A Statistical Analysis of Wind Power Density Based on the Weibull and Ralyeigh models of "Penjwen Region" Sulaimani/ Iraq

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Abstract

In the present study wind power density of the Penjwen region have been statistically analyzed during the period from January 2001 to December 2003 based on the monthly measured mean wind speed data. The annual and monthly wind speeds and wind density are estimated. The Weibull and Ralyeigh distribution functions have been derived from the available data and both Weibull and Rayleigh probability density functions are fitted to the measured probability distributions on yearly basis, it was shown that the Weibull distribution is fitting the measured monthly probability density distributions better than the Rayleigh distribution for the whole years. The wind power density of this region has been studied based on the Weibull function. Weibull distribution shows a good approximation for estimation of power density.

Keywords: Mean wind speed; Weibull and Rayleigh distribution functions; Wind power density

1. Introduction

Renewable energy sources, wind, solar, geothermal, hydro, biomass and ocean thermal energy have attracted increasing attention from all over the world due to their almost inexhaustible and non-polluting characteristics. Wind energy as one of these important sources is perhaps the most suitable, most effective and inexpensive sources for electricity production as a result, it is vigorously pursued in many countries [1].

Iraq in general and northern region especially has to make use of its renewable resources, such as solar, wind and geothermal energy, not only to meet increasing demand, but also for environmental reasons.

Penjwen region located in the eastern of Sulaimani at 35°37´ N and 45°56´ E and its height above sea level is 1320 meter.

The aim of the present study is to analyze wind speed at Penjwen region due to the important of statistical analysis of wind data to predict the power density in this area and the areas around it.

2. Theoretical Analysis and Formulation

2.1. Vertical extrapolation of wind speed:

Wind speed near the ground changes with height, this requires an equation that predicts wind speed at any height in terms of the measured speed at another height. The most common expression for the variation of wind speed with height is the power law having the following form [2].

$$\frac{v_2}{v_1} = \left(\frac{h_2}{h_1}\right)^{\alpha} \qquad (2\text{-}1)$$

Where v_1 (m/sec) is the actual wind speed recorded at height h_1 (m), and v_2 (m/sec) is the wind speed at the required or extrapolated height h_2 (m). The exponent α depends on the surface roughness and atmospheric stability numerically it lies in the range (0.05 to 0.5).

2.2. Wind speed probability distribution:

The wind speed data in time series format is usually arranged in the frequency distribution format since it is more convenient for statistical analysis, therefore the available time-series data were translated into frequency distribution format [3].

Two of the commonly used functions for fitting a measured wind speed probability distribution in a given location over a certain period of time are the Weibull and Rayleigh distributions. The probability density function of the Weibull of wind speed being v, $f_w(v)$ during any time interval is given, as following [2, 3].

$$f_w(v) = \left(\frac{k}{\alpha}\right)\left(\frac{v}{\alpha}\right)^{k-1} e^{-\left(\frac{v}{\alpha}\right)^k} \qquad (2\text{-}2)$$

Where (m/s) is the Weibull scaling parameter and is the dimensionless Weibull parameter. The shape and scale parameters can be estimated by using the Maximum Likelihood Method (MLH) as [4,5].

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$$k = \left(\frac{\sum_{i=1}^{n} v_i^{\,k} \ln(v_i)}{\sum_{i=1}^{n} v_i^{\,k}} - \frac{\sum \ln(v_i)}{n} \right)^{-1} \tag{2-3}$$

$$a = \left(\frac{1}{2} \sum_{i=1}^{n} v_i^{\,k} \right)^{1/2} \tag{2-4}$$

Where v_i is the wind speed in time stage i and n is the number of non-zero wind data points.

The Ralyeigh $f_R(v)$ distribution is a special case of the Weibull distribution in which the shape parameter k is assumed to be equal to 2. From Equation (2-2) the probability density functions of the Ralyeigh distribution given by [3].

$$f_R(v) = \frac{2v}{a^2} e^{-\left(\frac{v}{a} \right)^2} \tag{2-5}$$

2.3. Wind power density function:

The evaluation of the wind power per unit area is of fundamental importance in assessing wind power projects, it is well known that the power of the wind at speed V through the blade sweep area A increases as the cube of its velocity and is given by [2,6].

$$P_v = \frac{1}{2} \rho A v^3 \tag{2-6}$$

Where ρ (kg/m^3) is the mean air density, the value 1.069 kg/m^3 is used in this work [7]. This depends on altitude, air pressure and temperature.

The expected monthly or annual wind power density per unit area of a site based on a Weibull probability density function can be expressed as follows[8].

$$P_w = \frac{1}{2} \rho a^3 \Gamma \left(1 + \frac{3}{k} \right) \tag{2-7}$$

Where Γ is the gamma function and a is the Weibull scale parameter (m/s) given by:

$$P = \frac{V_m}{\Gamma \left(1 + \frac{1}{k} \right)} \tag{2-8}$$

The two significant parameters k, a are closely related to the mean value of the wind speed V_m.

By extracting a from Equation (2-8) and setting k equal to 2, the power density for the Rayleigh model is found to be [9].

$$P_R = \frac{3}{\pi} \rho V_m \tag{2-9}$$

Where

$$V_m = a\Gamma \left(1 + \frac{1}{2} \right) \tag{2-10}$$

The errors in calculating the power densities using the distribution models (Weibull and Rayleigh) in comparison to values of the Probability density distributions derived from measured values can be found using the following formula [2,3].

$$Error\% = \frac{P_{w,R} - P_{m,R}}{P_{m,R}} \tag{2-11}$$

Where $P_{W,R}$ (w/m^2) is the mean power density calculated from either the Weibull or Rayleigh function used in the calculation of the error, and $P_{m,R}$ is the wind power density for the probability density distribution, derived from measured values which serves as the reference mean power density.

3. Results and Discussion

Data for wind speed in the present calculation were obtained during the period 2001 to 2003 taken from the meteorological directorate center of Sulaimani. The main results obtained from the present study can be summarized as follows.

The monthly mean wind values estimated from the available data for the overall and individual three years are presented in Table (1). It is seen in Table (1) that the highest wind speeds 7.65 m/s occurs in April and June in year 2002. While lowest wind speed 4.88 m/s occur in August in year 2003. The variation of wind speeds often described using the Weibull two-parameter density function. This is statistical method which widely accepted for evaluation local wind local probabilities and considered as a standard approach.

Maximum Likelihood Method was used to calculate bothWeibull's parameters, scale and shape, as shown in Table (1), it is seen from the Table that, while the scale factor varies between 6.19 to 9.73 m/s, the shape factor ranges from 8.84 to 15.00 for location analyzed.

The annual probability density distributions obtained from the Weibull and Rayleigh models were compared to the measured distributions to study their suitability. The annual comparison shows that the Weibull model better than the Rayleigh model to fit the measured probability density distribution as shown in Figure (1), while Figure (2) shows the Weibull distribution of wind speeds all over the data for each year for the studied area.

Table 1: Monthly mean wind speeds, wind power density and the two Weibull parameters (shape and scale).

|	Mean Wind Speed (m/sec)			Wind Power Density (w/m²)		
	2001	2002	2003	Weibull	Rayleigh	measured
Jan	6.43	7.13	5.91	6.49	6.51	6.30
Feb	7.13	6.26	6.26	6.55	6.36	6.39
Mar	7.13	6.61	5.74	6.49	6.28	6.17
Apr	5.56	7.65	5.74	6.32	6.57	6.21
May	7.13	6.61	6.43	6.72	6.59	6.58
Jun	6.26	7.65	6.09	6.67	6.80	6.52
Jul	6.43	6.09	5.39	5.97	5.82	5.73
Aug	7.13	6.61	4.88	6.21	5.90	5.66
Sep	7.13	6.43	5.39	6.32	6.05	5.92
Oct	5.56	5.22	6.09	5.62	5.64	5.79
Nov	7.13	5.22	7.13	6.49	6.28	6.63
Dec	6.26	5.22	6.26	5.91	5.80	5.99
Shape	15	8.84	11.01			
Scale	6.86	6.75	6.19			
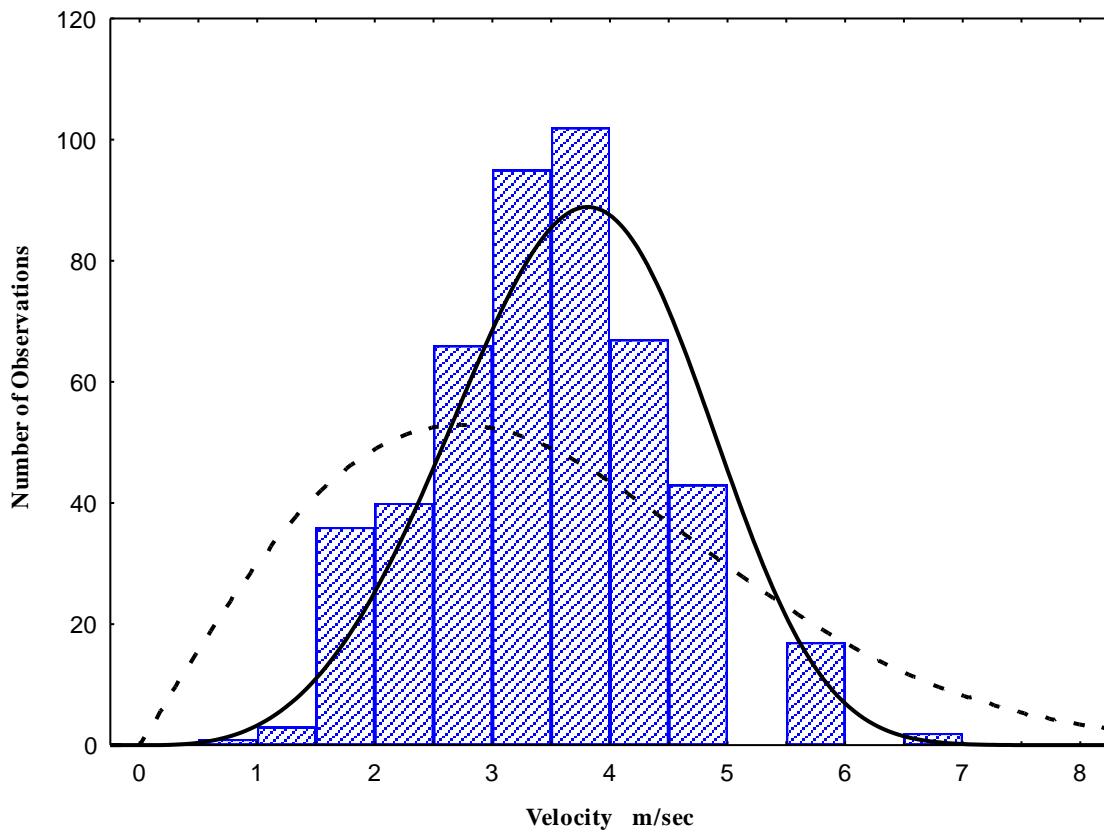
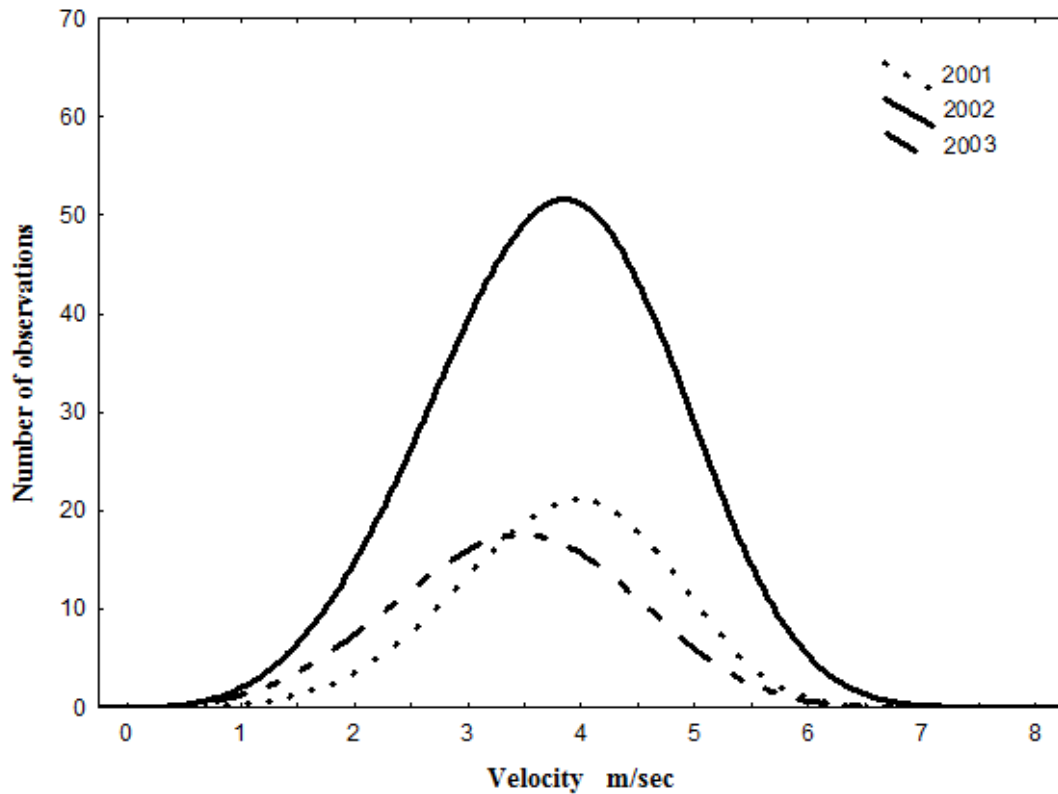Figure 1: Weibull and Rayleigh comparisons of the actual probability distribution of the wind.

Figure 2: Yearly Weibull probability density distribution of each year for the period of (2001- 2003).

The power densities calculated from the measured probability density distribution and those obtained from the models are shown in Figure (3). The power density shows a large month to month variation, the minimum power densities occur in the February and September (2003) with 14.55 and 14.45 w/m^2 respectively, it is interesting to note that the highest power density value occur in the Spring and Summer months of March, May (2002) and May, July (2001) with the maximum value of (53.49, 53.12) and (54.87, 50.83) w/m^2 respectively. The power densities in the remaining moths are between these two groups of low and high as shown in Figure (4).
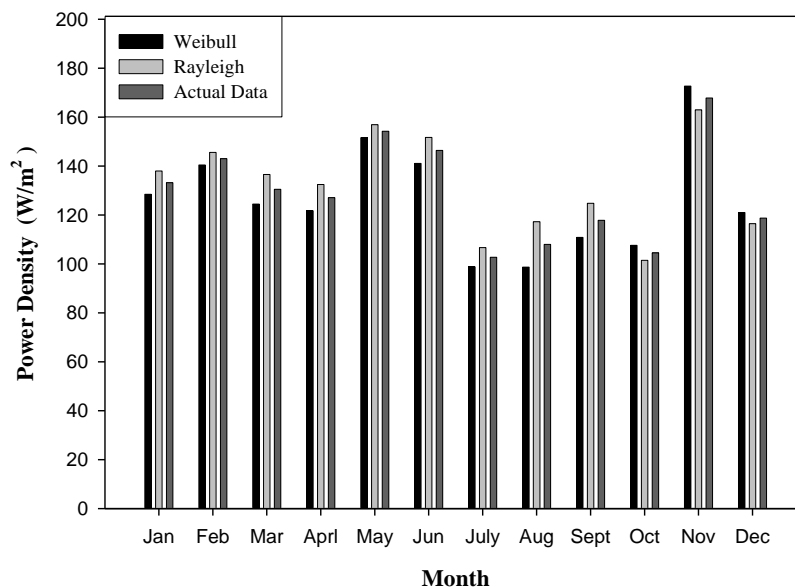

Figure 3: Wind power density obtained from the actual data versus those obtained from the Weibull and Rayleigh models on a monthly basis.
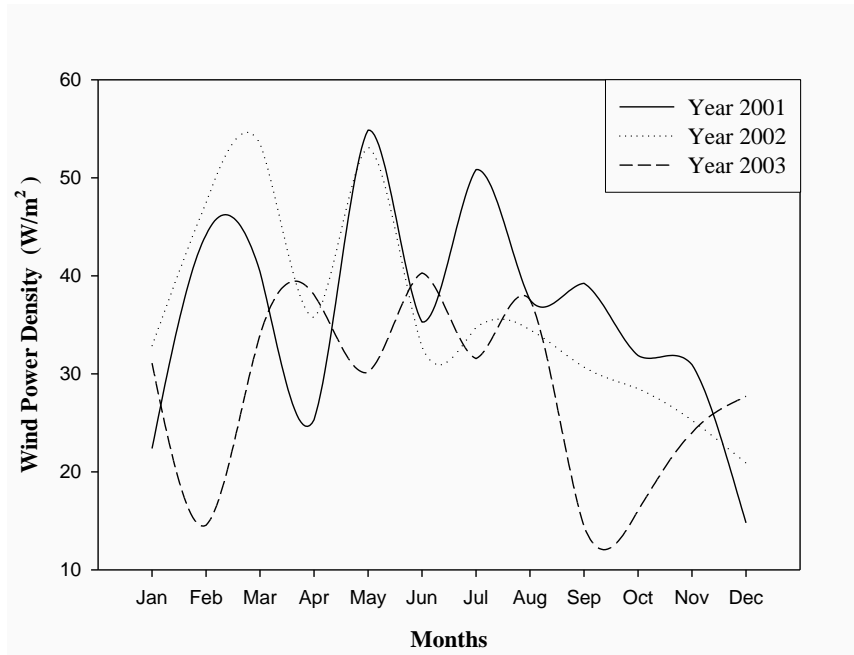
Figure 4: Monthly Weibull probability density distribution for the period of 2001- 2003 for the studied area.

Errors in calculating the power densities using the distributions (Weibull and Rayleigh models) in comparison to those using the measured probability density distributions are presented in Figure (5). The highest error values occur in August and September with 9.27% and 6.96% for the Weibull model respectively, the power density as estimated by the Weibull model has a very small error value 2.59% in February. The monthly analysis shows that the highest error value using the Rayleigh model occur in November with 4.86%, whereas the smallest error in the power density calculation using Raleigh model is 2.28% in December.
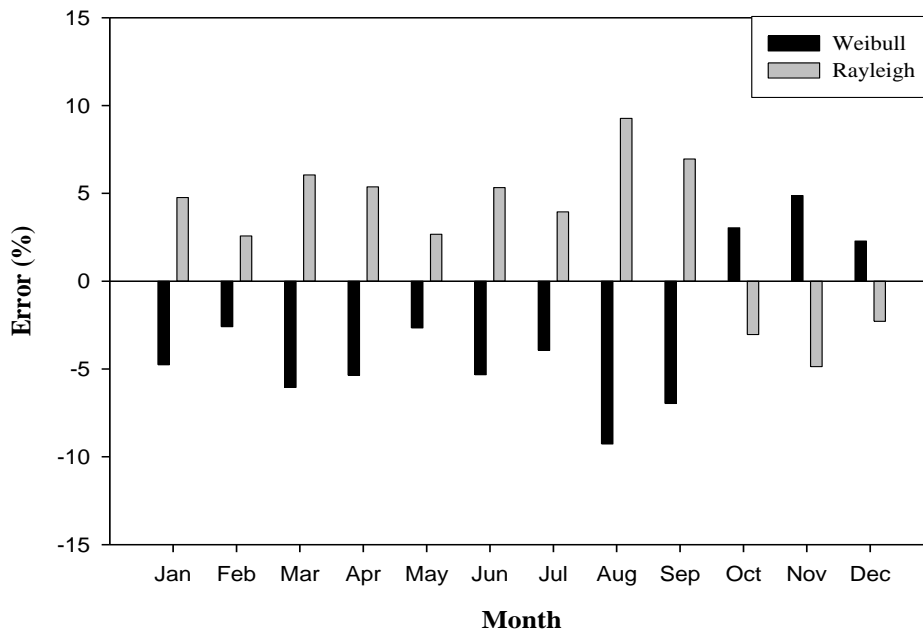

Figure 5: Error values in calculating the wind power density on monthly basis obtained from the Weibull and Rayleigh models in reference to the wind power density obtained from the measured data.

4. Conclusions

Wind characteristics of Penjwen have been analyzed statistically, wind speed data were collected for a period of three years (2001-2003). The probability density distributions and power density distributions were derived from the time series data. Two probability density functions have been fitted to the measured probability distributions on a monthly basis, based on the Weibull and

Rayleigh models. The most important outcomes of the study can be summarized as follows:

- The Weibull distribution is fitting the measured monthly probability density distributions better than the Rayleigh distribution for the whole years.

- The Rayleigh distribution provides better power density estimations in nine months than the Weibull distribution.
- The yearly average error values in calculating the power densities using the distributions in comparison to those using the measured probability density distributions shows a large month to month variation due to the difference of the shape parameters.

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