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Numerical and Theoretical Analysis of a Straight Bevel Gear Made from Orthotropic Materials

Haidar Fadhil AL-Qrimli [*a], Ahmed M. Abdelrhman [a], Karam S. Khiled [b]

[a] Department of Mechanical Engineering, Curtin University of Technology, Miri, Sarawak, CDT 250, 98009, Malaysia
[b] Department of Mechanical Engineering, Ibra College of Technology, Ibra, Sultanate Oman

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Abstract

The purpose of the present study is to present a clear fundamental thought for designing and investigating straight bevel gear made of composite material. Application of composites in modern engineering includes gear-drive components, such as robotic arms, and this is especially evident when metallic gears are incapable of coping with the vigorous requirements of the machines. The presentwork demonstrates an actual form of the straight bevel gear that is made out into a set of equation rearranged and derived from the classical theory of gearing. The gear is then modeled using CAD commercial software in order to generate the whole gear in three dimension coordinates. This analytical calculation was constructed to allow us to calculate the straight bevel gear profile points, which are crucial for modeling and fabricating the composite gear model and to numerically analyze this model using finite element method. The results were extensively compared. This comparison attempted to study the composite bevel gear teeth problem. The focus in this work is on how much the divergence in behavior between orthotropic material and isotropic material such as steel can be reduced. The results prove the capability or potential for composite materials to be used as component materials in power transmission gearing in robotic arms applications.

Keywords: *Orthotropic Materials; Composite; Gear; Torque.*

1. Introduction

Many researchers have studied and discussed the behavior of composite gears as a polymer matrix and glasses or graphite fiber, which named a Polymer Matrix Composite (PMC) used in engineering applications as gear due to their medium level strength and low density. Nozawa [1] studied the tribology of a metal spur gear and a hybrid gear, in order to reduce noise with greaseless metal gear and plastic gear with smaller rate of tooth failure. The test was at rotation speeds of 1000 rpm, and a torque range of 13–8 Nm. They discovered that the noise is suddenly amplified when a single polymer sheet was spontaneously removed from the gear's surface; this is probably due to its low adhesive strength against a shear. Kozo [2] investigated the effect of different types of composite material on the strength of plastic gears. They used glass and carbon fibers in the gear tooth surfaces as reinforcement in order to improve bending strength, epoxy resin as the matrix material. Experimental static and dynamic tests were carried out to evaluate the effect of fiber reinforcements, and it shows the reinforcement is effective in improving the strength of plastic gears.

Ezhil and Paul [3] reported on the corrosion characteristics of Al 7075/ basalt short fiber metal matrix composites as a function of percentage of reinforcement. The matrix Composites Al 7075 alloy demonstrated a higher corrosion rate compared to MMCs under the corrosive atmosphere. Al-Shhyab and Ahmet Kahraman [4] studied the torsional dynamic behavior of a multi-mesh gear train and reported that, the torsional stiffness of the shaft is influencing the modal characteristics of the system and the nonlinear response.

In 2007, Melick [5] examined the effect of steel and plastic gear transmission with numerical and analytical methods by studying the influence of the stiffness of the gear material on the bending of the gear teeth, the consequences on contact path, load sharing, stresses and kinematics. Mao [6] carried out experimental investigations and modeled polymer composite (glass fiber reinforced nylon with PTFE). The design method is based on the relationship between polymer composite gears wear rate and surface temperature. A similar test was conducted on non-lubricated metal gears, and it was discovered that the polymer gear wear rate dramatically increased when the load reaches a critical rate for a specific geometry. Furthermore, Zhu and Li in 2011 [7] established a finite element model on a straight bevel gear by drilling holes in gears to reduce its weight. Nevertheless, they concluded that straight bevel gears with holes would weaken the mesh impact beneath the vibration compared to the normal straight bevel gear under the same loading and environment conditions [8]. Simon [9] presents an optimal tooth modification for a spiral bevel gear that improves

* Corresponding author e-mail: haidar.fa@curtin.edu.my.

load distribution and decreases the maximum tooth contact pressure.

Fiber reinforcement results in high specific strength and stiffness compared to metal matrix composite, depending on different types of fiber and their arrangement in the matrix when acting as laminates, which are mostly used in weight-sensitive industry gear applications, offering the highest specific strength and stiffness. Pihtili [10] investigated the wear resistance and behavior of woven glass fiber and composite materials under different loads, speeds and sliding distances. Antonio and Marcos [11] used a polyethylene terephtalate to replace a spur gear. The module of the spur gear equals to 2 mm/tooth, with a pitch diameter 34mm, and a pressure angle of 20º and the width of 18 mm. Enormous efforts have been made by previous investigators to relate tensile fillet stresses observed in statically and dynamically loaded gear teeth to the geometric appearance of the tooth. There is a number of approaches used in the past to verify the stresses and deflection in the gear's teeth, Wilfred Lewis made where the first effort to find the tooth root stresses. He based his analysis on a cantilever beam, and assumed that failure will occur at the weakest point of the beam, with him assuming it to be at the cross-section at the base of the gear. At the same time, Heinrich Hertz researched the contact pressures of the teeth, and his study was based on the elastic contact of two cylindrical bodies that determines the contact pressure between a gear and a pinion. With this tendency and experimental studies on bending stress analysis for the gears, the American Gear Manufacturers Association (AGMA) published their own standards based on Lewis's equation [12-15]. It is more accurate, and calculates all the geometrical factors, which are important in calculating the bending stresses for the gears. These geometrical factors take into account the loading position and the fillet radius tooth of the tip and base. Hasan [16] analytically studied the elastic–plastic stress analysis on an orthotropic rotating annular disc. The disc is made from metal matrix curvilinear reinforced steel fibers, and they used different angular velocities to enable them to see the separation of the plastic region. The results showed that the radial displacements and the plastic flow at the inner surface have higher values than those at the outer surface. Huali and Ahmet [17] proposed two dynamic models in order to study the interaction between a gear's surfaces wear and its dynamic response. Arafa and Megahed [18] constructed an FEA model of a spur gear to gather more information on the gear's mesh stiffness. The analysis involves quasi-static meshing conditions, where its compliance is evaluated at discrete meshing positions; with it assumed to be homogeneously isotropic. However, there are very few studies reported on a straight bevel gear by using different types of orthotropic material. In the present study, the theoretical and numerical methodologies are discussed. The theoretical results are compared to the numerical results. In the numerical stage we used finite element method to solve the gear tooth problem.

2. Methodology and Procedures

2.1. Theoretical procedure (Analytical Calculation)

The development of designing gear teeth is a bit arbitrary as far as the specific applications in which the gear is used to determine several design parameters. As stated in previously the basis for the bending stress analysis of gears was founded by Wilfred Lewis using his own formulation. His formulation for the bending stress started with the basis that a gear can be as basic as a beam which is subjected to tension and compression effects. However, this equation was not applicable to nearly all types of gears and not accurate because he didn't include the relevant geometrical parameters which influence the bending stress. As a result we used the AGMA standard 2005-D03, Design Manual for Bevel Gear Teeth. It is a design standard that illustrates all aspects of bevel gear tooth design, starting from the preliminary design standards and moving towards to complete the design and to be ready for analysis.

The torque application to a bevel gear mesh brings on tangential, radial, and separating loads on the bevel gear teeth. To make it easy, these loads are assumed to operate as point loads applied at the middle of the width gear tooth face, while only the tangential load is to be considered. The radial and separating loads are dependent upon the direction of rotation, pressure angle and pitch angle. The tangential loads (Wt), radial loads (Wr) and the separating loads (Ws) are defined as,

$$Wt = 2T / dp - F \sin\gamma \tag{1}$$

$$Wr = Wt\ (\tan\emptyset\ \sin\gamma) \tag{2}$$

$$Ws = Wt\ (\tan\emptyset\ \cos\gamma) \tag{3}$$

where dp is considered as the pitch diameter, T is the torque, F is the face width, γ is the pitch angle, and $\emptyset$ is the pressure angle. In the present study, we use a straight bevel gear with a ratio of 1:1 so the pitch diameter and the pitch angle are the same for the gear and the pinion. To calculate the pitch angle, we apply the equation: $\tan\gamma = \sin\varepsilon/(1/i + \cos\varepsilon)$ where; $\varepsilon = 90^{\circ}$ for the straight bevel gear. Table 1 shows the gear parameters.

Design for pitting resistance is mainly administered by a failure form of fatigue on the gear teeth surface because of the influence of the contact stress between the mating gears. Design for bending strength ability is based on a failure form of breakage in the gear teeth caused by bending fatigue. Pitting resistance is allied to Hertzian contact (compressive) stresses flanked by the two mating gear teeth surfaces. The formulas were developed based on Hertzian theory of the contact pressure between two curved surfaces and load sharing between adjacent gear teeth as well as load concentration that might be a consequence from uncertainties in the built-up procedure. The contact stress is generally a function of the square root of the applied tooth load. The equation to calculate the compressive stress (pitting resistance) in a straight bevel gear tooth is given by:

$$\sigma c = Cp\sqrt{Wt\ Ko\ Kv\ Ks\frac{Km}{dpF}\frac{Cf}{I}} \quad (4)$$

where *Cp* is the elastic coefficient, *Wt* is the lateral tooth load, *Ko* is the overload factor, Kv is the speed factor, Ks size factor, *F* is gear face width, *dp* pitch diameter, *Km* load distribution factor, and *I* is the geometry factor. Cf is the surface condition factor has not been evaluated and it's always equal to 1. The elastic coefficient can be calculated by using the equation below:

$$Cp = \left[\frac{1}{\pi\left(\left(\frac{(1-vg^2)}{Eg}\right)+\left(\frac{(1-vp^2)}{Ep}\right)\right)}\right]^{\frac{1}{2}} \quad (5)$$

where *E* and *v* stands for the Young's modulus and Poisson's ratio of the material correspondingly.

Bending strength capacity ratings in bevel gear teeth are developed using a simplified approach to cantilever beam theory. This method accounts for a variety of factors including: the compressive stresses at the tooth roots caused by the radial component of the tooth load; stress concentration at the tooth root fillet; load sharing between adjacent contacting teeth; and lack of smoothness due to low contact ratio. Calculating the bending strength rating determines the acceptable load rating at which tooth root fillet fracture should not occur during the entirety of the life of the gear teeth under normal operation. The basic equation for bending stress in a bevel gear is given by:

$$\sigma b = Wt\ P\ Kv\ Ko\ Km\frac{Ks}{FJ} \quad (6)$$

Wt is the tangential load applied on the tooth as discussed previously, *F* symbolizes for the face width, *Ks* symbolizes for the size factor, *Km* is the load distribution factor, and *J* is the geometry factor. *P* stands for the diametric pitch and can be calculated from equation (7):

$$P = \frac{Number\ of\ teeth}{Pitch\ Diameter} \quad (7)$$

The dynamic factor or velocity factor *Kv*, used in the calculation of the pitting resistance factor, accounts for quality of gear teeth while working at a particular speed and load conditions. It is typically influenced by design effects, manufacturing effects, transmission error, dynamic response, and resonance. In a broader sense, the velocity factor makes allowance for high-accuracy gearing, which requires low-accuracy gearing and, at the same time, makes allowance for heavily loaded gearing, which requires less derating than lightly loaded gearing. When gearing is manufactured using very harsh processes and controls, resulting in very accurate gearing, typical values of *Kv* between 1.0 and 1.5 are used. For our industrial gear application, *Kv* value of 1.3 is used. This value based on AGMA equations. The overload factor, *Ko*, accounts for quick peak loads that are much higher than the normal operating conditions. The overload factor (*Ko*) and load distribution factor are extremely qualitative in nature. For the load distribution factor, *Km*, is a function of the rigidity of the mounting and reflects the degree of misalignment under load. It modifies the rating formulas in order to capture the non-standardized distribution of the load along the length of the gear tooth. The amount of the non-uniformity of the load distribution (*Km*) is a function of the gear tooth manufacturing accuracy, tooth contact and spacing, alignment of the gear in its mounting, bearing clearances, and face width of the gear teeth, and therefore all are considerations which affect the load distribution factor. Therefore, based on this current study for the straight bevel gear is supported by two ball bearings. The size factor (*Ks*) is a reflection of non-uniformity of material properties and is a function of the strength of the material. In addition to material properties, it depends primarily on tooth size, diameter of the part, face width, and ratio of tooth size to diameter of the part. The size factor can be quickly calculated using:

$$Ks = \frac{1}{p^{0.25}} \quad (8)$$

The geometry factor for resistance to pitting, *I*, evaluates the effects that the geometry of the gear tooth has on the stresses applied to the gear tooth. More in particular, it evaluates the relative radius of curvature of the mating tooth surfaces and the load sharing between adjacent pairs of teeth at the point on the tooth surfaces where the calculated contact pressure will reach its maximum value. The geometry factor may be calculated from. Successful minimization of this distance will result in the smoothest stress distribution across the gear tooth:

$$I = \frac{1}{r1} + \frac{1}{r2} \quad (9)$$

where r1 and r2 symbolized for the radius of the tooth surfaces curvature at the contact point and in the present study r1 equals r2.

$$r = \frac{dp\sin\emptyset}{2} \quad (10)$$

The geometry factor for bending strength (J) is also concerned with gear tooth geometry but it gives more consideration to the tooth shape and the concentration of the stress due to the root fillet geometric shape.

Table 1. Standard gear parameter

NO.	Design Parameter	Value	Options
1.	Pressure angle	20^o	Material model
2.	Module	6	
3.	Face width	28.5m	Glasses/Epoxy
4.	Addendum	1m	Carbon/Epoxy
5.	Dedendum	1.25m	Jute/Epoxy
6.	Shaft angle	90^o	Steel
7.	Root fillet radius	0.3m	
8.	Number of teeth	24	

2.2. Numerical Procedure (FEA)

The boundary conditions of any finite element problem need to be determined, and in the present work, the boundary conditions that we used are similar to what we do in the experimental phase. A torque is applied on one side of the gear, while all the points along the second gear are fixed in all the directions and have zero degrees of freedom. All the points and displacement in the first gear can move in the circumferential direction, which is the X-axis that we fixed the translation onto and make it rotate around while fixed in the radial directions (Y-axis=0, Zaxis=0) in order to have five-degrees of freedom. The pinion is fixed in all six directions (DoF=0), while the load condition looks to be an external torque applied on the gear. The magnitude of the applied torque is (17640 N.m), which is used in the present work, and is equal to the tangential force of the gear's tooth surface (245N). The damping ratio in the gears does not seriously affect the dynamic stress, so it is considered to be 0.004, based on literature. The liner bulk viscosity and the quadratic bulk viscosity are taken as default, with values of 0.06 and 1.2, respectively. All the discussions are built on the standard parameter for a straight bevel gear shown in Table 1.

3. Results and Discussion

3.1. AGMA

The analysis begins with calculation of the gear loads generated by the bevel mesh. Using Equation 1, Equation 2, and Equation 3, the tangential load (*Wt*), radial loads (*Wr*), and separating load (*Ws*), are calculated. After solving the three equations, the values of the gear tooth loads are shown in Table 2. Once the design of the gear is verified and calculating the applied load of the teeth is carried out, the Hertz stresses calculation is performed in order to gauge the ability of the gear teeth to resist pitting.

Table 2. Calculated bevel gear tooth load

Wt (N)	245
Wr (N)	63.05
Ws (N)	63.05

Equation 9 is employed to calculate the geometry factor for pitting resistance (I), which is then used to find the value for compressive stress acting on the gear teeth. Calculation of the geometry factor involves solving Equation 10 to find the curvature tooth surfaces radius at the contact point. Once the geometry factor calculated, the other factors required to calculate the Hertz stresses could also be finalized. After calculating all the factors based on the AGMA standard, AGMA 2005-D03, Design Manual for Bevel Gears, the value of the Hertz stresses in the gear teeth were calculated using Equation 4 and the values are shown in Table 3.

3.2. Comparison between FEA model and Analytical Calculation

Static analysis calculates the effects of steady loading conditions on a structure can be determined using static analysis. It is still effective even if the dynamic effects caused by time and inertia are ignored. It is a simple analysis, wherein the effect of an immediate change in the model is calculated without taking into account the longer-term response of the model to that change. The main advantage of static analysis is the fact that it can discover initial errors in the material, which sves time and effort due to the fact that beginning dynamic analysis directly takes time and a considerable amount of skills. The static analysis is only the first step in solving a finite element problem, with the next step being the dynamic analysis. In FEA, static analysis means fixed and ignoring runtime environment, while dynamic means action and change. The dynamic analysis involves the testing and evaluation of a straight bevel gear, and we applied static analysis to determine the stress and strain of the tooth's surface of the straight bevel gear. 245N was applied on the tooth's surface of the gear to satisfy the static loading condition. The FEA results, obtained from the present study, are compared to the analytical calculation, along with the static behavior and stress strain curves. The stress-strain curves of the theoretical calculation and the finite element is drawn in the same Figure, in order to compare it to the Figures below. From the Figures below, it is clear that the curves have similar trends. The major reason is that we used the same gear dimensions and boundary conditions. The analytical calculation and the finite element results of the straight bevel gear, and the results between the two stages show a good match. The theoretical results show a linear behavior, while the finite element model of the steel shows nonlinearity. The values of the results are very close to each other, and we concluded that the FEA model is correct, and can be used for further simulation.

For the composite material model, we used engineering constrains in order to add the orthotropic mechanical properties for the glass/epoxy, carbon/epoxy and jute/epoxy. The parameters and conditions applied on the steel model used for the orthotropic models are similar, and it is assumed that the direction of the material is one direction in the finite element model. After running the simulation in the ABAQUS solver stage, the stress-strain behavior of the models was compared to the analytical calculation shown in the Figures below (Figure 2 to Figure 4), and the behavior of all the models shows good match with the theoretical phase. The results of the glass fiber are almost the same, and it is very near to the transverse direction results, similar for jute fiber, but the results of the carbon fiber model shows lower values than the analytical calculation. Its behavior, however, is the same.

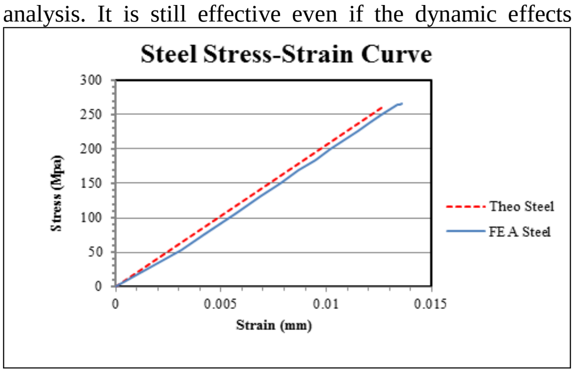

Figure 1. FEA and theoratical Stress Strain curve of straight bevel gear made from steel

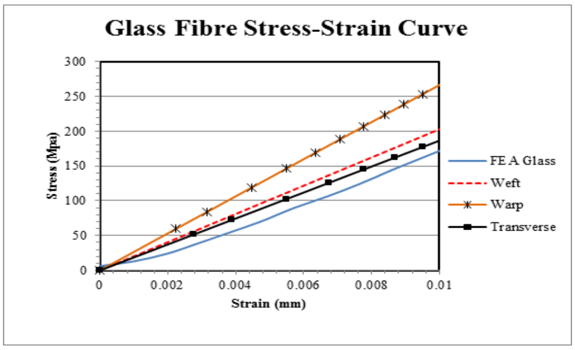

Figure 2. FEA and theoratical Stress Strain curve of straight bevel gear made from glass fiber

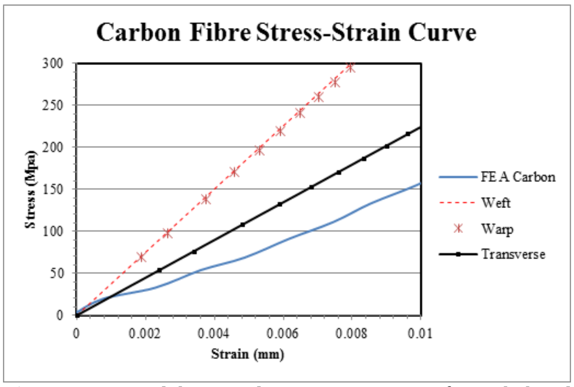

Figure 3. FEA and theoratical Stress Strain curve of straight bevel gear made from carbon fiber

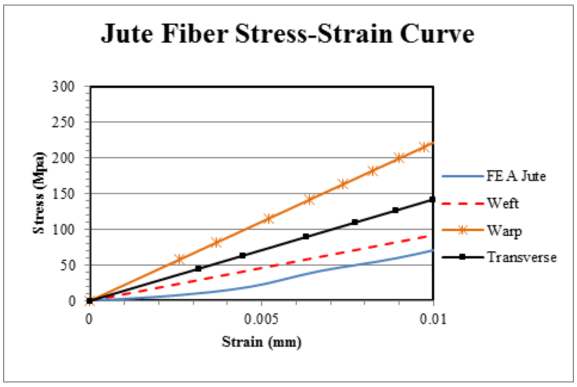

Figure 4. FEA and theoratical Stress Strain curve of straight bevel gear made from jute fiber

Table 3. Numerical and analytical results of the straight bevel gear for different type of materials

Stress/Material	Steel		Glass/Epoxy		Carbon/Epoxy		Jute/Epoxy	
	FEA	AGMA	FEA	AGMA	FEA	AGMA	FEA	AGMA
$\sigma 1$ (Mpa)	236.497	263.34	347.367	261.652	361.712	358.219	233.809	184.025
$\sigma 2$ (Mpa)				295.527		343.125		285.516
$\sigma 3$ (Mpa)				253.991		266.981		221.724

4. Conclusions

We derived an analytical model based on the AGMA standard, as has been discussed. We studied the standard straight bevel gear made of different type of materials (glasses/epoxy, Carbon/Epoxy, Jute/Epoxy and Mild steel) analytically by conventional approach. It considers the tooth as cyclic symmetry sector. We analyzed the stresses and deflection in a single point in the midpoint of the gear tooth surface where it is useful to verify the stresses and deflections that we determined and measured the present work. We studied the model numerically by running the straight bevel gear generation built using a commercial software to create points that describe the whole gear body, these points with composite mechanical properties are the input data for ABAQUS program to draw the bevel gear domain and contour the principle stresses inhabited in its teeth made of composite material, and draw the mode shape present due to the same load conditions. The validity of the FEM achieved by comparing its results (ABAQUS) for (steel, glass/epoxy, carbon/epoxy, jute/epoxy) with one analytical calculation load applied at the midpoint of the gear tooth surface. The comparison shows that there is a good agreement; therefore, the validity of the FEM is satisfied. Static stress analysis of these glass/epoxy, carbon/epoxy, jute/epoxy and mild steel materials gears are performed and their normal stresses in X, Y, Z directions are obtained. The same load of 245.25 N per mm length of face width of gear tooth was applied in all the cases.

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