Jordan Journal of Mechanical and Industrial Engineering

Accurate Modeling and Numerical Control Machining for Spiral Rotor of Double Rotor Flowmeter

Chunyan Tian, Hai Jiang^{*}, Chaochao Chen

School of Advanced Manufacturing Engineering, Hefei University, Hefei 230601, China

Received July 18 2020

Accepted Feb 20 2021

Abstract

Aiming at the design and manufacturing problems of the spiral rotor of dual-rotor flowmeter, the profile characteristics and forming principles of the spiral rotor are analyzed, and the function equation of its cross-section profile curve is derived. Based on the analysis, a mathematical model for machining screw rotor with spherical milling cutter is established and the accurate three-dimensional modeling of spiral rotor parts is realized by using UG software. The simulation of the two rotors proves that the modeling method is reasonable and effective. Then using automatic programming and manual programming methods, a multi-axis NC machining program for the parts is written. The macro variable is introduced for the main program to call many times, which greatly simplifies the program structure, reduces the amount of program, and is easy to modify. The NC machining experiment of the spiral rotor with a standard ball-end milling cutter on a four-axis milling machining center is conducted. The results show that the screw rotor machined by the mathematical model has good quality. This programming and manufacturing method is very suitable for processing spiral rotors in multi-variety small batch production, and thus a high machining accuracy can be achieved.

© 2021 Jordan Journal of Mechanical and Industrial Engineering. All rights reserved

Keywords: spiral rotor; functional equation; digital modeling; NC programming; multi-axis simultaneous machining;

1. Introduction

The dual-rotor flowmeter is an advanced volumetric flowmeter with the advantages of accurate measurement, no pulsation, low noise, and strong viscosity adaptability ^[1, 2]. The key functional part of the flowmeter is a pair of meshing spiral rotors. The spiral rotor is a special helical surface gear with a complex end profile, and its structural parameters directly affect the working performance of the flowmeter ^{[3,} ^{4]}. Based on the gearing theory, many researches relate to the geometry design, machining methods and machining tools of a screw rotor have been presented in references [5-^{7]}. Su et al. proposed the contact lines on rotor surfaces as design indices for rotor profile optimization ^[8]. Zhihuang Shen et al. put forward a digital graphic scanning (DGS) method based on computer graphics to generate the grinding profile of the rotor or the forming tool ^[9]. But these rotor profile design methods still have their technical limitations and the further research on geometric profile generation of the rotor should be carried out. Manufacturing of screw rotors is a complex process that can be achieved by using a special helical grinding machines and form tools [10-13]. Wu and Hsu proposed a general mathematical model to continuously machine screw rotors using worm-shaped tools ^[14]. However, such approaches are very expensive because the machine and the tools are designed specifically for the rotors. So, in general it is used for small varieties and large manufacturing batches.

Due to the reasons such as actual production conditions and manufacturing costs, forming tools are generally not used to process spiral rotors in multi-variety small batch production in China. Unlike the form cutter which is necessary to redesign and manufacture for new type parts, a standard ball-end milling cutter can manufacture different types of screw rotors. Therefor the standard ball-end milling cutter is a great substitute for CNC milling ^[15-17]. So, cooperating with the related companies, we carry out the research on the digital accurate modeling of spiral rotors and multi-axis NC machining programming methods.

2. Working principle and contour analysis of spiral rotor

The working principle of the dual-rotor flowmeter is shown in Figure 1. The two helical rotors mesh with each other without active or passive distinction, which have the same tooth shape parameters and structural dimensions as well as opposite rotation directions. Under the action of fluid pressure, they rotate smoothly and uniformly in the direction shown by the arrow, and the fluid flows from the space between the cogging and the housing to the right pipe at the same time. The fluid flow rate is proportional to the number of rotor revolutions. To make the two rotors rotate smoothly without interference, the working tooth surfaces of the two rotors should be a pair of conjugate surfaces which can meet the basic law of tooth profile meshing. In order to meet the sealing requirements between the two rotors of the volumetric flowmeter, the teeth of the two

^{*} Corresponding author e-mail: 2814884634@qq.com.

rotors should be meshed with no backlash and no root clearance ^[18-21].



Figure 1. Working principle of the dual-rotor flowmeter

There are many tooth shapes that can meet the above requirements according to the principle of meshing. Considering such factors as ease of manufacture and maximum volume space of the flowmeter, a composite tooth profile composed of involute curve and transition curve is designed and analyzed. The cross-sectional shape of the tooth profile is composed of a tooth top arc, involute curve, transition curve, and tooth root arc, as shown in Fig. 2. So, the spiral rotor can be described as a helical gear with a large modulus and a small number of teeth. In order to meet the sealing requirements of the two rotors during meshing rotation, the double-sided tooth profile, tooth top circle and root circle of the rotor must participate in the meshing which is different from general helical gears. Due to the small number of teeth (e.g. only 4 teeth), the undercut phenomenon of the gear teeth will occur during the meshing transmission and the transition curve in the profile is formed. The spiral surface of the rotor is formed by the spiral movement of the end face tooth because of the plane meshing of the two rotors. Therefore, the core problem to be solved in the design and manufacture of the rotor is to accurately obtain the cross-sectional profile curve.



Figure 2. Cross-sectional profile of spiral rotor

3. Digital accurate modeling of spiral rotor

According to the above analysis of the spiral rotor contour, a function equation expression can be established for each curve segment to obtain the profile curve of the spiral rotor section. The rectangular coordinate parameter equation is used for UG software modeling.

3.1. Establishment of involute equation of sectional profile

In the spiral rotor cross-sectional profile, the section adjacent to the tooth top arc is an involute curve. As shown in Fig. 3, the involute is a locus EPS formed by any point P on a straight line CP when the straight line makes a pure rolling along a base circle O with a radius r_b .



Figure 3. Establishment of involute equation of sectional profile

In the figure, θ is the spread angle of the involute and t is the angle between the occurrence line and the y-axis. When the involute starts from the base circle and the X-axis intersection E, the values of t and θ both increase from 0. It can be seen from the figure that the X coordinate of any point P on the involute EPS is as follows:

$$x = OA + BP = r_b \cos t + CP \sin t \tag{1}$$

Because the occurrence line cp makes a pure rolling along the basis circle o, the line CP is equal to the arc segment CE, that is:

$$CP = r_b \times t$$

Substituting the value of line CP into equation (1), the follow equation (2) can be obtained.

$$x = r_b \cos t + r_b t \sin t \tag{2}$$

Similarly, the y coordinate of any point P on the involute line is as follows:

$$y = AC - BC = r_b \sin t - CP \cos t = r_b \sin t - r_b t \cos t \quad (3)$$

Therefore, the rectangular coordinate parameter equation of the involute EPS is as follows:

$$\begin{cases} x = r_b(\cos t + t\sin t) \\ y = r_b(\sin t - t\cos t) \end{cases}$$
(4)

Correspondingly, the rectangular coordinate parameter equation of the involute EP_1S_1 in another direction symmetrical to the y-axis is:

$$\begin{cases} x = r_b(\cos t + t\sin t) \\ y = r_b(t\cos t - \sin t) \end{cases}$$
(5)

The parameter of the above curve function equation is t, $t \in [t_1, t_2]$

3.2. Establishment of the cross-sectional profile transition curve equation

In the cross-sectional profile of the spiral rotor, the section adjacent to the tooth root circle is a transition curve, which is formed by undercutting of the gear teeth when the helical gear with a small number of teeth meshes. As shown in Figure 2 above, according to the principle of processing conjugate tooth profiles using Generating method, two pitch

circles of equal diameter on the rotor are tangent and roll without sliding when the two rotors are engaged for transmission. It can be regarded that one rotor is fixed, and the pitch circle of the other rotor makes pure rolling along the pitch circle of the fixed rotor. Since both the rotor tooth top circle and the tooth root circle participate in meshing in this process, the trajectory generated by rolling the tooth tip edge point B of the rotor tooth is the transition curve formed by undercutting of the two rotors.



Figure 4. Establishment of the cross-sectional profile transition curve equation

The coordinate system is now established with the fixed rotor section center O as the origin, as shown in Fig. 4. The initial tangent point of the two rotor pitch circles on the rolling rotor is point P. Line AB is the connection between the center A of the rolling rotor and the edge point B of the tooth top, and line AO is the connection of the two-rotor center. The initial angle between line AB and line AO is δ which is half of the circumferential angle of the tooth top circle tooth thickness corresponding to the circumferential angle. When the rolling rotor pitch circle rotates through the angle t by pure rolling along the fixed rotor pitch circle, the original rolling rotor center point reaches point C, the original tangent point P reaches point K, the original tooth tip edge point B reaches point D, and the tangent point of the two rotor pitch circles becomes point N.

Because the two rotors have the same section shape, the base circle radius of both rotors can be set to r_b , the radius of the tip circle of both rotors is r_a , and the pitch radius of the two rotors is the average radius of the dividing circle r, therefor line AB and line CD are both equal to r_a . It can be seen from Figure 5 that arc segment NK is equal to arc segment NP, so angle NCK and angle NOP are both t. Correspondingly:

$$\theta = -(\pi - t - t - \delta) = -(\pi - (2t + \delta)) \tag{6}$$

Let the coordinates of the edge point D of the rolling rotor tooth tip be (x, y), and the geometric vector relationship in the figure is shown as follows:

$$OD = OC + CD \tag{7}$$

$$\overrightarrow{OD} = \left\{ x, y \right\} \tag{8}$$

$$\overrightarrow{OC} = \left\{ 2r\cos t, \quad 2r\sin t \right\} \tag{9}$$

$$\overline{CD} = \{r_a \cos\theta, r_a \sin\theta\}$$
(10)

Substituting equation (6) into equation (10), the equation (11) can be obtained:

$$CD = \left\{-r_a \cos(2t+\delta), -r_a \sin(2t+\delta)\right\}$$
(11)

Substituting equations (8), (9) and (11) into equation (7), the follow equations can be obtained:

$$\{x, y\} = \{2r\cos t - r_a\cos(2t+\delta), 2r\sin t - r_a\sin(2t+\delta)\}$$
(12)
$$\begin{cases} x = 2r\cos t - r_a\cos(2t+\delta) \\ y = 2r\sin t - r_a\sin(2t+\delta) \end{cases}$$
(13)

$$t \in [t_1, t_2]$$

The equation (13) is the rectangular coordinate parameter equation of the spiral rotor section profile transition curve, and the parameter of the function equation is t.

In addition, during the pure rolling process of the two rotor pitch circles, the tooth top arc segment of the rolling rotor forms the tooth root arc segment of the other rotor, which is tangent to the transition curve segment. At this point, the form of each curve segment of the rotor section profile has been determined.

3.3. Examples of spiral rotor modeling

Given a pair of intermeshing spiral rotors, the known basic parameters are as follows:

Table 1. Basic parameters of spiral rotors

number of rotor teeth	end modulus	normal pressure angle (°)	center distance between two rotors	lead	Length of rotor
4	13.5	20	54	269	134.5

According to the above basic parameters, it is easy to obtain other parameters required for modeling and designing the spiral rotor, such as the indexing circle radius r, the base circle radius r_b, the tooth tip circle radius r_a, the tooth root circle radius r_r, the tooth tip circle tooth thickness S_a , and the tooth tip circle tooth thickness corresponding to the semicircle angle δ . Then, follow the basic steps to complete modeling by using UG software.

1. The parametric equation (5) of the rotor cross-section profile involute and the parametric equation (13) of transition curve are both input through the "Expression" function of UG software, and an appropriate parameter value range is selected. Then high-precision function equation curves are generated by using the "law Curve" function. The corresponding tooth top circle, tooth root circle and necessary auxiliary lines are drawn; then a single tooth profile curve is obtained by trimming and rotating to a suitable position of the corresponding curve and arc segments etc.

18

- 3. And the complete rotor cross-section profile is obtained through the operations of arraying and trimming.
- 4. On this basis, a corresponding space spiral is constructed as a guide line, and a three-dimensional digital model of a complete spiral rotor is generated after the sweep operation, as shown in Fig. 5.

In the same way, another spiral rotor with opposite rotation can be modeled.





In order to verify the rationality of the above-mentioned modeling design method and results, the helical rotor digital model is simulated through UG software, as shown in Fig. 6. Using the interference analysis function of UG software, it can be confirmed firstly that there is no interference between the working tooth surfaces that mesh with each other during the movement of the dual rotors. Then the rotating effects of the dual rotors under the pressure difference are simulated. Simulation results show that the meshing operation of the two rotors can be performed tightly and smoothly without interference.



Figure 6. Meshing movement simulation of spiral rotor

4. CNC machining of spiral rotor part

4.1. CNC machining principle

The final finish machining of the spiral rotor is processed by using a standard ball-end milling cutter in a four-axis milling machining center according to its structural characteristics. Before finish machining, the blank of rotor has to go through the processes such as rough turning, finish turning and rough milling, so as to get the semi-finished product with finishing allowance which is approximately consistent with the final contour shape. Then the semifinished product is processed by NC finish milling, and finally the four grooves at the end face of the rotor are milled to complete the part. The basic principle of NC finish milling of the semi-finished product is shown in Fig. 7, in which the rotary table on the machine tool drives the workpiece to rotate around the X axis to realize the A axis movement.



Figure 7. NC milling principle by using a standard ball-end milling cutter

During machining, the workpiece is linearly fed at a constant speed relative to the tool along the X axis, rotating at an average speed along the A axis, and the corresponding proportional relationship between the linear movement distance and the rotation angle is maintained. In this way, the tool will process a space spiral. Then the workpiece quickly reset, and the tool quickly moves to the starting point of the next spiral through linkage in the YOZ plane to processes the next spiral in turn until the milling of the entire spiral surface is completed. The milling technology parameters selected in NC finish milling are shown in Table 2:

Table 2. Milling machining technology parameters

Cutter Diameter (mm)	Cutter teeth	Feed rate (mm/min)	Spindle speed (rpm)	Maximum allowable roughness Ra (μm)
10	3	300	1200	3.2

4.2. Compiling of CNC machining programs

The essence of the NC program generated by computeraided automatic programming is using a series of small straight-line segments to approximate the spatial profile along each processing path. And there is a certain difference from the formation mechanism of the rotor spiral surface which is not conducive to accuracy improvement. Therefore, manual programming combined with automatic programming is used to compile CNC programming. First, according to the previously generated digital model of the spiral rotor, tool point data of the tool at the starting spiral point when milling each spiral are calculated by using the CAM module of UG software. Then according to the abovementioned principle of X and A axis linkage processing of spirals, manual programming is used to introduce macro variables. The tool point data at the starting cutting point of the spiral is used as values of macro variables to write the subroutine processing of the space spiral which can be called multiple times by the main program. This greatly

simplifies the program structure, reduces the program volume, and is easily modified.

The planning of the machining path during programming can be based on the shape of the cogging and follows the principle of symmetry from the outside to the inside as much as possible. That is, the outermost profile spiral of the left and right sides of a cogging of the rotor is processed first separately, then the tool feeds a cutting row width along the Y and Z directions and processes the adjacent left and right profile spirals in sequence until a cogging cutting is completed. Then the same method is used to process the next cogging after rotating axis A for 90 degrees. On this basis, the corresponding CNC machining program is obtained. According to the program, the simulation operation is carried out by Yulong's multi-axis CNC machining simulation software. Then, the actual cutting experiment is carried out on the 4-axis machining center equipped with FANUC series 0i-MC CNC system. It takes about 180 minutes to complete the CNC finish milling of a rotor part. The accuracy and surface roughness of the finial part obtained can meet the requirements. The machined part is shown in Fig. 8. The machined crosssectional profile of the rotor is consistent with the designed one, as shown in Fig. 9. Experiments show that the programming method is simple, efficient and effective.



Figure 8. Machined part



Figure 9. machined cross-sectional profile of the rotor in comparison with the designed one.

5. Conclusions

In this paper, the principle of forming the profile of the spiral rotor part is analyzed. The function equation of its cross-sectional profile curve is derived based on gear meshing theory and the three-dimensional accurate numerical model is constructed by using UG software. The automatic programming combined with the manual programming is used to write NC machining program for spiral rotors. A machining method with standard ball-end milling cuter is used in NC machining according to the characteristic of the helical surface, and the results show that the method fully meets the accuracy requirements of screw rotor. The simulation and actual numerical control machining experiments proves the correctness and efficiency of the mathematical model. The method provided in this paper can better solve the technical problems of processing the spiral rotor with standard tools under the existing production conditions, and has a certain application prospect.

Compared with the previous rotor profile design and machining methods, the method provided in this paper has the following advantages:

- 1. A three-dimensional accurate numerical model of new type rotor can be conveniently generated by changing the basic parameters of the rotor in the software using the parametric model constructed in this paper.
- The macro variable is introduced for the main program to call many times, which greatly simplifies the program structure, reduces the amount of program, and is easy to modify.
- The multi-variety small batch screw rotors can be manufactured with low cost by using standard milling cutters whose manufacturability and exchangeability are better.
- According to the surface measurement results of the trial product, the machining technology parameters can be easily adjusted through the cutter length and radius compensation.

But the cutting efficiency of this method is lower than that of other methods such as grinding or machining with forming cutter, which is its technical limitation.

6. Acknowledgement

This work was financially supported by the natural science foundation project and quality engineering projects provided by Anhui provincial education department, and the project numbers are KJ2017A359, 2018sjjd030 and 2019hfjxtd08.

Appendix

```
The part code of NC machining program for spiral rotor
parts is shown below:
  O5002
  G0G54Z100G90
  X-5.Y0
  M3S3500
  G10L2P1X-396.Y-236.05Z-365.98
  N1
  G10L2P1A0
  #101=24.633
  #102=37.234
  M98P502
  \#101 = 24.633
  #102=36.634
  M98P502
  \#101{=}24.633
  \#102{=}36.034
  M98P502
  \#101{=}24.633
  #102=34.943
  M98P502
  \#101 = 23.650
  #102=34.906
  M98P502
  \#101 = 23.022
  #102=34.869
  M98P502
```

.....

.....

References

- Gu KJ. Application test of dual-rotor flowmeter in the measurement of crude oil transportation. Petrochemical Automation, 2008, 2: 59-61.
- [2] Gao S, Wang L, Feng W, Chen YG, Xu L, Gong HW, Yang YW. Application analysis and discussion of double spiral flowmeter in crude oil measurement. Petroleum and Petrochemical Energy Saving, 2016, 6(07): 38-39.
- [3] Yu G, Li F, Yu YM, Zai SX. Active profiling and polishing model and validation based on rotor spiral groove. Chinese Journal of Mechanical Engineering, 2018, 31(28): 1-10.
- [4] Hayamizu Y, Yamamoto K, Yanase S, Hyakutake T, Morita S, Ohtsuka S. Experimental study of the viscous pump with a helical channel rotor: Torsion effect of the channel. Journal of Thermal Science, 2010, 19(2): 154–159.
- [5] Stosic N. A geometric approach to calculating tool wear in screw rotor machining. International Journal of Machine Tools & Manufacture, 2006, 46: 1961–1965.
- [6] Zheng F, Hua L, Han X, Li B, Chen D. Linkage model and manufacturing process of shaping non-circular gears. Mechanism & Machine Theory, 2016, 96: 192–212.
- [7] Wang J, Cui F, Wei SH, Sha RD, Liu HJ. Study on a novel screw rotor with variable cross- section profiles for twin-screw vacuum pumps. Vacuum, 2017, 145: 299–307.
- [8] Su SH, Tseng CH. Synthesis and optimization for rotor profiles in twin rotor screw compressors. Journal of Mechanical Design, 2000, 122(4): 543–552.
- [9] Shen ZH, Yao B, Teng WB, Feng W, Sun WF. Generating Grinding Profile between Screw Rotor and Forming Tool by Digital Graphic Scanning (DGS) Method. International Journal of Precision Engineering and Manufacturing, 2016, 17(1): 35-41.
- [10] Zhang ZX, Fong ZH. A novel tilt form grinding method for the rotor of dry vacuum pump. Mechanism and Machine Theory, 2015, 90: 47–58.
- [11] Wang A, Liang J, Liu Y. Application of improved particle swarm optimization in gear fault diagnosis of automobile

transmission. Jordan Journal of Mechanical and Industrial Engineering, 2020, 14(1): 175-182.

- [12] Shakoor M, Qureshi MR, Jaber S. Applying management principles of lean manufacturing for enhancing efficiency and effectiveness of emergency department rooms. Jordan Journal of Mechanical and Industrial Engineering, 2019, 13(4): 291-299.
- [13] Wei J, Zhang GH. A precision grinding method for screw rotors using CBN grinding wheel. The International Journal of Advanced Manufacturing Technology, 2010, 48: 495–503.
- [14] Wu YR, Hsu WH. A general mathematical model for continuous generating machining of screw rotors with wormshaped tools. Applied Mathematical Modelling, 2014, 38: 28– 37.
- [15] Stosic N, Smith IK, Kovacevic A, Mujic E. Geometry of screw compressor rotors and their tools. Applied Physics & Engineering, 2011, 12(4): 310-326.
- [16] Stosic N. A geometric approach to calculating tool wear in screw rotor machining. International Journal of Machine Tools & Manufacture, 2006, 46: 1961–1965.
- [17] Wei J, Zhang GH. A precision grinding method for screw rotors using CBN grinding wheel. The International Journal of Advanced Manufacturing Technology, 2010, 48: 495–503.
- [18] He X, Pan C, Wu M, Ji X, Zhang R. A twin-screw rotor profile design and computational fluid dynamic simulation method. Materials Research Innovations, 2015, 19(8): 721-726.
- [19] Liu ZM, Tang Q, Liu N, Song J. A profile error compensation method in precision grinding of screw rotors. The International Journal of Advanced Manufacturing Technology, 2019, 100: 2557-2567.
- [20] Ilie K, Subic A. Parametric modelling of helical rotors for efficient design of twin-screw superchargers. Mechanical Engineering Science, 2007, 221: 267-272.
- [21] Yu DY, Ding Z. Geometric characteristics analysis and parametric modeling for screw rotor precision machining. The International Journal of Advanced Manufacturing Technology, 2020, 107: 3615–3623.