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MHD Heat and Mass Transfer Free Convection Flow near The Lower Stagnation Point of an Isothermal Cylinder Imbedded in Porous Domain with the Presence of Radiation

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Abstract

Heat and mass transfer characteristics and the flow behavior on MHD flow near the lower stagnation point of a porous isothermal horizontal circular cylinder have been studied. The equations of conservation of mass, momentum, energy and concentration which govern the case study of heat and mass transfer flow have been obtained. These equations have been transformed into a system of non-dimensional coupled non-linear ordinary differential equations by using similarity transformations and finally solved by Runge-Kutta and shooting method. It has been assumed that the fluid is incompressible, absorbing-emitting radiation and viscous, with temperature dependent viscosity and temperature dependent thermal conductivity in the presence of radiation. Velocity profiles, temperature distributions and concentration distributions for the flow have been presented for various values of radiation parameter, viscosity variation parameter, thermal conductivity variation parameter, Prandtl number and Schmidt number. The skin friction factor, local Nusselt number and Sherwood number are also calculated for all the parameters involved in the problem. It has been observed that with the increase in Schmidt number skin friction and Nusselt number decrease, while Sherwood number increases.

Keywords: MHD heat mass transfer; free convection; isothermal circular cylinder; radiation effect; variable thermal conductivity and viscosity; Runge-Kutta shooting technique

Nomenclature

a	Curvature
b	Inertial drag coefficient
B_o	Magnetic intensity
C	Concentration
C_f	Skin friction
C_p	Specific heat at constant pressure
D	Mass diffusivity
f	Non-dimensional reduced stream function
Gm	Modified Grashoff number
Gr	Grashoff number
K	Porosity parameter
k	Thermal conductivity
k′	Permeability of porous media
M	Magnetic parameter
N	radiation parameter
Nf	Forchhiemer inertial porous parameter
Nu	Nusselt number
O	Stagnation point
Pr	Prandtl number
q_r	Radiative heat flux
q_w	Rate of heat transfer
Sc	Schmidt number
Sh	Sherwood number
So	Soret number
s_w	Rate of mass transfer
T	Temperature
u, v	Velocity components along X, Y directions
X, Y	distances along and perpendicular to the surface

Greek symbols

μ	Viscosity of the fluid
k_1	Mean absorption coefficient
β	Coefficient of thermal expansion
ε	Variable viscosity parameter
η	Dimensionless distance
θ	Non-dimensional temperature
ν	kinematic viscosity of the fluid
ρ	Density of the fluid
σ	Electrical conductivity
σ_1	Stefan Boltzmann constant
Φ	Non-dimensional concentration
ψ	Stream function
ω	Variable thermal conductivity parameter

Subscripts

w	wall of cylinder
∞	Distance far away from the surface

Superscript

′ Differentiation with respect to η

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1. Introduction

The study of flow problems which involve the interaction of several phenomena, has a wide range of applications in the field of Science and Technology. One such study is related to the effect of free convection MHD flow, which plays an important role in Agriculture, Engineering and Petroleum industries. The problem of free convection under the influence of magnetic field has attracted many researchers in view of its application in Geophysics, Astrophysics, geological formations, thermal recovery of oil, and in assessment of aquifers, geothermal reservoirs and underground nuclear waste storage site, etc. The heat transfer in porous media has great practical importance in geophysics and energy related engineering problems. These include the utilization of geothermal energy, the control of pollutants in ground water, solar power collectors, high performance insulations of buildings, food processing, casting and welding of a manufacturing process, etc.

The effect of temperature dependent viscosity on natural convection of fluid from heated vertical wavy surface was studied by [1]. In case of vertical cone, this effect was studied by [2]. Nazar et al. [3] studied the free convection boundary layer on an isothermal horizontal circular cylinder in a micropolar fluid. In case of horizontal cylinder the radiation-conduction interaction on mixed convection was investigated by [4]. Kafoussius et al. [5] studied the combined free and force convection laminar boundary layer past a vertical isothermal flat plate with temperature dependent viscosity. In porous media the effect of viscosity variation was considered by [6] and [7]. Free convection boundary layer on cylinders of elliptic cross section was studied by [8]. Harris et al. [9] studied the transient free convection near the lower stagnation point of a cylindrical surface subjected to a sudden change in surface temperature. Effect of aligned magnetic field on steady viscous flow past a circular cylinder was studied by [10]. Free convection and mixed convection about a circular cylinder was studied by the authors [11] and [12] respectively. The effect of variable viscosity on the fluid flow past a horizontal cylinder was also investigated by [13]. The combined heat and mass transfer along a vertical moving cylinder was studied by [14]. In this analysis both uniform wall temperature and uniform heat flux cases have been included. Bhargava et al. [15] found the finite element solution for non-newtonian pulsatile flow in a non-darcian porous medium conduit, they used the Darcy-Forchhiemer model to formulate the problem. Transient analysis of heat and mass transfer by natural convection in power law fluid past a vertical plate immersed in a porous medium is studied by [16]. Rashad [17] studied the effect of thermal radiationon the steady laminar flow past a verticalplate immersed in a porous medium. He used the Rosseland approximation to incorporate the effect of radiation, in the mathematical model of the problem.

It is observed that MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal horizontal circular cylinder in presence of radiation and temperature dependent fluid properties has given a very scant attention in the literature. Hence in the present study the effect of radiation with temperature dependent thermal conductivity and temperature dependent viscosity on MHD heat and mass transfer free convection flow near the lower stagnation point of a porous, isothermal horizontal circular cylinder has been considered.

2. Formulation

Consider a two dimensional MHD free convection flow of a viscous, incompressible, electrically conducting fluid absorbing-emitting radiation, over a uniformly heated circular cylinder of radius "r". It is assumed that the surface temperature of the porous cylinder is Tw and $T\infty$ is the ambient temperature of the fluid. A uniform radial magnetic field of strength B0 is applied perpendicular to the surface of the cylinder. A locally orthogonal co-ordinate system is choosen with origin O, at lower stagnation point and X and Y denoting the distances measured along and perpendicular to the surface respectively. If "a" is the curvature of the body surface, then by the choice of axes, "a" is the principal curvature at O. The physical model and coordinate system is shown in the fig. 1.

We assume that (i) the fluid has constant kinematic viscosity and the Boussinesq approximation may be adopted for the steady laminar boundary layer flow, (ii) the magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field, (iii) the cylinder is considered to be non- electrically conducting and the hall effect has been neglected, (iv) the joule heating effect has been neglected, and (v) the fluid is considered to be gray absorbing-emitting radiations but non scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the x-direction is considered negligible in comparison to y-direction. This approximation is valid at points far from the boundary surface, and is good for intensive absorption, that is, for an optically thick boundary layer. The Darcy- Forchhiemer model is used to describe the flow in porous media. Under the usual Bousinesq approximation, the equations that govern the flow are:

Equation of Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation of Momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)ax+g\beta_c(C-C_\infty)ax+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu}{\rho}\frac{u}{k\prime}-bu^2 \tag{2}$$

Equation of Energy:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)-\left(\frac{\partial q_r}{\partial y}\right) \tag{3}$$

Equation of Diffusion:

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v denote the fluid velocity components in the x and y directions respectively, T is the fluid temperature, C is fluid concentration, g is the magnitude of acceleration due to gravity, β is the coefficient of thermal expansion, ρ is the density of the fluid, σ is the fluid electrical conductivity, B0 is the strength of applied magnetic field, k′ is the permeability of porous medium, b is the Forchhiemer geometrical (inertial drag) coefficient, Cp is specific heat at constant pressure, μ(T) is the temperature dependent viscosity of the fluid, k(T) is the temperature dependent thermal conductivity and D is mass diffusivity. The term $g\beta(T-T_\infty)ax$ in the momentum equation arises from the component of buoyancy force in the x direction in the vicinity of O and the last term qr in the energy equation represent the radiative heat flux in y direction.

The radiative heat flux qr under Rosseland approximation by Brewster [18] has the form:

$$q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ1 is Stefan-Boltzmann constant and k1 is the mean absorption coefficient.

We assume that the temperature differences within the flow are so small that T4 can be expressed as a linear function of T∞. This is obtained by expending T4 in a Taylor series about T∞ and neglecting the higher order terms. Thus we get:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

The initial and boundary conditions are:

$$u=0,\ v=0,\ T=T_w,\ C=C_w \text{ at } y=0 \tag{7a}$$

$$u\to 0,\ T\to T_\infty,\ C\to C_\infty, \text{as } y\to\infty \tag{7b}$$

It is assumed that the viscosity μ(T) and thermal conductivity k(T) varies with temperature as follows:

$$\mu(T) = \frac{\mu_\infty}{1+\gamma(T-T_\infty)} \tag{8}$$

$$k(T) = k_\infty(1+b(T-T_\infty)) \tag{9}$$

The system of partial differential equations (1)-(4) and initial and boundary conditions (7) after introducing equations (5-6) and (8-9) can be reduced to a system of semi-similar equations by employing the following transformations:

$$\psi = Gr^{\frac{1}{4}}a\nu x f(\eta),\quad \eta = Gr^{\frac{1}{4}}ay,\quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \tag{10a}$$

$$\phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, Gr = \frac{g\beta(T_w-T_\infty)}{a^3\nu^2}, Gm = \frac{g\beta_c(C_w-C_\infty)}{a^3\nu^2} \tag{10b}$$

where ψ is the stream function, f is non-dimensional reduced stream function, θ is non-dimensional reduced temperature, C is non-dimensional reduced concentration, Gr is Grashoff number and Gm is modified Grashoff number.

Thus, the reduced equations in non-dimensional are:

$$f''' = (1+\epsilon\theta)\begin{bmatrix}(1-Nf)f'^2 - ff'' + \frac{\epsilon}{(1+\epsilon\theta)^2}\theta' f'' \\ +\frac{1}{\sqrt{Gr}}\left\{M + \left(\frac{1}{(1+\epsilon\theta)K}\right)\right\}f' - \theta - \frac{Gm}{Gr}\phi\end{bmatrix} \tag{11}$$

$$\theta'' = -\frac{3N}{4+3N(1+\omega\theta)}\left[Pr.f.\theta' + \omega\theta'^2\right] \tag{12}$$

$$\phi'' = -Sc.f.\phi' \tag{13}$$

Here, $\epsilon = \gamma(T_w - T_\infty)$ is variable viscosity parameter, $\omega = b(T_w - T_\infty)$ is variable thermal conductivity parameter, $N = \frac{k_1 k_\infty}{4\sigma_1 T_\infty^3}$ is radiation parameter, $M = \frac{\sigma B_0^2 a^2}{\mu_\infty}$ is magnetic parameter, $K = k' a^2$ is porosity parameter, $Nf = bx$ is Forchhiemer inertial porous parameter, $Pr = \frac{k_\infty}{\mu_\infty c_p}$ is Prandtl number, $Sc = \frac{\nu}{D}$ is Schmidt number and prime (′) denote the differentiation with respect to η.

The corresponding initial and boundary conditions are:

$$f(0) = 0, f'(0) = 0, \theta(0) = 1, \phi(0) = 1 \tag{14a}$$

$$f'(\infty) \to 0, \theta'(\infty) \to 0, \phi'(\infty) \to 0 \tag{14b}$$

In the absence of magnetic field, radiation and porosity and at Gm=0, eq. (11) and eq.(12) reduce to the equations given by Md. Mamun Molla et al. [13] as follows:

$$f''' = (1+\epsilon\theta)\left[f'^2 - ff'' + \frac{\epsilon}{(1+\epsilon\theta)^2}\theta' f'' - \theta\right] \tag{15a}$$

$$\theta'' = -Pr.f.\theta' \tag{15b}$$

Keeping in view of engineering aspects, the most important characteristics of the flow are local surface heat flux (Nusselt number), local surface mass flux (Sherwood number) and skin-friction, which can be written as

$$Nu = \frac{Gr^{-1/4}}{ak_\infty(T_w-T_\infty)}q_w, Mu = \frac{Gr^{-1/4}}{aD(C_w-C_\infty)}S_w, C_f = \frac{Gr^{-3/4}}{a^3 x\nu_\infty}\tau_w \tag{15}$$

Where

$q_w = -\left(k\frac{\partial T}{\partial y}\right)_{y=0}$ is rate of heat transfer,

$S_w = -\left(D\frac{\partial C}{\partial y}\right)_{y=0}$ is rate of mass transfer and,

$\tau_w = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0}$ is local wall shear stress.

Using the variables equations (8)-(10) and initial and boundary conditions (14a, 14b), we get the following expressions for the Nusselt number, Sherwood number and skin-friction:

$$Nu = -(1+\omega)\theta'(0), Sh = -\Phi'(0), C_f = \frac{1}{(1+\epsilon)}f''(0) \tag{16}$$

3. Results and Discussion

The equations (11-13) with initial and boundary conditions (14) have been solved using Runge-Kutta and Shooting method. Taking Δη=0.05 shooting technique has been applied for getting missing boundary conditions. The value of dependent variable is calculated at the terminal point by adopting fourth-order Runge-Kutta method within an admissible tolerance viz., of order 10^{-6}.

In the absence of magnetic field, porosity and radiation and at Gm=0, ε =0 and Nf=0 the value of -θ′(0) is 0.4212, the value of -θ′(0) found by Merkin [10] was 0.4214, by Nazar [14] it was found to be 0.4214 and by Md. Mamun Molla [10] it has been calculated to 0.4241. This shows that our results are in good agreement with these three solutions.

For several values of the dimensionless parameters, values of dimensionless velocity f′(η) and dimensionless temperature θ(η) have been computed and are presented in figures (2)-(7). Figures (2) and (3) show the effects of variable viscosity parameter and radiation parameter on velocity and temperature respectively. It is seen from the figure (2) that the velocity increases with the increase in viscosity parameter, but after a certain distance from the surface of cylinder it decreases. It is also noticed that temperature decreases uniformly with an increase in viscosity parameter. Figure (3) depicts the effect of radiation parameter and results that velocity and temperature both decrease with the increase in radiation parameter. The effects of thermal conductivity parameter and Schmidt number on velocity as well as temperature are shown in figures (4) and (5). It is noticed that velocity and temperature both increase with the increase in thermal conductivity parameter. This is because as thermal conductivity parameter ω increases, the thermal conductivity of the fluid increases. This increase in the fluid thermal conductivity increases the fluid temperature and accordingly its velocity. Moreover, it is obvious that neglecting the variation of fluid thermal conductivity for high temperature differences introduces a substantial error. This error has been shown by plotting the dimensionless velocity and temperature for ω=0. On increasing the Schmidt number the velocity decreases but temperature increases. The effects of Prandtl number on dimensionless velocity and temperature have been shown in figure (6). It is clear that thermal boundary layer thickness decreases sharply with the increase in Prandl number. Also the momentum boundary layer thickness decreases with the increase in Prandtl number from Pr=0.71 to Pr=7.0, but for Pr=70.0 the velocity is smaller than in the case of Pr=7.0 in the neighborhood of the cylinder and afterwards it increases. Figure (7) shows the velocity distribution for various values of *Nf* i.e. Forchhiemer parameter. A rise in *Nf* increases the velocity near the surface of the cylinder, but if we move longitudinally far away from the cylinder a rise in *Nf* depresses the velocity slightly and there is a slight depression in temperature for an increase in the Nf value for all the distances.

The effects of various dimensionless parameters on dimensionless concentration Φ (η) are shown in figures (8)-(10). From all these figures it is clear that the concentration decreases sharply as we move away from the surface. The effects of Prandtl number and thermal conductivity parameter are shown in figure (8), which depicts that the concentration boundary layer thickness increases with the increase in Prandtl number and there is a slight decrease in the concentration with the increase in thermal conductivity parameter. The variation of concentration with the change in the values of Schmidt number and viscosity parameter has been shown in figure (9). It is seen that with the increase in Schmidt number concentration boundary layer thickness decreases. Also it depicts that the dimensionless concentration decreases with an increase in viscosity parameter. Figure (10) shows the effect of radiation parameter and Forchhimer parameter on concentration distribution and results that concentration increases with the increase in radiation parameter, and there is a slight depression with with the increase in *Nf*, but this depression is negligible.

The numerical values of f″(0), -θ′(0) and -Φ′(0) have been presented in tabular form in table 1 for different values of various dimensionless parameters ε, M, K, ω, N, Sc, Nf and Pr at Gr=Gm=1.0. It is observed that dimensionless wall velocity gradient f″(0) increases as ε, K, Nf and ω increase, while it decreases with the increase in M, N, Sc and Pr. Moreover, the value of -θ′(0) decreases with the increase in M, Sc and ω, while it increases with the increase in K, ε, N, Nf and Pr. Also it is seen that the value of -Φ′(0) increases with the increase in ε, ω, K, Nf and Sc and it decreases with the increase in N, M and Pr.

4. Conclusion

In this work we used darcy Forchhiemer model to formulate the problem. The effect of Radiation, Porosity, Variable thermal conductivity, Variable Viscosity, Magnetic field and Prandtl number has been included in this analysis. The governing nonlinear equations have been solved by using Runge-Kutta and Shooting method. It was found that:

- Skin friction factor increases with the increasing porosity and thermal conductivity, while this is reduced with the increase in applied magnetic field, viscosity and radiation.
- Rate of heat transfer (Nusselt number) increases with the increase in the porosity, radiation and Prandtl number, while it decreases with the increase in Magnetic field, viscosity and thermal coducitivity.
- Rate of mass transfer (Sherwood number) increases with the increase in thermal conductivity and porosity, while it decreases with the increase in viscosity, applied magnetic field and radiation.

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