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Melting Heat Transfer in Boundary Layer Stagnation Point Flow of MHD Micro - polar Fluid towards a Stretching / Shrinking Surface

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Abstract

The present study investigates the fluid flow and heat transfer characteristics occurring during the melting process due to a stretching / shrinking surface in micropolar fluid. A uniform magnetic field is applied normally to the surface. The governing equations representing fluid flow were transformed into nonlinear ordinary differential equations using similarity transformation. The equations thus obtained were solved numerically using the Runge–Kutta-Fehlberg fourth-fifth order method with shooting technique. The effects of the magnetic parameter on the fluid flow, couple stress coefficient and heat transfer characteristics, are illustrated graphically and discussed in detail. Significant changes were observed in the fluid flow, couple stress coefficient and heat transfer with respect to magnetic parameter.

Keywords: *Boundary Layer, Melting Heat, MHD, Micropolar Fluid, Stagnation-Point, Stretching Sheet*.

1. Introduction

The micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can affect the hydrodynamics of the flow. It has many practical applications, like, for example, analyzing the behaviour of exotic lubricants, colloidal suspensions, solidification of liquid crystals, extrusion of polymer fluids, cooling of metallic plate in a bath, animal blood, body fluids, among others. Eringen [1] introduced the theory of micropolar fluids that is capable to describe those fluids by taking into account the effect arising from local structure and micromotions of the fluid elements. Gamal and Rahman [2] studied the effect of MHD on thin films of a micropolar fluid and they observed that the rotation of the microelements at the boundary increase the velocity when compared with the case when there is no rotation at the boundary. Das [3] investigated the effect of the first order chemical reaction and thermal radiation on hydro-magnetic free convection heat and mass transfer flow of a microplar fluid through a porous medium. Satya Naraya *et al.* [4] investigated the effects of Hall current and radiation absorption on MHD free convection mass transfer flow of a micropolar fluid in a rotating frame of reference. Srinivasacharya [5] analyzed the heat and mass transfer characteristic of the forced convection on a vertical wall temperature and concentration in a doubly stratified micropolar fluid. Das [6] studied the effect of partial slip on steady boundary layer stagnation point flow of an electrically conducting micropolar fluid impinging normally through a shrinking sheet in the presence of a uniform transverse magnetic field. The unsteady MHD boundary layer flow of a micropolar fluid near the stagnation point of a two-dimensional plane surface through a porous medium was studied by Nadeem *et al.* [7]. Ishak *et al.* [8] investigated the heat transfer over a stretching surface with variable heat flux in micropolar fluid. Wang [9] investigated the shrinking flow where the velocity of boundary layer moves toward a fixed point and he found an exact solution of Navier-Stokes equations. A good list of references for micropolar fluids is available in Łukaszewicz [10]. Tien and Yen [11] investigated the effect of melting on forced convection heat transfer between a melting body and surrounding fluid. Epstein and Cho [12] analyzed the melting heat transfer of the steady laminar flows over a flat plate. The steady laminar boundary layer flow and heat transfer from a warm, laminar liquid flow to a melting surface moving parallel to a constant free stream has been studied by Ishak *et al.* [13]. Rosali *et al.* [14] studied micropolar fluid flow towards a permeable stretching /shrinking sheet in a porous medium numerically. Yacob *et al.* [15] investigated a model to study the heat transfer characteristics occurring during the melting process due to a stretching / shrinking sheet and they studied the effects of the material parameter, melting parameter and the stretching /shrinking parameter on the velocity, temperature, skin friction coefficient and the local Nusselt number. Cheng and Lin [16] analyzed the melting effect on transient mixed convective heat transfer from a vertical plate in a liquid saturated porous medium.

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In the present work, we consider the boundary layer stagnation-point flow and melting heat transfer of a MHD micropolar fluid towards a stretching / shrinking surface. To the best of our knowledge, this problem has not been considered before, so that the results are new.

2. Mathematical formulation

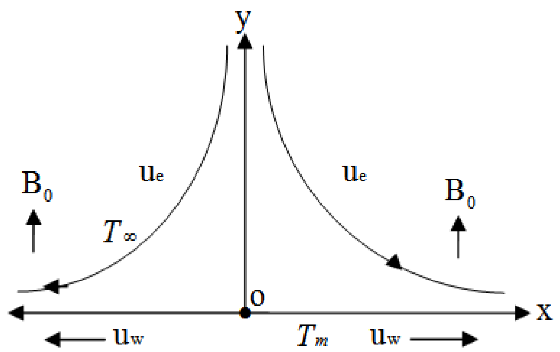

Figure 1: Flow configuration and Coordinate system

The graphical model of the problem is given along with flow configuration and coordinate system. The system deals with two dimensional stagnation point steady flow of micropolar fluids towards a stretching / shrinking surface and subject to a constant transverse magnetic field B_0. The velocity of the external flow is $u_e(x) = ax$ and the velocity of the stretching surface is $u_w(x) = cx$, where a is a positive constant and c is a positive (stretching surface) or a negative (shrinking surface) constant, x is the coordinate measured along the surface. It is also assumed that the temperature of the melting surface and free stream condition is T_m and T_∞, where $T_\infty > T_m$. The viscous dissipation and the heat generation or absorption has assumed to be negligible. Under these assumptions, the governing equations representing flow are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{(\mu+k)}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}(u_e-u) \quad (2)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (4)$$

with the following boundary conditions:

$$u=u_w(x)=cx\,;\; N=-n\frac{\partial u}{\partial y}\,;\; T=T_m \quad at\; y=0$$

$$u=u_e(x)=ax\,;\; N=0\,;\; T=T_\infty \quad as\; y\to\infty \quad (5)$$

and $\kappa\left(\frac{\partial x}{\partial y}\right)_{y=0}=\rho\left[\lambda+c_s(T_m-T_0)\right]v(x,0)$

here u and v are the velocity component along the x and y axis, respectively. Further, μ is dynamic viscosity, k is vortex viscosity, σ is electrical conductivity of the fluid, ρ is fluid density, T is fluid temperature, j is micro inertia density, N is microrotation, γ is spin gradient viscosity, α is thermal diffusivity, κ is the thermal conductivity, λ is the latent heat of the fluid and C_s is the heat capacity of the solid surface. We note that n is a constant such that $0 \le n \le 1$. The case when $n = 0$, is called strong concentration which indicates that no microtation near the wall. In case $n = 1/2$ it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and the case $n = 1$ is used for the modeling of turbulent boundary layer flows by Yacob *et al.* [15].

$\gamma=\left(\mu+\frac{k}{2}\right)l=\mu\left(1+\frac{K}{2}\right)l$, where $K=\frac{\kappa}{\mu}$ is the micro polar or material parameter and $l=\frac{\upsilon}{a}$ as reference length. The total spin N reduces to the angular velocity.

3. Problem solution

Equations (2) - (4) can be transformed into a set of nonlinear ordinary differential equations by using the following similarity variables:

$$\gamma=\left(\mu+\frac{k}{2}\right)l=\mu\left(1+\frac{K}{2}\right)l,\; K=\frac{k}{2},\; l=\frac{\upsilon}{a},$$

$$\psi=\left(a\upsilon\right)^{1/2}xf(\eta),\; N=xa\left(\frac{a}{\upsilon}\right)^{1/2}g(\eta), \quad (6)$$

$$\theta(\eta)=\frac{T-T_m}{T_\infty-T_m},\qquad \eta=\left(\frac{a}{\upsilon}\right)^{1/2}y$$

The transformed ordinary differential equations are:

$$\left(1+K\right)f'''+ff''+1-f'^2+Kg'+M\left(1-f'\right)=0 \quad (7)$$

$$\left(1+K/2\right)g''+fg'-f'g+K\left(2g+f''\right)=0 \quad (8)$$

$$\theta''+\Pr f\theta'=0 \quad (9)$$

where primes denote differentiation with respect to η and $Pr = \upsilon/\alpha$ is Prandtl number. The boundary conditions (5) become:

$$f'(0)=\varepsilon,\; g(0)=-nf''(0),$$

$$\Pr f(0)+m\theta'(0)=0,\;\theta(0)=0 \quad (10)$$

$$f'(\infty)=1,\; g(\infty)=0,\; \theta(\infty)=1.$$

where $\varepsilon = c/a$ is the stretching ($\varepsilon > 0$) or shrinking ($\varepsilon < 0$) parameter, m is the dimensionless melting parameter and M is magnetic parameter which are defined as:

$$m=\frac{c_f\left(T_\infty-T_m\right)}{\lambda+c_s\left(T_m-T_0\right)},$$

$$M=\frac{\sigma B_0^2}{a\rho} \qquad (11)$$

The physical parameters of interest are the skin friction coefficient C_f, local Couple stress coefficient C_m and the local Nusselt number Nu_x, which are defined as:

$$C_f=\frac{\tau_w}{\rho u_e^2},$$

$$C_m=\frac{C_w}{x\rho u_e^2}, \qquad (12)$$

$$Nu_x=\frac{xq_w}{\kappa\left(T_\infty-T_m\right)}$$

where τ_w, c_w and q_w are the surface shear stress, the local couple stress and the surface heat flux respectively, which are given by:

$$\tau_w=\left(\mu+k\right)\left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$C_w=\gamma\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad (13)$$

$$q_w=-\kappa\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

hence using (6), we get:

$$\mathrm{Re}_x^{1/2}C_f=\left[1+\left(1-n\right)K\right]f''(0),$$

$$\mathrm{Re}_x\,C_m=\left(1+\frac{K}{2}\right)g'\left(0\right), \qquad (14)$$

$$\mathrm{Re}_x^{-1/2}Nu_x=-\theta'\left(0\right)$$

where $\mathrm{Re}_x=u_e\left(x\right)x/\upsilon$ is the local Reynolds number.

4. Results and discussion

The transformed equations (7) - (9), subject to boundary conditions (10), were solved numerically using the Runge-Kutta-Fehlberg fourth-fifth order method with shooting technique to obtain the missing values of $f''(0)$, $g'(0)$ and $\theta'(0)$ for some values of the magnetic parameter M, micropolar parameter K, melting parameter m and the stretching / shrinking parameter ε, while the Prandtl number Pr is fixed to unity and we take $n=0.5$ (weak concentration).

In order to validate the numerical results obtained, we compared our results with those reported by Ishak *et al.* [8], Wang [9], and Yacob *et al.* [15], as shown in Table 1; and they are found to be in a favorable agreement.

Figures 2, 3 and 4 show the variations of the skin friction coefficient $f''(0)$, the local couple stress coefficient $g'(0)$ and the local Nusselt number $-\theta'(0)$, respectively with ε for different value of M when $m=1$, $K=1$. It is also seen from these figures that for the shrinking case ($\varepsilon<0$), the solution exists up to a critical value of ε (say ε_c) beyond which no solution exists. The values of $f''(0)$ are positive when $\varepsilon<1$, and become negative when $\varepsilon>1$. Physically, positive value of $f''(0)$ means the fluid exerts a drag force on the solid surface and negative value means the solid surface exerts a drag force on the fluid. The zero skin friction when $\varepsilon=1$, since for this case the stretching velocity is equal to the free stream velocity. However, for this case, the heat transfer rate at the surface $-\theta'(0)\neq 0$ means there is a heat transfer between the fluid-solid interfaces (even when the friction is zero). The couple stress coefficient $g'(0)$ (Figure 3) shows similar behaviour as that of skin friction coefficient for the variation of the magnetic parameter M with the Stretching parameter ε. The negative value of $-\theta'(0)$, presented in Figure 4, shows that the heat is transferred from the warm fluid to cool solid surface. It is evident from Table 2 and Figure 2 that an increase in magnetic parameter M leads to a decrease in the value of $f''(0)$ absolute sense. It is clear form Table 2 and Figure 3 that the value of local couple stress coefficient $g'(0)$ decreases with the increase in the value of magnetic parameter M for $\varepsilon<1$, whereas the value of local couple stress coefficient $g'(0)$ increases with increasing value of magnetic parameter M for $\varepsilon>1$. This result in the decreasing manner of the heat transfer rate at the fluid-solid interface $|-\theta'(0)|$ for $\varepsilon<1$, but opposite behaviours are observed for $\varepsilon>1$.

It is observed from velocity profiles $f'(\eta)$ in Figure 5 that the value of $f'(\eta)$ decreases as M increases and from the angular velocity profiles $g(\eta)$ in Figure 6 show that the value of $g(\eta)$ initially increases as M increases and then changing the behaviour for large η the value of $g(\eta)$ decreases with M, thus due to the increase in magnetic parameter M the boundary layer thickness increases. For temperature (Figure 7), the change in magnetic parameter M there is a small change in the temperature $\theta(\eta)$. Consequently, thermal boundary layer undergoes negligible change with M. Finally, from all the figures (Figures 5–9) above, it can be easily seen that the far field boundary conditions are satisfied asymptotically and it signifies the correctness of the numerical scheme used.

Table 1. Comparison between *f ''(0)* and - *θ'(0)* calculated by the present method, Ishak *et al.* [8], Wang [9] and Yacob *et al.* [15] for various values of *m, ε, K* when *M=0*.

M	*ε*	*M*	*K*	Ishak et al. [8]	Wang [9]	Yacob et al. [15]		Present Result	
				f ''(0)	*f ''(0)*	*f ''(0)*	*-θ'(0)*	*f ''(0)*	*-θ'(0)*
	0	0	0	1.2326	1.232588	1.232588	-0.570465	1.232588	-0.570466
			1			1.006404	-0.544535	1.006404	-0.544535
		1	0			1.037003	-0.361961	1.037003	-0.361962
0			1			0.879324	-0.347892	0.879324	-0.347892
	0.5	0	0		0.7133	0.713295	-0.692064	0.713295	-0.692065
			1			0.582403	-0.680176	0.582403	-0.680176
		1	0			0.59909	-0.438971	0.59909	-0.438971
			1			0.506342	-0.432443	0.506342	-0.432443

Table 2 The values of Skin friction coefficient *f ''(0)*, local couple stress coefficient *g ' (0)* and local nusselt number – *θ ' =(0)* for various values of *M*, *ε* when *m*=1 and *K*=1.

M	***ε***	***f''(0)***	***g'(0)***	**- *θ'(0)***
0	0.75	0.26818	0.10379	-0.470624
0.5	0.75	0.23458	0.086155	-0.468501
1	0.75	0.19584	0.06638	-0.466002
0	1.5	-0.61692	-0.29904	-0.572001
0.5	1.5	-0.5596	-0.26514	-0.574801
1	1.5	-0.49625	-0.22813	-0.57793
0.5	- 0.25	0.76177	0.11516	-0.27859
0.5	-0.5	0.72995	0.03944	-0.27859
1	-0.25	0.3965	-0.01038	-0.23432
1	-0.5	0.136	-0.09535	-0.14615

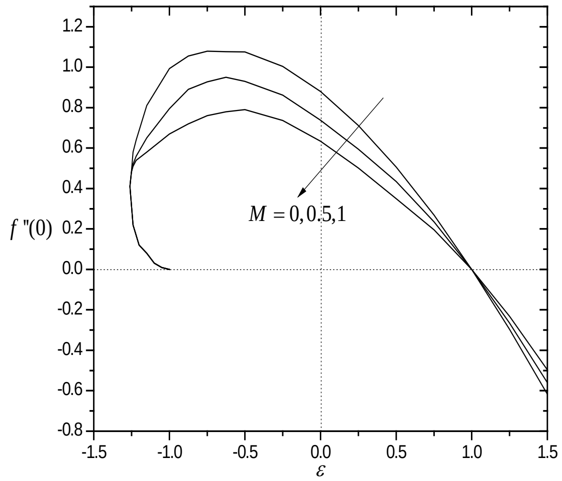

Figure2. Skin friction coefficient *f ''(0)* with *ε* for several value of *M* when *m=1, K=1*

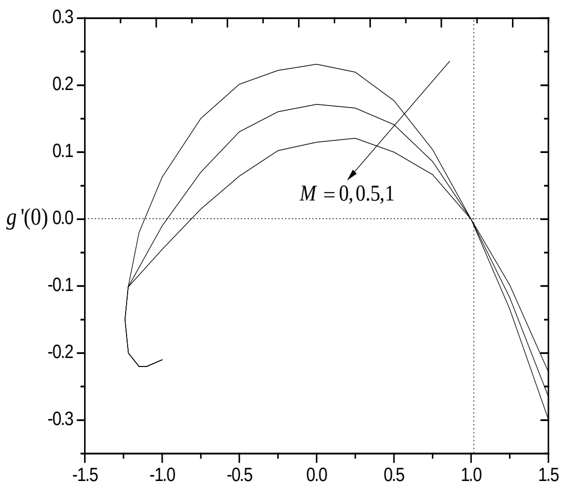

Figure3. Local Couple stress coefficient *g'(0)* with *ε* for several value of *M* when *m = 1, K = 1*

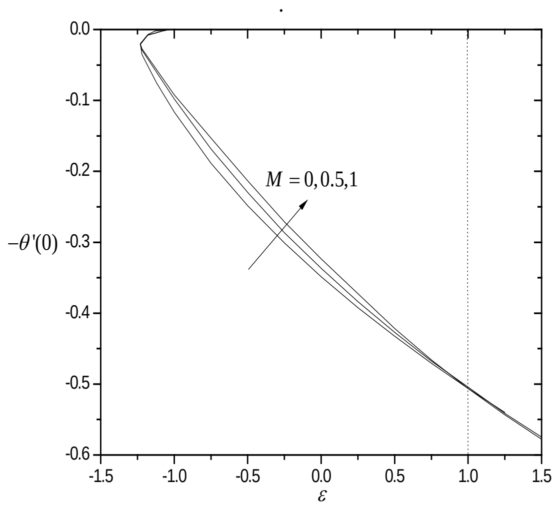

Figure 4. Heat transfer coefficient -*θ′(0)* with *ε* for several value of *M* when *m=1, K= 1.*

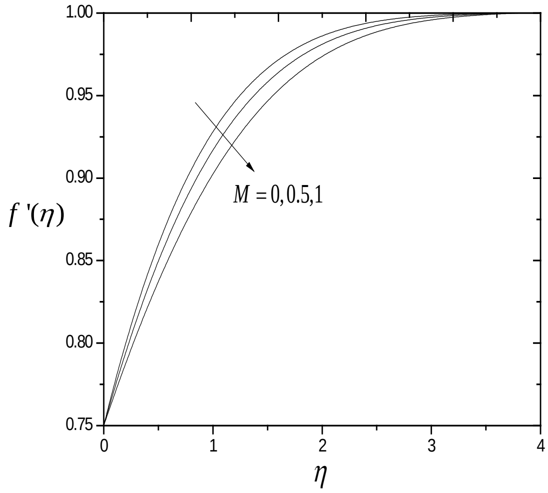

Figure 5. Velocity profiles *f ′(η)* for several value of *M* when *m=1, K=1*and *ε =0.75*

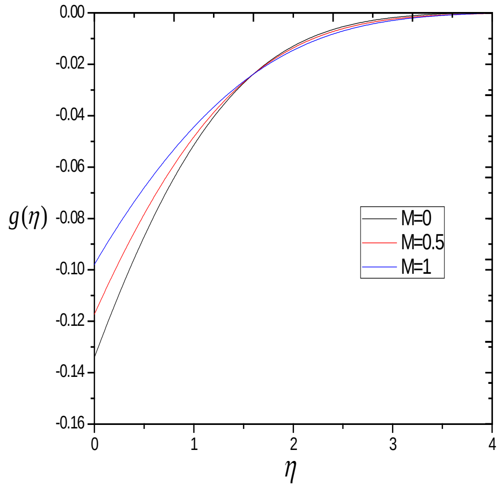

Figure 6. Angular velocity profiles *g(η)* for several value of *M* when *m=1, K=1*and *ε = 0.75*

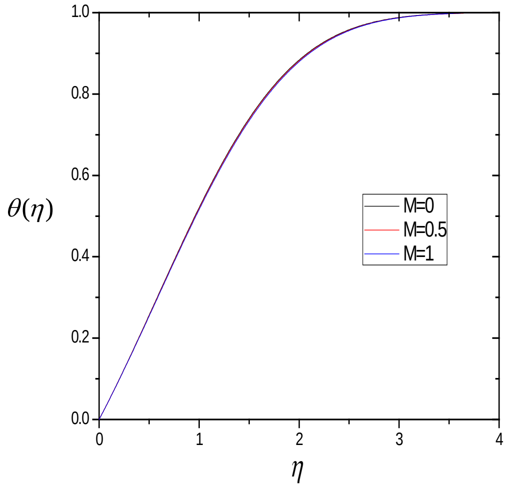

Figure 7. Temperature profiles *θ(η)* for several value of *M* when *m=1, K=1*and *ε = 0.75*

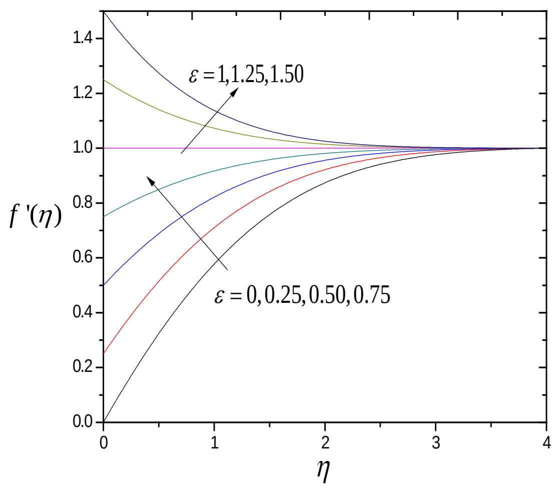

Figure 8. Velocity profile *f ′(η)* for different values of *ε* when *M=0.5, K=1* and *m=1*.

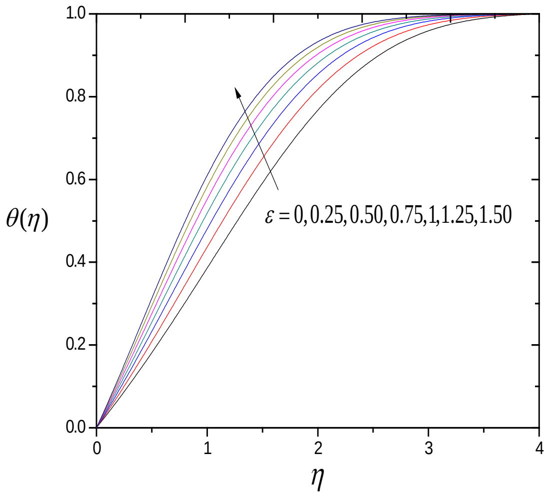

Figure 9. Temperature profile *θ(η)* for different value of ε when *M=0.5, K=1* and *m=1*

5. Conclusions

We have studied the effects of the magnetic parameter on skin friction coefficient, couple stress coefficient and local Nusselt number (which represents the heat transfer rate at the surface) for the steady laminar boundary layer stagnation point flow and heat transfer from a warm micropolar fluid to a melting solid surface of the same material. It has been found that the skin friction coefficient, the couple stress coefficient and the heat transfer coefficient decreases with increase in magnetic parameter.

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