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Parallel Translating Mechanism Process-Oriented Mathematical Model and 3-D Model for Cylindrical Gears with Curvilinear Shaped Teeth

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Abstract

For the main purpose of studying the tooth surface equation and undercutting conditions of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, the tooth surface equation and meshing condition of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism were deduced by the method of differential geometry and coordinate transformation, based on considering cutting tool shape and installation position error. The undercutting line of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism was also calculated. The manufacturing process of the parallel translating mechanism was simulated in three-dimensional software and then the three dimensional solid model of cylindrical gears with curvilinear shaped teeth was obtained. The studies referred to in the present paper have a certain reference value for research, development and design of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism.

Keywords: *Cylindrical gears with curvilinear shaped teeth; Tooth surface equation; Undercutting; Envelope; Translational Processing Device..*

1. Introduction

Gear is the most important and the most commonly key parts used in industrial production. With the development of productivity, the development of high speed, overloading and compact structure gear pair has a strong demand. The improvement of the gear has two aspects: tooth profile curve and tooth line curve.

For the study of tooth line curve, a lot of valuable studies have been done by the scholars. From the study, the advantages of longer contact line, tooth line symmetry, transmission more stable, higher bearing capacity, good lubrication performance and none axial force in cylindrical gears with curvilinear shaped teeth have been found [1-7]. The two common methods used in the processing of cylindrical gears with curvilinear shaped teeth are the method of rotating knife dish and the method of parallel translating mechanism. The tooth surface equation and characteristics are different by the different method. Di Yutao and professor Chen Ming [1, 2] analyzed the forming principle, meshing performance and bearing capacity of the cylindrical gears with curvilinear shaped teeth processed by the rotating knife dish. Wang Shaojiang and Xiao Huajun [3-6] studied the tooth surface equation, machining process of cylindrical gears with curvilinear shaped teeth processed by the rotating knife dish by the computer simulation. Tseng and his partner [7-9] established the mathematic model of cylindrical gears with curvilinear shaped teeth processed by the rotating knife dish by vector method and the contact characteristics were studied in the research papers.

In the studied of cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, Song Aiping from Yang Zhou University did many interesting work. The tooth surface equation, bending stress and modification methods under ideal parameters were studied and a translational processing device with parallel linkage was put forward by Song Aiping [10-15]. Then, Sun Zhijun [17] from Sichuan University raised the translational processing device by planetary gear train. But there are rare studies on the actual machining process. However, the tooth surface equation, bending stress, modification methods and undercutting in the actual machining process is the most important.

In the present paper, to study the tooth surface equation and undercutting of cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, the tooth surface equation and meshing conditions were deduced by the method of coordinate transformation based on the translational processing device referred in literature [16] and [17]. The computer simulation was done in 3-D software and the 3-D model was obtained. The present study has some reference value for the design of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism.

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2. Two Translational Processing Device and Simplified Mechanism

A translational processing device, invented by Song Aiping [16], is shown in Figure 1, and another translational processing device, invented by Sun Zhijun [17], is shown in Figure 2.

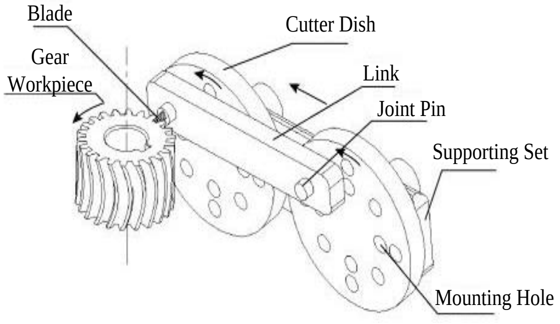

Fig. 1. Translational processing device by parallel linkage

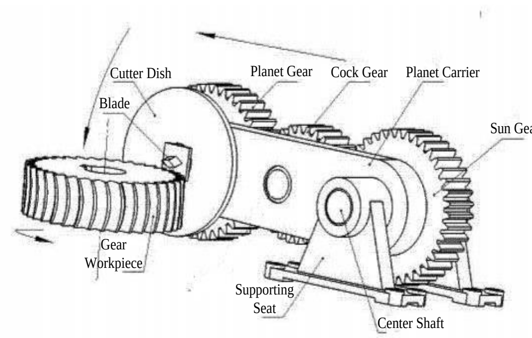

Fig. 2. Translational processing device by planetary gear train

In the processing device 1, the parallel linkage was adopted. The tool was installed on the translational connecting rod. When the rotation connecting rod rotated with the prime motor, the tool moved translational. Then the side of the blade moved with the same radius. With the feed movement of the gear workpiece, the cylindrical gears with curvilinear shaped teeth could be obtained.

In the processing device 2, the planetary gear train was obtained as the knife rest. Through analysis, there are some special points in the planetary gear train shown in Figure 2. With the planetary gear train moved, the tool moved translational too. When the gear workpiece rotated and the planetary gear train does translational movement, the gear was generated. In the same time, the device 2 could avoid the vibration caused by the unbalanced quality in the device 1.

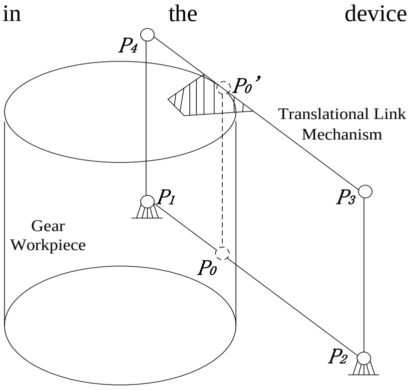

Fig. 3. The mechanism diagram of translational processing device

The mechanism in device 1 and device 2 was simplified just as the parallel translating mechanism shown in Figure 3. In Figure 3, the point O is the center of the gear workpiece, then P_1 , P_2 , P_3 and P_4 are the four hinges of the parallel translating mechanism. P_1P_2 is the static link, P_3P_4 is the translational link. P_1P_4 and P_2P_3 are the rotation links. Let the $P_0P_0{}'$ as the virtual link and represented by imaginary line, then the point P_0 and $P_0{}'$ are the virtual hinges of the virtual link.

3. Tooth Surface Equation

3.1. The Coordinate Systems

To study the forming process and forming tooth surface of cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, a coordinate system was set up in the translational processing system.

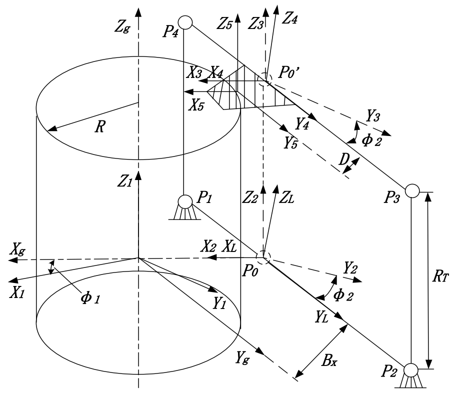

Fig. 4. The coordinate system settings of the translational processing system

Taking the mechanism diagram of translational processing device as the research object shown in Figure 3, the coordinate systems were set up as shown in Figure 4. The static coordinate systems are $S_g(O_gX_gY_gZ_g)$ and $S_l(O_lX_lY_lZ_l)$, and the other coordinate systems are moving coordinate systems. The coordinate system $S_l(O_lX_lY_lZ_l)$ was connected with the static link rack P_1P_2 of the parallel linkage, O_l is located on the endpoint P_0 of the virtual link, the axle O_lY_l is along the direction of static link rack and the axle O_lZ_l has the same direction of the gear workpiece. The direction of the three axle of the coordinate system $S_g(O_gX_gY_gZ_g)$ has the same direction of the coordinate system $S_l(O_lX_lY_lZ_l)$, but the origin of coordinate O_g is located at the center of the gear workpiece. When the gear workpiece moves, the coordinate system $S_g(O_gX_gY_gZ_g)$ moved parallel, but there is no rotational motion in the system $S_g(O_gX_gY_gZ_g)$.

The coordinate system $S_1(O_1X_1Y_1Z_1)$ is connected with the gear workpiece, and does the parallel and rotational motion with the gear workpiece at the same time. The axle O_gZ_g and O_1Z_1 is overlapping and has the same direction, the origin of coordinate is the center of the gear workpiece. The coordinate system $S_2(O_2X_2Y_2Z_2)$ is connected with the virtual rotation connecting rod $P_0P_0{}'$, the origin of coordinate O_2 is located at point P_0, then O_2X_2 and O_lX_l are overlapped to each other. The coordinate system $S_3(O_3X_3Y_3Z_3)$ is also connected with the virtual rotation connecting rod $P_0P_0{}'$ and the directions of the three axles have the same direction with coordinate system $S_2(O_2X_2Y_2Z_2)$, but the origin of coordinate O_3 is located at point $P_0{}'$ which is the other end of the virtual link. The coordinate system $S_4(O_4X_4Y_4Z_4)$ is connected with the translational connecting rod of the parallel linkage, the origin of coordinate O_4 is the point $P_0{}'$ and the axle O_4X_4 is the same to the axle O_3X_3. At last, the coordinate system $S_5(O_5X_5Y_5Z_5)$ is connected on the cutter blade to measure the geometric parameters, the axles of coordinate system $S_5(O_5X_5Y_5Z_5)$ have the same direction to $S_4(O_4X_4Y_4Z_4)$, and the axle O_5X_5 is the same to the axle O_4X_4, then $\overline{O_4O_5} = D$.

The installation position error of cutter is inevitable, in which includes angle deviation and position deviation. There are just position deviation between the rack and the gear workpiece (the position deviation between point O_g and O_l) being considered. The relative projection position between point O_g and O_l is shown in Figure 5.

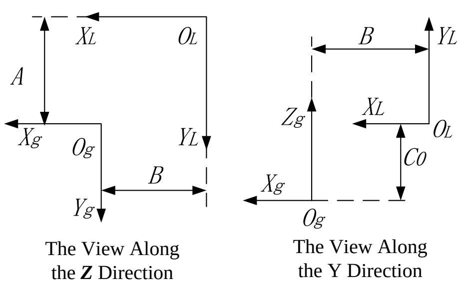

Fig. 5. The relative projection position between point O_g and O_l

In Figure 5, A、B、C is the distance between point O_g and O_l at Y、X、Z direction, respectively.

A_0、B_0、C_0 were the static initial installation error, and B_x is the ideal installation size at X direction. Then:

$$B_x = R + D = R + \overline{O_4O_5} \tag{1a}$$

$$A = A_0 + V_T = A_0 + R\varphi_1 \tag{1b}$$

Where, φ_1 is the rotational angle of the gear workpiece; V_T is the feed displacement, $V_T = R\varphi_1$.

3.2. The Cross-Section Shape of Cutting Tool

In the gear machining, the tool geometry have big influence for the forming of the gear tooth surface. The tooth surface equation is determined by the tool geometry and relative motion relationship between the cutting tooth and the gear workpiece. In common research, the effective cutting tool section of the cutting tool was taken to study the forming process of the gears.

The cutting tool section was regarded as with sharp points in some studies [3-4] when the geometry parameter of cutting tool was measured as Figure 6 shows, but in other studies [7-8], the cutting tool section was with rounded corners as shown in Figure 7.

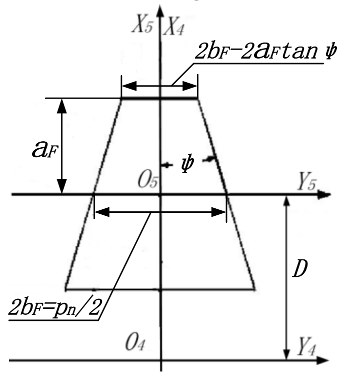

Fig. 6. The shape and parameter of cutting tool with sharp points

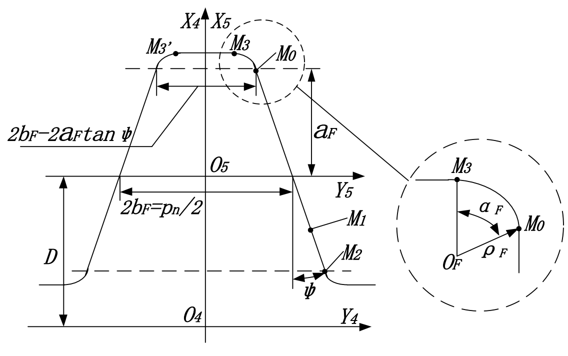

Fig. 7. The shape and parameter of cutting tool with rounded corners

In the actual production process, the cutting tool with sharp points doesn't exist, and the rounded corners always exist due to the factor of design and friction. To close to the actual production, the cutting tool with rounded corners was adopted to study the tooth surface equation and undercutting conditions of the cylindrical gears with curvilinear shaped teeth. The cutting tool geometry parameters are shown in Figure 7.

From Figure 7, the equation of cutting tool blade side could be shown as Eq. (2a) in the coordinate system $S_5(O_5X_5Y_5Z_5)$:

$$\begin{cases} \vec{r}_5 = x_5\vec{i}_5 + y_5\vec{j}_5 + z_5\vec{k}_5 \\ x_5 = -l\cos\psi + a_F \\ y_5 = \pm(l\sin\psi + b_F - a_F\tan\psi) \\ z_5 = 0 \end{cases} \tag{2a}$$

The chisel edge of cutting tool could be shown as Eq. (2b) in the coordinate system $S_5(O_5X_5Y_5Z_5)$ ·

$$\begin{cases} \vec{r}_5' = x_5'\vec{i}_5 + y_5'\vec{j}_5 + z_5'\vec{k}_5 \\ x_5' = a_F + \rho_F(1-\cos\alpha_F) \\ y_5' = l'[a_F + \rho_F(1-\cos\alpha_F)] \\ z_5' = 0 \end{cases} \tag{2b}$$

The rounded corners of cutting tool could be shown as Eq. (2b) in the coordinate system $S_5(O_5X_5Y_5Z_5)$ ·

$$\begin{cases} \vec{r}_5'' = x_5''\vec{i}_5 + y_5''\vec{j}_5 + z_5''\vec{k}_5 \\ x_5'' = a_F + \rho_F\cos\alpha_F + \rho_F\cos\theta'' \\ y_5'' = \pm(b_F - a_F\tan\psi - \rho_F\sin\alpha_F) + \rho_F\sin\theta'' \\ z_5'' = 0 \end{cases} \tag{2c}$$

where, the l , l' and θ'' are expression parameter in the parameter-vector equation(2a)-(2c). The range of l , l' and θ'' are $0 \le l \le 2a_F$, $|l'| \le 1$ and $\frac{\pi}{2} - \alpha_F \le \theta'' \le \frac{\pi}{2}$, respectively.

3.3. Tooth Surface Composition of Cylindrical Gears with Curvilinear Shaped Teeth

From the literature [4, 19], there are three main surface in the production process. The three surfaces are work surface, tooth bottom surface and dedendum surface, respectively. The work surface is processed by the cutting tool blade side M_0M_2 , the tooth bottom surface is processed by the chisel edge of cutting tool M_3M_3', and the dedendum surface is processed by the rounded corners M_0M_3 and $M_0'M_3'$.

3.4. Surface Equation of the Working Tooth Surface

From section 2.3, the work surface of the cylindrical gears with curvilinear shaped teeth was machined by the cutting tool blade side M_0M_2 . When the cutting tool blade side M_0M_2 rotates with the rack as the speed ω_2 , the cutting tool generatrix $\sum_1$ will be generated. At the same time, the gear workpiece rotates as the speed ω_1 and moves with feed speed $v_1 = R\omega_1$, then the cutting tool generatrix $\sum_1$ will envelope out the work surface $\sum_2$. When the cutting tool generatrix $\sum_1$ moves, the surface $\sum_1$ will form a family of surfaces $\sum_{\varphi 1}$ in the coordinate system $S_1(O_1X_1Y_1Z_1)$ by coordinate transformation from coordinate system $S_l(O_lX_lY_lZ_l)$ to $S_1(O_1X_1Y_1Z_1)$. The surface $\sum_2$ is the envelope of the family of surfaces $\sum_{\varphi 1}$ due to the work surface $\sum_2$ is tangent to $\sum_{\varphi 1}$ everywhere.

3.4.1. The Cutting Tool Generatrix Equation

From the above state, the cutting tool generatrix $\sum_1$ could be obtained by coordinate transformation from $S_5(O_5X_5Y_5Z_5)$ to $S_l(O_lX_lY_lZ_l)$.

$$\vec{r}_l = M_{l5}\vec{r}_5 \tag{3}$$

Where, M_{l5} is the coordinate transformation matrix from coordinate system $S_5(O_5X_5Y_5Z_5)$ to $S_l(O_lX_lY_lZ_l)$:

$$M_{l2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_2 & -\sin\varphi_2 & 0 \\ 0 & \sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4a}$$

$$M_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & R_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4b}$$

$$M_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_2 & \sin\varphi_2 & 0 \\ 0 & -\sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4c}$$

$$M_{45} = \begin{bmatrix} 1 & 0 & 0 & D \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4d}$$

$$M_{l5} = M_{l2}M_{23}M_{34}M_{45}$$
$$= \begin{bmatrix} 1 & 0 & 0 & D \\ 0 & 1 & 0 & -R_T\sin\varphi_2 \\ 0 & 0 & 1 & R_T\cos\varphi_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4e}$$

Take the formula (2a) and (4e) into (3), and the cutting tool generatrix $\sum_1$ could be got as:

$$\begin{cases} \vec{r}_l = x_l \vec{i}_l + y_l \vec{j}_l + z_l \vec{k}_l \\ x_l = -l\cos\psi + a_F + D \\ y_l = \pm(l\sin\psi + b_F - a_F \tan\psi) - R_T \sin\varphi_2 \\ z_l = R_T \cos\varphi_2 \end{cases} \quad (5)$$

3.4.2. Surface Family Equation

When the gear workpiece moved relative to the rack at parameter φ_1, the cutting tool generatrix $\sum_1$ will generate a family of surfaces $\sum_{\varphi 1}$ in the coordinate system $S_1(O_1X_1Y_1Z_1)$. The family of surfaces $\sum_{\varphi 1}$ could be got by the following formula (6):

$$\vec{r}_1 = M_{1l}\vec{r}_l \quad (6)$$

Where, M_{1l} is the coordinate transformation matrix from the coordinate system $S_l(O_lX_lY_lZ_l)$ to the coordinate system $S_1(O_1X_1Y_1Z_1)$.

$$M_{1g} = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7a)$$

$$M_{gl} = \begin{bmatrix} 1 & 0 & 0 & -(B_0 + R + D) \\ 0 & 1 & 0 & -(A_0 + R\varphi_1) \\ 0 & 0 & 1 & C_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7b)$$

Mark $B = -(B_0 + R + D)$, $A = -(A_0 + R\varphi_1)$ and $C = C_0$, where B_0、A_0、C_0 is the static installation error between the coordinate system $S_l(O_lX_lY_lZ_l)$ with the coordinate system $S_g(O_gX_gY_gZ_g)$.

$$M_{1l} = M_{1g}M_{gl}$$
$$= \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & B\cos\varphi_1 - A\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & B\sin\varphi_1 + A\cos\varphi_1 \\ 0 & 0 & 1 & C \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7d)$$

Take the formula (7d) and (5) into (6), then the equation for family of surfaces $\sum_{\varphi 1}$ could be obtained as formula (8) shows:

$$\begin{cases} \vec{r}_1 = x_1 \vec{i}_1 + y_1 \vec{j}_1 + z_1 \vec{k}_1 \\ x_1 = Q_1\cos\varphi_1 - Q_2\sin\varphi_1 + R_T \sin\varphi_1 \sin\varphi_2 \\ y_1 = Q_1\sin\varphi_1 + Q_2\cos\varphi_1 - R_T\cos\varphi_1\sin\varphi_2 \\ z_1 = R_T\cos\varphi_2 + C \end{cases} \quad (8)$$

Where,

$$Q_1 = -l\cos\psi + a_F + D + B \quad (9a)$$

$$Q_2 = A \pm (l\sin\psi + b_F - a_F\tan\psi) \quad (9b)$$

3.4.3. Envelope Condition/Meshing Equation

The family of surfaces $\sum_{\varphi 1}$ is a single parameter curved surface with the movement parameter φ_1, and the surface parameter coordinate of $\sum_1$ are l and φ_2. So the family of surfaces $\sum_{\varphi 1}$ was expressed as:

$$\vec{r}_1 = \vec{r}_1(l, \varphi_2, \varphi_1) \quad (10)$$

The envelope condition or meshing function could be obtained as the formula (11).

$$\left(\frac{\partial \vec{r}_1}{\partial l}, \frac{\partial \vec{r}_1}{\partial \varphi_2}, \frac{\partial \vec{r}_1}{\partial \varphi_1}\right) = 0 \quad (11)$$

For the differences of the two sides of generatrix, mark the cutting tool generatrix of the right cutting tool blade side in Figure 7 is $\Sigma_1^{(1)}$, and the cutting tool generatrix of the left cutting tool blade side is $\Sigma_1^{(2)}$ to convenience study. Cutting tool generatrix $\Sigma_1^{(1)}$ forms the family of surfaces $\sum_{\varphi 1}^{(1)}$ in coordinate system $S_1(O_1X_1Y_1Z_1)$ and $\Sigma_1^{(1)}$ forms the family of surfaces $\sum_{\varphi 1}^{(2)}$.

In the present study, just envelope condition for the family of surfaces $\sum_{\varphi 1}^{(1)}$ was deduced. The envelope condition of $\sum_{\varphi 1}^{(2)}$ could be obtained just repeat the derivation process again. For the family of surfaces $\sum_{\varphi 1}^{(1)}$:

$$\frac{\partial \vec{r}_1^{(1)}}{\partial l} = -\cos(\psi - \varphi_1)\vec{i}_1 + \sin(\psi - \varphi_1)\vec{j}_1 \quad (12a)$$

$$\frac{\partial \vec{r}_1^{(1)}}{\partial \varphi_2} = R_T(\sin\varphi_1\cos\varphi_2\vec{i}_1 - \cos\varphi_1\cos\varphi_2\vec{j}_1 - \sin\varphi_2\vec{k}_1) \quad (12b)$$

$$\begin{aligned} \frac{\partial \vec{r}_1^{(1)}}{\partial \varphi_1} &= [(-Q_1 + R)\sin\varphi_1 - Q_2\cos\varphi_1 + R_T\cos\varphi_1\sin\varphi_2]\vec{i}_1 \\ &\quad + [(Q_1 - R)\cos\varphi_1 - Q_2\sin\varphi_1 + R_T\sin\varphi_1\sin\varphi_2]\vec{j}_1 \end{aligned} \quad (12c)$$

So, the envelope condition or meshing function of the family of surfaces $\sum_{\varphi 1}^{(1)}$ could be expressed as:

$$\begin{aligned} \Phi^{(1)} &= \left(\frac{\partial \vec{r}_1}{\partial l}, \frac{\partial \vec{r}_1}{\partial \varphi_2}, \frac{\partial \vec{r}_1}{\partial \varphi_1}\right) \\ &= R_T\sin\varphi_2[(R - Q_1)\cos\psi + (Q_2 - R_T\sin\varphi_2)\sin\psi] \\ &= R_T\sin\varphi_2[(R + l\cos\psi - a_F - D - B)\cos\psi + (-A_0 \\ &\quad - R\varphi_1 + l\sin\psi + b_F - a_F\tan\psi - R_T\sin\varphi_2)\sin\psi] \\ &= 0 \end{aligned} \quad (13)$$

3.4.4. The Tooth Surface Equation

From formula (8) and formula (13), the tooth surface equation of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism could be expressed as:

$$\begin{cases} \vec{r}_1 = \vec{r}_1(l, \varphi_2, \varphi_1) \\ \Phi^{(1)} = 0 \end{cases} \tag{14}$$

When the installation error is ignored, take the B_0、A_0、C_0 as 0 and the tooth surface equation in the ideal installation parameters could be obtained.

4. Undercutting Condition of Cylindrical Gears with Curvilinear Shaped Teeth Processed by Parallel Translating Mechanism

From literature [7, 19], there will be the condition of undercutting when the installation position of cutting tool is not correct. And through selecting the installation position, the undercutting of the gear could be avoided.

In the parameters of the tooth surface equation for the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, the parameters l, φ_2 are surface parameters of the cutting tool generatrix $\sum_1$, while the parameter φ_1 is the motion envelope parameters in the process processing. From the derivations in [7, 18], there are:

$$-\vec{v}_l^{(l1)} = \frac{\partial \vec{r}_l}{\partial l}\frac{dl}{dt} + \frac{\partial \vec{r}_l}{\partial \varphi_2}\frac{d\varphi_2}{dt} \tag{15}$$

Find the derivative of envelope condition shown in formula (13), we could get:

$$\frac{\partial \Phi}{\partial l}\frac{dl}{dt} + \frac{\partial \Phi}{\partial \varphi_2}\frac{d\varphi_2}{dt} = -\frac{\partial \Phi}{\partial \varphi_1}\frac{d\varphi_1}{dt} \tag{16}$$

The formula (15) and formula (16) could determine a curve L in the coordinate system $S_l(O_l X_l Y_l Z_l)$. Through limiting the position of the cutting tool generatrix $\sum_1$ by the curve L, the surface $\sum_1$ could avoid to cause singular points on the surface $\sum_2$, so the undercutting is avoided.

The curve L could be determined by the formula (17):

$$\begin{cases} \vec{r}_l = \vec{r}_l(l, \varphi_2) \\ \Phi = \Phi(l, \varphi_2, \varphi_1) = 0 \\ F = F(l, \varphi_2, \varphi_1) = 0 \end{cases} \tag{17}$$

Where, $F(l, \varphi_2, \varphi_1) = \Delta_1^2 + \Delta_2^2 + \Delta_3^2 = 0$ is the necessary and sufficient condition of appearing the undercutting and singular points existing on the surface $\sum_2$. Δ_1、Δ_2、Δ_3、Δ_4 could be expressed, respectively, as:

$$\Delta_1 = \begin{vmatrix} \frac{\partial x_l}{\partial l} & \frac{\partial x_l}{\partial \varphi_2} & -v_{xl}^{(l1)} \\ \frac{\partial y_l}{\partial l} & \frac{\partial y_l}{\partial \varphi_2} & -v_{yl}^{(l1)} \\ \frac{\partial \Phi}{\partial l} & \frac{\partial \Phi}{\partial \varphi_2} & -\frac{\partial \Phi}{\partial \varphi_1}\frac{d\varphi_1}{dt} \end{vmatrix} \tag{18a}$$

$$\Delta_2 = \begin{vmatrix} \frac{\partial x_l}{\partial l} & \frac{\partial x_l}{\partial \varphi_2} & -v_{xl}^{(l1)} \\ \frac{\partial z_l}{\partial l} & \frac{\partial z_l}{\partial \varphi_2} & -v_{zl}^{(l1)} \\ \frac{\partial \Phi}{\partial l} & \frac{\partial \Phi}{\partial \varphi_2} & -\frac{\partial \Phi}{\partial \varphi_1}\frac{d\varphi_1}{dt} \end{vmatrix} \tag{18b}$$

$$\Delta_3 = \begin{vmatrix} \frac{\partial y_l}{\partial l} & \frac{\partial y_l}{\partial \varphi_2} & -v_{yl}^{(l1)} \\ \frac{\partial z_l}{\partial l} & \frac{\partial x_l}{\partial \varphi_2} & -v_{zl}^{(l1)} \\ \frac{\partial \Phi}{\partial l} & \frac{\partial \Phi}{\partial \varphi_2} & -\frac{\partial \Phi}{\partial \varphi_1}\frac{d\varphi_1}{dt} \end{vmatrix} \tag{18c}$$

$$\Delta_4 = \begin{vmatrix} \frac{\partial x_l}{\partial l} & \frac{\partial x_l}{\partial \varphi_2} & -v_{xl}^{(l1)} \\ \frac{\partial y_l}{\partial l} & \frac{\partial y_l}{\partial \varphi_2} & -v_{yl}^{(l1)} \\ \frac{\partial z_l}{\partial l} & \frac{\partial x_l}{\partial \varphi_2} & -v_{zl}^{(l1)} \end{vmatrix} \tag{18d}$$

5. 3-D Model of Cylindrical Gears with Curvilinear Shaped Teeth

To study the machining process, tooth surface molding method and the tooth surface features of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism, the relevant program was compiled by Open Grip and operated in 3-D software. The parameters were shown in Table 1. Figure 8 is a 3-D model of the gear pair.

Table. 1. Gear parameters in simulation processing

Parameters	Pinion	Gear
Number of teeth	25	36
Modulus	4	4
Pressure angle	20	20
Tooth width	60	60
Tooth line radius	150	150

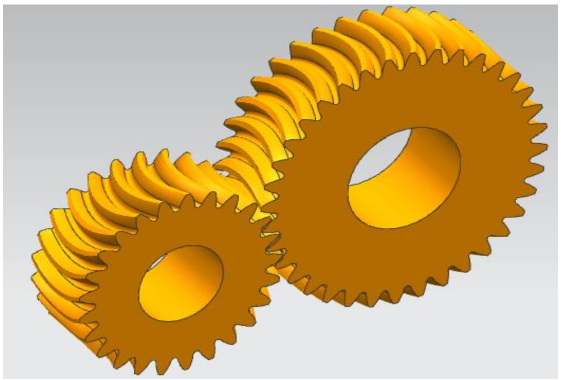

Fig. 8. 3-D model of gear pair

From the model in Figure 8, the mathematical model correctness of the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism could be verified. Using the interference inspection tool in the software, there is no interference phenomenon in the gear pair shown in Figure 8, and the meshing transmission requirements could be achieved.

6. Conclusions

In the present paper, taking the cylindrical gears with curvilinear shaped teeth processed by parallel translating mechanism as the research object, the parallel translating mechanism was taken as a parallel linkage mechanism, then the virtual rotation connecting rod was considered between the static link and the cutting tool.

1. The relevant coordinate systems were established and the tooth surface equation of the cylindrical gears with curvilinear shaped teeth was deduced by differential geometry and the method of matrix transformation.
2. The envelope condition of the tooth surface equation was analyzed. Based on the research in the literature [19], the undercutting condition of the cylindrical gears with curvilinear shaped teeth was revealed.
3. The process of the machining process for the cylindrical gears with curvilinear shaped teeth was simulated by Grip programming language. The 3-D model of cylindrical gears with curvilinear shaped teeth was got.

From the present paper, some theoretical basis could be provided to the processing error analysis of the cylindrical gears with curvilinear shaped teeth.

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