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Three-Dimensional Investigations of Stress Intensity Factors in a Rotating Thick-Walled FGM Cylinder

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Abstract

The present study focuses on three-dimensional analyses of a thick walled functionally graded material cylinder, containing a longitudinal semielliptical internal surface flaw, and is subjected to an internal pressure and a rotational speed. The cylinder is assumed to be isotropic with exponentially varying elastic modulus through the thickness. The effect of wall thickness on the distribution of stress intensity factor is also studied. The results which are normalized for the advantage of non-dimensional analysis show that the material gradation, the crack geometry and wall thickness have a significant influence on the amount and distribution of stress intensity factors. Numerical results are given to assess the safety of the FGM and homogeneous cracked cylinder. The study is valuable to engineering applications.

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Keywords: *Functionally graded materials; 3D finite element analysis; Stress intensity factor, rotating thick-walled cylinder*.

NOMENCLATURE

a	= depth of the deepest point on the crack front
c	= semi axis of the elliptical crack
E	= elastic modulus
K_I, K_{II}, K_{III}	= conventional SIFs, modes 1, 2 and 3
$K_{i,non}$	= non dimensional SIFs where i=I,II,III
K_O	= nominal stress intensity factor
P_{in}	= inner pressure of cylinder
Q	= shape factor
R_{in}	= inner radius of cylinder
R_{out}	= outer radius of cylinder
β	= constant of material non-homogeneity
δ	= small distance
ζ	= non-dimensional normalized coordinates
η	= relative depth of the crack
μ	= shear modulus
υ	= Poisson's ratio
ξ	= aspect ratio
σ_{ij}	= stress tensor
ϕ	= parametric angle
ς, χ, ψ	= the local coordinate systems at the crack tip
ω	= constant of thermal non-homogeneity
Δu	= the crack tip opening displacement
APDL	= ANSYS Parametric Design Language
COD	= Crack Opening Displacements
FGM	= Functionally Graded Material
ID	= Inner Diameter of the cylinder
OD	= Outer Diameter of the cylinder
SIF	= Stress Intensity Factor
TBC	= Thermal Barrier Coating

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1. Introduction

Rotating thick-walled cylinders are common in industrial applications, such as shaft of machine, multi-barrel rotary guns in weapon, engine of aircraft and rotating pipeline of petrochemical equipment, etc. Cracks are easy to appear in the internal bore of the cylinder because of the flaw of production and usage [1]. The theory of thick tubes (Lame's theory) shows that the longitudinal cracks located on the internal face of the cylinder are most dangerous [2]. The stress intensity factor calculation on the crack face is of the most important for study of the crack propagation and fatigue life.

Parallel to new industrial developments, it seems that the use of conventional materials in rotating thick-walled cylinders is inadequate. Recent developments in the space, automobile and many modern industries have placed demands on Functionally Graded Material (FGM) cylinders suitable for unusual conditions of pressure, temperature and environment [3].

FGMs have attracted much interest primarily as an alternative to Thermal Barrier Coatings (TBCs), which are used in aerospace and high temperature applications. The possibility of tailoring the desired thermo-mechanical properties holds a wide range application potential for FGMs.

FGMs are multiphase materials in which the volume fractions of the constituents vary continuously as a function of position. Therefore, the mismatch of thermos-

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mechanical properties near the bond line is minimized. Another application area of FGMs include their use as interfacial zone between two different layers, improves the bonding strength [4], and reduces the residual stresses, interfacial delamination [5] and stress concentration or stress intensity factors [6,7]. Because of their outstanding advantages over conventional composites and monolithic materials, these materials have received a special attention from engineers and researchers of various fields of interest. Kim and Paulino [8] addressed a wide variety of FGMs applications. Tutuncu [9] considered power series solution for the stresses and displacements in functionally-graded cylindrical vessels subjected to internal pressure alone using the infinitesimal theory of elasticity. Stress analysis of thick-walled tubes, due to the nature of functions, are chosen to describe the inhomogeneous properties of cylinder material, are studied by Fukui and Yamanaka [10].

So far, the effects of material distribution on the characteristics of these materials under various loading conditions and for various geometries have been investigated from different points of view.

Although FGM cylinders can be used in these unusual working conditions, generation of defects such as holes, cavity, and cracks in the material substructures during manufacturing or in-service conditions is inevitable. The fatigue failure of such components often develops from the propagation of surface defects. Therefore, the consideration of fracture mechanic criterion in the design process of this equipment is essential for reliable application in the above mentioned industries. Crack analysis of FGMs is an indispensable task in the optimization, reliable and durable design of functionally graded materials and structures in innovative engineering applications. For crack problems in FGMs with general geometrical and loading conditions, efficient and accurate numerical simulation tools are required due to the high mathematical complexity of the arising governing partial differential equations. In this regards, there has been extensive research focusing on predicting response of FGM cylinders containing cracks. Afsar and Anisuzzaman [3] considered a thick walled FGM cylinder with two diametrically opposed edge cracks emanating from the inner surface and solved the problem of the Stress Intensity Factor (SIF) by numerical procedures. The FGM cylinder was radially divided into layers of infinitesimal thickness with constant material properties at each layer. Chen [11] determined the SIFs for an internal and an external fully circumferential crack in FGM cylinders subjected to a longitudinal tension. Nami and Eskandari [12] studied three-dimensional investigations of stress intensity factors in a thermo-mechanically loaded cracked FGM hollow cylinder. The cylinder was assumed to be isotropic with exponentially varying elastic modulus through the thickness. The effect of non-uniform coefficient of thermal expansion on the distribution of stress intensity factor was also studied. The problem of the stress intensity factor analysis for cracks located at an arbitrary position in rotating FGM disks is studied by Eskandari [13]. A semi-analytical solution for the purpose of thermo-elastic analysis of functionally graded rotating thick cylindrical pressure vessels with variable thickness subjected to the temperature gradient and internal non-uniform pressure has been performed by Jabbaria and Zamani Nejad [14] by using higher-order shear deformation theory and multi-layer method.

In the present paper, a longitudinal semielliptical internal surface flaw in a rotating thick walled FGM cylinder of thickness t and internal radius Rin subjected to internal pressure Pin is considered (Fig. 1). The effect of wall thickness of the cylinder on the distribution of stress intensity factor is also studied. The elliptical-arc defect is described by two dimensionless parameters, $\xi = a/c$ and $\eta = a/t$, the so-called aspect ratio and relative depth of the crack, respectively.

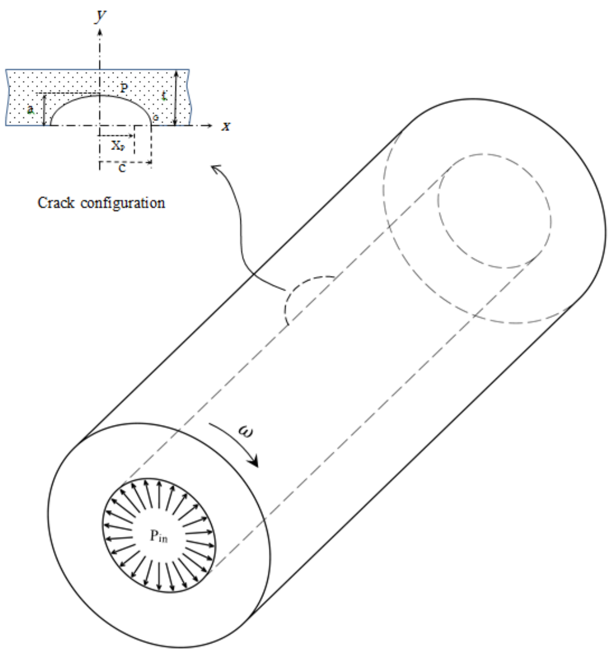


Figure 1. Longitudinal internal semi-elliptical surface crack in a rotating thick walled cylinder.

2. Stress Intensity Factor of the Functionally Graded Cylinder

2.1. Problem Definition

A problem which is encountered in three dimensional finite element analyses is the large number of elements and as a result, a remarkable and time consuming computation. Also, because of very rapid changes in the geometrical parameters around the crack front region, mesh generation of this region must be done with a great care. This may lead to increase the run time which makes it difficult to reach valid results and conclusion. The sub-modeling technique is an advanced numerical tool which solves the problem of analyzing the complex finite element models. This method is also named as the cut-boundary displacement method or the specified boundary displacement method [15]. If the boundary of the sub-model is reasonably selected, and a fine mesh is used in it, the high accuracy results can be achieved [15]. Very rapid changes in the geometrical parameters cause to employ a higher mesh density in the vicinity of the crack front. For such a case, a sub-model technique is employed to overcome the problem. In this technique, a coarse mesh is generated at the first step, and then the region around the geometrical discontinuity, i.e., crack region, is cut. Then the boundary conditions on this cut boundary are extracted from the coarse model and transferred to the cut boundary or 'sub-model'. Finally this sub-model can be analyzed using higher mesh density and accuracy [15].

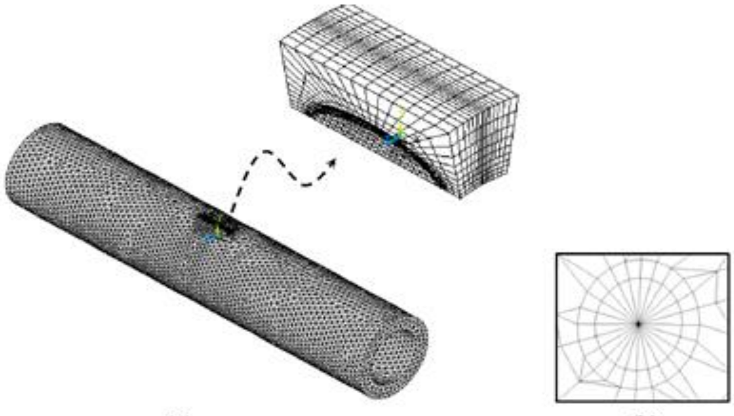
The present paper studies the stress intensity factor in a rotating thick walled FGM cylinder subjected to the internal pressure. Also, the effects of diameter ratio of cylinder on the SIF's are studied. The results obtained by the 3-D finite element analysis. The special effort is made to condense the SIF data into a compact dimensionless form and to study the effect of material gradation, Diameter Ratio (RD) and the crack depth on the stress intensity factor for points on the crack front. A cylinder of the inside diameter of 45 mm and a total length of 700 mm with different diameter ratios, i.e., $\frac{D_{out}}{D_{in}} = \frac{R_{out}}{R_{in}} = 1.5, 2, 2.5, 3$ is considered. The problem has also been examined through the commercial software, ANSYS, which has been used as a finite element solver. The location of a semi-elliptical surface crack is shown in Fig. 1. The geometry of crack is identified by two dimensionless parameters, a/c and a/t which called aspect ratio and relative depth of crack, respectively. Any arbitrary point on the crack front defined by a non-dimensional normalized coordinates as follow; $\zeta = X_P/X_G$ where XP and XG are the x-coordinate of the arbitrary point P which generally located on the crack front and the corner point G, respectively (Fig. 1).

2.2. Finite Element Modeling

The finite element modeling of cylinder is shown in Figs. (2a-2b). The higher order 3-D 20-node isoparametric brick elements are used everywhere except near the crack front and the singularity elements are applied around the crack front (Fig. (2b)). The singularity elements have square-root terms in their assumed displacement distribution and, therefore, produce a singular stress field at the crack front. The mesh around the crack front is refined enough (i.e., 0.4 mm) to achieve the stress and strain distribution accurately.

The crack front region has been constructed by sweeping an auxiliary area around the crack front line. The auxiliary area is meshed using singular elements around the crack front node and non-singular elements in the remaining part of the area. After this, the half of sub-model is constructed. Then, the next half of sub-model is generated by reflecting the model about the crack plane. By merging the nodes, except those on the crack faces, it becomes possible to model the crack in the cylinder. The whole finite element model has nearly 60,000 elements for the FGM cylinder.

A finite element code can be used to account for spatial variation in material property of FGM cylinder. There are different ways of incorporating changes in material properties into a finite element program. Walter et al. [16] describe two commonly-used methods. An element base method where the desired spatial material property of each element based on its location is achieved through a finite element code. Another way is to compute the material property at each integration point for element stiffness matrix via the spatially varying in material property function.



(a) (b)

Figure 2. Finite element model of cylinder: (a) the whole model and the corresponding sub-model, (b) the singular elements near the crack front

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2.3. Functionally Graded Cylinder

In the present study, a finite element code using the ANSYS Parametric Design Language (APDL) is used to account the material property changes for each element via its location. This section describes the details of the finite element formulation for stress and fracture analyses of rotating thick walled FGM cylinder. The material is assumed to be isotropic with exponentially varying elastic modulus through the thickness is as follow:

$$E(r) = E_1 e^{\beta (r-r_i)} \quad (1)$$

where, r_i is the inner radius of cylinder and β is the constant of material non-homogeneity which is defined as:

$$\beta = \frac{1}{w} \ln(\frac{E_2}{E_1}) \quad (2)$$

where, w denotes the thickness of cylinder, E_1 and E_2 are the values of elastic modulus at the inner and outer radius of cylinder, respectively. For a simple traceable solution, the dependency on the Poisson's ratio is neglected and it is assumed constant throughout the cylinder.

2.4. Crack Tip Fields in FGMs

Material non-homogeneity has a significant influence on SIFs. Eischen [17] established the general form of the stress and displacement fields near a crack tip in a nonhomogeneous material with a spatially varying material property. He solved the problem for materials with continuous, bounded, and differentiable property variations and showed that the asymptotic fields for a crack in a FGM with continuous mechanical properties are same as those of a crack embedded in a homogeneous material. In addition, the asymptotic displacement expressions for the homogeneous materials can be used for FGMs on condition that the material properties are calculated at the crack-front location.

Since the nature of the stress singularity for continuously non-homogenous, isotropic and linear elastic solid is precisely the same as the well-known form applicable to homogeneous materials, irrespective of the particular form of the Young's modulus variation [17], the stress intensity factors can be obtained from Crack-Opening-Displacements (CODs) as [18]:

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \frac{\mu_{tip}\sqrt{2\pi}}{4(1-\nu)} \lim_{\delta\to 0}\frac{1}{\sqrt{\delta}} \begin{Bmatrix} \Delta u_{\varsigma}(\delta) \\ \Delta u_{\chi}(\delta) \\ (1-\nu)\Delta u_{\psi}(\delta) \end{Bmatrix}$$

where K_I, K_{II} and K_{III} are opening, sliding and tearing modes of SIFs, μ_{tip} is the shear modulus at the crack front, δ which approaches zero is a small distance between specified node at crack-surface and a node at crack-front, and Δu_I (X) = [u_I(X ∈ upper crack surface) − u_I(X ∈ lower crack surface)] in which $I = \varsigma, \chi$ and ψ are the CODs in the local coordinate systems.

The stress intensity factor for cylinder is considered in the non-dimensional form and is defined as:

$$K_{I,non} = \frac{K_I}{K_0} \quad (3)$$

In which K_I is the calculated values of the stress intensity factors and K_0 is the nominal stress intensity factor. The nominal stress intensity factor for FGM cylinders are used as:

$$K_0 = \frac{2\,P_{in}\,(R_{out})^2}{(R_{out})^2 - (R_{in})^2}\sqrt{\frac{\pi a}{Q}} \qquad \text{for plane stress} \quad (4)$$

where the shape factor, Q, is approximated by

$$Q = 1 + 1.464\,(\frac{a}{c})^{1.65} \quad (5)$$

It should be noted that the nodes near the surface of the plate are in the plane stress conditions. Through thickness nodes, far from the plate surface are in plane strain situations. Plan strain stress intensity factor differs by the coefficient of $\frac{1}{1-\nu}$ from the plane stress ones.

3. Numerical Results

3.1. The Validation of the Method

To justify the reliability of the method, a semi-elliptical surface crack in a FGM plate under tension is considered. Figure 3 shows the configuration and location of the crack in the plate. It is assumed that the material gradient is in the y-direction, i.e. $E(y) = E_1 e^{\beta(y)}$. The crack parameters are supposed as $\xi = \frac{1}{3}$ and $\eta = 0.8$ and the ratio of Young modulus in FGM plate is considered as E(y=t) /E(0) = 0.2. The constant value of 0.3 is used for Poisson's ratio throughout the material. The corresponding homogenous case is also considered and compared to the FGM solution.

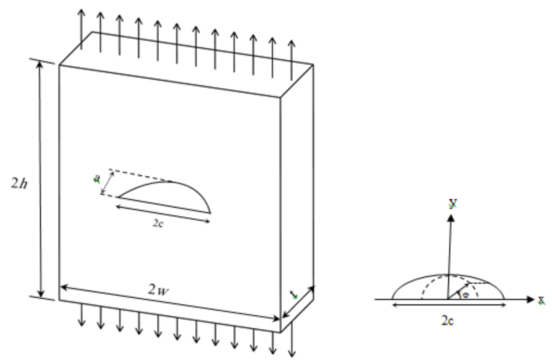

Figure 3. Semi-elliptical surface crack in a FGM plate under tension

The distribution of normalized stress intensity factor through the semi-elliptical surface crack with a/c = 0.33 and a/t = 0.8 as a function of the parametric angle for both the FGM and homogenous plates under uniform tension loading is shown in Figure 4. In this case, it is assumed that the FGM plate has the material gradation equal to 0.2. The results are compared with those reported by Yildirim *et al.* [19] showing to be in a good agreement for both cases. It is noteworthy that, for symmetry, only one-half of crack front is considered.

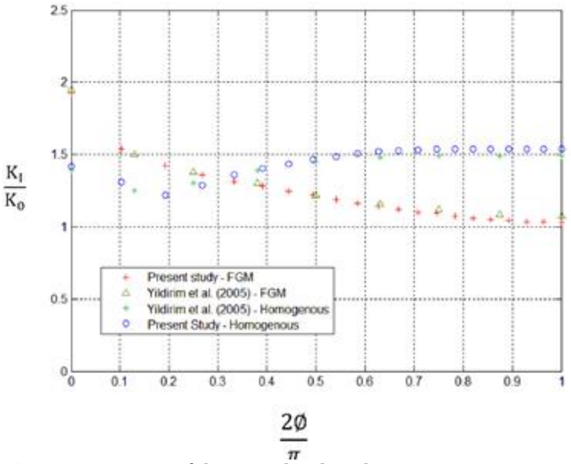

Figure 4. Variation of the normalized mode-I stress intensity factor on the crack front in a FGM plate with $\xi = \frac{1}{3}$, $\eta = 0.8$, $\nu = 0.25$, $E(r_o)/E(r_i) = 0.2$ and homogeneous material.

3.2. Results and Discussion

3.2.1. Pressurized Rotating Thick Walled FGM Cylinder

In the present study, we considered a semi-elliptical surface crack in a rotating thick-walled FGM cylinder subjected to the internal pressure of 5 MPa. The constant value of 0.30 is used for Poisson's ratio throughout the material. The rotation speed of cylinder with inner diameter of 90 mm and different wall thicknesses is considered as 1300 rad/s.

The distributions of mode I stress intensity factors along the crack front as a function of the normalized coordinate (ζ), for a FGM and corresponding homogeneous rotating cylinder are compared in Figs. (5a-5e). Each curve is plotted for a certain values of relative crack depth (η), material gradation ($RE=E_2/E_1$), i.e., 0.2,2,5,10 and for different values of aspect ratio (ξ). The results are compared for corresponding homogeneous material ($E_2/E_1=1$) in Fig. (5d).

It can be observed from Fig. 5a that for η = 0.1 and RE = 0.2, the higher values of aspect ratios (ξ), the smaller values of K_I in midpoints of the crack fronts. In other words, for corner points of the crack fronts, the higher values of aspect ratios, the higher values of K_I . The effects of material gradation on variation of first mode of the stress intensity are studied in Fig. 6. The following results can be seen from the figures:

- It can be seen that the distribution of the K_I along the crack front is symmetric on both sides of the deepest point, and the stress intensity factors at the corner points are the same. It may be concluded that, regarding the probable crack growth, the corner points start to propagate simultaneously. Moreover, the same crack growth rates are expected to be seen at both sides of the crack.
- The distribution of the SIF on the crack front tends to a parabola like shape and the critical point on the crack front in rotating FGM cylinders with low material gradation($RE=E_2/E_1< 1$) depends on the value of aspect ratio.
- For rotating FGM cylinders with small values of material gradation ($RE=E_2/E_1<1$), as the aspect ratio increases, the deepest point of the crack front tends to be safer than corners in probable crack growth. In other words, in cracks with small aspect ratios the deepest point will propagate sooner than corners. Cracks with large values of aspect ratio apt to grow in corners sooner than others.
- For large values of material gradation (RE>>1), as the aspect ratio decreases, the critical point happens at the deepest points and the corner points experiences minimum value of the stress intensity factor.
- The points on the crack fronts in rotating cylinders with high material gradation and large values of aspect ratio grow simultaneously in same manner.
- For homogeneous rotating cylinder, the critical point of the stress intensity factor depends on the aspect ratio of the crack. For small values of the aspect ratio, the critical point is always one between the two corner points. The maximum stress-intensity factor occurs at the deepest point.
- In a certain value of material gradation, higher the aspect ratio, higher the K_I at corners. For large values of the aspect ratio, K_I increases dramatically in corners.
- The points far from corners on the crack front in a homogenous rotating cylinder have the same values of SIF and the distribution of K_I on the crack front is nearly linear. The gradation in material cause to change it to a parabola like shape.
- Higher the material gradation, smaller the stress intensity factor in corners. In other words, increasing the material gradation will decrease the risk of crack propagation on the deepest point.

The effect of Diameter Ratio (RD) on the distribution of the K_I along a crack front with constant aspect ratio and relative crack depth ($\eta = \xi = 0.4$) for different gradation of materials, i.e. 0.2,1, 2, 5 and 10 are studied in Fig. (7a-7e). The effects of internal pressure and rotational speed of the cylinder on the distribution of the mode-I stress intensity factor along the crack front are depicted in Figs. 8-9.A general investigation of graphs demonstrates that the wall thickness variation causes the following results:

- Higher the diameter ratio, higher the KI along the crack front. In other words, in probable crack growth, cylinders with smaller diameter ratios are more reliable than others.
- In a certain value of wall thickness, higher the material gradation, smaller the K_I at corners. In other words, for large gradation of materials (RE>1) with constant diameter ratio, smaller the material gradation, smaller the K_I at the deepest point.
- Smaller the material gradation, higher the effects of diameter ratio on the values of stress intensity factor on the crack fronts. It means that in a constant value of diameter ratio, the maximum SIF occurs on cylinders with smaller gradation of materials.
- Higher the internal pressure, higher the values of the SIFs along the crack front.
- The rotational speed of the cylinder significantly affects on the values of the stress intensity factors along the crack fronts. As seen, with increasing the rotational speed of the cylinder the stress intensity is increased.

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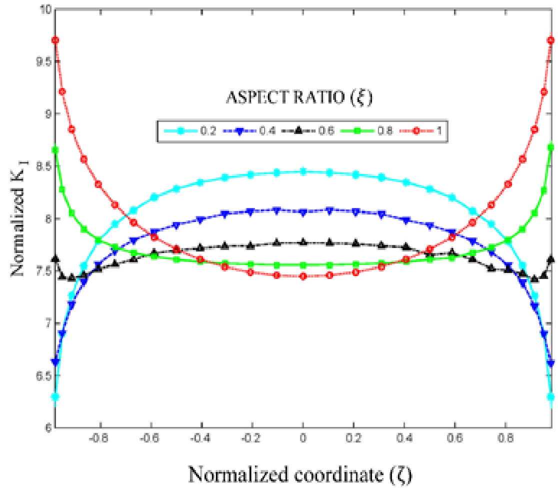

Fig. 5a

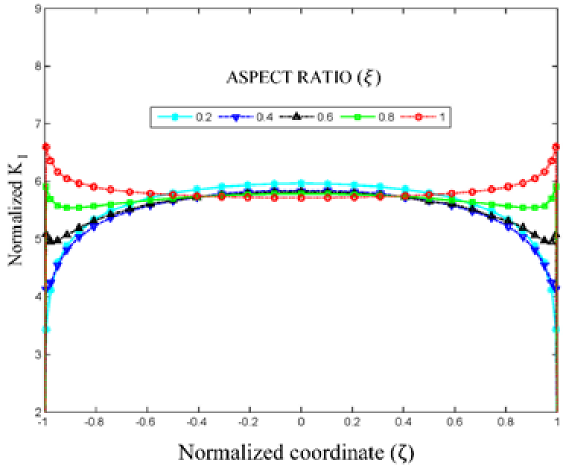

Fig. 5b

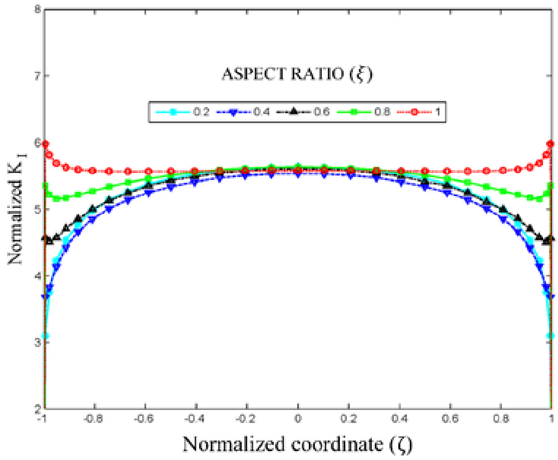

Fig. 5c

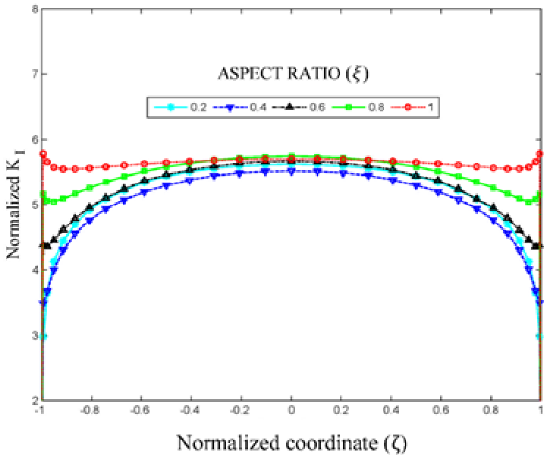

Fig. 5d

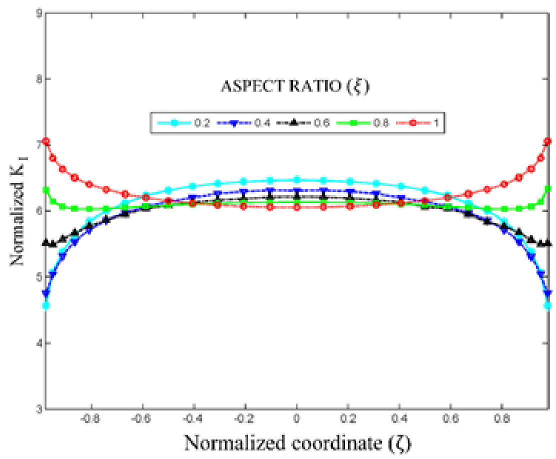

Fig. 5e

Figure 5. Distribution of the first mode stress intensity factor along the crack front in a rotating FGM and homogeneous cylinder with $\eta = 0.1$, $\nu = 0.3$, diameter ratio (RD) = 1.5 and for different values of aspect ratios (ξ) with material gradation of: a) RE=0.2; b) RE=2; c) RE=5; d) RE=10 e) RE=1(homogeneous material).

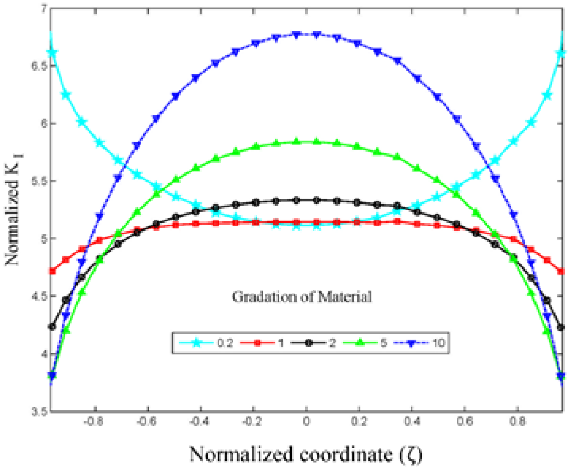

Figure 6. Distribution of the normalized K_I along the crack front in a rotating FGM cylinder with different values of material gradation for constant $\eta = \xi = 0.4$, $\nu = 0.30$ and diameter ratio (RD) of 1.5

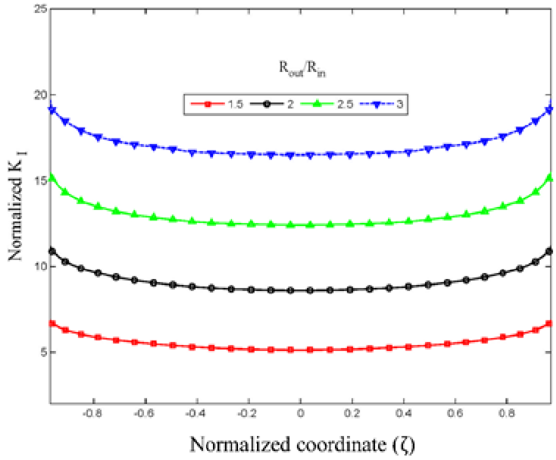

Fig. 7a

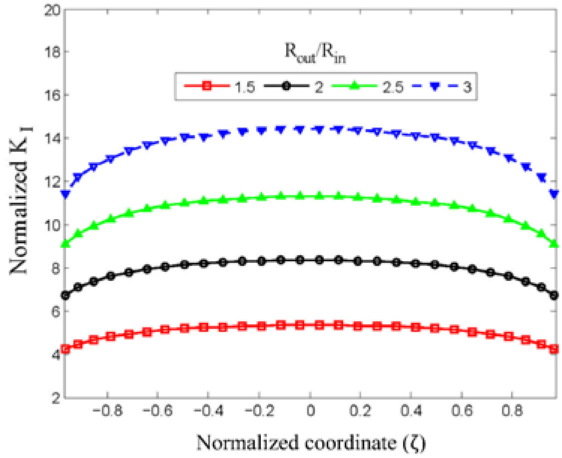

Fig. 7b

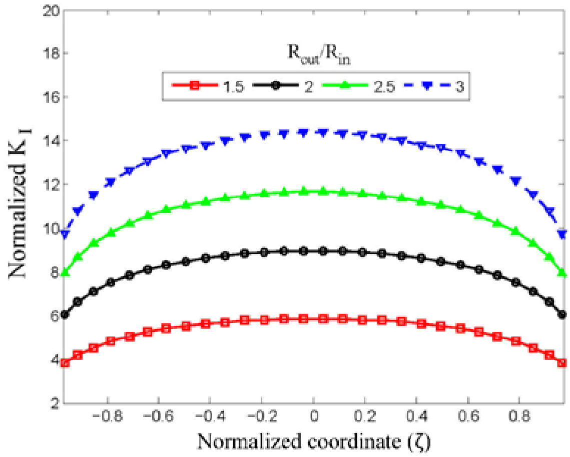

Fig. 7c

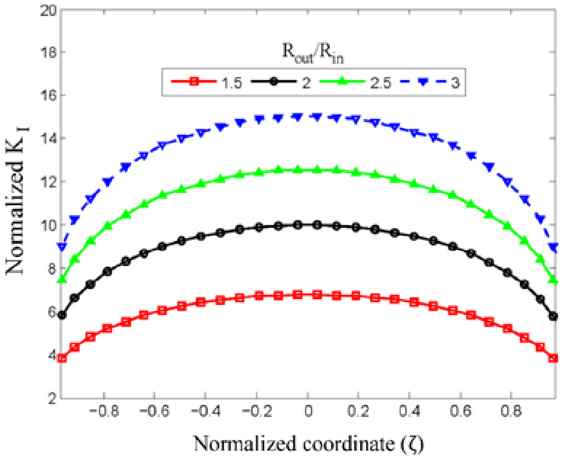

Fig. 7d

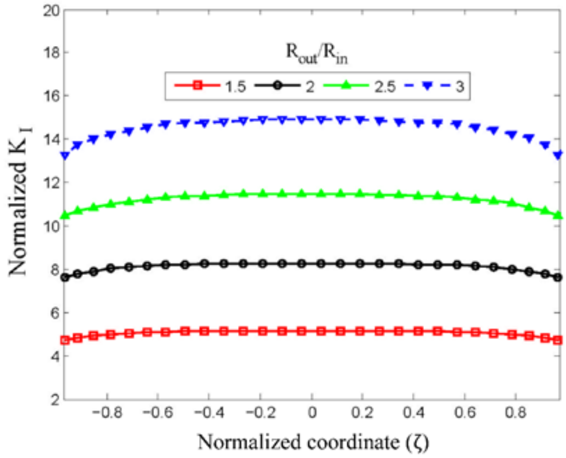

Fig. 7e

Figure 7. Distribution of the normalized K_I along the crack front in a rotating FGM and homogeneous cylinder for different values of diameter ratios (RD) with $\eta = \xi = 0.4$, $\nu = 0.3$ and different gradation of materials; a) RE=0.2; b) RE=2; c) RE=5; d) RE=10 e) RE=1(homogeneous material).

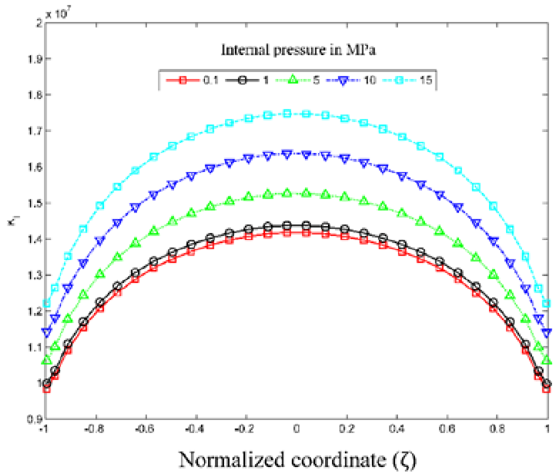

Figure 8. Variation of the K_I along the crack front for different internal pressure in a rotating FGM cylinder with $\eta = \xi = 0.4$, RE=2, RD=2.

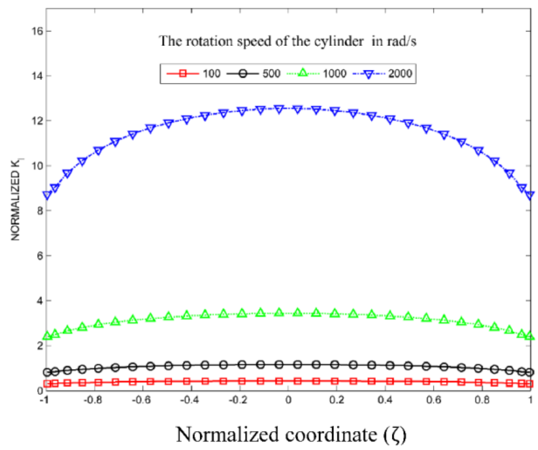

Figure 9. Variation of the normalized K_I along the crack front in a rotating FGM cylinder for different rotational speed with $\eta = \xi = 0.4$, RE=2, RD=2.

4. Summary and Conclusions

In this study, three dimensional finite element analyses of a rotating FGM and homogeneous cylinder containing a semi longitudinal elliptical surface crack under internal pressure is carried out. At first, only the effect of exponentially varying elastic modulus through the thickness is studied. Then the effect of diameter ratio on K_I is also considered.

In both cases, the distribution of SIF's along the crack front is studied and the effect of aspect ratio of crack (a/c) is investigated. It can be seen that the distribution of the K_I along the crack front is symmetric on both sides of the deepest point, and the stress intensity factors at the corner points are the same.

Depends on the material gradation, diameter ratio and the crack geometry, the values of the SIF at the corners or the deepest point are the critical ones.

5. Acknowledgments

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