Reliability Modelling of a Computer System with Priority to H/W Repair over Replacement of H/W and Up-gradation of S/W Subject to MOT and MRT

Ashish Kumar* a, S.C. Malik b

aDepartment of Mathematics, Manipal University, Jaipur-303007, Rajasthan (India)
bDepartment of Statistics, M.D. University, Rohtak-124001, Haryana (India)

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Abstract

In the present paper, some reliability measures of a computer system of two cold standby identical units are obtained using a semi-Markov process and a regenerative point technique. For this purpose, a reliability model is proposed by considering independent h/w and s/w failure. Initially, one unit is operative and another is kept as spare in cold standby. A single server is provided to conduct preventive maintenance, repair, replacement and up-gradation of the components upon their failure. Then, a maximum operation time system undergoes preventive maintenance, directly from normal mode. If the repair of the h/w is not possible by the server up to a pre-specific time, it is replaced by a new one with some replacement time. However, only up-gradation of the software by a new one is made whenever s/w fails to execute the desired function properly. Priority to h/w repair is given over h/w replacement and s/w up-gradation. The failure time distribution of the h/w and s/w follows a negative exponential while the distributions of preventive maintenance, repair, up-gradation and replacement time are taken as arbitrary with different probability density functions. To depict the importance of the study, graphs are drawn for a particular case with fixed values of other parameters and costs.

Keywords: Computer System, H/W and S/W Failure, Maximum Operation and Repair Times, Priority and Preventive Maintenance.

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Nomenclature

| E          | The set of regenerative states |
| NO         | The unit is operative and in normal mode |
| Cs         | The unit is cold standby |
| a/b        | Probability that the system has hardware / software failure |
| λ1/λ2      | Constant hardware / software failure rate |
| α0         | Maximum Operation Time |
| β0         | Maximum Repair Time. |
| Pm/PM      | The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state |
| WPm/WPM    | The unit is waiting for preventive Maintenance/ waiting for preventive maintenance from previous state |
| HFur/HFUR | The unit is failed due to hardware and is under repair / under repair continuously from previous state |
| HFarp/HFURP | The unit is failed due to hardware and is under replacement / under replacement continuously from previous state |
| HFwr / HFWR | The unit is failed due to hardware and is waiting for repair/ waiting for repair continuously from previous state |
| SFarp/SFURP | The unit is failed due to the software and is under up-gradation/ under up-gradation continuously from previous state |
| SFwrp/SFWRP | The unit is failed due to the software and is waiting for up-gradation/ waiting for up-gradation continuously from previous state |

pdf / cdf of up-gradation time of unit due to software
pdf / cdf of repair time of the hardware
pdf / cdf of replacement time of the hardware
pdf / cdf of the time for PM of the unit
pdf / cdf of Probability density function/ Cumulative density function

* Corresponding author. e-mail: ashishbarak2020@gmail.com.
1. Introduction

In our modern society, there is an increased reliance on computer systems due to the rise of technology in everyday life. These systems control everything from banking to telecommunications, to nuclear power plants, to air traffic. Their failure can be catastrophic. Therefore, ensuring the reliability and performance of such systems has become a growing need for engineers and academicians. Currently, redundancy is one of the best methods of improving the reliability of any operating system. During the past few years, some reliability models of computer systems with independent h/w and s/w failures have been suggested by some researchers, including Malik and Anand [1] and Malik and Kumar [3].

Also, it is a well known fact that preventive maintenance can slow down the deterioration process of a computer system and restore the system as new. Thus, the method of preventive maintenance can be used to improve the reliability and the profit of the system. The concept of preventive maintenance was used by Malik and Nandal [2] while discussing a cold standby system with a maximum operation time system undergoes preventive maintenance directly from the normal mode. If the repair of the h/w is not possible by the server up to a pre-specific time, it is replaced by a new one with some replacement time. However, only up-gradation of the software by new one is made whenever s/w fails to execute the desired function properly. Priority to h/w repair is given over h/w replacement and s/w up-gradation. The failure time h/w and s/w follows exponential distribution while the distributions of preventive maintenance, repair, up-gradation and replacement time are taken as arbitrary with different probability density functions. Various system effectiveness measures, such as mean time to system failure, availability, busy period of the server due to preventive maintenance, busy period of the server due to repair, busy period of the server due to hardware replacement, busy period of the server due to software up-gradation, expected number of software up-gradations, expected number of hardware replacement and expected number of visits of the server are derived by using semi-Markov process and regenerative point technique. All random variables are statistically independent and uncorrelated. Switch devices are perfect. The graphical study of the results for a particular case has also been made to highlight the importance of the results.

2. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions by taking $A = a\lambda_1 + b\lambda_2 + \alpha_0$, $B = a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0$ for the non-zero elements.

\[
p_{ij} = Q_i(x) = \int_0^\infty q_j(t) \, dt
\]

As

\[
p_{16} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0}, \quad p_{06} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{18} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0}, \quad p_{08} = \frac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{113} = \frac{a\lambda_1 + b\lambda_2 + \alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0)) \, dt, \quad p_{113} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)) \, dt, \quad p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0},
\]

\[
p_{01} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{126} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{13} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{10} = \frac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{03} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{113} = \frac{a\lambda_1 + b\lambda_2 + \alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0)) \, dt, \quad p_{113} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)) \, dt, \quad p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0},
\]

\[
p_{01} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{126} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{13} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{10} = \frac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{03} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{113} = \frac{a\lambda_1 + b\lambda_2 + \alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0)) \, dt, \quad p_{113} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0},
\]

\[
p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0} \cdot \int_{-}^{x} f'((a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)) \, dt, \quad p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0},
\]
It can be easily verified that
\[ p_{01} + p_{02} + p_{03} = p_{10} + p_{16} + p_{18} + p_{1.13} = 1 \]
\[ p_{20} + p_{24} + p_{25} + p_{2.11} + p_{2.12} + p_{30} + p_{37} + p_{39} + p_{3.10} = 1 \]
\[ p_{40} + p_{4.17} + p_{4.18} + p_{4.19} + p_{5.1} + p_{5.16} = p_{62} = p_{73} = p_{83} = p_{91} = 1 \]
\[ p_{10.3} = p_{11.3} + p_{11.14} = p_{12.2} + p_{12.15} = p_{13.1} = p_{14.1} = p_{15.2} = 1 \]
\[ p_{16.1} = p_{17.1} = p_{18.3} = p_{19.4} = p_{10} + p_{12.6} + p_{11.13} + p_{13.8} = 1 \]
The mean sojourn times ($\mu_i$) is the state $S_i$ are

\[
\mu_0 = \frac{1}{(a\lambda_1 + b\lambda_2 + \alpha_0)}, \quad \mu_1 = \frac{1}{(a\lambda_1 + b\lambda_2 + \alpha_0 + \alpha)}, \quad \mu_2 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \theta + \beta_0}, \quad \mu_3 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta}, \quad \mu_4 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha_0 + \gamma},
\]

\[
\begin{align*}
\mu_5 &= \frac{\beta_1^2 + a\lambda_1\beta_1 + (b\lambda_2 + \alpha_0)(a\lambda_1 + b\lambda_2 + \alpha_0)}{\beta(a\lambda_1 + b\lambda_2 + \alpha_0 + \theta + \beta_0)^2}, \\
\mu_6 &= \frac{1}{\gamma} + \frac{(\alpha + \theta)^2(\beta_0 + \theta_0)^2 + \gamma\theta(B) + \beta_0(\beta_0 + \theta)(\theta + B)(B)}{(B + \theta)^2 + \gamma(\theta + B)^2(\beta_0 + \beta_0)^2}, \\
\mu_7 &= \frac{1}{\theta}
\end{align*}
\]

(4)

3. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the CDF of first passage time from the regenerative state $i$ to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

\[
\phi_i(t) = \sum_{j} Q_{ij}(t)\phi_j(t) + \sum_{k} Q_{ik}(t)
\]

(5)

Where $j$ is an un-failed regenerative state to which the given regenerative state $i$ can transit and $k$ is a failed state to which the state $i$ can transit directly.

Taking LT of the above relation (5) and solving for $\phi_i(s)$.

We have

\[
R^*(s) = 1 - \frac{\bar{\phi}_i(s)}{s}
\]

(6)

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

\[
\text{MTSF} = \lim_{s \to 0} \frac{1 - \bar{\phi}_i(s)}{s} = \frac{N_i}{D_i}
\]

(7)

Where

\[
N_i = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{24}p_{03}\mu_4
\]

And $D_i = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} - p_{02}p_{24}p_{40}$

4. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant $t$ given that the system entered regenerative state $i$ at $t = 0$. The recursive relations for $A_i(t)$ are given as

\[
A_i(t) = M_i(t) + \sum q_{ij}(t)A_j(t)
\]

(8)

Where $j$ is any successive regenerative state to which the regenerative state $i$ can transit through $n$ transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time $t$ without visiting to any other regenerative state, we have

\[
M_0(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}, \\
M_1(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \alpha)t}G(t), \\
M_2(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t}H(t), \\
M_3(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \gamma)t}M(t)
\]

(9)
Taking LT of the above relations (8) and solving for $A_{i0}(s)$, the steady state availability is given by

$$A_{i0}(s) = \lim_{s \to 0} s A_{i0}(s) = \frac{N_2}{\beta},$$

Where

$$N_2 = \{p_2\} [p_{12.6} p_{31.9} + p_{41.17} (1 - p_{33.10} - p_{37.73})] - p_4 \{p_{43.18} p_{31.9} + (1 - p_{33.10} - p_{37.73}) p_{41.17}\} + p_5 \{p_{50.1} p_{43.18} p_{31.9} + (1 - p_{33.10} - p_{37.73}) p_{41.17}\} + p_3 \{p_{50.1} p_{43.18} p_{31.9} + (1 - p_{33.10} - p_{37.73}) p_{41.17}\} + p_3.$$ 

5. Busy Period Analysis for Server

5.1. Due to Preventive Maintenance (PM)

Let $B_{i0}^p(t)$ be the probability that the server is busy in the preventive maintenance of the unit at an instant ‘t’ given that the system entered state i at $t = 0$. The recursive relations for $B_{i0}^p(t)$ for are as follows:

$$B_{i0}^p(t) = W_{i0}(t) + \sum_{j} q_{ij}^{(n)}(t) \odot B_{j0}^p(t)$$

Where $j$ is any successive regenerative state to which the regenerative state $i$ can transit through $n$ transitions. $W_{i0}(t)$ is the probability that the server is busy in steady state $i$ due to the preventive maintenance up to time $t$ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_{i0} = e^{-a_i q + b_i q + a_q}(t) + (a_i e^{-a_i q + b_i q + a_q}) \odot \Xi(t) + (a_i e^{-a_i q + b_i q + a_q}) \odot \Xi(t),$$

where $q = a_i q + b_i q + a_q$.

5.2. Due to Hardware Failure

Let $B_{i0}^h(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant ‘t’ given that the system entered state i at $t = 0$. The recursive relations for $B_{i0}^h(t)$ are as follows:

$$B_{i0}^h(t) = W_{i0}(t) + \sum_{j} q_{ij}^{(n)}(t) \odot B_{j0}^h(t)$$

Where $j$ is any successive regenerative state to which the regenerative state $i$ can transit through $n$ transitions. $W_{i0}(t)$ is the probability that the server is busy in steady state $i$ due to hardware failure up to time $t$ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_{i0} = e^{-(a_i q + b_i q + a_q)\Xi(t) + (a_i e^{-(a_i q + b_i q + a_q)\Xi(t)}) \odot \Xi(t) + (a_i e^{-(a_i q + b_i q + a_q)\Xi(t)}) \odot \Xi(t)},$$

5.3. Due to Up-Gradation of the Software

Let $B_{i0}^s(t)$ be the probability that the server is busy due to up-gradation of the software at an instant ‘t’ given that the system entered the regenerative state $i$ at $t = 0$. We have the following recursive relations for $B_{i0}^s(t)$:

$$B_{i0}^s(t) = W_{i0}(t) + \sum_{j} q_{ij}^{(n)}(t) \odot B_{j0}^s(t)$$

Where $j$ is any successive regenerative state to which the regenerative state $i$ can transit through $n$ transitions. $W_{i0}(t)$ is the probability that the server is busy in steady state $i$ due to up-gradation of the software up to time $t$ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_{i0} = e^{-(a_i q + b_i q + a_q)\Xi(t) + (a_i e^{-(a_i q + b_i q + a_q)\Xi(t)}) \odot \Xi(t) + (a_i e^{-(a_i q + b_i q + a_q)\Xi(t)}) \odot \Xi(t)}.$$
Taking the LT of the above relations (12) to (15), and solving for $B_0^H(s)$ and $B_0^S(s)$, the time for which server is busy due to repair and replacements, respectively, is given by

$$
B_0^H = \lim_{s \to 0} sB_0^H(s) = \frac{N_2^H}{D_2} \\
B_0^S = \lim_{s \to 0} sB_0^S(s) = \frac{N_2^S}{D_2}
$$

The recursive relations for $N_2^P$, $N_2^R$, $N_2^S$ and $N_2^{HRp}$ are already mentioned.

6. Expected Number of H/W Replacements and S/W Up-Gradations

6.1. Due to Hardware Failure

Let $R_{ij}^H(t)$ be the expected number of replacements of the failed hardware components by the server in $(0, t]$ given that the system entered the regenerative state $i$ at $t = 0$. The recursive relations for $R_{ij}^H(t)$ are given as

$$
R_{ij}^H(t) = \sum_j Q_{ij}^{(m)}(t) \delta_j + R_{ij}^H(t)
$$

Where $j$ is any regenerative state to which the given regenerative state $i$ transits and $\delta_j = 1$, if $j$ is the regenerative state where the server does the job afresh, otherwise $\delta_j = 0$.

6.2. Due to Software Failure

Let $R_{ij}^S(t)$ be the expected number of up-gradations of the failed software by the server in $(0, t]$ given that the system entered the regenerative state $i$ at $t = 0$. The recursive relations for $R_{ij}^S(t)$ are given as

$$
R_{ij}^S(t) = \sum_j Q_{ij}^{(m)}(t) \delta_j + R_{ij}^S(t)
$$

Where $j$ is any regenerative state to which the given regenerative state $i$ transits and $\delta_j = 1$, if $j$ is the regenerative state where the server does the job afresh, otherwise $\delta_j = 0$. Taking LT of the relations (17) and (18), and solving for $\tilde{R}_{ij}^H(s)$ and $\tilde{R}_{ij}^S(s)$, the expected numbers of replacements per unit time to the hardware and software failures are respectively given by

$$
\tilde{R}_{ij}^H(\infty) = \lim_{s \to 0} s\tilde{R}_{ij}^H(s) = \frac{N_2^H}{D_2} \\
\tilde{R}_{ij}^S(\infty) = \lim_{s \to 0} s\tilde{R}_{ij}^S(s) = \frac{N_2^S}{D_2}
$$
Where $D_2$ is already mentioned.

$$N_4^H = (p_{40} + p_{41.17} + p_{43.18})(p_{24})(p_{01}(1 - p_{33.10} - p_{37}P_{73})p_{12.6} + p_{02})$$
$$[(1 - p_{11.13})(1 - p_{33.10} - p_{37}P_{73}) - p_{31.9}P_{13.8}]+p_{03}p_{31.9}P_{12.6}]$$
$$+ (p_{22.12.15} + p_{21.5.16} + P_{23.11.14}(1 - p_{9.4}P_{4.19}))(p_{01}(1 - p_{33.10} - p_{37}P_{73})p_{12.6} + p_{02})$$
$$[(1 - p_{11.13})(1 - p_{33.10} - p_{37}P_{73}) - p_{31.9}P_{13.8}]+p_{03}p_{31.9}P_{12.6}]$$

$$N_4^S = (p_{30} + p_{33.10} + p_{31.9})(p_{24})[(p_{01}P_{43.18}P_{12.6} - p_{03}p_{41.17}P_{12.6} + p_{02} (1 - p_{11.13}))(p_{43.18} - P_{41.17}P_{13.8})]$$
$$+ (1 - p_{19.4}P_{4.19})p_{01}p_{31.8}[(1 - p_{22.12} - P_{22.12.15}) + P_{23.11} + p_{23.11.14})]$$

7. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state $i$ at $t = 0$. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j N_j(t) \delta_j + N_i(t)$$

Where $j$ is any regenerative state to which the given regenerative state $i$ transits and $\delta_j = 1$, if $j$ is the regenerative state where the server does the job afresh, otherwise $\delta_j = 0$. Taking LT of the relation

$$(20)$$

and solving for $\tilde{N}_i(s)$. The expected number of visit per unit time by the server are given by

$$N_i(\infty) = \lim_{s \to 0} s\tilde{N}_i(s) = \frac{N_i}{D_2}$$

$$(21)$$

$N_4 = (p_{24})p_{12.6}\{p_{43.18}p_{31.9} + (1- p_{33.10}p_{41.17}) + (1 - p_{41.19}p_{9.4}))(1 - p_{11.13})p_{31.8} - p_{31.9}p_{22.12} - p_{22.12.15} - P_{23.11} + (1 - p_{23.11.14}) + p_{31.8}p_{21.5} + p_{21.5.16}]$$

8. Profit Analysis

The profit incurred to the system model in a steady state can be obtained as

$$P = K_o - K_A \cdot \gamma - \frac{K_o}{B_o^o} - K_B \cdot B_o^o - K_C \cdot B_o^o$$

$$(22)$$

$K_0 =$Revenue per unit up-time of the system

$K_A =$Cost per unit time for which server is busy due preventive maintenance

$K_B =$Cost per unit time for which server is busy due to hardware failure

$K_C =$Cost per unit replacement of the failed software component

$K_D =$Cost per unit replacement of the failed hardware component

$K_1 =$Cost per unit replacement of the failed hardware

$K_2 =$Cost per unit replacement of the failed software component

9. Conclusion

In the present study, the numerical results considering a particular case $g(t) = \beta e^{-\alpha t}$, $h(t) = \beta e^{-\alpha t}$ and $m(t) = \gamma e^{-\alpha t}$ are obtained for some of the reliability measures of a computer system of two identical units with h/w and s/w components. The graphs of the mean time of the system failure (MTSF), the availability and the profit are drawn with respect to preventive maintenance rate ($\alpha$) for fixed values of other parameters including $a=.7$ and $b=.3$ as shown, respectively, in Figure 2, 3 and 4. These figures reveal that MTSF, availability and profit increase with the increase of the preventive maintenance rate ($\alpha$), h/w repair rate ($\theta$) and by interchanging the values of $a$ and $b$, i.e., $a=.3$ and $b=.7$. But the values of these measures decrease with the increase of the maximum operation time ($\alpha_0$) and maximum repair time ($\beta_0$). Thus, on the basis on the results obtained for a particular case, it is suggested that the reliability and profit of a system in which the chances of h/w failure are high can be improved by

- By taking one more computer system in cold standby.
- By performing preventive maintenance after a maximum operation time.
- By making replacement of the outdated s/w by new one immediately.

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Figure 1: State transition diagram

Figure 2: MTSF Vs. Preventive Maintenance Rate
References


