in Figure (5). It is noted that a breakeven point of x = 0.24 is found for the 'make or buy' decision making. For instance, if the random defective rate is less than 0.24, then the decision of 'make' is in favor in terms of cost savings.

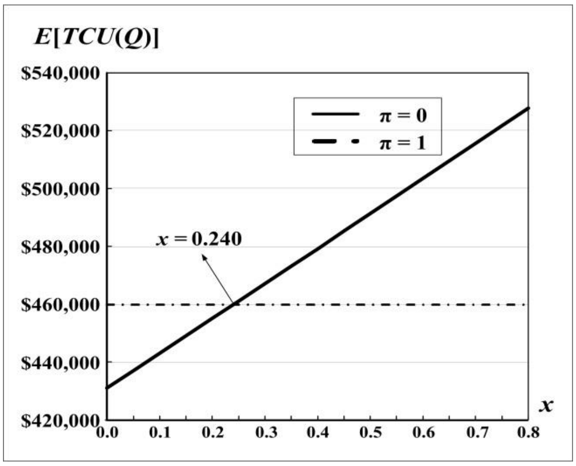

Figure 5. Analysis of the breakeven point of defective rate in the proposed system

Applying Equation (11) with both $\pi = 0$ and $\pi = 1$, and different values of C_R, the analytical result of the breakeven point of ratio of rework cost C_R/C for $\beta_2 = 0.1$ in the proposed system can be obtained, as shown in Figure (6). It is noted that a breakeven point of $C_R/C = 0.722$ is obtained for the 'make or buy' decision making. For example, if the ratio of rework cost C_R/C is less than 0.722, then the decision of 'make' is in favor in terms of cost savings.

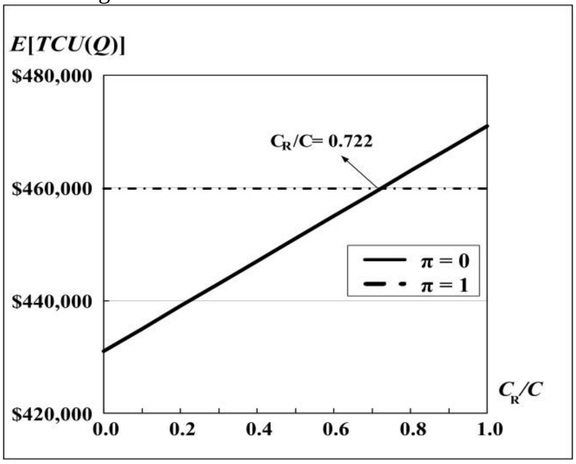

Figure 6. Analysis of the breakeven point of ratio of rework cost C_R/C for $\beta_2 = 0.1$ in the proposed system

Applying Equation (11) with different values of π and β_2, the joint effects of different π and β_2 on the expected system cost $E[TCU(Q)]$ are found and displayed in Figure (7). It is noted that there is a breakeven point on β_2 (i.e., C_π/C -1), when β_2 goes beyond the breakeven point, as π increases, the expected system cost $E[TCU(Q)]$ rises significantly; and as β_2 increases, $E[TCU(Q)]$ also goes up significantly.

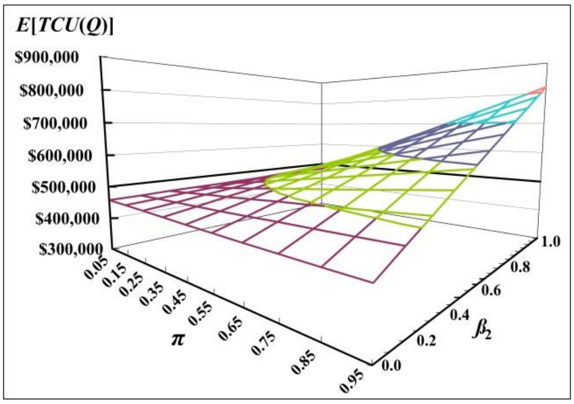

Figure 7. Joint effects of different ratios of π and $\beta_2$on the expected system cost $E[TCU(Q)]$

Applying Equation (11) with both $\pi = 0$ and $\pi = 1$, and different values β_2, the analytical result of the breakeven point of β_2 in the proposed system is obtained and depicted in Figure (8). It is noted that a breakeven point of β_2= 0.086 is found for the 'make or buy' decision making. For instance, if β_2 (i.e., C_π/C - 1) is less than 0.086; thus, the decision of 'buy' is in favor in terms of cost savings.

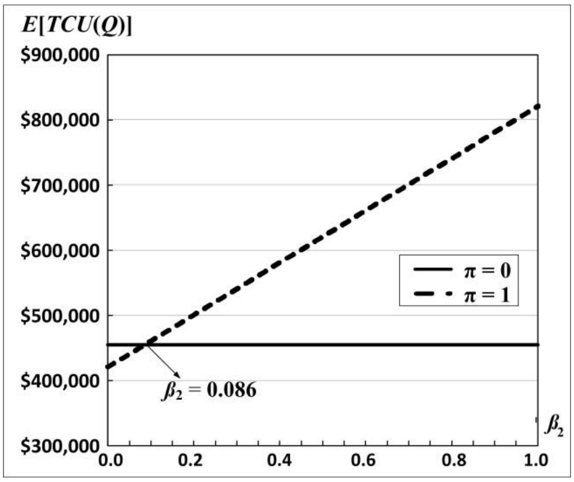

Figure 8. Analytical result of the breakeven point of β_2 in the proposed system

5. Conclusions

The present study developed an exact model for solving a replenishment batch size problem considering rework and a partial outsourcing plan. We not only successfully achieved the goal of deriving the optimal batch size that minimizes overall production and outsourcing costs, but also provided a decision-support-system type of tools to help managerial decision makings (refer to Figures (4) to (8)). For instances, various critical system performance indicators, such as the breakeven points of unit outsourcing cost (see Figure (8)), in-house defective rate (refer to Figure (5)), and unit reworking cost (see Figure (6)), etc., can now be revealed for assisting diverse decision-makings. One interesting direction for future studies will be incorporating a stochastic product demand rate into the present system and examining its effect on the optimal batch size.

Acknowledgements

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References

[1] E. W. Taft, "The most economical production lot". Iron Age, Vol.**101**, 1918, 1410–1412.

[2] W. Shih, "Optimal inventory policies when stock-outs result from defective products". *International Journal of Production Research*, Vol. 18, No.6, 1980, 677-686.

[3] M. Henig, Y. Gerchak, "Structure of periodic review policies in the presence of random yield". Operations Research, Vol. 38, No. 4, 1990, 634-643.

[4] T. Boone, R. Ganeshan, Y. Guo, J.K. Ord, "The impact of imperfect processes on production run times". Decision Sciences, Vol. 31, No. 4, 2000, 773-785.

[5] Y-S. P. Chiu, K-W. Chiang, S. W. Chiu, M-S. Song, "Simultaneous determination of production and shipment decisions for a multi-product inventory system with a rework process". Advances in Production Engineering & Management, Vol. 11, No. 2, 2016, 141-151.

[6] C-C. Chern, P. Yang, "Determining a Threshold Control Policy for an Imperfect Production System with Rework Jobs". Naval Research Logistics, Vol. 46, No. 2-3, 1999, 273-301.

[7] A. Grosfeld-Nir, Y. Gerchak, "Multistage production to order with rework capability". Management Science, Vol. 48, No. 5, 2002, 652-664.

[8] B. J. Yum, E. D. McDowell, "Optimal Inspection Policies in a Serial Production System including scrap, rework and repair: An MILP approach". International Journal of Production Research, Vol. 25, No. 10, 1987, 1451-1464.

[9] P. Biswas, B.R. Sarker, "Optimal batch quantity models for a lean production system with in-cycle rework and scrap". International Journal of Production Research, Vol. 46, No. 23, 2008, 6585-6610.

[10] A. Coman, B. Ronen, "Production outsourcing: A linear programming model for the theory-of-constraints". International Journal of Production Research, Vol. 38, No. 7, 2000, 1631-1639.

[11] S. W. Chiu, C-C. Huang, K-W. Chiang, M-F. Wu, "On intra-supply chain system with an improved distribution plan, multiple sales locations and quality assurance". SpringerPlus, Vol. 4, No. 1, 2015, Article No. 687.

[12] Y-S. P. Chiu, P-C. Sung, S. W. Chiu, C-L. Chou, "Mathematical modeling of a multi- product EMQ model with an enhanced end items issuing policy and failures in rework". SpringerPlus, Vol. 4, No. 1, 2015, Article No. 679.

[13] P. Jawla, S. R. Singh, "Multi-item economic production quantity model for imperfect items with multiple production setups and rework under the effect of preservation technology and learning environment". International Journal of Industrial Engineering Computations, Vol. 7, No. 4, 2016, 703-716.

[14] A. Coman, B. Ronen, "Production outsourcing: A linear programming model for the theory-of-constraints". International Journal of Production Research, Vol. 38, No. 7, 2000, 1631-1639.

[15] J. Amaral, C.A. Billington, A.A. Tsay, "Safeguarding the promise of production outsourcing". Interfaces, Vol. 36, No. 3, 2006, 220-233.

[16] S. Kumar, A.S. Arbi, "Outsourcing strategies for apparel manufacture: A case study". Journal of Manufacturing Technology Management, Vol. 19, No. 1, 2008, 73-91.

[17] H-E. Chiao, H-M. Wee, M.V. Padilan, "A model to outsource deteriorating items using two outsourcers with different deteriorating rates and costs". International Journal of Computer Integrated Manufacturing, Vol. 25, No. 6, 2012, 536-549.

[18] S-D. Lee, S-C. Lan, "Production lot sizing with a secondary outsourcing facility". International Journal of Production Economics, Vol. 141, No. 1, 2013, 414-424.

[19] B. Leavy, "Outsourcing strategy and a learning dilemma". Production and Inventory Management Journal, Vol. 37, No. 4, 1996, 50-54.

[20] G.P. Cachon, P.T. Harker, "Competition and outsourcing with scale economies". Management Science, Vol. 48, No. 10, 2002, 1314-1333.

[21] S. Narayanan, V. Jayaraman, Y. Luo, J.M. Swaminathan, "The antecedents of process integration in business process outsourcing and its effect on firm performance". Journal of Operations Management, Vol. 29, No. 1-2, 2011, 3-16.

[22] K. R. Balachandran, H.W. Wang, S-H. Li, T. Wang, "In-house capability and supply chain decisions". Omega, Vol. 41, No. 2, 2013, 473-484.

[23] R. Malhotra, G. Taneja, "Comparative study between a single unit system and a two-unit cold standby system with varying demand". SpringerPlus, Vol. 4, No. 1, 2015, article no. 705.

[24] G.J. Hahn, T. Sens, CDecouttere, N.J. Vandaele, "A multi-criteria approach to robust outsourcing decision-making in stochastic manufacturing systems". Computers & Industrial Engineering, Vol. 98, 2016, 275-288.

[25] J. Gómez, I. Salazar, P. Vargas, "Sources of information as determinants of product and process innovation". PLoS ONE, Vol. 11, No. 4, 2016, article no. e0152743.

[26] C. Ma, X. Liu, H. Zhang, Y. Wu, "A green production strategies for carbon-sensitive products with a carbon cap policy". Advances in Production Engineering & Management, Vol. 11, No. 3, 2016, 216-226.

[27] V.V.S. Prasad Patnaik, D.P. Patnaik, R.K. Srinivasa, "A Numerical Study on Deterministic Inventory Model for Deteriorating Items with Selling Price Dependent Demand and Variable Cycle Length". Jordan Journal of Mechanical and Industrial Engineering, Vol. 9, No. 3, 2015, 223-240.

[28] L. Ocampo, E. Clark, "A Sustainable Manufacturing Strategy Decision Framework in the Context of Multi-Criteria Decision-Making. Jordan Journal of Mechanical and Industrial Engineering, Vol. 9, No. 3, 2015, 177-186.

[29] H. Kaylani, A. Almuhtady, A. M. Atieh, "Novel approach to enhance the performance of production systems using lean tools". Jordan Journal of Mechanical and Industrial Engineering, Vol. 10, No. 3, 2016, 215-229.