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Dynamic Analysis and Design of Steel-Ball Grinding Machines Based on No-Slip Cases

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Abstract

Based on no-slip cases between steel balls and lapping discs while a horizontal grinding machine is working, the motion equations of the ball lapping are established and the law of motion is revealed by the MATLAB software. This paper shows that the difference of the trace spacing is reduced by increasing the disc diameter, which can make the trace distribution of the steel ball surface more uniform. In order that balls do not slip when rotary disc drives them to rotate, the ranges of the lapping pressure and lapping disc speed is deduced by using the dynamics analysis considering from the vertical trench surface and along the groove surface, respectively. Finally, this paper takes the selected material as an example to analyze the process parameters that affect the lapping curves, and explains the selection principles of alpha and beta when the trench has some bias, which can provide a reference to improve the quality of the steel balls surface.

Keywords: *: Grinding Machine, Lapping, Steel Ball, Dynamics, Design, MATLAB Software .*

1. Introduction

Bearings are one kind of vital parts of modern machinery and equipment. They are widely used in many industries. Their main function is to support the mechanical rotating body and reduce the coefficient of load friction in the transmission process. As shown in Figure 1, steel balls are important component of ball bearings, which processing technology has unique specialized characteristics and the processing quality will largely affect the life and reliability of the ball bearings [1-3].

Figure 1. Ball bearing

Lapping is the last step in steel balls processing technology. In the meantime, steel ball-billets are squeezed, scraped by lapping discs and abrasive, which can remove the machining allowance of the ball surface and improve steel balls sphericity as well as reduce the roughness of the surface [4, 5]. The sphericity is a main technical index of steel balls and it has a great impact on bearing performance (such as accuracy, noise, vibration, etc.) [6-8]. About lapping, the conventional view is that rotary disc drives steel balls revolution around the disc axis, and the linear velocity difference of the arc between the spherical surface and groove makes balls rotation. The revolution and rotation complete the lapping of steel balls together [9]. This interpretation is relatively simple because the revolution speed is much greater than the speed of the rotation, which will make balls become ellipsoid in the lapping process. Zhu [10] proposed that the lapping disc grinds along the three circles on the steel ball surface repetitively through the contact points under the lapping method of two discs along a coaxial. It has been proved that the lapping trajectory is a trace circle rounding an axis of the ball center when a steel ball is in motion.

This paper presents the motion equations of the lapping model without slipping, and the angular velocity of rotation, angular velocity of revolution as well as angle of deflection are calculated by using the MATLAB software. Based on the analysis of the trace distribution of the steel ball surface, it is shown that reducing the difference of two trace circles can make the trace distribution of the steel ball surface more uniform. In order for the balls not to slip when the rotary disc drives them to rotate, the ranges of the lapping pressure and lapping disc speed are deduced by using the dynamics analysis considering from the vertical

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trench surface and along the groove surface, respectively. Finally, this paper takes the lapping disc material HT300, steel balls material GCr15 as an example to analyze the fact that process parameters affect the apping curves, and explains the selection principles of alpha and beta when the trench has some bias, which can provide a reference to improve the quality of the steel balls surface.

2. Dynamic Analysis of Ball Lapping

2.1. Motion Equation

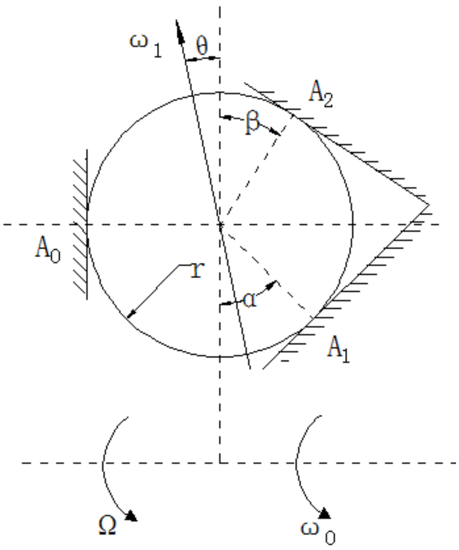

Figure 2. The motion analysis of a steel ball

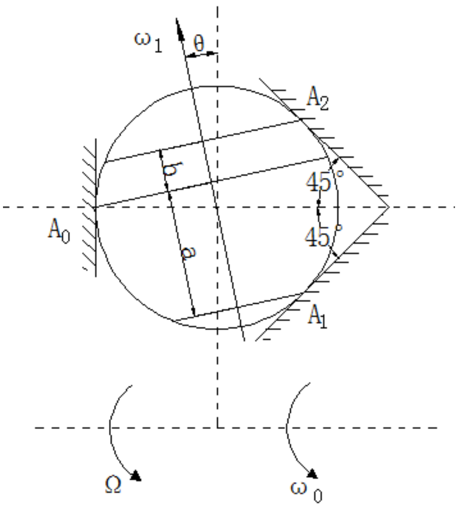

Figure 3. The trace distribution of lapping

Balls and lapping discs are regarded as rigid bodies when a horizontal grinding machine is working and the contacts between them are idealized as point contacts. As shown in Figure 2, A_0, A_1 , A_2 are three contact points between balls and lapping discs, the revolution radius of the balls are R_0, R_1, R_2. The motion of balls in the groove can be divided into ω_0 that revolves around the center of discs and ω_1 that rotates around its own sphere center. Generally, the rotation ω_1 can also be decomposed into pivot motion and roll motion [11]. Without considering the contact deformation and applying the general principle of rigid bodies, the equations of lapping motion without slipping at three contact points is expressed as follows:

$$\begin{cases} \omega_0 R_0 + \omega_1 r\cos\theta = \Omega R_0 \\ \omega_0 R_1 - \omega_1 r\sin(\alpha-\theta) = 0 \\ \omega_0 R_2 - \omega_1 r\sin(\beta+\theta) = 0 \end{cases} \tag{1}$$

where Ω denotes the angular speed of rotary disc, θ is the angle of deflection, and r is the radius of steel ball.

Lapping parameters are calculated, respectively, as:

$$\begin{cases} \tan\theta = \dfrac{-R_1\sin\beta + R_2\sin\alpha}{R_1\cos\beta + R_2\cos\alpha} \\ \omega_0 = \dfrac{R_0\sin(\alpha+\beta)\Omega}{R_0\sin(\alpha+\beta) + R_1\cos\beta + R_2\cos\alpha} \\ \omega_1 = \dfrac{R_0 R_2 \Omega}{r\left[R_0\sin(\beta+\theta) + R_2\cos\theta\right]} \end{cases} \tag{2}$$

In which $R_1=R_0-r\cos\alpha$, $R_2=R_0+r\cos\beta$.

The above formulae provide the no slipping laws of lapping motion. When the position of steel balls in the groove and the speed of rotary disk are determined, θ, ω_0 andω_1 will have a unique solution, which means the motion can be uniquely identified. It also shows that ω_0, ω_1 are proportional to Ω. Since θ is generally not equal to 0, ω_1 will have a normal component and a tangential component. The corresponding movement of normal component and tangential component take a part mainly in grinding steel balls and rolling steel balls, respectively. The lapping of steel balls is implemented by the movements together.

2.2. Structure Optimization of Lapping Disc

There exists the angle of deflection θ remaining unchanged when the trench truncate $\alpha=\beta=45^0$ is taken into consideration. In the same circulation with different rotational loops, the lapping traces are three circles around the axis of rotation. The final machining is completed by using the repeated lapping.

As shown in Figure 3, the distance among three traces are a and b, which can be expressed as:

$$\begin{cases} a = r\cos(45^\circ - \theta) + r\sin\theta \\ b = r\cos(45^\circ + \theta) - r\sin\theta \\ \tan\theta = \dfrac{-R_1 + R_2}{R_1 + R_2} \end{cases} \tag{3}$$

From the above formulae, we can get

$$\frac{a}{b} = \frac{(1+\sqrt{2})\dfrac{r}{R_0} + \sqrt{2}}{-(1+\sqrt{2})\dfrac{r}{R_0} + \sqrt{2}} \tag{4}$$

There are three trace circles on the surface of steel balls, where A_0 is in the middle of A_1 and A_2. It can be seen from Eq. (4) that the interval ratio of trace circles are only relevant to r/R_0, and the relation curve of which is shown in Figure 4. It is showed by tests that the value of a/b influences the lapping balls quality and efficiency, and the accuracy of balls surface will be improved when the value is close to 1 [12].

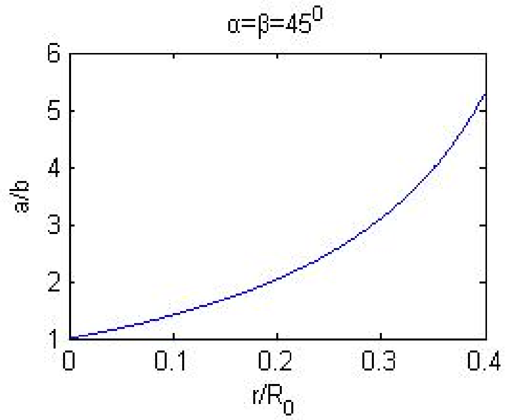

Figure 4. The curve of r/R_0-a/b

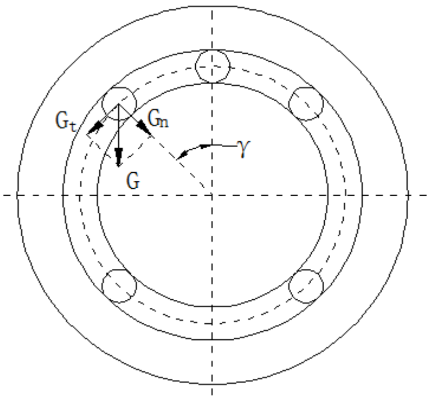

Figure 5. The gravity decomposition of steel balls

When the ordinate is equal to 1, the abscissa will be close to 0 infinitely, which means that the radius of the revolution is much larger than the ball radius. The abscissa of outside groove can be reduced by increasing the disc diameter, and the quality of balls surface can be improved. However, there are some problems when increasing the diameter. The linear velocities inside and outside the groove will inevitably lead to a great difference, which means that it cannot make all balls get in and out of the groove smoothly at the same time, and then the steel ball surface will be destroyed. Thus, it is essential to consider the above two aspects when the value of r/R_0 is selected.

3. Dynamic Analysis of Lapping Ball

Figure 5 presents the ball distribution in the same trench for horizontal lapping machine. Supposing the weight of a single ball is G, the components of four quadrants are calculated as:

$$\begin{cases} G_n = G\cos\gamma \\ G_t = \text{-}G\sin\gamma \end{cases}, \quad \begin{cases} G_n = G\cos\gamma \\ G_t = G\sin\gamma \end{cases}$$
$$\begin{cases} G_n = \text{-}G\cos\gamma \\ G_t = G\sin\gamma \end{cases}, \quad \begin{cases} G_n = \text{-}G\cos\gamma \\ G_t = \text{-}G\sin\gamma \end{cases} \tag{5}$$

3.1. Dynamic Equation of the Vertical Trench Surface

Without taking into account the ball interaction and the impact of lapping fluid, a single steel ball without slipping is chosen to be the research object. In order for the balls not to slip in all directions, the vertical trench surface and along the groove surface are analyzed, respectively, as shown in Figure 6 and Figure 7.

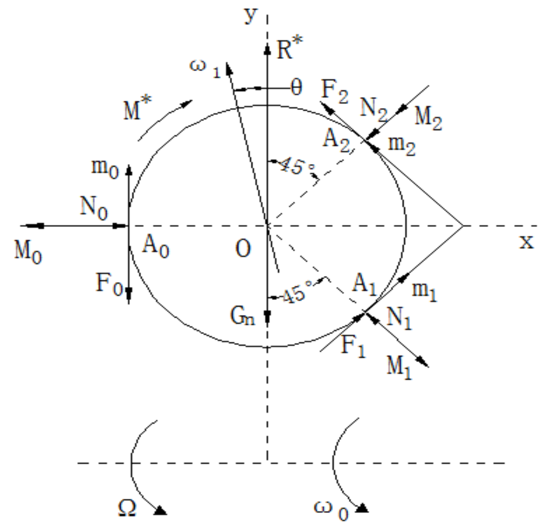

Figure 6. The stress analysis of the vertical trench surface

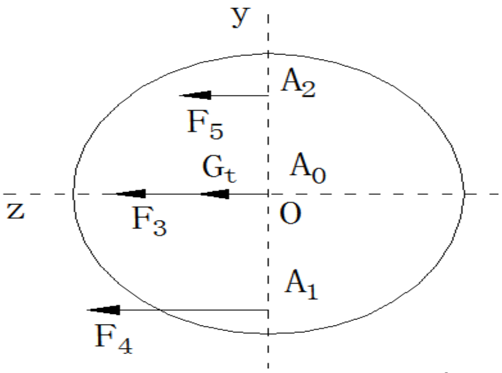

Figure 7. The stress analysis along the groove surface

In case that steel balls are not allowed to slip around the direction of z-axis in the V-shaped groove, the conditions that the dynamic equation has to satisfy are as follows:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_z(F) = 0 \tag{6}$$

It can be also written in the form as follows:

$$\begin{cases} N_0 - N_1\sin 45° - N_2\sin 45° + F_1\cos 45° - F_2\cos 45° = 0 \\ N_1\sin 45° - N_2\sin 45° + R^* - F_0 - G_n + F_1\cos 45° + F_2\cos 45° = 0 \\ (F_0 + F_1 + F_2)*r - M^* = 0 \end{cases} \tag{7}$$

Where N_0, N_1, N_2 represent the pressures at three contact points, F_0, F_1, F_2 denote the forces of the sliding friction at three contact points, R^* and M^* are the inertia force and couple of a ball, respectively.

Suppose that the critical pressures of three contact points are $[N_0]$, $[N_1]$, $[N_2]$, and the critical forces of the sliding friction are $[F_0]$, $[F_1]$, $[F_2]$, respectively, their relation can be expressed as follows:

$$[F_0]=f\,[N_0],\ [F_1]=f\,[N_1],\ [F_2]=f\,[N_2] \tag{8}$$

Combining Eqs.(7) and (8), it can be obtained as

$$\begin{cases} [N_0]=\dfrac{(-2+\sqrt{2})}{2}\left[\dfrac{-2R^*f}{1+f^2}+\dfrac{2G_n f}{1+f^2}-\dfrac{\sqrt{2}M^*}{rf}\right] \\ [N_1]=\dfrac{(-2+\sqrt{2})}{2}\left[\dfrac{(1+\sqrt{2}+f)R^*}{1+f^2}-\dfrac{(1+\sqrt{2}+f)G_n}{1+f^2}-\dfrac{M^*}{rf}\right] \\ [N_2]=\dfrac{(2-\sqrt{2})}{2}\left[\dfrac{(1+\sqrt{2}-f)R^*}{1+f^2}-\dfrac{(1+\sqrt{2}-f)G_n}{1+f^2}+\dfrac{M^*}{rf}\right] \end{cases} \tag{9}$$

Each of critical pressures contains R^*, G_n and M^*. Generally, R^*, G_n, M^*/r have the same order of magnitude, and the sliding friction coefficients are relatively small. Thus, Eq. (9) can be expressed approximately as:

$$[N_0]=\frac{(\sqrt{2}-1)M^*}{rf},\ [N_1]=\frac{(2-\sqrt{2})M^*}{2rf},\ [N_2]=\frac{(2-\sqrt{2})M^*}{2rf} \tag{10}$$

To the motion balls, the rotational inertia J and inertia couple M* are calculated as:

$$J=\frac{2}{5}mr^2, \qquad M^*=J\omega_1\times\omega_0 \tag{11}$$

The critical pressures need to less than the actual lapping pressures at three contact points in order not to slip, which can be written as:

$$[N_0]<N_0, \quad [N_1]<N_1, \quad [N_2]<N_2 \tag{12}$$

Using Eqs.(10), (11) and (12), the lower limit inequality of N0 can be derived as

$$N_0>\frac{2(\sqrt{2}-1)mr\omega_0\omega_1\cos\theta}{5f} \tag{13}$$

3.2. Dynamic Equation along the Groove Surface

Analogously, to prevent steel balls from slipping around the direction of x-axis and y-axis, the conditions have to be satisfied as follows:

$$\sum F_z=0,\ \sum M_x(F)=0,\ \sum M_y(F)=0 \tag{14}$$

Which can be written in the following form:

$$\begin{cases} F_3+F_4+F_5+G_t=0 \\ (-F_4\cos 45^0+F_5\cos 45^0)r+m_x=0 \\ (F_3-F_4\sin 45^0-F_5\sin 45^0)r+m_y=0 \end{cases} \tag{15}$$

where F_3, F_4, F_5 represent the forces of the sliding friction at three contact points, M_0, M_1, M_2 denote the pivot frictional moments at three contact points, m_0, m_1, m_2 are the rolling frictional moments at three contact points, m_x, m_y refer to the algebraic sums of projection that pivot frictional and rolling frictional moments work on x-axis and y-axis, respectively, and in which

$$m_x=-M_0+(M_1-M_2)\cos 45^0+(m_1-m_2)\cos 45^0$$

$$m_y=-(M_1+M_2)\sin 45^0+m_0+(m_1+m_2)\sin 45^0$$

Because the rolling frictional moment is far less than pivot frictional moment at the contact points, one can wish to omit the rolling frictional moment, which means only to consider the pivot frictional moment. Then the above expressions can be simplified as:

$$m_x=-M_0+(M_1-M_2)\cos 45^0,$$

$$m_y=-(M_1+M_2)\sin 45^0 \tag{16}$$

By Eq. (15) and (16), it can be obtained as:

$$\begin{cases} F_3=\dfrac{(\sqrt{2}-1)(M_1+M_2-rG_t)}{r} \\ F_4=\dfrac{(2-\sqrt{2})}{2r}\left[-(\sqrt{2}+1)M_0+M_1-(1+\sqrt{2})M_2-rG_t\right] \\ F_5=\dfrac{\sqrt{2}}{2r}\left[M_0-M_1+(\sqrt{2}-1)M_2+(1-\sqrt{2})rG_t\right] \end{cases} \tag{17}$$

Supposing that the elastic modulus and poisson's ratio of steel balls are E_g, υ_g, the elastic modulus and poisson's ratio of lapping discs are E_y, υ_y. According to the elastic contact theory [13], the pivot frictional moments can be calculated as follows:

$$\begin{cases} M_0=\dfrac{3\pi f}{2}N_0\sqrt[3]{\dfrac{3rN_0}{4}\left(\dfrac{1-\upsilon_g{}^2}{E_g}+\dfrac{1-\upsilon_y{}^2}{E_y}\right)} \\ M_1=\dfrac{3\pi f}{2}N_1\sqrt[3]{\dfrac{3rN_1}{4}\left(\dfrac{1-\upsilon_g{}^2}{E_g}+\dfrac{1-\upsilon_y{}^2}{E_y}\right)} \\ M_2=\dfrac{3\pi f}{2}N_2\sqrt[3]{\dfrac{3rN_2}{4}\left(\dfrac{1-\upsilon_g{}^2}{E_g}+\dfrac{1-\upsilon_y{}^2}{E_y}\right)} \end{cases} \tag{18}$$

Analyzing the statics of steel balls in the groove, it can be found that the ratio of lapping pressure is approximately equal to the ratio of critical pressure. Thus, it is reasonable that the relative relation of actual lapping pressure is:

$$N_0:N_1:N_2=\sqrt{2}:1:1 \tag{19}$$

Substituting Eq.(18) and Eq.(19) into Eq.(17), it is obtained as follows

$$\begin{cases} F_3=\dfrac{\sqrt[3]{2}(\sqrt{2}-1)}{r}M_0-(\sqrt{2}-1)G_t \\ F_4=\dfrac{-(2-\sqrt{2})(\sqrt{2}+1+2^{-\frac{1}{6}})}{2r}M_0-\dfrac{2-\sqrt{2}}{2}G_t \\ F_5=\dfrac{\sqrt{2}+\sqrt[3]{2}-2^{\frac{5}{6}}}{2r}M_0-\dfrac{2-\sqrt{2}}{2}G_t \end{cases} \tag{20}$$

In the lapping process, the rotary disk has to overcome the pivot frictional moment and rolling frictional moment, and then drives steel balls to rotate. To stop balls from slipping when they are rotated, the conditions that have to be satisfied are as follows

$|F_3|<N_0 f$, $|F_4|<N_1 f$, $|F_5|<N_2 f$ (21)

From Eq. (20), it can be seen that F_4 is greater than the other two forces of the sliding friction. Selecting F_4 as the represented formula and substituting Eq. (20) into Eq. (21), one obtains

$$\frac{1}{f}[\frac{1+2^{-\frac{1}{6}}(\sqrt{2}-1)}{r}M_0+(\sqrt{2}-1)G_t]<N_0 \quad (22)$$

3.3. Selection of the Pressure and Speed

It can be drawn from Eq. (18) that M_0 is proportional to $N_0^{4/3}$, and it is obtained that N_0 is less than a specified value, referred to as B by substituting Eq. (18) into Eq. (22). Similarly, to solve the first and third inequalities of Eq. (21), it can deduce that N_0 is less than A and C, respectively.

Therefore, the following inequality has to be satisfied if lapping balls do not slip in all directions:

$$\frac{2(\sqrt{2}-1)mr\omega_0\omega_1\cos\theta}{5f}<N_0<\min\{A,B,C\} \quad (23)$$

Substituting Eq. (2) into Eq. (23) gives that the expression of the lower limit is proportional to Ω^2. Since A, B, C are constant and independent of the rotary disc speed, it can deduce $\Omega<D$, which means D is the maximum allowable speed of the rotary disc.

4. Influencing Parameter

4.1. Process Parameter

When the steel balls rotate without slipping in the groove, the radius of the revolution affects the lapping pressure and the maximum allowable speed. Here select HT300 as the material of lapping discs, GCr15 as the material of steel balls, and 315mm as the revolution radius of R_0.

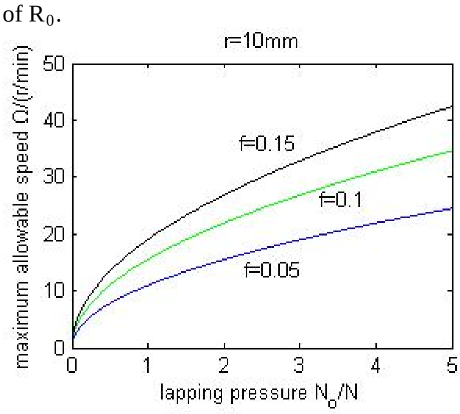

Figure 8. Coefficients of friction that work on lapping curves

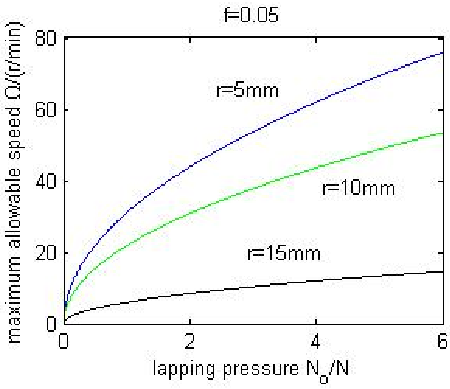

Figure 9. Sizes of balls that work on lapping curves

As shown in Figure 8, it is the relation curve between the lapping pressure and maximum allowable speed. The curve shows that with the increase of the lapping pressure, the maximum allowable speed rises. The maximum allowable speed of the rotary disc can achieve a higher value when the coefficient of the friction increases, which generates that steel balls are less likely to slip in the lapping process. In Figure 9, it can be seen that the maximum allowable speed of the rotary disc decreases while the sizes of the steel balls are increasing, which illustrates that the steel balls will slip easily. The lapping pressure will be increasing and the speed of the rotary disc will be reducing at this moment.

4.2. Trench Bias

There are many factors that affect the surface quality of the ball in the lapping process, such as the groove shape, machine accuracy, mechanical properties of lapping disc, etc. The groove shape is relatively easy to control and change, which can be achieved by changing the parameters α and β. As shown in Figure 10, the normal angular velocity and angular tangential velocity at three points are as follows:

$$\begin{cases} \omega_0^n=\omega_1\sin\theta, & \omega_0^t=\omega_1\cos\theta \\ \omega_1^n=\omega_1\cos(\alpha-\theta), & \omega_1^t=\omega_1\sin(\alpha-\theta) \\ \omega_2^n=\omega_1\cos(\beta+\theta), & \omega_2^t=\omega_1\sin(\beta+\theta) \end{cases} \quad (24)$$

From the above formulae, it can be seen that the normal component and tangential component are the negative relations, and there exist different lapping conditions at the contact points. In Figure 11, it is the relation curve between α and tanθ, which shows that tanθwill be equal to 0 when α is near to the value of π/4, and at this time the difference of the lapping condition is relatively smaller. Thus, αand βshould comply with the certain principle when the trench is bias. They should be unequal, the difference between them is very small, and their sum is around π/2. It can make the trace circles have a good distribution and improve the efficiency of the ball lapping simultaneously.

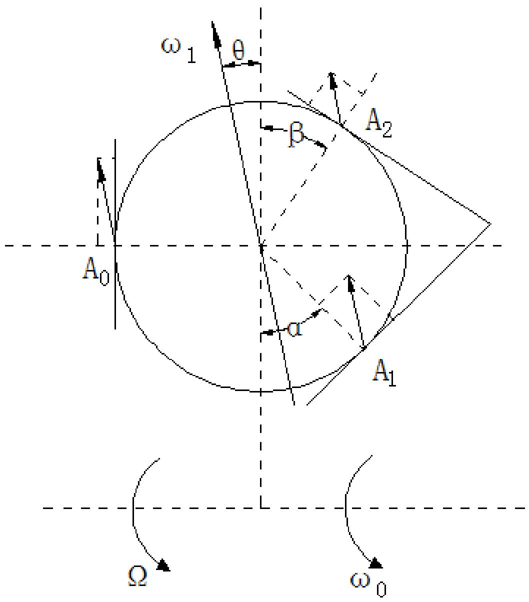

Figure 10. The relative rotation of three contact points.

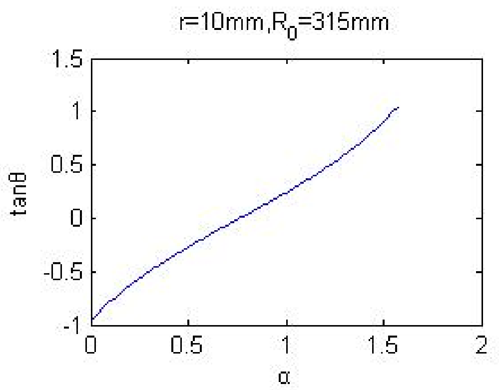

Figure 11. The curve of α-tanθ

5. Conclusions

This paper reveals the motion law of grinding balls through establishing the motion equations. Based on the vertical trench surface and along the groove surface, respectively, the dynamics analysis of a single steel ball is achieved and the influence on the quality of the steel ball surface by different parameters is compared. It can be drawn as follows:

(1) The motion law can be uniquely identified when steel balls do not slip in the groove, and the angular velocities of the revolution and rotation are both proportional to the rotational speed of the rotary disc.

(2) When steel balls are ground by a horizontal grinding machine, in order to prevent them from slipping, the lapping pressure has to satisfy a certain range, and the rotary speed of the rotary disc also has to be less than the maximum allowable speed.

(3) The possibility of slip cases can be reduced by increasing the lapping pressure or the coefficient of the friction. Lapping big steel balls are easier to slip than the small ones, and the ways of increasing the lapping pressure and decreasing the rotary speed can solve this problem effectively.

(4) A satisfactory distribution of trace circles can be achieved if the difference between αand β is not big as well as their sum is approximatively equal to 90^0. It will provide a better foundation for the further research.

Acknowledgments

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