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Parametric Coordination and Simulation Study on Nonstandard Spur Gears

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Abstract

This paper presents the parametric collaborative rules of nonstandard spur gear in order to improve the bending strength. For numerical work, equation between fillet radius and pressure angle is established according to the gearing theory. Also, the mechanism of action that pressure angle, fillet radius and addendum coefficient have on the bending strength is investigated. So it is shown how pressure angle, fillet radius and addendum coefficient affect the bending strength. Based on that, coordinated figures among the gear parameters and the collaborative design for stronger bending strength of nonstandard spur gears are proposed. Furthermore, bending stress of gears with different parameters are simulated and compared with the theoretical computing values to support the conclusion.

Keywords: *Spur Gears; Bending Strength; Pressure Angle; Fillet Radius; Addendum Coefficient.*

Nomenclature

α :pressure angle (°)
h_a :addendum
h^*_a :addendum coefficient
m :module(mm)
Z :number of teeth
r_p : pitch radius (mm)
c : clearance
c^* : clearance coefficient
r_ρ : fillet radius(mm)
F_n :normal net force on the involute curve(N)
F_t : tangential force(N)
F_r : radial force (N)
K : load coefficient
b :tooth width(mm)
Y_F :tooth form factor
Y_S : stress correction factor
α_p : pressure angle of rank cutter (°)
$r_{\rho\max}$: maximum of fillet radius(mm)

1. Introduction

Nonstandard spur gears have broader applications compared to standard gears due to the flexible design and manufacture. Furthermore, standard gears cannot meet the needs of various operating conditions because of the limited carrying capacity on tooth root. So in many cases they need to be designed into a nonstandard form in order to improve the mechanical performance, such as improving addendum, changing tooth thickness, making pressure angle not be 20 degree, etc. The performance of nonstandard spur gears will be far more excellent than that of the standard gears if properly designed.

Recently, there have been some efforts made to widen the application of nonstandard spur gears. Mabie H. [1] investigated the mathematical model of processing nonstandard spur gears by hob. Wu Jize [2], Fang Zongde [3] and Jiang Xiaoyi [4] studied the equations of fillet curves, the stress concentration and the influence which fillet has on the tooth bending strength. C. Spitas and Spitas V.[5; 6] investigated the method of processing nonstandard spur gears by standard cutter, and analyzed the effect the achieved circular fillet and trochoid fillet curve have on tooth bending strength. ODA Satoshi [7; 8] developed the rule of tooth deformation under standard pressure angle based on the experimental method. Handschuh R.F. [9], Thirumurugan R. [10] and F.M Khoshnaw [11] investigated the performance between different pressure angle gears. Hidaka [13] and Shuting Li [14] investigated the effect of the addendum change on contact strength, bending strength and basic performance

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based on photoelastic and finite element analysis. S. Baglioni and F. Cianetii [15] analyzed spur gears' efficiency through two different approaches for friction coefficient calculation along the line of action. The authors always considered only a single factor when studying the relations between parameter and bending strength in their research, ignoring the relations between the parameters.

This paper proposes a collaborative design method for strengthening the bending strength of nonstandard spur gears by investigating the effect that pressure angle, fillet radius and addendum coefficient have on bending strength. It is hoped that the study will provide guidance for nonstandard spur gears design.

2. Effect of Parameters on Bending Strength

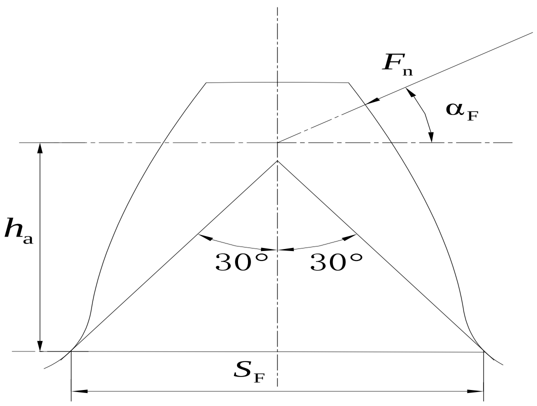
2.1. Bending Stress Calculation Model

The ISO method is used to calculate the bending stress of nonstandard spur gears, the dangerous section is determined by the $30°$ tangent method, as shown in Figure 1. If we assume gear as a cantilever beam, the normal force which acts on the involute can be divided into two parts: The tangential force that makes gear bending, and the radial force that makes gear shearing, during the power transmission.

strength of gear is as follow:

$$\sigma = \frac{KF_t Y_F Y_S}{bm} \quad (1)$$

Where, Y_F and Y_S are complicated to calculate for the many intermediate parameters contained [4]. Some of the intermediate parameters need that the iterative computing be solved. So the program calculation method is used.



Figuer 1. Computing model of bending strength

2.2. Effect of Pressure Angle on Bending Strength

For gears, tooth profile will differ with the shape of the rack cutter. The pressure angle of standard rack cutter is 20 degree, but it is only a compromise value. For the gear design of some special purpose (design of nonstandard spur gear), it is not the best value.

According to the computing equation of tooth bending stress (as show in Eq.(1)) and combined with the stress calculation model (as shown in Figure 1), bending strength is calculated under pressure angles $(14.5°, 16°, 22.5°, 25°, 28°)$ and a suitable range of normal loads. In order to have a president value to confirm that the computing results are suitable, the parameters of the gear are chosen according to reference [16] as shown in Table 1.

Table 1. Parameters of nonstandard spur gears

Module (mm)	Tooth width (mm)	Load coefficient	Number of teeth
6	17	1.3	32

Based on the MATLAB software, a program for bending stress calculation is written. Bending stress that varies with the normal load is devised under 5 sets of pressure angle as shown in Figure 2. The vertical line in the graph expresses the rule that bending stress varies with the pressure angle. It can be drawn from the chart that bending stress increases with the increase of the load, and the changing trend does not change when the pressure angle differs. The one with smaller pressure angler has larger gradient. This means that the load has a stronger influence on bending stress under high pressure angler. Bending stress decreases with increasing the pressure angle and the variation curve is almost linear; thus, increasing the pressure angle helps to improve the bending strength. When the load is large, it obviously increses the bending strength, if increasing of pressure angle is applied.

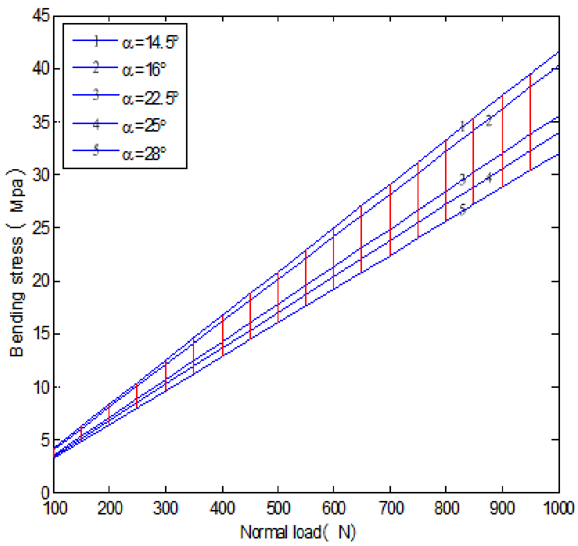


Figure 2. Relation of bending stress and normal load

2.3. Effect of Fillet on Bending Strength

There are several kinds of fillets for nonstandard spur gears. Different shape of fillet of rack cutter processes out different gear fillet. The standard value of cutter fillet radius is 0.38*m; otherwise the processed gear is a nonstandard gear. Because the dangerous section occurs in the gear fillet, so it has a great significance for improving the bending strength.

Parameters of standard rack cutter are shown in Figure 3.

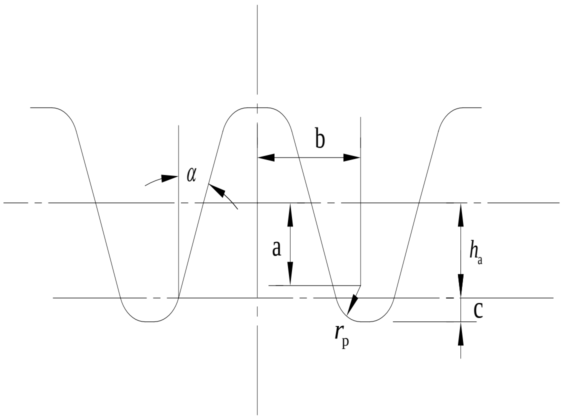


Figure 3. Tooth profile of standard rack cutter.

According to the geometrical relationship of gear [2], we can learn that the relation between each parameter is as follows.

$$a = h^*_a m + c^* m - r_\rho \tag{2}$$

$$b = \frac{\pi m}{4} + h^*_a m \tan\alpha + r_\rho \cos\alpha \tag{3}$$

$$r_\rho = \frac{c^* m}{1 - \sin\alpha} \tag{4}$$

In a standard rack cutter, the fillet arc is tangent to both the tooth profile and the addendum line. So if the tip clearance is given, the fillet radius will be a fixed value and improper selection may affect the gear bending strength. So it is necessary to explore the relation between the cutter fillet radius and the pressure angle. From reference [2], we can learn that the equation that expresses top clearance with pressure angle and cutter fillet radius is given by Eq. (5).

$$c = r_\rho (1 - \sin\alpha) \tag{5}$$

Figure 4 shows the relation between bending stress and cutter fillet. It is achieved by calculating the bending stress with a proper range of the cutter fillet radius based on the MATLAB software. Three curves in Figure 4 indicate that there are three different pressure angle values. From the graph, we can see that the variation trend is consistent between the bending stress and the cutter fillet radius. The bending stress decreases with the increase of the cutter fillet radius, and the reduction slows down with the increase of the cutter fillet radius (decreases in the radius of curvature of curve), which illustrates that the increase of the cutter fillet radius can enhance the bending strength of the processed gear. But the increase of the cutter fillet radius leads to the increase of the tip clearance that causes raising the tooth height which can lead to an increase in the bending arm. It will weaken the mechanical properties of gear. Therefore, a blind pursue of the high fillet radius is not recommended.

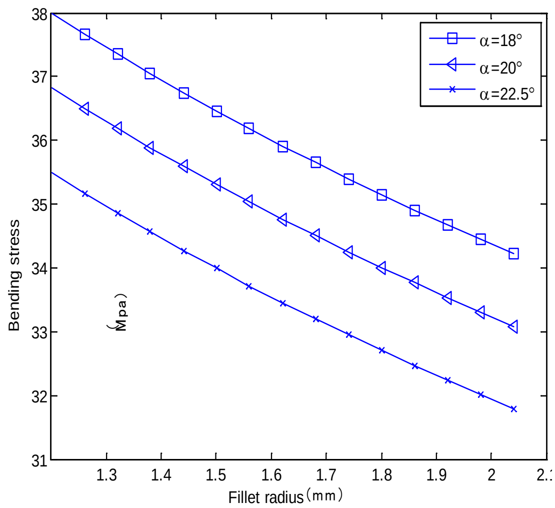


Figure 4. Relation of the bending stress and fillet radius of rack cutter.

3. Bending Strength of Nonstandard Spur Gears

3.1. Parameters Collaborative Rule of Nonstandard Spur Gears

Based on the above analysis, the bending strength of nonstandard spur gears has a close contact among rack cutter fillet radius, pressure angle and addendum coefficient. The gear shape and the mechanical properties will change with these three parameters. So coordination between these three parameters is helpful for improving the carrying capacity of gear transmission.

According to geometry of tooth profile of rack cutter (as shown in Figure 3), the cutter fillet radius will reach the maximum value when the cutter fillet becomes round. Change of the pressure angle will affect the fillet radius of rack cutter when the module is in certain cases, and the pitch remains unchanged. The greater pressure the angle has, the smaller tooth width of the addendum circle is; the limiting of fillet radius will also decrease.

From the geometrical relationship of gear as (shown in Figure 3), when the cutter fillet becomes round, fillet radius value of rack cutter can be computed with:

$$r_{\rho\max} = \frac{\pi m - 4h^*_a m \tan\alpha}{4\cos\alpha} \tag{6}$$

According to Eq.(6), the bending strength is computed by MATLAB software program under limited value of the fillet radius. The relation graph between the bending stress and pressure angle is acquired under the maximum value of the fillet radius, as shown in Figure 5. It can be derived from Figure 5 that the gear has the strongest bending strength when the pressure angle is 23.5 degree. If the pressure angle keeps increasing, the bending strength will decrease. Comparing the maximum fillet radius with the standard case, it can be found that distinct differences exist between the two cases: the case of maximum fillet radius, on the one hand, bending strength increased because of the increase of the fillet radius which, on the other hand, causes the increase of the tooth height, and then weakens the gear bending strength. That is why the bending strength weakens with the increase of the pressure angle after 23.5 degree. So for the small pressure angle gear, it can effectively improve the bending strength with rational allocation of the addendum coefficient.

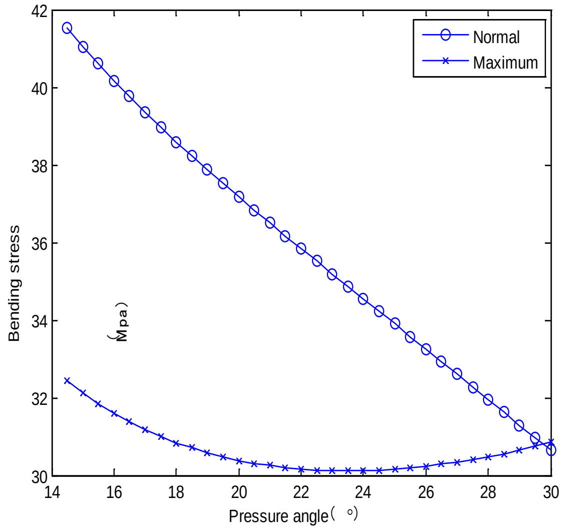


Figure 5. Relation between pressure angle and bending stress with standard and maximum fillet

3.2. Parameters Coordination of Nonstandard Spur Gears

In sum, the effect of the pressure angle on bending stress is a complex process based on the findings of the present study. The change of pressure angle will influence many other parameters, which further affects the bending strength. So the coordination of pressure angle, cutter fillet radius and the addendum coefficient is investigated. Bending stress decreases with the decrease of the addendum coefficient, but then it increases with the decrease of the addendum coefficient under a small pressure angle ($\alpha = 14.5°$, $\alpha = 16°$) and fillet radius based on MATLAB software calculation results. For big pressure angle ($\alpha = 22.5°$, $\alpha = 25°$, $\alpha = 28°$), the bending stress decreases with the increase of the addendum coefficient. The recommended value of collaboration of the pressure angle, fillet radius and addendum coefficient is given in Figure 6 and Figure 7.

Figure 6 shows the coordinated relation of addendum coefficient and cutter fillet radius in small pressure angle, if the pressure angle is 14.5 degree, when the fillet radius is 0.4*m and the addendum coefficient is recommended to 0.9. We also proposed that in the case of small pressure angle, a larger value of fillet radius is recommended. Figure 7 shows the relation between fillet radius and addendum coefficient under high pressure angle when the tooth height is compensated to standard value. The range of the fillet radius will decrease in a large pressure angle. The recommended parameters of the pressure angle, fillet radius and addendum coefficient (Figure 6 and Figure 7) can improve the gear bending strength.

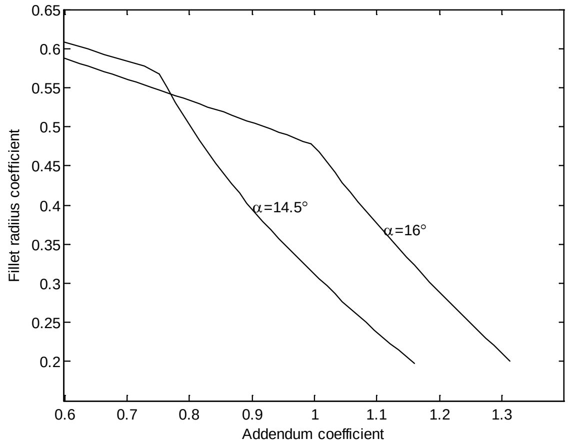


Figure 6. Relation of fillet radius and addendum coefficient in small pressure angle.

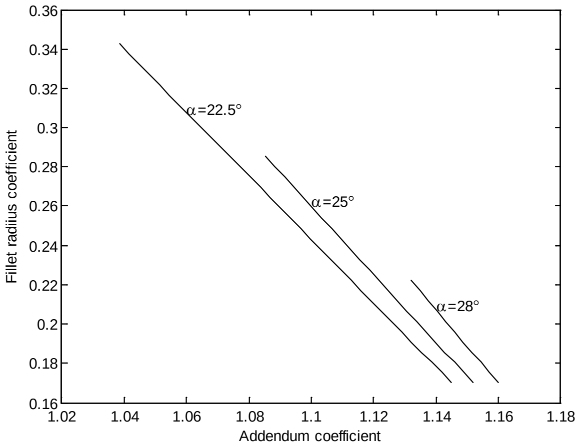


Figure 7. Relation of fillet radius and addendum coefficient in big pressure angle.

4. FEM Analysis

On the choice of pressure angle, cutter fillet radius and addendum coefficient, the orthogonal experiment method, which involves using a neatly arranged orthogonal table to integrate design, for comprehensive comparisons and statistical analyses in order to achieve better production conditions through a few number of experiments, is adopted. There may be many factors, and every factor can have several different values that influence the parts we studied. The number of the values is experiment levels. Fifteen finite element analysis models for bending stress computing are established by three factors and five levels orthogonal table [12], made by mathematicians. Then, according to the orthogonal table, different combinations of gear parameters are determined. The factor level of the orthogonal experiment is shown in Table 2.

Table 2. Basic parameters for gear design

Level	Factor		
	Pressure angle (°)	Fillet radius (mm)	addendum coefficient
1	14.5	0.45*m	0.85
2	16	0.4*m	1.08
3	20	0.38*m	1
4	22.5	0.3*m	1.07
5	25	0.26*m	1.1

The simulation analysis model of nonstandard gear bending stress is established as shown in Figure 8. To achieve the FEM model, a static/general step is used in Abaqus software. Element type is C3D8R and 32616 elements are meshed. According to the finite element analysis, the stress nephograms is derived as shown in Figure 9.

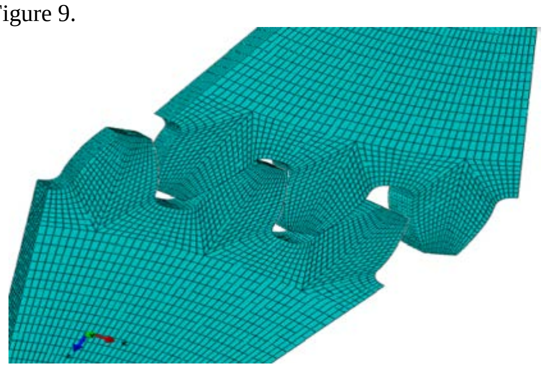


Figure 8. FEM model of nonstandard spur gears.

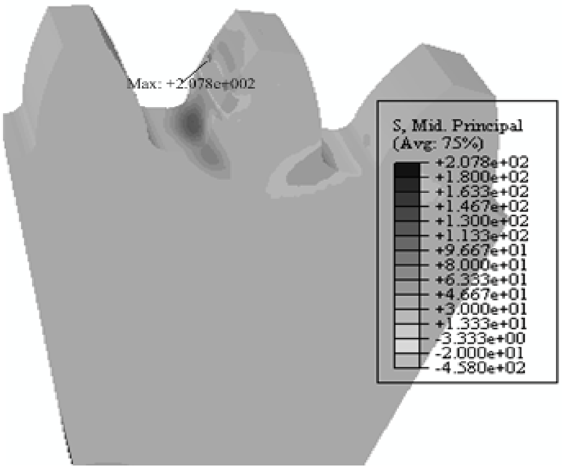


Figure 9. Bending stress nephogram of nonstandard spur gears.

From the FEA results of the bending stress simulation and theoretical calculation, we can conclude that gears with a larger pressure angle have a relatively smaller bending stress, and those with bigger fillet radius have better mechanical properties. The bending stress of nonstandard spur gears, with which their pressure angle, fillet radius and addendum coefficient are chosen according to Figure 6 and Figure 7, are smallest than the others. For example, the bending stress of which the pressure is 16 degree, fillet radius is 0.4*m and addendum coefficient is 1.08, reduced almost 31.2% compared with the same series of gears with the same pressure angle.

Table 3. Design parameters and FEA results of nonstandard spur gears

Sequence number	Pressure angle (°)	Fillet radius (mm)	Addendum coefficient	Bending strength（MPa）		Relative error (%)
				FEA results	Theoretical calculation results	
1	14.5	0.45*m	0.85	235.3	233	0.99
2	16	0.4*m	1.08	192.1	188	2.19
3	20	0.38*m	1	207.8	211	1.52
4	22.5	0.3*m	1.07	136.9	123	11.3
5	25	0.26*m	1.1	152.5	161	5.28
6	14.5	0.4*m	1	290.1	275	5.49
7	16	0.38*m	1.07	266.7	262	1.79
8	20	0.3*m	1.1	231.2	219	5.57
9	22.5	0.26*m	0.85	154.2	148	4.19
10	14.5	0.38*m	1.1	301.0	282	6.74
11	16	0.3*m	0.85	285.4	273	4.54
12	20	0.26*m	1.08	241.5	259	6.76
13	14.5	0.3*m	1.08	304.1	300	1.36
14	16	0.26*m	1	241.9	246	1.67
15	14.5	0.26*m	1.07	328.6	320	2.69

5. Conclusion

From the research on the relation of parameters and bending stress and the analysis of the collaborative relations, it is concluded that:

(1) Increasing the fillet radius can enhance the bending strength during the nonstandard spur gears design, but it causes a tooth height increase, which will have a negative impact on the bending strength. Parameter collaborating of nonstandard spur gears is needed to significantly improve the mechanical properties.

(2) Bending stress decreases with the decrease of the addendum coefficient, but it then increases with the decrease of the addendum coefficient under small pressure angle. But under high pressure angle, the bending stress decreases with the increase of the addendum coefficient.

(3) Parametric coordination of the pressure angle, fillet and addendum can improve the bending strength of nonstandard spur gears. Furthermore, the values of the parameters which can improve the carrying capacity of nonstandard spur gears are given.

(4) The FEA model is established based on the collaborative relation graph. The comparison between FEA results and theoretical values displays the availability of the results which demonstrate the feasibility of parameter collaborating.

Acknowlengment

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