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Buckling Analysis of Nonlocal Embedded Shear Deformable Functionally Graded Piezoelectric Nanoscale Beams

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Abstract

Buckling behavior of nonlocal Functionally Graded Piezoelectric (FGP) beams on elastic substrate via a higher order beam model is examined. The present beam model takes into consideration the shear deformation effects needless of any shear correction factor. Employing power-law function, the gradation of material properties of the beam is described. Incorporation of small scale parameter is carried out using nonlocal elasticity theory. Implementing an analytical approach which satisfies simply-supported boundary conditions, the governing equations derived from Hamilton's principle are solved. The obtained results are compared with those provided in the literature. It is indicated that the buckling behavior of piezoelectric Nanobeams is significantly influenced by elastic foundation parameters, external voltage, scale parameter, power-law index and slenderness ratio.

Keywords: *Functionally graded piezoelectric nanobeam; Elastic foundation; Buckling; Nonlocal elasticity theory; Third-order beam theory.*

1. Introduction

Functionally Graded Materials (FGMs) are well known as an alternative materials, which are extensively applied in chemical, mechanical, electronic, civil, automotive and optical industries due to possessing supreme mechanical performance compared to classical composite materials. FGMs have microscopically heterogeneous structure, and material properties change with continuous composition gradation from one surface to another [1-9]. Nowadays, functionally graded materials have been extensively utilized in micro-electromechanical and nano-electromechanical systems and devices. On the other hand, microstructures, such as micro and nano scale beams are key components in many structures so that of mechanical behavior analysis of micro/nano beams received striking attention from research communities [10]. The classical continuum theory can properly applied in the mechanical analysis of the macroscopic structures, but it is unable to consider the size effect on the mechanical behaviors on micro/nano structures. While, due to the classical continuum theory is scale-independent, it needs to incorporate a higher-order continuum theory to capture size effects.

Thus, this can be attained by the nonlocal elasticity theory proposed by Eringen [11-13] in which a stress state at a reference point is suggested as a function of the strain of all neighbor points. It is noted that some studies are published on mechanical behavior of size-dependent FG beams. As one the first attempt to study this case, Simsek and Yurtcu [14] investigated bending and buckling behavior of size-dependent FG nanobeam using analytical method and Timoshenko and Euler-Bernoulli beam models. Also, the static and stability behavior of FG nanobeams based on nonlocal continuum theory is studied by Eltaher *et al.* [15]. Nonlinear free vibration of functionally graded nanobeams within the framework of Euler–Bernoulli beam model including the von Kármán geometric nonlinearity studied by Sharabiani and Yazdi [16]. Also, forced vibration analysis of Functionally Graded (FG) nanobeams based on the nonlocal elasticity theory and using Navier method for various shear deformation theories studied by Uymaz [17]. Zemri *et al.* [18] investigated the size effects on vibration of nanobeams based on nonlocal refined beam model. Nonlinear free vibration of FG nanobeams with fixed ends, i.e., simply supported-simply supported (SS) and simply Supported-Clamped (SC), using the nonlocal elasticity within the frame work of EBT with von kármán type nonlinearity is studied by Nazemnezhad and Hosseini-Hashemi [19]. Also, Ebrahimi et al. [20, 21] examined the applicability of differential transformation method in investigations on vibrational characteristics of FG size-dependent nanobeams. In another work, Ebrahimi and Salari [22] presented a semi-analytical method for vibrational and buckling analysis of FG nanobeams considering the position of neutral axis. An exact solution for the nonlinear forced vibration of functionally graded nanobeams in thermal environment based on surface elasticity theory is presented by Ansari *et al.* [23]. Recently, Rahmani and Jandaghian [24] presented Buckling analysis of functionally graded nanobeams based on a nonlocal third-order shear deformation theory. Also, vibration behavior of functionally graded nanoscale plates using a zeroth-order theory is examined by Bounouara *et al.* [25]. Vibration of Axially Functionally Graded

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Material (AFGM) nanobeam is investigated by Zeighampour and Beni [26] using strain gradient theory.

Piezoelectric materials are certain class of solid materials which convert mechanical load to electrical energy and vice versa. Such materials are applied in a wide range of systems and devices, such as sensors, actuators, speakers and timing devices. So, the application of piezoelectric materials for the vibration diminution and shape control is a useful tool in smart devices and systems design [27]. Mechanical responses of a FGP cantilever beam under loads are investigated by Shi and Chen [28]. Doroushi *et al*. [29] investigated the free and forced vibration characteristics of an FGPM beam subjected to thermo-electro-mechanical loads using the higher-order shear deformation beam theory. Kiani, Y. *et al*. [30] analyzed buckling behavior of functionally graded material (FGM) beams with or without surface-bonded piezoelectric layers subjected to both thermal loading and constant voltage. Komijani *et al*. [31] studied free vibration of Functionally Graded Piezoelectric Material (FGPM) beams with rectangular cross sections under in-plane thermal and electrical excitations in pre/post-buckling regimes. Lezgy-Nazargah *et al*. [32] suggested an efficient three-nodded beam element model for static, free vibration and dynamic response of functionally graded piezoelectric material beams. Large amplitude free flexural vibration of shear deformable Functionally Graded Materials (FGMs) beams with surface-bonded piezoelectric layers subjected to thermo-piezoelectric loadings with random material properties presented by Shegokar and Lal [33]. Therefore it could be noted that the most important imperfection of above researches is that the small size effects is not considered in these studies. A few studies are conducted to analyze mechanical behavior of size-dependent FGP beams. Among them, vibration behavior of piezoelectric microbeams is studied by Ansari *et al*. [34] on the basis of the modified couple stress theory. Also, Sahmani and Bahrami [35] investigated size-dependent dynamic stability analysis of microbeams actuated by piezoelectric voltage based on strain gradient theory.

Electro-mechanical buckling problem of functionally graded piezoelectric nanobeams supported by Winkler–Pasternak elastic foundation subjected to electric voltage is analyzed via the higher order beam model. The electro-mechanical material properties of the beam are supposed to be graded in the thickness direction according to the power law distribution. The small size effect is captured using Eringen's nonlocal elasticity theory. Coupled governing equations for the buckling of embedded FGP nanobeams have been derived via Hamilton's principle and they are solved using Navier type method. Finally, several numerical results are provided investigating the influences of elastic foundation, external electric voltage, nonlocal parameter, power-law index and slenderness ratio on buckling behavior of embedded FGP nanobeams.

2. Theoretical Formulations

2.1. The Material Properties of FGP Nanobeams

Assume a functionally graded nanobeam composed of PZT-4 and PZT-5H piezoelectric materials exposed to an electric potential $\Phi(x,z,t)$, with length L and uniform thickness h, as shown in Figure 1. The effective material properties of the FGPM nanobeam are supposed to change continuously in the *z*-axis direction (thickness direction) based on the power-law model. So, the effective material properties, *P*, can be stated in the following form [1]:

$$P = P_2 V_2 + P_1 V_1 \tag{1}$$

In which P_1 and P_2 denote the material properties of the bottom and higher surfaces, respectively. Also V_1 and V_2 are the corresponding volume fractions related by:

$$V_2 = (\frac{z}{h}+\frac{1}{2})^p, \quad V_1 = 1 - V_2 \tag{2}$$

Therefore, according to Eqs. (1) and (2), the effective electro-mechanical material properties of the FGP beam is defined as:

$$P(z) = (P_2 - P_1)\left(\frac{z}{h}+\frac{1}{2}\right)^p + P_1 \tag{3}$$

where p is power-law exponent which is t non-negative and estimates the material distribution through the thickness of the nanobeam and *z* is the distance from the mid-plane of the graded piezoelectric beam. It must be noted that, the top surface at $z = +h/2$ of FGP nanobeam is assumed PZT-4 rich, whereas the bottom surface ($z = -h/2$) is PZT-5H rich.

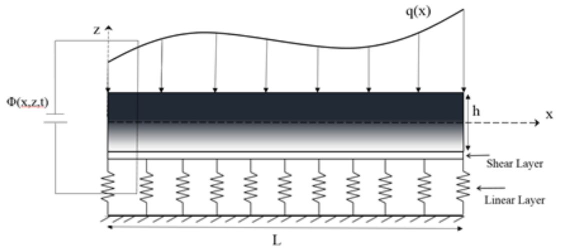

Figure1. Configuration of an embedded functionally graded piezoelectric nanobeam.

2.2. Nonlocal Elasticity Theory for the Piezoelectric Materials

Contrary to the constitutive equation of classical elasticity theory, Eringen's nonlocal theory notes that the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For a nonlocal homogeneous piezoelectric solid the basic equations with zero body force may be defined as:

$$\sigma_{ij} = \int_V \alpha\left(|x'-x|,\tau\right)\left[C_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x')\right]dV(x') \tag{4a}$$

$$D_i = \int_V \alpha\left(|x'-x|,\tau\right)\left[e_{ikl}\varepsilon_{kl}(x') + k_{ik}E_k(x')\right]dV(x') \tag{4b}$$

where σ_{ij}, ε_{ij}, D_i and E_i denote the stress, strain, electric displacement and electric field components, respectively; C_{ijkl}, e_{kij} and k_{ik} are elastic, piezoelectric and dielectric constant, respectively; $\alpha\left(|x'-x|,\tau\right)$ is the nonlocal kernel function and $|x'-x|$ is the Euclidean distance. $\tau = e_0 a / l$ is defined as scale coefficient, where e_0 is a material constant which is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics; and *a* and *l* are the internal and external characteristic length of the nanostructures, respectively. Finally it is possible to represent the integral constitutive relations given by Eq. (4) in an equivalent differential form as:

$$\sigma_{ij} - (e_0\, a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k \tag{5a}$$

$$D_i - (e_0\, a)^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} + k_{ik}E_k \tag{5b}$$

where ∇^2 is the Laplacian operator and $e_0 a$ is the nonlocal parameter revealing the size influence on the response of nanostructures.

2.3. Nonlocal FG Piezoelectric Nanobeam Model

Based on parabolic third order beam theory, the displacement field at any point of the beam is supposed to be in the form:

$$u_x(x,z) = u(x) + z\psi(x) - \alpha z^3 (\psi + \frac{\partial w}{\partial x}) \tag{6a}$$

$$u_z(x,z) = w(x) \tag{6b}$$

in which u and w are displacement components in the mid-plane along the coordinates x and z, respectively, while ψ denotes the total bending rotation of the cross-section.

To satisfy Maxwell's equation in the quasi-static approximation, the distribution of electric potential along the thickness direction is supposed to change as a combination of a cosine and linear variation as follows:

$$\Phi(x,z,t) = -\cos(\xi z)\phi(x,t) + \frac{2z}{h}V \tag{7}$$

where $\xi = \pi / h$. Also, V is the initial external electric voltage applied to the FGP nanobeam; and $\phi(x,t)$ is the spatial function of the electric potential in the *x*-direction. Considering strain–displacement relationships on the basis of parabolic beam theory, the non-zero strains can be stated as:

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)} \tag{8}$$

$$\gamma_{xz} = \gamma_{xz}^{(0)} + z^2\gamma_{xz}^{(2)} \tag{9}$$

where

$$\varepsilon_{xx}^{(0)} = \frac{\partial u}{\partial x},\ \varepsilon_{xx}^{(1)} = \frac{\partial \psi}{\partial x},\ \varepsilon_{xx}^{(3)} = -\alpha(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}) \tag{10}$$

$$\gamma_{xz}^{(0)} = \frac{\partial w}{\partial x} + \psi,\ \gamma_{xz}^{(2)} = -\beta(\frac{\partial w}{\partial x} + \psi) \tag{11}$$

And $\beta = \frac{4}{h^2}$.

According to the defined electric potential in Eq. (7), the non-zero components of electric field (E_x, E_z) can be obtained as:

$$E_x = -\Phi_{,x} = \cos(\xi z)\frac{\partial \phi}{\partial x},$$
$$E_z = -\Phi_{,z} = -\xi \sin(\xi z)\phi - \frac{2V}{h} \tag{12}$$

The Hamilton's principle can be stated in the following form to obtain the governing equations of motion,:

$$\int_0^t \delta(\Pi_S + \Pi_W)\, dt = 0 \tag{13}$$

where Π_S is strain energy and Π_W is work done by external applied forces. The first variation of strain energy Π_S can be calculated as:

$$\delta\Pi_S = \int_0^L \int_{-h/2}^{h/2} (\sigma_{xx}\,\delta\varepsilon_{xx} + \sigma_{xz}\,\delta\gamma_{xz} - D_x\,\delta E_x - D_z\,\delta E_z)\, dz dx \tag{14}$$

Substituting Eqs. (8) and (9) into Eq. (14) yields:

$$\delta\Pi_S == \int_0^L (N\delta\varepsilon_{xx}^{(0)} + M\delta\varepsilon_{xx}^{(1)} + P\delta\varepsilon_{xx}^{(3)} + Q\delta\gamma_{xz}^{(0)} + R\delta\gamma_{xz}^{(2)})\, dx$$
$$+ \int_0^L \int_{-h/2}^{h/2} \left(-D_x \cos(\xi z)\,\delta\left(\frac{\partial \phi}{\partial x}\right) + D_z \xi \sin(\xi z)\,\delta\phi \right) dz dx \tag{15}$$

in which N, M and Q are the axial force, bending moment and shear force resultants, respectively. Relations between the stress resultants and stress component used in Eq. (15) are defined as:

$$N = \int_A \sigma_{xx}\, dA,\ M = \int_A \sigma_{xx} z\, dA,\ P = \int_A \sigma_{xx} z^3 dA$$
$$Q = \int_A \sigma_{xz}\, dA,\ R = \int_A \sigma_{xz} z^2\, dA \tag{16}$$

The work done due to external electric voltage, Π_W, can be written in the form:

$$\Pi_W = \int_0^L ((N_E + N_b)\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + q\,\delta w + f\,\delta u - N\delta\varepsilon_{xx}^{(0)} - \hat{M}\frac{\partial \delta\psi}{\partial x}$$
$$+ \alpha P \frac{\partial^2 \delta w}{\partial x^2} - \hat{Q}\delta\gamma_{xz}^{(0)} - k_w \delta w + k_p \frac{\partial^2 \delta w}{\partial x^2})dx \tag{17}$$

where $\hat{M} = M - \alpha P,\ \hat{Q} = Q - \beta R$ and $q(x)$ and $f(x)$ are the transverse and axial distributed loads and k_w and k_w are foundation parameters and N_E is normal forced due to external electric voltage (V) which is defined as:

$$N_E = -\int_{-h/2}^{h/2} e_{31} \frac{2V}{h} dz \tag{18}$$

For a FGPM nanobeam exposed to electro-mechanical loading in the one dimensional case, the nonlocal constitutive relations (5a) and (5b) may be rewritten as:

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = c_{11}\varepsilon_{xx} - e_{31}E_z \tag{19}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = c_{55}\gamma_{xz} - e_{15}E_x \tag{20}$$

$$D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = e_{15}\gamma_{xz} + k_{11}E_x \tag{21}$$

$$D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31}\varepsilon_{xx} + k_{33}E_z \tag{22}$$

Inserting Eqs. (15) and (17) in Eq. (13) and integrating by parts, and gathering the coefficients of δu, δw, $\delta\psi$ and $\delta\phi$, the following governing equations are obtained:

$$\frac{\partial N}{\partial x} + f = 0 \tag{23}$$

$$\frac{\partial M}{\partial x} - Q = 0 \tag{24}$$

$$\frac{\partial \bar{Q}}{\partial x} + q - (N_E + N_b)\frac{\partial^2 w}{\partial x^2} + \alpha \frac{\partial^2 P}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \tag{25}$$

$$\int_{-h/2}^{h/2} \left(\cos(\xi z)\frac{\partial D_x}{\partial x} + \xi \sin(\xi z) D_z \right) dz = 0 \tag{26}$$

By integrating Eqs. (19)-(22), over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal third order Reddy FGP beam theory can be obtained as follows:

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \psi}{\partial x} - \alpha E_{xx}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + A_{31}^e \phi - N_E \tag{27}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \psi}{\partial x} - \alpha F_{xx}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + E_{31}\phi \tag{28}$$

$$P - \mu \frac{\partial^2 P}{\partial x^2} = E_{xx}\frac{\partial u}{\partial x} + F_{xx}\frac{\partial \psi}{\partial x} - \alpha H_{xx}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + F_{31}\phi \tag{29}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = (A_{xz} - \beta D_{xz})\left(\frac{\partial w}{\partial x} + \psi\right) - E_{15}\frac{\partial \phi}{\partial x} \tag{30}$$

$$R - \mu \frac{\partial^2 R}{\partial x^2} = (D_{xz} - \beta F_{xz})\left(\frac{\partial w}{\partial x} + \psi\right) - F_{15}\frac{\partial \phi}{\partial x} \tag{31}$$

$$\int_{-h/2}^{h/2} \left\{ D_x - \mu \frac{\partial^2 D_x}{\partial x^2} \right\} \cos(\xi z) dz = (E_{15} - \beta F_{15})\left(\frac{\partial w}{\partial x} + \psi\right) + F_{11}\frac{\partial \phi}{\partial x} \tag{32}$$

$$\int_{-h/2}^{h/2} \left\{ D_z - \mu \frac{\partial^2 D_z}{\partial x^2} \right\} \xi \sin(\xi z) dz = A_{31}^e \frac{\partial u}{\partial x} + (E_{31} - \alpha F_{31})\frac{\partial \psi}{\partial x} - \alpha F_{31}\frac{\partial^2 w}{\partial x^2} - F_{33}\phi \tag{33}$$

where $\mu = (e_0 a)^2$ and quantities used in above equations are defined as:

$$\{A_{xx}, B_{xx}, D_{xx}, E_{xx}, F_{xx}, H_{xx}\} = \int_{-h/2}^{h/2} c_{11}\{1, z, z^2, z^3, z^4, z^6\} dz \tag{34}$$

$$\{A_{xz}, D_{xz}, F_{xz}\} = \int_{-h/2}^{h/2} c_{55}\{1, z^2, z^4\} dz \tag{35}$$

$$\{A_{31}^e, E_{31}, F_{31}\} = \int_{-h/2}^{h/2} e_{31}\{\xi \sin(\xi z), z\xi \sin(\xi z), z^3 \xi \sin(\xi z)\} dz \tag{36}$$

$$\{E_{15}, F_{15}\} = \int_{-h/2}^{h/2} e_{15}\{\cos(\xi z), z^2 \cos(\xi z)\} dz \tag{37}$$

$$\{F_{11}, F_{33}\} = \int_{-h/2}^{h/2} \{k_{11}\cos^2(\xi z), k_{33}\xi^2 \sin^2(\xi z)\} dz \tag{38}$$

The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq.(23) into Eq.(27) as follows:

$$N = A_{xx}\frac{\partial u}{\partial x} + K_{xx}\frac{\partial \psi}{\partial x} - \alpha E_{xx}\frac{\partial^2 w}{\partial x^2} + A_{31}^{e}\phi - N_E + \mu(-\frac{\partial f}{\partial x}) \tag{39}$$

Omitting $\hat{Q}$ from Eqs. (24) and (25), we obtain the following equation:

$$\frac{\partial^2 \bar{M}}{\partial x^2} = -\alpha\frac{\partial^2 P}{\partial x^2} - q + (N_E + N_b)\frac{\partial^2 w}{\partial x^2} + k_w w - k_p\frac{\partial^2 w}{\partial x^2} \tag{40}$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of M from Eq. (24) into Eq. (28) and using Eqs. (28) and (29) as follows:

$$\hat{M} = K_{xx}\frac{\partial u}{\partial x} + I_{xx}\frac{\partial \psi}{\partial x} - \alpha J_{xx}(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}) + (E_{31} - \alpha F_{31})\phi + \mu(-\alpha\frac{\partial^2 P}{\partial x^2} - q$$
$$+ \frac{\partial}{\partial x}((N_E + N_b)\frac{\partial w}{\partial x}) + k_w w - k_p\frac{\partial^2 w}{\partial x^2}) \tag{41}$$

where

$$K_{xx} = B_{xx} - \alpha E_{xx},\; I_{xx} = D_{xx} - \alpha F_{xx},\; J_{xx} = F_{xx} - \alpha H_{xx} \tag{42}$$

By substituting for the second derivative of $\hat{Q}$ from Eq. (25) into Eq. (30), and using Eqs. (30) and (31) the following expression for the nonlocal shear force is derived:

$$\hat{Q} = \bar{A}_{xz}(\frac{\partial w}{\partial x} + \psi) - (E_{15} - \beta F_{15})\frac{\partial \phi}{\partial x} + \mu((N_E + N_b)\frac{\partial^3 w}{\partial x^3} - \alpha\frac{\partial^3 P}{\partial x^3} - \frac{\partial q}{\partial x} + k_w\frac{\partial w}{\partial x} - k_p\frac{\partial^3 w}{\partial x^3}) \tag{43}$$

Where

$$\bar{A}_{xz} = A^{*}_{xz} - \beta I^{*}_{xz},\; A^{*}_{xz} = A_{xz} - \beta D_{xz},\; I^{*}_{xz} = D_{xz} - \beta F_{xz} \tag{44}$$

Now we use M and Q from Eqs. (41) and (43) and the identity

$$\alpha\frac{\partial^2}{\partial x^2}(P - \mu\frac{\partial^2 P}{\partial x^2}) = \alpha(E_{xx}\frac{\partial^3 u}{\partial x^3} + F_{xx}\frac{\partial^3 \psi}{\partial x^3} - \alpha H_{xx}(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^4 w}{\partial x^4}) + F_{31}\frac{\partial^2 \phi}{\partial x^2}) \tag{45}$$

It must be cited that inserting Eq. (26) into Eqs. (32) and (33), does not provide an explicit expressions for D_x and D_z . To overcome this problem, by using Eqs. (32) and (33), Eq. (26) can be re-expressed in terms of u , w , ψ and ϕ . For a higher order FGP nanobeam by substituting for N , M and Q from Eqs. (39),(41) and (43) into Eqs. (23)-(25) the nonlocal governing equations can be written as:

$$A_{xx}\frac{\partial^2 u}{\partial x^2} + K_{xx}\frac{\partial^2 \psi}{\partial x^2} - \alpha E_{xx}\frac{\partial^3 w}{\partial x^3} + A_{31}^{e}\frac{\partial \phi}{\partial x} + \mu(-\frac{\partial^2 f}{\partial x^2}) + f = 0 \tag{46}$$

$$K_{xx}\frac{\partial^2 u}{\partial x^2} + I_{xx}\frac{\partial^2 \psi}{\partial x^2} - \alpha J_{xx}(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}) - \bar{A}_{xz}\left(\varphi + \frac{\partial w}{\partial x}\right) + (E_{31} - \alpha F_{31})\phi$$
$$+(E_{15} - \beta F_{15})\frac{\partial \phi}{\partial x} = 0 \tag{47}$$

$$\bar{A}_{xz}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \mu((N_E + N_b)\frac{\partial^4 w}{\partial x^4} - \frac{\partial^2 q}{\partial x^2} + k_w\frac{\partial^2 w}{\partial x^2} - k_p\frac{\partial^4 w}{\partial x^4}) + q - (N_E + N_b)\frac{\partial^2 w}{\partial x^2}$$
$$-(E_{15} - \beta F_{15})\frac{\partial \phi}{\partial x} - k_w w + k_p\frac{\partial^2 w}{\partial x^2} + \alpha(E_{xx}\frac{\partial^3 u}{\partial x^3} + J_{xx}\frac{\partial^3 \psi}{\partial x^3} - \alpha H_{xx}\frac{\partial^4 w}{\partial x^4} + F_{31}\frac{\partial^2 \phi}{\partial x^2}) = 0 \tag{48}$$

$$(E_{15} - \beta F_{15})(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi}{\partial x}) + F_{11}\frac{\partial^2 \phi}{\partial x^2} + A_{31}^{e}\frac{\partial u}{\partial x} + (E_{31} - \alpha F_{31})\frac{\partial \psi}{\partial x} - \alpha F_{31}\frac{\partial^2 w}{\partial x^2} - F_{33}\phi = 0 \tag{49}$$

3. Solution Procedure

Here, on the basis the Navier method, an analytical solution of the governing equations for buckling of a simply supported FGP nanobeam is presented. To satisfy governing equations of motion and the simply supported boundary condition, the displacement variables are adopted to be of the form:

$$u(x,t)=\sum_{n=1}^{\infty}U_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (50)$$

$$w(x,t)=\sum_{n=1}^{\infty}W_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (51)$$

$$\psi(x,t)=\sum_{n=1}^{\infty}\Psi_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (52)$$

$$\phi(x,t)=\sum_{n=1}^{\infty}\Phi_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (53)$$

where U_n, W_n, Ψ_n and Φ_n are the unknown Fourier coefficients to be determined for each n value. The boundary conditions for simply supported FGP beam can be identified as:

$$u(0)=0,\quad \frac{\partial u}{\partial x}(L)=0,\quad w(0)=w(L)=0 \quad (54)$$
$$\frac{\partial \psi}{\partial x}(0)=\frac{\partial \psi}{\partial x}(L)=0,\quad \phi(0)=\phi(L)=0$$

Inserting Eqs. (50)-(53) into Eqs. (46)-(49), respectively, yields:

Inserting Eqs. (50)-(53) into Eqs. (46)-(49), respectively, yields:

$$(-A_{xx}(\frac{n\pi}{l})^2)U_n+(-K_{xx}(\frac{n\pi}{l})^2)\psi_n+(\alpha E_{xx}(\frac{n\pi}{l})^3)W_n+(-A^e_{31}(\frac{n\pi}{l}))\phi_n=0 \quad (55)$$

$$(-K_{xx}(\frac{n\pi}{l})^2)U_n+(-I_{xx}(\frac{n\pi}{l})^2+\alpha J_{xx}(\frac{n\pi}{l})^2-\bar{A}_{xz})\psi_n+(\alpha J_{xx}(\frac{n\pi}{l})^3-\bar{A}_{xz}(\frac{n\pi}{l}))W_n$$
$$+((E_{31}-\alpha F_{31})+(E_{15}-\beta F_{15})(\frac{n\pi}{l}))\phi_n=0 \quad (56)$$

$$(\alpha E_{xx}(\frac{n\pi}{l})^3)U_n+(-\bar{A}_{xz}(\frac{n\pi}{l})+J_{xx}(\frac{n\pi}{l})^3)\psi_n+(N_E+N_b)(\frac{n\pi}{l})^2(1+\mu(\frac{n\pi}{l})^2)-\bar{A}_{xz}(\frac{n\pi}{l})^2-\alpha^2H_{xx}(\frac{n\pi}{l})^4$$
$$-k_w(1+\mu(\frac{n\pi}{l})^2)-k_p(\frac{n\pi}{l})^2(1+\mu(\frac{n\pi}{l})^2))W_n+(-(E_{15}-\beta F_{15})(\frac{n\pi}{l})-F_{31}(\frac{n\pi}{l})^2)\phi_n=0 \quad (57)$$

$$\left(-A^e_{31}(\frac{n\pi}{L})\right)U_n-\left(((E_{15}-\beta F_{15})-\alpha F_{31})(\frac{n\pi}{L})^2\right)W_n-\left((E_{15}-\beta F_{15})+(E_{31}-\alpha F_{31})(\frac{n\pi}{L})\right)\Psi_n$$
$$-\left(F_{11}(\frac{n\pi}{L})^2+F_{33}\right)\Phi_n=0 \quad (58)$$

By setting the determinant of the coefficient matrix of the above equations, the nontrivial analytical solutions can be obtained from the following equations:

$$\{[K]\}\begin{Bmatrix} U_n \\ W_n \\ \Psi_n \\ \Phi_n \end{Bmatrix} = 0 \qquad (59)$$

where $[K]$ denotes the stiffness matrix. By setting this polynomial to zero, we can find buckling loads of the FGP nanobeam exposed to electrical loading.

4. Results and Discussion

This section is devoted to explore the buckling behavior of nanoscale FGP beams on elastic substrate incorporating shear deformation effects. Effects of applied voltage, geometrical parameters, scale parameter and foundation constants will on buckling load will be discussed. Here, a FGP beam composed of PZT-4 and PZT-5H, with electro-mechanical material properties listed in Table 1, is supposed. The beam geometry has the following dimensions: L (length) = 10 nm and h (thickness) = varied. Also, the following relation is described to calculate the non-dimensional buckling loads as well as foundation parameters:

$$N_{bcr} = N_b \frac{L^2}{(c_{11}I)_{PZT-4}},\ K_w = k_w \frac{L^4}{(c_{11}I)_{PZT-4}},\ K_p = k_p \frac{L^2}{(c_{11}I)_{PZT-4}} \qquad (60)$$

In which $I = \mathrm{h}^3/12$ denote the moment of inertia of the cross section of the nanobeam. Table 2 compares dimensionless buckling loads of the present model with those of nonlocal Reddy beams, because there are no available data for the buckling loads of FGP nanobeams based on the nonlocal elasticity theory. For comparison study, the material selection is performed as follows: E_m = 70 GPa, ν_m = 0.3, for Steel and E_c = 390 GPa and ν_c = 0.3 for Alumina.

Tables 3-6, present the influences of elastic foundation parameters (K_w, K_p), nonlocal parameter (μ), electric voltage (V), gradient index (p) and slenderness ratio on the non-dimensional buckling load of the S-S FGP nanobeams. It must be cited that nonlocal parameter weakens the nanobeam structure, so it has a remarkable decreasing influence on the non-dimensional buckling loads. Also, it is found that existence of elastic foundation makes the beam more rigid and hence the dimension buckling loads rise. In addition, it is observed from these tables that the buckling load results for negative voltage are higher than positive voltages. In a special case, when electric voltage is equal to 0V the variation of buckling loads with slenderness ratio is not considerable. But for negative and positive voltages, with the rise of slenderness ratio the buckling loads increase and decrease, respectively.

In Figs. 2-3 the effects of Winkler and Pasternak foundations on the variations of the non-dimensional buckling load of FG piezoelectric nanobeams with the power-law index are illustrated for various values of external voltages ($V = -0.5, -0.25, 0, +0.25, +0.5$) at nonlocal parameter $\mu = 2$ and slenderness ratio $L/h = 20$. It is observable from these figures that for all values of Winkler or Pasternak parameter the non-dimensional buckling load decreases dramatically for lower values of gradient indexes, and then continues to decline with non-sensitive variation for higher values of power-law index. Moreover, it is seen that the reduction in the dimensionless buckling load for positive voltages occurs more significantly compared to negative voltages which shows notability of the sign of external electric voltage. Also, it should be mentioned that with an increase in Winkler or Pasternak parameter, non-dimensional buckling load begins to decline with higher values versus the power-law index.

Figs. 4 and 5 shows the influence of electric voltage on the variations of the non-dimensional buckling load with respect to Winkler and Pasternak foundation parameters, respectively, for different nonlocal parameters at slenderness ratio L/h=20 and power-law index p=1. It is seen that for both negative and positive voltages as the Winkler or Pasternak parameter rises the buckling load increases due to the stiffening influence of elastic foundation on the structure of the beam. Also, it is found that at a fixed Winkler or Pasternak parameter when the voltage changes from $V = -0.5$ to $V = +0.5$ the dimensionless buckling load of FGP nanobeams decreases. Also, it can be seen that at a constant electric voltage the rise of the dimensionless buckling load with respect to Pasternak parameter is more significant than Winker parameter.

The variations of the dimensionless buckling loads of FGM piezoelectric nanobeams with respect to electric voltage for various foundation and nonlocal parameters at L/h=20 and p=1 are plotted in Figs. 6-7. As a result, for all values of Winkler or Pasternak parameters when the electric voltage rises from $V = -1$ to $V = +1$ the dimensionless buckling load reduce. It is also observed from the figures that, with the rise of Winkler and Pasternak parameters the variations of buckling loads decrease because the flexibility of the beam reduces.

The effect of material composition (power-law exponent) on the variation of dimensionless buckling load of FGP nanobeams versus external electric voltage with and without elastic foundations at $L/h = 20$ and $\mu = 2(\mathrm{nm})^2$ is depicted in Fig.8. As an important observation, at the negative voltages, for example V=-1, the values of buckling loads for various gradient indexes are very close together but at positive electric voltages the differences of buckling loads rise. Due to the fact that increase of power-law exponent makes the beam more flexible, the reduction of buckling loads for higher power-law exponents is more significant than lower ones.

The influence of slenderness ratio and elastic foundation on the dimensionless buckling load of FGP nanobeams with various gradient indices when $\mu = 2(\mathrm{nm})^2$ and gradient index p=0.5 is presented in Fig.9. It can be seen that slenderness ratio has an important influence on the non-dimensional buckling loads especially for its higher values. Moreover, an important observation is that the negative/positive electric voltages increases /decrease the buckling loads of the piezoelectric nanobeams. This is due to the axial compressive and tensile forces produced in the FGP nanobeams via the applied positive and negative voltages, respectively. So, zero external electric voltage $V=0$ makes no compressive or tensile force and will not affect the dimensionless buckling load with changing of slenderness ratio.

Table 1. Electro-mechanical coefficients of material properties for PZT-4 and PZT-5H [29].

Properties	PZT-4	PZT-5H
c_{11} (GPa)	81.3	60.6
c_{55} (GPa)	25.6	23.0
e_{31} (Cm^{-2})	-10.0	-16.604
e_{15} (Cm^{-2})	40.3248	44.9046
k_{11} ($C^2 m^{-2} N^{-1}$)	0.6712e-8	1.5027e-8
k_{33} ($C^2 m^{-2} N^{-1}$)	1.0275e-8	2.554e-8

Table 2. Comparison of the non-dimensional buckling load for a FG nanobeam with various power-law index (L/h =20).

p	Nonlocal parameter							
	μ =1		μ =2		μ =3		μ =4	
	RBT [24]	Present	RBT [24]	Present	RBT [24]	Present	RBT [24]	Present
0	8.9258	8.925759	8.1900	8.190046	7.5663	7.566381	7.0309	7.030978
0.1	9.7778	9.777865	8.9719	8.971916	8.2887	8.288712	7.7021	7.702196
0.2	10.3898	10.389845	9.5334	9.533453	8.8074	8.807489	8.1842	8.184264
0.5	11.4944	11.494448	10.5470	10.547009	9.7438	9.743863	9.0543	9.054379
1	12.3709	12.370918	11.3512	11.351234	10.4869	10.486847	9.7447	9.744790
2	13.1748	13.174885	12.0889	12.088934	11.1683	11.168372	10.3781	10.378089
5	14.2363	14.236343	13.0629	13.062900	12.0682	12.068171	11.2142	11.214218

Table 3. Influence of elastic foundation and external electric voltage on the non-dimensional buckling load of a FGP nanobeam ($\mu=0(nm)^2$).

(K_p, K_w)	L/h	V=-0.5			V=0			V=+0.5		
		p=0.2	p=1	p=5	p=0.2	p=1	p=5	p=0.2	p=1	p=5
(0,0)	20	11.7966	11.3101	11.0785	10.4858	9.73938	9.24789	9.17502	8.16866	7.41724
	25	13.0536	12.8144	12.8300	10.4935	9.74657	9.25456	7.93337	6.67876	5.67906
	30	14.9216	15.0517	15.4367	10.4977	9.75050	9.25820	6.07381	4.44933	3.07975
(5,25)	20	19.3296	18.8431	18.6116	18.0188	17.2724	16.7809	16.7081	15.7017	14.9503
	25	20.5866	20.3474	20.3631	18.0265	17.2796	16.7876	15.4664	14.2118	13.2121
	30	22.4546	22.5847	22.9697	18.0307	17.2835	16.7912	13.6068	11.9824	10.6128
(5,50)	20	21.8626	21.3762	21.1446	20.5519	19.8054	19.3139	19.2411	18.2347	17.4833
	25	23.1197	22.8804	22.8961	20.5595	19.8126	19.3206	17.9994	16.7448	15.7451
	30	24.9876	25.1177	25.5027	20.5638	19.8166	19.3243	16.1399	14.5154	13.1458
(10,25)	20	24.3296	23.8431	23.6116	23.0188	22.2724	21.7809	21.7081	20.7017	19.9503
	25	25.5866	25.3474	25.3631	23.0265	22.2796	21.7876	20.4664	19.2118	18.2121
	30	27.4546	27.5847	27.9697	23.0307	22.2835	21.7912	18.6068	16.9824	15.6128
(10,50)	20	26.8626	26.3762	26.1446	25.5519	24.8054	24.3139	24.2411	23.2347	22.4833
	25	28.1197	27.8804	27.8961	25.5595	24.8126	24.3206	22.9994	21.7448	20.7451
	30	29.9876	30.1177	30.5027	25.5638	24.8166	24.3243	21.1399	19.5154	18.1458

Table 4. Influence of elastic foundation and external electric voltage on the non-dimensional buckling load of a FGP nanobeam ($\mu=1(nm)^2$)

(K_p,K_w)	L/h	V=-0.5			V=0			V=+0.5		
		p=0.2	p=1	p=5	p=0.2	p=1	p=5	p=0.2	p=1	p=5
(0,0)	20	10.8546	10.4352	10.2478	9.54386	8.86449	8.41715	8.23308	7.29377	6.58650
	25	12.1110	11.9388	11.9987	9.55085	8.87103	8.42322	6.99074	5.80323	4.84773
	30	13.9786	14.1758	14.6050	9.55468	8.87461	8.42654	5.13080	3.57345	2.24809
(5,25)	20	18.3877	17.9682	17.7808	17.0769	16.3975	15.9502	15.7661	14.8268	14.1195
	25	19.6440	19.4719	19.5317	17.0839	16.4041	15.9562	14.5238	13.3363	12.3808
	30	21.5116	21.7088	22.1380	17.0877	16.4076	15.9596	12.6638	11.1065	9.78111
(5,50)	20	20.9207	20.5013	20.3139	19.6099	18.9305	18.4832	18.2991	17.3598	16.6526
	25	22.1770	22.0049	22.0648	19.6169	18.9371	18.4893	17.0568	15.8693	14.9138
	30	24.0446	24.2418	24.6710	19.6207	18.9407	18.4926	15.1969	13.6395	12.3141
(10,25)	20	23.3877	22.9682	22.7808	22.0769	21.3975	20.9502	20.7661	19.8268	19.1195
	25	24.6440	24.4719	24.5317	22.0839	21.4041	20.9562	19.5238	18.3363	17.3808
	30	26.5116	26.7088	27.1380	22.0877	21.4076	20.9596	17.6638	16.1065	14.7811
(10,50)	20	25.9207	25.5013	25.3139	24.6099	23.9305	23.4832	23.2991	22.3598	21.6526
	25	27.1770	27.0049	27.0648	24.6169	23.9371	23.4893	22.0568	20.8693	19.9138
	30	29.0446	29.2418	29.6710	24.6207	23.9407	23.4926	20.1969	18.6395	17.3141

Table 5. Influence of elastic foundation and external electric voltage on the non-dimensional buckling load of a FGP nanobeam ($\mu=2(nm)^2$)

(K_p,K_w)	L/h	V=-0.5			V=0			V=+0.5		
		p=0.2	p=1	p=5	p=0.2	p=1	p=5	p=0.2	p=1	p=5
(0,0)	20	10.0680	9.70454	9.55401	8.75720	8.13382	7.72336	7.44642	6.56311	5.89271
	25	11.3237	11.2076	11.3044	8.76362	8.13983	7.72893	6.20350	5.07202	4.15343
	30	13.1910	13.4443	13.9104	8.76713	8.14311	7.73197	4.34325	2.84195	1.55352
(5,25)	20	17.6010	17.2376	17.0870	16.2902	15.6669	15.2564	14.9795	14.0961	13.4257
	25	18.8568	18.7407	18.8374	16.2966	15.6729	15.2620	13.7365	12.6051	11.6865
	30	20.7240	20.9773	21.4435	16.3002	15.6761	15.2650	11.8763	10.3750	9.08655
(5,50)	20	20.1340	19.7706	19.6201	18.8233	18.1999	17.7894	17.5125	16.6292	15.9588
	25	21.3898	21.2737	21.3705	18.8297	18.2059	17.7950	16.2696	15.1381	14.2195
	30	23.2571	23.5103	23.9765	18.8332	18.2092	17.7980	14.4093	12.9080	11.6196
(10,25)	20	22.6010	22.2376	22.0870	21.2902	20.6669	20.2564	19.9795	19.0961	18.4257
	25	23.8568	23.7407	23.8374	21.2966	20.6729	20.2620	18.7365	17.6051	16.6865
	30	25.7240	25.9773	26.4435	21.3002	20.6761	20.2650	16.8763	15.3750	14.0866
(10,50)	20	25.1340	24.7706	24.6201	23.8233	23.1999	22.7894	22.5125	21.6292	20.9588
	25	26.3898	26.2737	26.3705	23.8297	23.2059	22.7950	21.2696	20.1381	19.2195
	30	28.2571	28.5103	28.9765	23.8332	23.2092	22.7980	19.4093	17.9080	16.6196

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Table 6. Influence of elastic foundation and external electric voltage on the non-dimensional buckling load of a FGP nanobeam($\mu = 3(nm)^2$)

(K_p, K_w)	L/h	V=-0.5			V=0			V=+0.5		
		p=0.2	p=1	p=5	p=0.2	p=1	p=5	p=0.2	p=1	p=5
(0,0)	20	9.40113	9.08516	8.96588	8.09035	7.51444	7.13523	6.77957	5.94372	5.30458
	25	10.6564	10.5878	10.7159	8.09627	7.51999	7.14038	5.53616	4.45218	3.56488
	30	12.5234	12.8242	13.3216	8.09952	7.52302	7.14319	3.67564	2.22186	0.96473
(5,25)	20	16.9342	16.6182	16.4989	15.6234	15.0475	14.6683	14.3126	13.4768	12.8376
	25	18.1894	18.1208	18.2489	15.6293	15.0530	14.6734	13.0692	11.9852	11.0979
	30	20.0564	20.3572	20.8547	15.6326	15.0561	14.6762	11.2087	9.75489	8.49777
(5,50)	20	19.4672	19.1512	19.0319	18.1564	17.5805	17.2013	16.8456	16.0098	15.3706
	25	20.7224	20.6539	20.7819	18.1623	17.5860	17.2064	15.6022	14.5182	13.6309
	30	22.5895	22.8902	23.3877	18.1656	17.5891	17.2092	13.7417	12.2879	11.0308
(10,25)	20	21.9342	21.6182	21.4989	20.6234	20.0475	19.6683	19.3126	18.4768	17.8376
	25	23.1894	23.1208	23.2489	20.6293	20.0530	19.6734	18.0692	16.9852	16.0979
	30	25.0564	25.3572	25.8547	20.6326	20.0561	19.6762	16.2087	14.7549	13.4978
(10,50)	20	24.4672	24.1512	24.0319	23.1564	22.5805	22.2013	21.8456	21.0098	20.3706
	25	25.7224	25.6539	25.7819	23.1623	22.5860	22.2064	20.6022	19.5182	18.6309
	30	27.5895	27.8902	28.3877	23.1656	22.5891	22.2092	18.7417	17.2879	16.0308

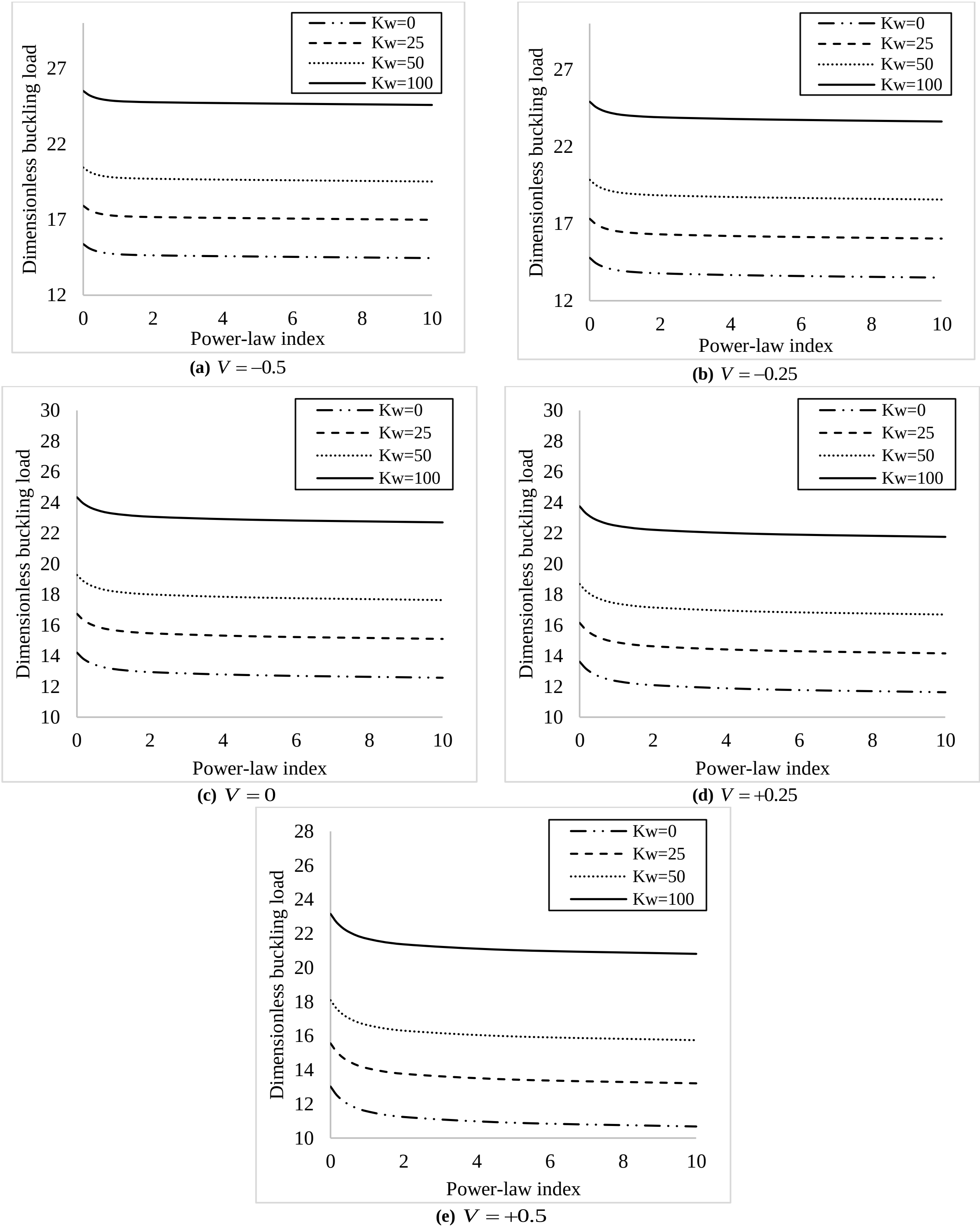
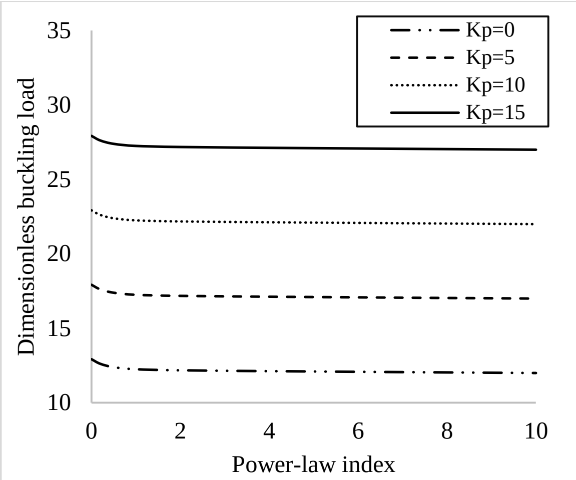
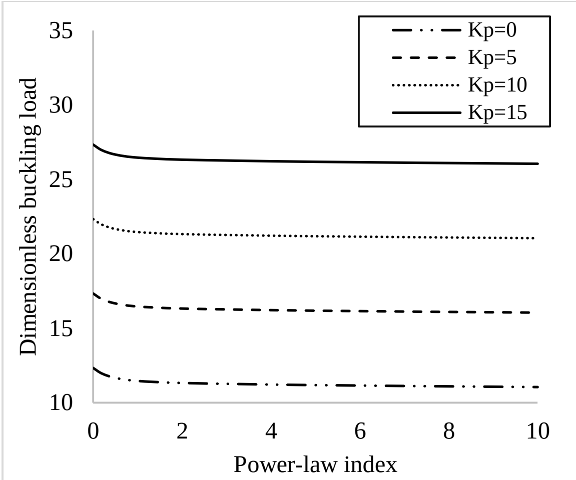


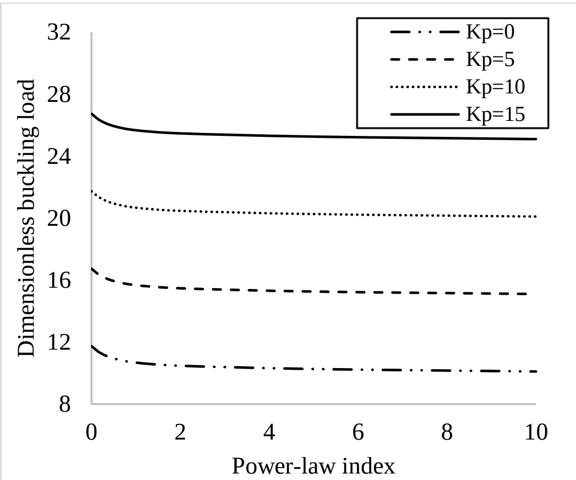
Fig. 2. Effect of Winkler parameter on the variation of dimensionless buckling load of the FGP nanobeam with respect to power-law index for different values of electric voltage ($L/h=20, K_p=5, \mu=2$).



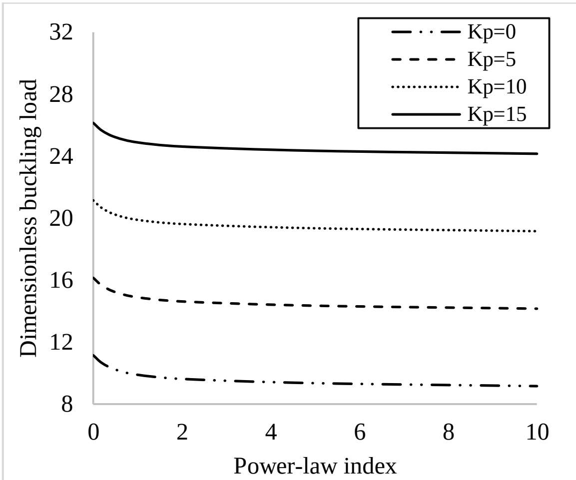
(a) $V = -0.5$



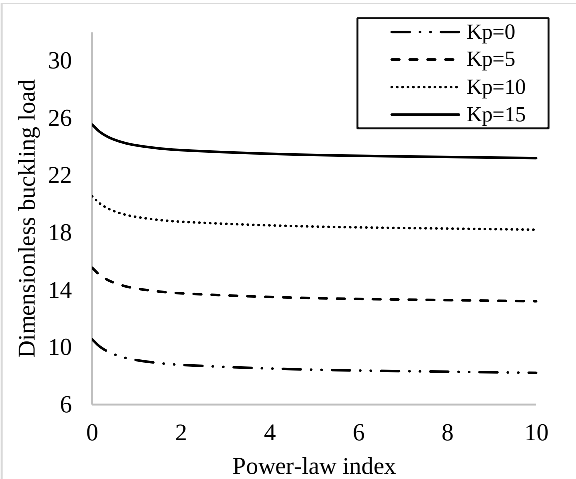
(b) $V = -0.25$



(c) $V = 0$

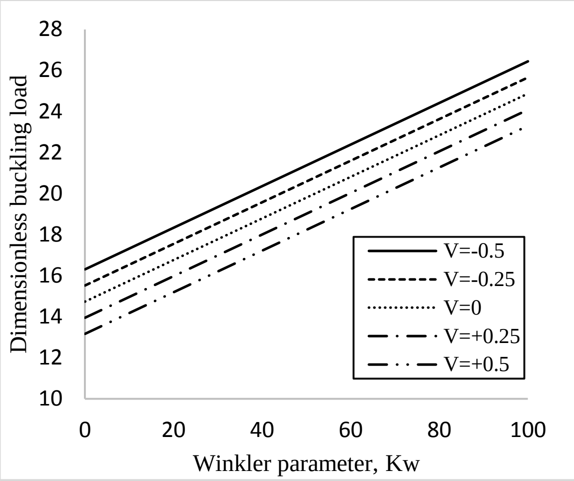


(d) $V = +0.25$

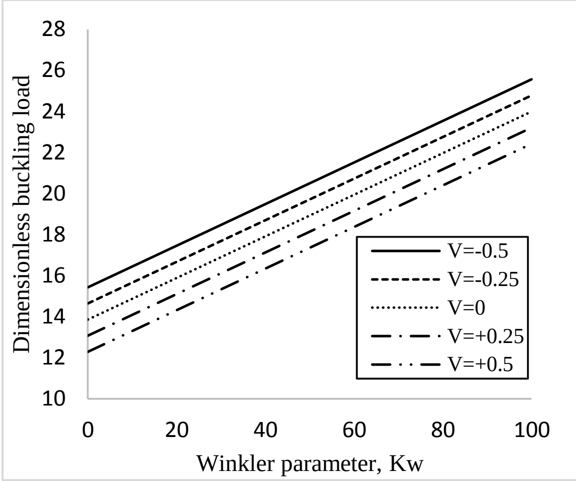


(e) $V = +0.5$

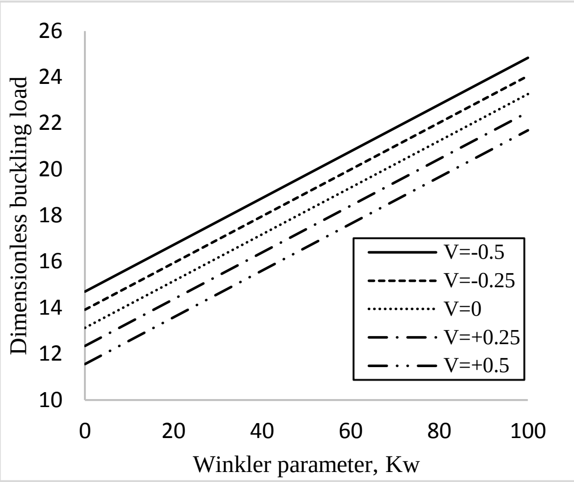
Fig. 3. Effect of Pasternak parameter on the variation of dimensionless buckling load of the FGP nanobeam with respect to power-law index for different values of electric voltage ($L/h = 20, K_w = 25, \mu = 2$).



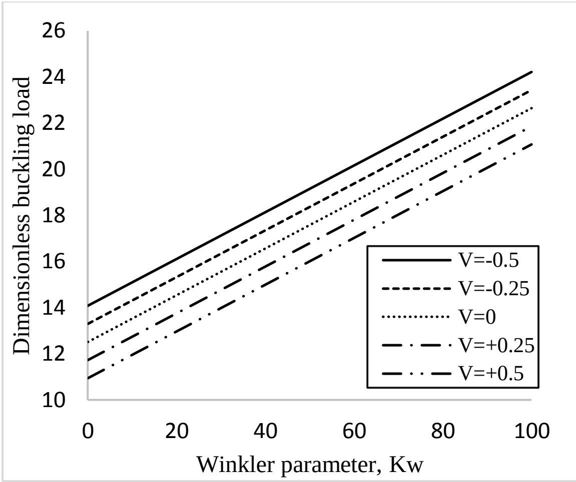
(a) $\mu = 0(\text{nm})^2$



(b) $\mu = 1(\text{nm})^2$

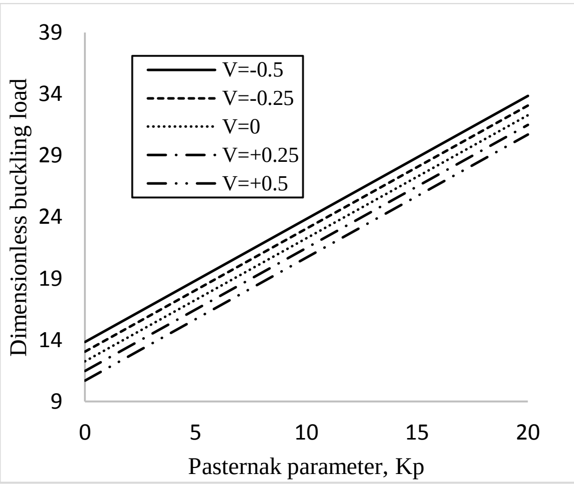


(c) $\mu = 2(\text{nm})^2$

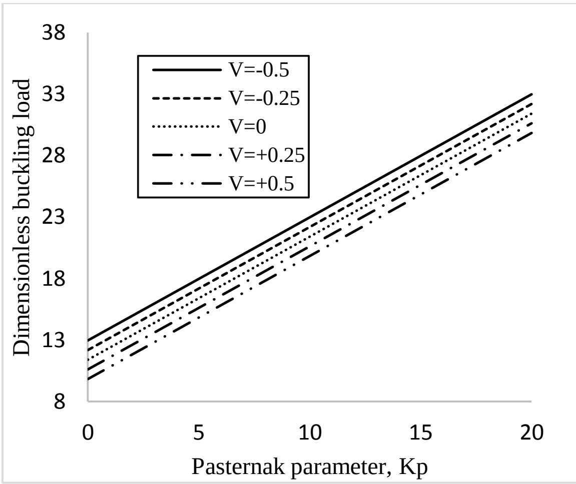


(d) $\mu = 3(\text{nm})^2$

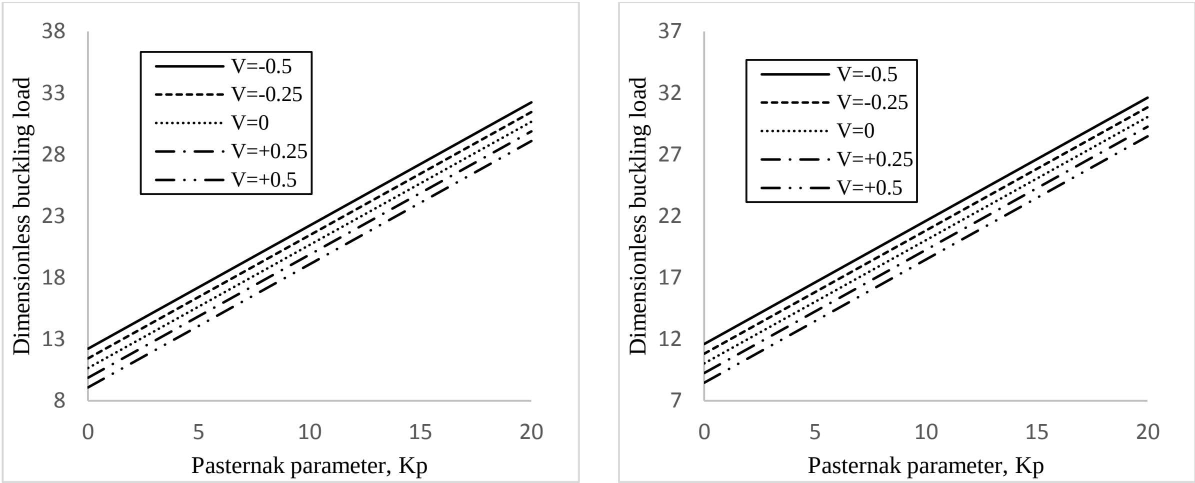
Fig. 4. Effect of electric voltage on the variation of dimensionless buckling load of the FGP nanobeam with respect to Winkler parameter for different values nonlocal parameter ($L/h = 20, K_p = 5, p = 1$).



(a) $\mu = 0(\text{nm})^2$

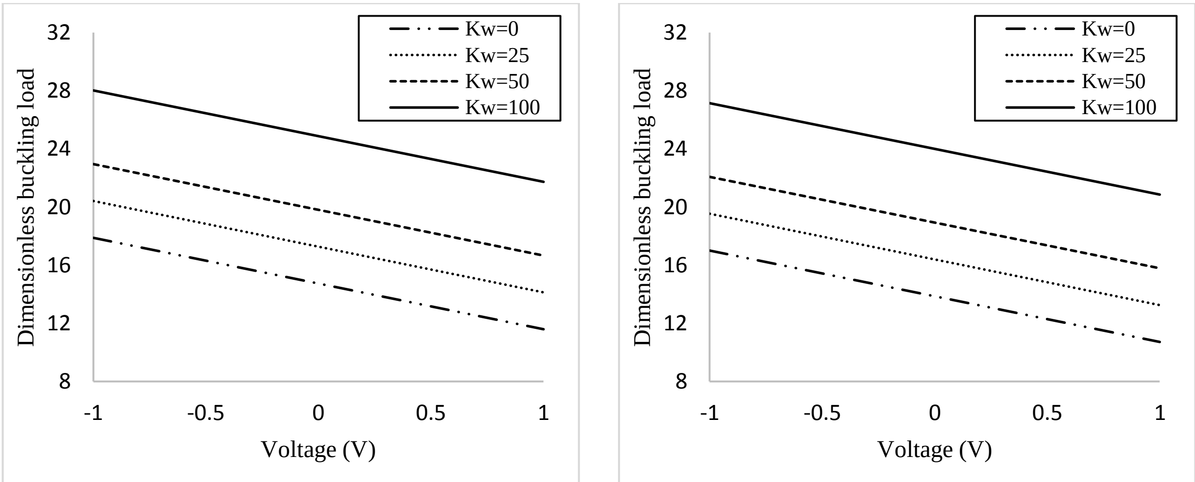


(b) $\mu = 1(\text{nm})^2$

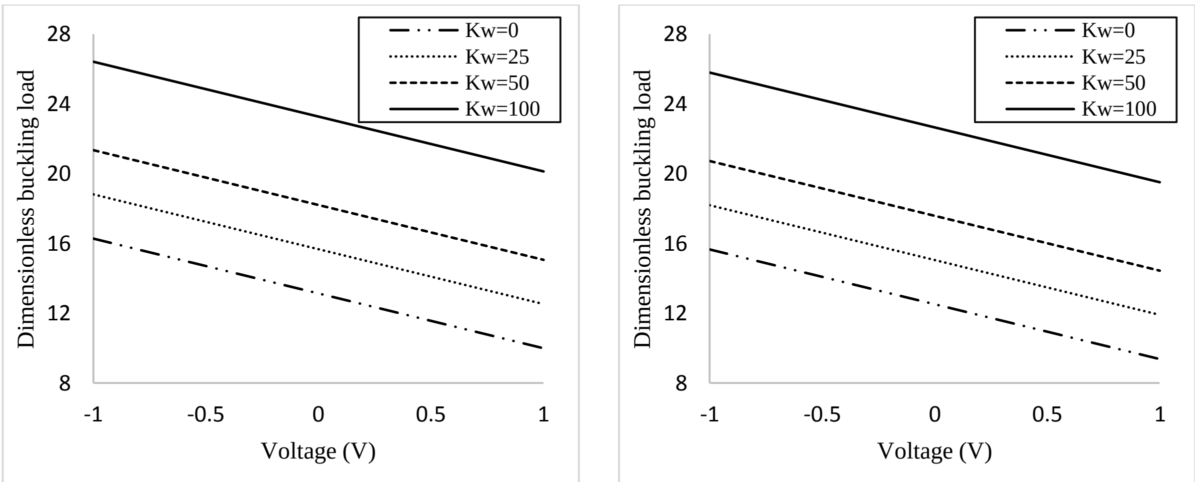


(c) $\mu = 2(\text{nm})^2$ (d) $\mu = 3(\text{nm})^2$

Fig. 5. Effect of electric voltage on the variation of dimensionless buckling load of the FGP nanobeam with respect to Pasternak parameter for different values nonlocal parameter ($L/h = 20, K_W = 25, p = 1$).

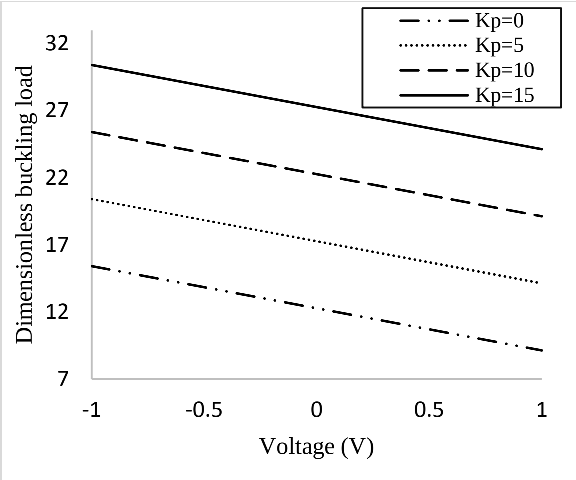


(a) $\mu = 0(\text{nm})^2$ (b) $\mu = 1(\text{nm})^2$

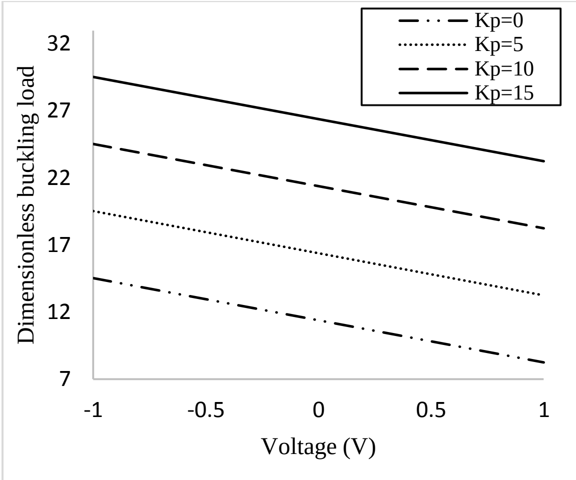


(c) $\mu = 2(\text{nm})^2$ (d) $\mu = 3(\text{nm})^2$

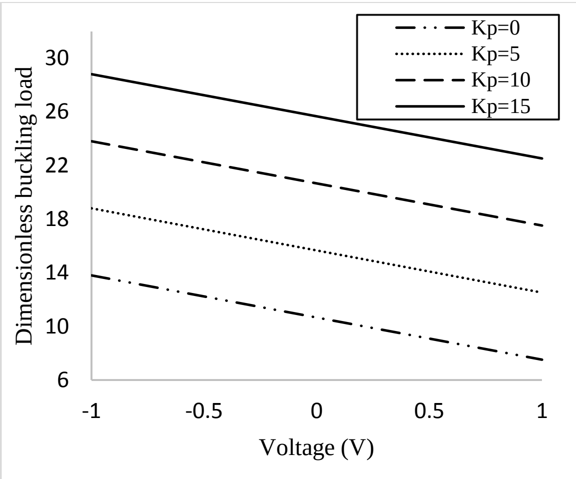
Fig. 6. Effect of Winkler parameter on the variation of dimensionless buckling load of the FGP nanobeam with respect to electric voltage for different values nonlocal parameter ($L/h = 20, K_p = 5, p = 1$).



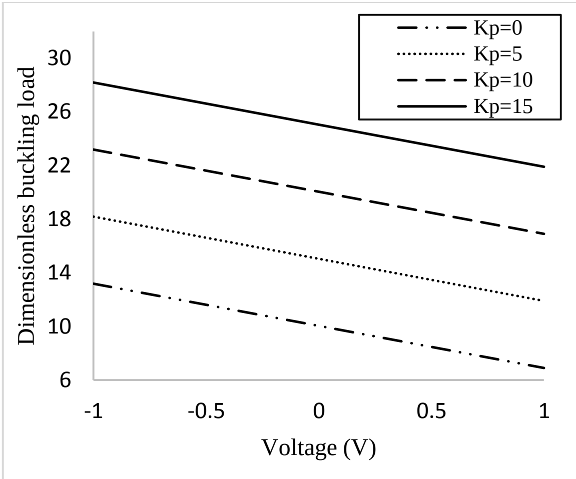
(a) $\mu = 0(\text{nm})^2$



(b) $\mu = 1(\text{nm})^2$

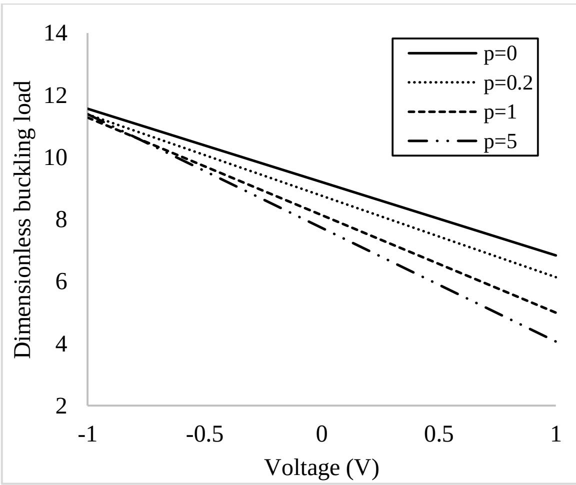


(c) $\mu = 2(\text{nm})^2$

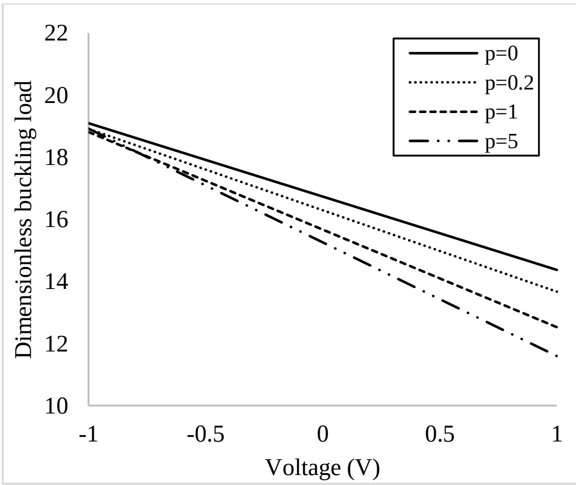


(d) $\mu = 3(\text{nm})^2$

Fig. 7. Effect of Pasternak parameter on the variation of dimensionless buckling load of the FGP nanobeam with respect to electric voltage for different values nonlocal parameter ($L/h = 20, K_w = 25, p = 1$).



(a) $K_w = K_p = 0$



(b) $K_w = 25, K_p = 5$

Fig. 8. Effect of material composition on the variation of dimensionless buckling load of FGP nanobeam versus electric voltage with and without elastic foundation ($L/h = 20,\ \mu = 2$).

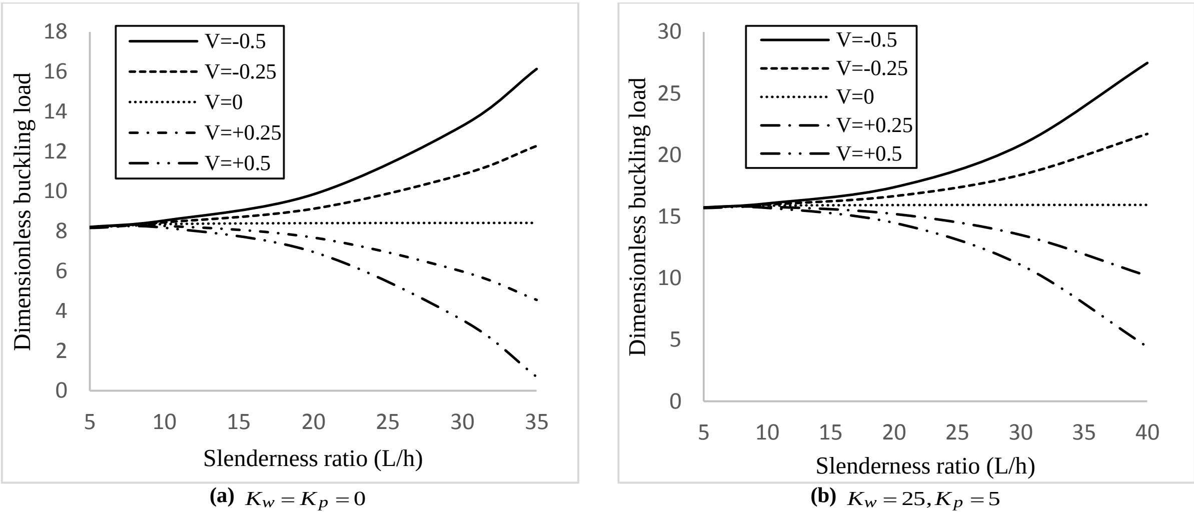


Fig. 9. Effect of slenderness ratio on the variation of dimensionless buckling load of the FGP nanobeam for different values of electric voltage with and without elastic foundation ($L/h = 20, \mu = 2, p = 0.5$).

5. Conclusions

Buckling behavior of nonlocal functionally graded piezoelectric (FGP) beams on elastic substrate via a higher order beam model is examined. The present beam model takes into consideration the shear deformation effects needless of any shear correction factor. Employing power-law function, the gradation of material properties of the beam is described. A detailed parametric study is conducted to study the influences of the elastic foundation, nonlocal parameter, external electric voltage, material composition and slenderness ratio on the size-dependent buckling characteristics of the FG piezoelectric nanobeams. It is found that for all values of elastic foundation parameters nonlocality and power-law exponent yields in reduction on both rigidity of the nanobeam structure and buckling loads. But with the rise in magnitude of Winkler or Pasternak constants the rigidity of the FGP nanobeam as well as the buckling load results increase. Moreover, it is deduced that the electric voltage value has an important influence on the buckling loads of FGP nanobeams. A change in the external electric voltage from a negative value to a positive value yields reduction in the buckling loads and bending rigidity.

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