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A New Model for Predicting Crack Initiation Life in Thin Walled Tubes under Multiaxial Proportional Loading

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Abstract

The theory of fracture mechanics is unable to estimate crack initiation life, but the theory of damage mechanics can do it well. The application of critical plane method in multiaxial fatigue has made certain progress. According to the law of thermodynamics, a new damage model is proposed in this paper to predict the crack initiation life under multiaxial proportional loading condition based on damage mechanics and critical plane method. The maximum shear strain amplitude and the normal strain on the maximum shear strain plane are the components of this model. Finally, the crack initiation life is predicted with the proposed model, which is damage mechanics-critical plane method. The predicted results of using the new model comply with the experimental results.

Keywords: *: Damage Mechanics; Critical Plane Method; Damage Evolution Equation; Equivalent Stress; Crack Initiation Life.*

1. Introduction

The mechanical structures in practical application mostly work under cyclic loading. The loading mode may be uniaxial cyclic loading, or likely to be multiaxial cyclic loading. One of the main forms of failure is the fatigue fracture. The loading procedure will continuously produce damage, which often results in micro crack formation and propagation of damage accompanied by a large amount of plastic deformation. The mathematical description of the damage variable is introduced in damage mechanics, and it is applied to structural analysis. The fatigue life predicted model is put forward with the help of the concept of effective stress [1-3]. The critical plane method considers the physical meaning of fatigue fracture, which is widely used currently in the predicted of multiaxial fatigue life. Brown and Miller [4-7] believed that the maximum shear strain helps crack nucleation and normal strain helps crack extension. So a new multiaxial nonlinear damage model based on damage mechanics and critical plane method is proposed in this article. The essence of this model is to replace the control parameters of the uniaxial nonlinear damage model with the largest equivalent strain [8-9]. It was proved that the multiaxial nonlinear damage model is available and practicable by the comparative analysis of the predicted results with the experimental data.

2. Damage Variable and Effective Stress

The premise of analysis of materials and components mechanical property by damage theory is to choose the proper damage variables to describe the damage state [10-11]. The concept of continuum damage mechanics is proposed by Kachanov when he studied metal creep problems, he considered that the main mechanism of material degradation is caused by the decrease of effective bearing area [12-13], then the concept of continuous degree is proposed to describe the damage of materials and it can be defined as follows: a representative hexahedron element is selected and the total sectional area, which is perpendicular to the direction of n, is assumed A (Figure 1).

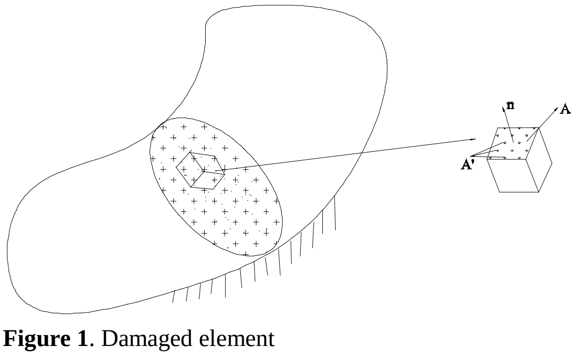

Figure 1. Damaged element

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The continuous degree is defined as:

$$\varphi=\frac{A'}{A} \tag{1}$$

where A' is the actual effective section area and A is the cross section area which doesn't have damage.

The damage degree D, the supplement parameter of continuous degrees, is introduced to describe the damage by Rabotnov [14].

$$D=1-\varphi \tag{2}$$

Combining Eq. (1) with Eq. (2), the following equation can be given as:

$$A'=\left(1-D\right)A \tag{3}$$

The effective stress can be defined as the ratio of the load *F* to the effective bearing area:

$$\sigma'=\frac{F}{A'}=\frac{F}{\left(1-D\right)A} \tag{4}$$

Eq. (4) is a classical expression of damage variable, which has been widely accepted. The principle of equivalent strain can be described as: the strain in the case of the effective stress equal to that in the case of no damage. According to this principle, the constitutive relation of damaged material can be obtained by replacing the nominal stress with the effective stress of damaged material. In the case of one dimensional elastic [15-16]:

$$\sigma=\left(1-D\right)E\varepsilon \tag{5}$$

It is widely accepted that fatigue crack initiation involves a localized plastic deformation in persistent slip bands in the low cycle fatigue region. It has been experimentally observed that the direction of these persistent slip bands is very closely aligned with that of the maximum shear strain direction and fatigue cracks have always been found to initiate on the maximum shear strain planes under different loading situations. This justifies the belief that the fatigue initiation process is predominantly controlled by the maximum shear strain. Comparing the torsion data with the uniaxial data based on the maximum shear strain, most investigators have found the torsion data to lie above the uniaxial data as was also the case in this investigation [17-19]. This suggests that a second parameter is involved in the fatigue damage process. As mentioned previously, Brown and Miller take this parameter to be the normal strain on the maximum shear plane. They argue that his normal strain influences fatigue ductility which in turn is related to the fatigue strength and then conclude that the normal strain across the maximum shear strain plane assists in crack propagation. One advantage of these theories is their physical interpretation of the fatigue damage accumulation. The equivalent strain amplitudes can be obtained based on von Mises rule.

3. Uniaxial Fatigue Damage Model

In the fatigue damage theory, the damage is often expressed as a function of load cycles. Under normal circumstances, the fatigue damage evolution equation can be represented as the following form:

$$dD=f\left(...\right)dN \tag{6}$$

The variables of function $f\left(...\right)$ can be stress, strain and damage variable. At the same time, in order to describe the nonlinear damage accumulation and the loading sequence effect, loading parameters and damage variable are inseparability. The damage evolution of materials or components is a kind of irreversible thermodynamics process. Lemaitre *et al*, describe the fatigue damage evolution equation as the following equation [6]:

$$dD=\left(1-D\right)^{-p}\left[\frac{\sigma_{\max}-\sigma_{\mathrm{m}}}{M\left(\sigma_{\mathrm{m}}\right)\left(1-D\right)}\right]^{\beta}dN \tag{7}$$

where $\sigma_{\max}$ is the largest stress amplitude, σ_{m} is the average stress, p and β are the parameters associated with the loading form and material constant.

When $D=0,\ N=0$; and when $D=1,\ N=N_f$. The following equations can be obtained by definite integration of Eq. (7):

$$\int_0^1\left(1-D\right)^{p+\beta}dD=\left[\frac{\sigma_{\max}-\sigma_{\mathrm{m}}}{M\left(\sigma_{\mathrm{m}}\right)}\right]^{\beta}\int_0^{N_f}dN \tag{8}$$

$$N_f=\frac{1}{1+p+\beta}\left[\frac{\sigma_{\max}-\sigma_{\mathrm{m}}}{M\left(\sigma_{\mathrm{m}}\right)}\right]^{-\beta} \tag{9}$$

$$D=1-\left(1-\frac{N_R}{N_f}\right)^{\frac{1}{1+p+\beta}} \tag{10}$$

Where, p, β and M are constant, which are concerned with material and the way of loading, $\sigma_{\max}-\sigma_{\mathrm{m}}=\frac{\Delta\sigma}{2}$, $M\left(\sigma_{\mathrm{m}}\right)=M_0\left(1-b\sigma_{\mathrm{m}}\right)$, N_R is actual life.

This paper mainly studies the fatigue damage problem under a symmetric constant amplitude loading, so Eq.(9) can be represented as:

$$N_f=\frac{M_0}{1+p+\beta}\left(\frac{\Delta\sigma}{2}\right)^{-\beta} \tag{11}$$

It can be seen that the main parameter for uniaxial fatigue model is $\frac{\Delta\sigma}{2}$, which can be obtained from the existing test and theory analysis. The fatigue property of material under proportional loading is consistent with that under uniaxial fatigue condition, thus $\frac{\Delta\sigma}{2}$ can be replaced by the equivalent stress amplitude under the condition of proportional multiaxial loading, namely the multiaxial nonlinear fatigue cumulative damage model can be got by the above method.

According to the strain hardening laws:

$$\frac{\Delta\sigma}{2}=K\left(\Delta\varepsilon_p/2\right)^n \tag{12}$$

where K and n are the material constants.
So Eq. (11) can be rewritten as:

$$N_f=\frac{{M_0}^{\beta}}{1+p+\beta}\left(K\left(\Delta\varepsilon_p/2\right)^n\right)^{-\beta} \tag{13}$$

4. Multiaxial Fatigue Damage Model

A thin wall pipe is generally selected in the multiaxial fatigue test. The most serious damage plane is vertical to the free surface [20-23], which is the plane we care about. The stress and strain state under pull-torsion loading can be expressed as:

$$\sigma=\begin{bmatrix}\sigma_{xx} & \sigma_{xy} & 0\\ \sigma_{xy} & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \varepsilon=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & 0\\ \varepsilon_{xy} & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz}\end{bmatrix} \tag{14}$$

In this paper, the loading form can be expressed as:

$$\varepsilon_{xx}=\varepsilon_a\sin\omega t \tag{15}$$

$$\gamma_{xy}=\lambda\varepsilon_a\sin\left(\omega t-\varphi\right) \tag{16}$$

Where λ is the ratio of shear strain to axial strain, φ is phase difference.

The strain state of the plane which is canted by θ to the axis of the specimens can be expressed as:

$$\varepsilon_\theta=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta+\frac{1}{2}\gamma_{xy}\sin 2\theta \tag{17}$$

$$\frac{\gamma_{xy}}{2}=\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta-\frac{1}{2}\gamma_{xy}\cos 2\theta \tag{18}$$

where $\varepsilon_y=-\nu\varepsilon_x$.

So Eq.(17) and Eq.(18) can be represented as:

$$\varepsilon_\theta=\frac{1-\nu}{2}\varepsilon_x+\frac{1+\nu}{2}\varepsilon_x\cos 2\theta+\frac{1}{2}\gamma_{xy}\sin 2\theta \tag{19}$$

$$\frac{\gamma_{xy}}{2}=\frac{1+\nu}{2}\varepsilon_x\sin 2\theta-\frac{1}{2}\gamma_{xy}\cos 2\theta \tag{20}$$

At the time, when $\varphi=0$

$$\varepsilon_\theta=\frac{1-\nu}{2}\varepsilon_a\sin\omega t+\frac{1+\nu}{2}\varepsilon_a\sin\omega t\cos 2\theta+\frac{1}{2}\lambda\varepsilon_a\sin\omega t s \tag{21}$$

$$\frac{\gamma_{xy}}{2}=\frac{1+\nu}{2}\varepsilon_a\sin\omega t\sin 2\theta-\frac{1}{2}\lambda\varepsilon_a\sin\omega t\cos 2\theta \tag{22}$$

Because the critical plane is defined as the plane of the maximum shear strain, thus,

$$\frac{\partial\gamma_{xy}}{\partial\theta}=0 \tag{23}$$

Though Eq.(23), the θ range can be obtained:

$$\theta=\frac{1}{2}\tan^{-1}\frac{-(1+\nu)}{\lambda} \tag{24}$$

In the range between –π/2 and π/2, there are two θ ($\theta_{\max},\theta_{\min}$) range which makes ε_θ reach to extreme value, but only $\theta_{\max}$ makes ε_θ reach to maximal value. So normal strain and shear strain of the critical plane can be represented as:

$$\overline{\varepsilon_\theta}=\frac{1-\nu}{2}\varepsilon_a\sin\omega t+\frac{1+\nu}{2}\varepsilon_a\sin\omega t\cos 2\overline{\theta}+\frac{1}{2}\lambda\varepsilon_a\sin\omega t\sin 2\overline{\theta} \tag{25}$$

$$\frac{\overline{\gamma_{xy}}}{2}=\frac{1+\nu}{2}\varepsilon_a\sin\omega t\sin 2\overline{\theta}-\frac{1}{2}\lambda\varepsilon_a\sin\omega t\cos 2\overline{\theta} \tag{26}$$

The equivalent strain can be represented as:

$$\varepsilon_{eq}=\left[\overline{\varepsilon_\theta}+\frac{1}{3}\left(\frac{\overline{\gamma_\theta}}{2}\right)^2\right]^{\frac{1}{2}} \tag{27}$$

Combining Eq.(13) with Eq.(27), the multiaxial fatigue damage model can be given as:

$$N_f=\frac{M_0}{1+p+\beta}\left\{K\left\{\left[\overline{\varepsilon_\theta}+\frac{1}{3}\left(\frac{\overline{\gamma_\theta}}{2}\right)^2\right]^{\frac{1}{2}}\right\}^n\right\}^{-\beta} \tag{28}$$

5. Experiments and Results

The material under investigation was 06Cr19Ni10 steel, a kind of widely used material in engineering. All indexes satisfy property requirement of 06Cr19Ni10 steel and the data will be the reference of the parameter of fatigue test.

In this paper, the reported fatigue life corresponds to the moment when a visible crack was found on the specimen surface. An Instron hydraulic tension-torsion loading frame (Figure 2) was used for the uniaxial and multiaxial proportional fatigue tests. The testing system was equipped with the Instron 8800 electronic control, computer control, and data acquisition.

Figure 2. Fatigue testing machine

The uniaxial results were listed in Table 1.

Table 1: Fatigue life of uniaxial experiment

ε(%)	0.48	0.59	0.69	0.8
N_f(cycles)	3881	2830	1962	1403

The parameters of Eq. (13) can be fitted using the uniaxial experiment data:

$$\frac{M_0^{\ \beta}}{1+p+\beta}=7.3294\times10^{42},\ \beta=13.5488$$

The shape of multiaxial fatigue sample is shown in Figure 3:

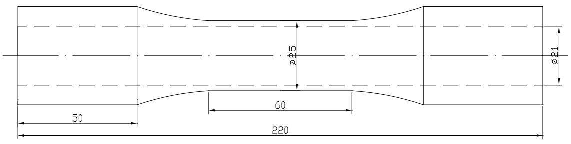

Figure 3. Shape of sample

The proposed model is used respectively to predict the crack initiation life when

$$\lambda_1=\sqrt{3},\ \lambda_2=\sqrt{3}/2,\ \lambda_3=1/2.$$

The loading conditions are listed in Table 2:

Table 2. Amplitude of axial and torsional loading (%)

λ_1	σ	0.370	0.490	0.566	0.670	0.800
	γ	0.64	0.84	0.981	1.16	1.39
λ_2	σ	0.370	0.490	0.566	0.670	0.800
	γ	0.32	0.424	0.49	0.58	0.693
λ_3	σ	0.370	0.490	0.566	0.670	0.800
	γ	0.185	0.245	0.283	0.335	0.400

The way of loading is shown in Figure 4.

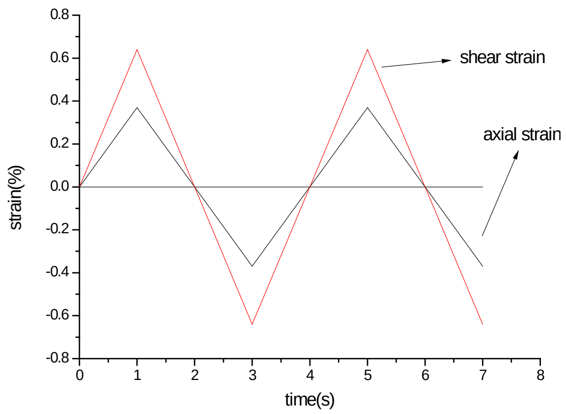

Figure 4. The way of loading

The experimental results and the predicted result are shown in Table 3:

Table 3. The data of multiaxial fatigue (cycles)

λ=√3	P	1751	1156	823	618	482
	E	1801	1209	856	632	493
λ=√3/2	P	1480	1613	1820	2209	2750
	E	1537	1594	1893	2284	2791
λ=1/2	P	1050	1187	1410	1721	2305
	E	971	1102	1371	1708	2277

P—predicted life; E—experimental life.

The predicted results using multiaxial nonlinear fatigue cumulative damage model are compared with the experimental results, and the comparison is shown in Figure5:

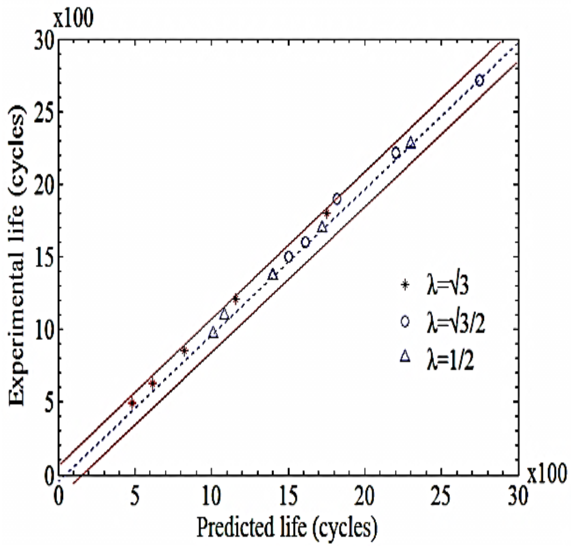

Figure 5. The comparison between predicted results and experimental results

Figure 5 shows the plots of the predicted life and experimental life when

$$\lambda_1=\sqrt{3},\ \lambda_2=\sqrt{3}/2,\ \lambda_3=1/2.$$

It is obvious from the figure that life predictions based on the proposed approach were conservative within a factor of 8 for proportional loading. In this paper, the damage mechanics is applied to predict the crack initiation life that the fracture mechanics is unable to predict. At the same time, the physical significance of the critical plane method is considered. The multiaxial nonlinear fatigue cumulative damage model proposed in this paper makes use of the advantage of the above two method. It can be seen from the comparative analysis that this method can predict the crack initiation life under proportional loading well.

The parameters of the proposed model, such as the material constants and uniaxial fatigue data, can be easily obtained through theoretical analysis and the existing experimental data. On the basis of these parameters the crack initiation life can be well predicted. Thus, the new model can avoid conducting the multiaxial test which is time-consuming, money-consuming and troublesome and it is easy to apply in engineering.

Conclusion

The proposed method has been verified in comparison with the predicted results and experimental data. The method of combining damage mechanics with critical plane method can commendably predict the crack initiation life and it has a more practical value because the multiaxial fatigue damage model can predict the crack initiation life using the material constant and uniaxial fatigue data only.

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