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Economic Design of Joint $\bar{X}$ and R Control ChartsUsing Differential Evolution

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Abstract

Benefits of economic designs can be realized to the full extent only by employing appropriate optimization techniques for minimizing the so called loss-cost functions or the total cost functions. Approximate methods employed to find the best control chart parameters may not be effective in obtaining the intended cost benefits. In the present work, differential evolution (DE), a population based evolutionary optimization technique has been employed to design joint $\bar{X}$ and R control charts. The optimum costs obtained are compared with the earlier designs which are based on conventional optimization techniques. It has been observed that the designs obtained using DE are very effective and in majority of the cases remarkable improvements are obtained in cost reductions.

Keywords*:* Economic Control Chart Design; $\bar{X}$, R Control Charts; Statistical Process Control; Differential Evolution.

1. Introduction

The simultaneous use of $\bar{X}$ chart to control the process mean and R chart to control the process variability gives good control of the process. The power of joint $\bar{X}$ and R charts is much greater than that of $\bar{X}$ or R chart alone. Therefore, in practice, $\bar{X}$ and R control charts are usually employed together to monitor the processes. The economic design of joint $\bar{X}$ and R control charts involves the determination of economically optimal sample size, sampling interval, and control limit coefficient for each chart, so as to minimize the total expected cost of controlling the process.

The economic design of joint $\bar{X}$ and R charts has been studied by various authors. Saniga [1] developed an expected cost model and performed a sensitivity analysis of the model for a process whose mean and variance are controlled by $\bar{X}$ and R charts. Saniga [2] investigated the effects of the types of process models on the joint economic design of $\bar{X}$ and R charts and suggested that accurate process model selection is an important determinant of the quality of joint $\bar{X}$ and R control chart design. Jones and Case [3] developed an economic model which determines the design of joint $\bar{X}$ and R charts to minimize costs and reported that the joint economic design can result in considerable savings over the traditional design of $\bar{X}$ and R charts. Rahim [4] developed a computer program for the optimal economic design of joint $\bar{X}$ and R charts based on the cost model of Saniga and Montgomery [5]. Chung and Chen [6] presented a simplified algorithm for the determination of optimal design parameters of joint $\bar{X}$ and R control charts. Costa [7] developed a model for joint economic design of $\bar{X}$ and R control charts, where two assignable causes are allowed to occur independently according to exponential distributions and found that the cost surface is convex to the model considered. Gelinas and Lefrancois [8] proposed a heuristic approach for the economic design of $\bar{X}$ and R control charts. Costa and Rahim [9] developed a cost model to determine the design parameters of joint $\bar{X}$ and R charts by adopting a non-uniform sampling interval scheme. A sensitivity analysis of the model is conducted and the cost savings associated with the use of non-uniform sampling intervals instead of constant sampling intervals are evaluated. Gelinas [10] presented a power approximation model for the joint determination of $\bar{X}$ and R control chart parameters based on three regression equations which are used to estimate the sample size and the control limits for the $\bar{X}$ chart and the R chart and the method's performance is tested using a set of previously studied problems. Use of evolutionary computational algorithms has become the need of the day to solve complicated objective functions in search of global solutions. Chou *et al*. [11] proposed joint economic design of $\bar{X}$ and R charts with variable sampling intervals using genetic algorithm. Minimizing the risk of using the uncertain cost and process parameters in the economic designs of $\bar{X}$ control chart has been dealt by Vommi and Seetala [12,13] employing genetic algorithm as a search tool. The present paper proposes the application of Neoteric Differential Evolution algorithm for the economic

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design of joint $\bar{X}$ and R charts based on the cost model of Saniga and Montgomery [5].

2. Cost Model

The process is assumed to start in a state of statistical control. The measurable quality characteristic of the process is assumed to be normally distributed with mean μ_0 and variance σ_0^2. The process is subject to a single assignable cause of variation. The time lapse between successive occurrences of the assignable cause is assumed to follow a negative exponential distribution with parameter λ. The occurrence of the assignable cause shifts the process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$, where δ is a positive constant. Furthermore, it is assumed that with the change in the process mean, the process variance σ_0^2 changes to σ_1^2, ($\sigma_1^2 \geq \sigma_0^2$) and also the process is shut down during the search for the assignable cause. The production cycle for the process model then consists of four possible periods: (a) the in-control period, (b) the out-of-control period due to the occurrence of the assignable cause, (c) the search period due to a false alarm, and (d) the search and repair period due to a true alarm. The cost model incorporates the fixed and variable costs of sampling, the cost of searching for the assignable cause when it exists, any adjustment or repair costs and the cost of searching for an assignable cause that does not exist.

Notation used in the formulation of loss-cost function:

- n = sample size
- h = sampling interval
- τ_s = expected search time for false alarm
- K_s = expected search cost for false alarm
- τ_r = expected search and adjustment time for true alarm
- K_r = expected search and adjustment cost for true alarm
- V_0 = profit per hour when the process is in control
- V_1 = profit per hour when the process is out of control
- L = average loss-cost per hour of the process
- b = fixed cost of sampling
- c = variable cost of sampling
- α = probability that the control charts for $\bar{X}$ or R or both indicate a false alarm (Type I error)
- $\alpha_{\bar{X}}$ = probability of Type I error of $\bar{X}$ chart
- $P_{\bar{X}}$ = power of $\bar{X}$ chart
- α_R = probability of Type I error of R chart
- P_R = power of R chart
- Φ(x) = standard normal cumulative distribution function
- P = probability that the control charts for $\bar{X}$ or R or both indicate a true alarm
- τ = average time within an interval before the assignable cause occurs
- K_1 = control limit coefficient for $\bar{X}$ chart
- K_2 = control limit coefficient for R chart

Saniga and Montgomery [5] presented the expected loss-cost per hour of operation as:

$$L = \frac{\lambda U B_1 + V B_0 + \lambda W + (b+cn)(1+\lambda B_1)/h}{1 + \lambda B_1 + \tau_s B_0 + \lambda \tau_r} \quad (1)$$

Where

$$U = V_0 - V_1 \quad (2)$$

$$V = K_s + V_0 \tau_s \quad (3)$$

$$W = K_r + V_0 \tau_r, \quad (4)$$

$$B_0 = \alpha(1 - \lambda\tau)/h, \quad (5)$$

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})},$$

$$\tau = \frac{1}{\lambda} - \frac{h}{e^{\lambda h}-1}, \quad (6)$$

$$B_1 = \frac{h}{P} - \tau \quad (7)$$

Chung and Chen [6] approximated the expression $1/(e^{\lambda h} - 1)$ to $(1/\lambda h) - (1/2)$ and the loss-cost function had been modified to $\bar{L}$ given as under. In order to compare the optimum solutions obtained by Chung and Chen [6] with the solutions obtained in the present work by applying DE technique, the same modified loss cost function $\bar{L}$ has been used.

$$\bar{L} = \frac{\lambda U\left(\frac{1}{P}-\frac{1}{2}\right)h + \lambda\left[W+(b+cn)\left(\frac{1}{P}-\frac{1}{2}\right)-\frac{V\alpha}{2}\right] + [V\alpha+(b+cn)]/h}{1+\lambda\left(\frac{1}{P}-\frac{1}{2}\right)h + \tau_s\left(\frac{\alpha}{h}-\frac{\alpha\lambda}{2}\right)+\lambda\tau_r}. \quad (8)$$

Hence, the present objective is to minimize the loss-cost function, $\bar{L}$ with respect to the design parameters n, h, K_1, and K_2. However, $\bar{L}$ also depends on α and P, which, in turn, involve the normal probability distribution function and the probability integral of the distribution of the range. The expressions for α and P is presented as follows:

Denoting by $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ a random sample of n observations, arranged in an ascending order of magnitude, drawn from a normal population having mean μ_0 and variance σ_0^2, the sample range R can be written as $X_{(n)} - X_{(1)}$.

The cumulative distribution function for the standardized range, $W_0 = R/\sigma_0$ can be expressed as

$$F_n(w_0) = \left(\int_{-w_0/2}^{w_0/2} \varphi(x)dx\right)^n + 2n\int_{w_0/2}^{\infty} \Phi(w)\left(\int_{w-w_0}^{w} \varphi(x)dx\right)^{n-1} dw \quad (9)$$

where

$$\varphi(x) = (2\pi)^{-1/2} e^{-x^2/2} \quad (10)$$

The upper and lower control limits respectively for the $\bar{X}$ chart are

$$UCL_{\bar{X}} = \mu_0 + K_1\sigma_0/\sqrt{n} \quad (11)$$

and

$$LCL_{\bar{X}} = \mu_0 - K_1\sigma_0/\sqrt{n} \quad (12)$$

where $K_1 \geq 0$, $\sigma_0 \geq 0$.

Also, the upper and lower control limits respectively for the R chart are

$$UCL_R = K_2\sigma_0 \quad (13)$$

And

$$LCL_R = 0 \quad (14)$$

where $K_2 \geq 0$.

The expressions for the joint probability of false alarm (Type I error) and the joint probability of true alarm (power) for $\bar{X}$ and R charts are as follows:

$$\alpha_{\bar{X}} = 2[1 - \Phi(K_1)] \quad (15)$$

and

$$P_{\bar{X}} = \Phi\left(+\frac{\delta\sqrt{n}\sigma_0}{\sigma_1} - K_1\sigma_0/\sigma_1\right) + \Phi\left(-\frac{\delta\sqrt{n}\sigma_0}{\sigma_1} - K_1\sigma_0/\sigma_1\right) \quad (16)$$

where $\sigma_1 \geq \sigma_0$.

$$\alpha_R = P(W_0 \geq K_2) = 1 - F_n(K_2) \quad (17)$$

and

$$P_R = 1 - F_n(K_2\sigma_0/\sigma_1). \quad (18)$$

Thus, the joint probability of false alarm for the $\bar{X}$ and R charts is

$$\alpha = \alpha_{\bar{X}} + \alpha_R - \alpha_{\bar{X}}.\alpha_R. \quad (19)$$

Similarly, the joint probability of true alarm for the $\bar{X}$ and R charts is

$$P = P_{\bar{X}} + P_R - P_{\bar{X}}.P_R. \quad (20)$$

3. . Application of Differential Evolution to Joint Economic Design of $\bar{X}$ and R Charts

Differential Evolution is a population-based, direct-search algorithm for globally optimizing the complicated objective functions. For the present joint economic design, Neoteric Differential Evolution algorithm suggested by Feoktistov [14] has been used. Storn and Price [15] first proposed classical Differential Evolution algorithm which forms the base for the present Neoteric Differential Evolution.

In Differential Evolution, the individuals of population contain design parameters and represent potential optimal solutions. The population is initialized by randomly generating individuals within the lower and higher boundary limits of the design parameters. Each individual of the initial population is evaluated by the cost function. In order to obtain next generation from the initial population, any one individual is chosen as the current best individual. Then, the initial population is subjected to repeated generations of differentiation, crossover and selection. Differentiation and crossover operations are used to create one trial or child individual for each target or parent individual. In order to perform the differentiation, a set of individuals, mutually different and also different from the current target individual, are randomly chosen from the current population. The search strategies of differentiation are designed on the basis of these individuals. In the crossover, by recombining the trial and target individuals, the trial individual inherits parameters of the target individual with certain probability. Next, boundary limits of the trial individual parameters are verified. If any parameter exceeds the limits, the parameter is reset by re-initialization. This trial individual is evaluated by the cost function. Afterwards, selection is fulfilled by comparing the cost function values of target and trial individuals. If the trial individual has an equal or lower cost to the target individual, it replaces its target individual in the population. If the trial individual has higher cost than the target one then the target individual is retained. Then, if the new trial individual of the population is better than the current best individual, the current best individual's index is updated.figure1 shows how the differential evolution is applied for joint economic design of $\bar{X}$ and R control charts.

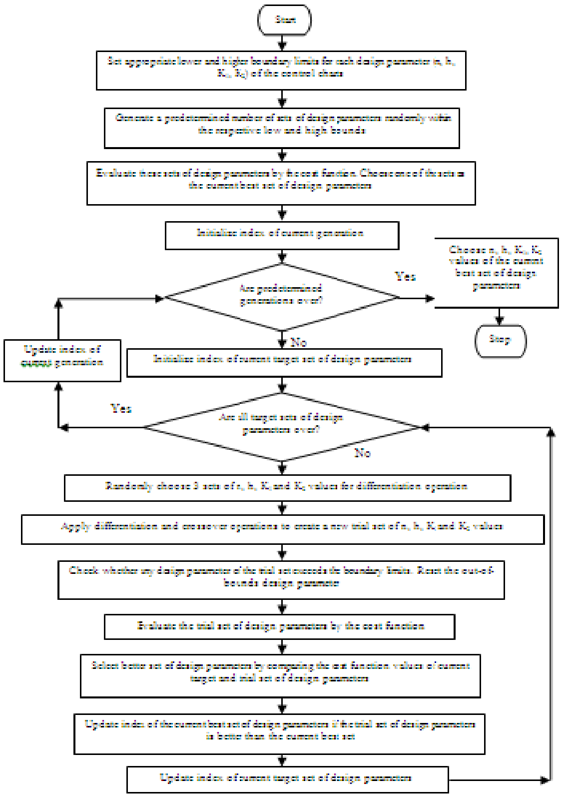

Figure 1. Procedure for Economic Design of Joint and R Control Charts Using Differential Evolution

In the present work, an individual of the population represents a set of design parameters of joint $\bar{X}$ and R control charts, namely n, h, K_1, and K_2. To define the limits of search space, feasible values are taken as lower and higher boundary limits of design parameters by considering the published economic designs on joint $\bar{X}$ and R control charts. Table1 contains the boundary constraints taken on the parameters of the control charts.

Table 1: Boundary constraints of $\bar{X}$ and R charts parameters used in present Differential Evolution algorithm

$\bar{X}$ and R Charts Parameters	Low – High Boundary Limits
n	2 – 33
h	0.25 – 12.00
K_1	1.00 – 6.00
K_2	1.00 – 6.00

Once the search space has been defined, the next step is to find the best parameters of the evolutionary algorithm. Parametric tuning has been carried out to find the effective control parameters for the algorithm namely population size, constant of differentiation, and constant of crossover. A few loss-cost function evaluations have been made using different combinations of control parameters, generations and search strategies. Different population sizes in multiples of 10, number of generations in multiples of 50 and search strategies of differentiation as suggested in Feoktistov [14] have been tested. For refining the selection of constant of differentiation (F) and constant of crossover (Cr), different values in multiples of 0.05 have been chosen in the intervals [0,1] for F and [0,1] for Cr. By using the feedback provided by these function evaluations and following the practical guidelines given in Feoktistov [14], the parameters for the differential evolution algorithm have been finalized. Table2 shows the parameters of DE employed in the present work to obtain the optimum control chart design parameters.

Table 2: Parameters used in present Differential Evolution algorithm.

Description of the Differential Evolution algorithm parameters	Magnitude/ method
No. of design parameters in a set	4
Population size	80
Search strategy	Rand3
Constant of Differentiation, F	0.85
Type of Crossover	Combinatorial
Constant of Crossover, Cr	0.50
Selection scheme	Elitist
Number of generations	300

As the performance of any evolutionary algorithm is best represented by the probability distribution of the best objective value (Kuo *et al*. [16]), it is required to run algorithm for a number of times, in order to check its consistency in providing the best solution. While obtaining robust $\bar{X}$ chart designs using genetic algorithm, Vommi and Seetala [13] has run the algorithm for 300 times and obtained the statistics of the best objective values. In the present case, to check the consistency of the algorithm in providing the best solutions, the algorithm has been run for 300 times for 40 sets of randomly selected cost and process parameters. All the 300 runs yielded the same best design parameters as reported in the present paper for each of the selected cost and process parameters sets. Since there is no variation in the best values of the objective function, single solution corresponding to each input data set for $\bar{X}$ and R control charts has been tabulated. The joint economic designs of Chung and Chen [6] are considered for comparison of optimum designs obtained by the present algorithm for the same cost and process parameters. A comparison of the results is presented in table 3.

Table 3.Comparison of Results

Cost and process parameters Chung and Chen's results Differential Evolution results % of Reduction in Loss-cost

e.g. No.[c]	δ	λ	V_0	V_1	K_r	K_s	τ_r	τ_s	b	c	σ_0	σ_1	n	h	K_1	K_2	L	n	h	K_1	K_2	L	
1.	0.50	0.01	50	25	2	1	0.3	0.1	0.5	0.1	0.02	0.03	10	3.46	2.07	4.14	1.4869	10	3.4682	2.0792	4.15	1.4781	0.5918
2.	0.50	0.05	50	25	2	1	0.4	0.1	0.5	0.1	0.02	0.03	10	1.66	2.04	4.09	3.8380	10	1.6720	2.0428	4.10	3.8134	0.6410
3.	0.50	0.01	50	25	15	5	0.3	0.1	0.5	0.2	0.02	0.03	8	4.04	2.05	3.96	1.9794	8	4.0837	2.0605	3.95	1.9691	0.5204
4.	0.50	0.01	50	25	15	5	0.3	0.1	0.5	0.1	0.02	0.03	13	3.71	2.24	4.58	1.7224	13	3.7201	2.2551	4.60	1.7105	0.6909
5.	0.50	0.05	50	25	15	5	0.6	0.1	0.5	0.8	0.02	0.03	2	4.10	0.56	1.00	6.5776	2	3.5593	1.0000	1.00	6.5416	0.5473
6.	0.50	0.05	50	25	15	5	0.5	0.1	0.5	0.4	0.02	0.03	4	2.15	1.70	2.96	5.9003	5	2.4010	1.7160	3.15	5.8771	0.3932
7.	0.50	0.05	50	25	15	5	0.4	0.1	0.5	0.2	0.02	0.03	7	1.88	2.00	3.79	5.1683	8	2.0247	2.0132	3.90	5.1387	0.5727
8.	0.50	0.05	50	25	15	5	0.4	0.1	0.5	0.1	0.02	0.03	12	1.74	2.21	4.48	4.6643	13	1.8238	2.2248	4.55	4.6337	0.6560
9.	0.50	0.01	150	50	2	1	0.4	0.1	0.5	0.4	0.02	0.03	6	2.26	2.01	3.70	5.0149	6	2.2664	2.0140	3.70	5.0004	0.2891
10.	0.50	0.01	150	50	2	1	0.4	0.1	0.5	0.2	0.02	0.03	10	2.08	2.22	4.36	4.3160	10	2.0644	2.2274	4.40	4.2907	0.5862
11.	0.50	0.01	150	50	2	1	0.6	0.1	0.5	0.1	0.02	0.03	16	1.93	2.39	4.93	3.9883	16	1.9385	2.4002	4.95	3.9619	0.6619
12.	0.50	0.05	150	50	2	1	0.6	0.1	0.5	0.4	0.02	0.03	6	1.09	1.97	3.64	13.5670	6	1.0899	1.9688	3.65	13.5256	0.3052
13.	0.50	0.05	150	50	2	1	0.6	0.1	0.5	0.2	0.02	0.03	10	0.99	2.19	4.32	12.1752	10	0.9861	2.1973	4.35	12.1150	0.4944
14.	0.50	0.05	150	50	2	1	0.6	0.1	0.5	0.1	0.02	0.03	15	0.88	2.36	4.86	10.9316	15	0.8891	2.3798	4.85	10.8705	0.5589
15.	0.50	0.01	150	50	15	5	0.6	0.1	0.5	0.4	0.02	0.03	7	2.39	2.10	3.93	5.6217	7	2.3826	2.1022	3.95	5.6011	0.3664
16.	0.50	0.01	150	50	15	5	0.3	0.1	0.5	0.2	0.02	0.03	11	2.14	2.29	4.53	4.4449	11	2.1406	2.3013	4.55	4.4160	0.6502
17.	0.50	0.01	150	50	15	5	0.6	0.1	0.5	0.1	0.02	0.03	17	1.95	2.45	5.06	4.2163	17	1.9467	2.4577	5.10	4.1877	0.6783
18.	0.50	0.05	150	50	15	5	0.6	0.1	0.5	0.4	0.02	0.03	7	1.15	2.06	3.88	14.5951	7	1.1498	2.0636	3.90	14.5395	0.3809
19.	0.50	0.05	150	50	15	5	0.6	0.1	0.5	0.2	0.02	0.03	11	1.02	2.26	4.49	13.0856	11	1.0282	2.2744	4.50	13.0175	0.5204
20.	0.50	0.05	150	50	15	5	0.6	0.1	0.5	0.1	0.02	0.03	17	0.92	2.43	5.04	11.7576	17	0.9268	2.4410	5.05	11.6905	0.5707
21.	0.50	0.01	150	50	2	1	0.6	0.1	5	0.2	0.02	0.03	17	4.29	2.03	4.46	5.9667	17	4.2836	2.0393	4.50	5.9337	0.5531
22.	0.50	0.01	150	50	2	1	0.6	0.1	5	0.1	0.02	0.03	27	4.06	2.21	5.06	5.4536	27	4.0557	2.2132	5.10	5.4268	0.4914
23.	0.50	0.05	150	50	2	1	0.6	0.1	5	0.2	0.02	0.03	16	2.01	1.99	4.35	15.0411	17	2.0395	2.0057	4.45	14.9361	0.6981
24.	0.50	0.05	150	50.	2	1	0.4	0.1	5	0.1	0.02	0.03	26	1.89	2.18	4.99	12.7627	26	1.8889	2.1786	5.025	12.6716	0.7138
25.	0.50	0.01	150	50	15	5	0.3	0.1	5	0.2	0.02	0.03	19	4.35	2.12	4.67	5.7943	19	4.3497	2.1236	4.70	5.7587	0.6144
26.	0.50	0.01	150	50	15	5	0.3	0.1	5	0.1	0.02	0.03	30	4.12	2.29	5.24	5.2376	29	4.0814	2.2797	5.25	5.2092	0.5422
27.	0.50	0.05	150	50	15	5	0.4	0.1	5	0.2	0.02	0.03	18	2.04	2.08	4.57	14.6625	18	2.0437	2.0815	4.60	14.5491	0.7734
28.	0.50	0.05	150	50	15	5	0.4	0.1	5	0.1	0.02	0.03	29	1.93	2.26	5.18	13.5381	28	1.9142	2.2500	5.175	13.4431	0.7017
29.	0.50	0.05	150	50	15	5	0.6	0.1	5	0.1	0.02	0.03	28	1.93	2.25	5.14	14.7888	28	1.9277	2.2472	5.175	14.6951	0.6336
30.	1.0	0.01	50	25	2	1	0.3	0.1	0.5	0.4	0.02	0.03	4	4.07	1.62	3.36	1.7241	4	4.0816	1.6157	3.35	1.7212	0.1682
31.	1.0	0.01	50	25	2	1	0.3	0.1	0.5	0.2	0.02	0.03	6	3.55	1.87	4.09	1.4732	6	3.5525	1.8706	4.10	1.4694	0.2579
32.	1.0	0.01	50	25	2	1	0.3	0.1	0.5	0.1	0.02	0.03	9	3.23	2.09	4.73	1.2563	9	3.2484	2.1011	4.70	1.2523	0.3184
33.	1.0	0.05	50	25	2	1	0.6	0.1	0.5	0.8	0.02	0.03	2	2.37	1.06	1.90	5.1965	2	2.5118	0.9910	1.75	5.1747	0.4195
34.	1.0	0.05	50	25	2	1	0.6	0.1	0.5	0.4	0.02	0.03	3	1.78	1.52	2.97	4.7272	4	2.0257	1.5556	3.25	4.7120	0.3215
35.	1.0	0.05	50	25	2	1	0.6	0.1	0.5	0.2	0.02	0.03	6	1.72	1.84	4.03	4.2393	6	1.7376	1.8332	4.00	4.2243	0.3538
36.	1.0	0.05	50	25	2	1	0.6	0.1	0.5	0.1	0.02	0.03	9	1.54	2.07	4.69	3.8083	9	1.5466	2.0643	4.70	3.7942	0.3702
37.	1.0	0.01	50	25	5	5	0.6	0.1	0.5	0.4	0.02	0.03	5	4.29	1.85	3.90	2.1147	5	4.2883	1.8429	3.90	2.0147	4.7288

[c] The values of cost and process parameters are same as Chung and Chen[6].

Table 3 (cont.): Comparison of Results

Cost and process parameters Chung and Chen's results Differential Evolution results % of Reduction in Loss-cost

e.g. No.[c]	δ	λ	V0	V1	Kr	Ks	πr	πs	b	c	σ0	σ1	n	h	K1	K2	L	n	h	K1	K2	L	
38.	1.0	0.01	50	25	5	5	0.3	0.1	0.5	0.2	0.02	0.03	8	3.93	2.08	4.62	1.6806	8	3.9160	2.0784	4.65	1.5787	6.0633
39.	1.0	0.01	50	25	5	5	0.4	0.1	0.5	0.2	0.02	0.03	7	3.67	2.07	4.49	1.7273	8	3.9200	2.0782	4.65	1.6255	5.8936
40.	1.0	0.01	50	25	5	5	0.3	0.1	0.5	0.1	0.02	0.03	11	3.44	2.29	5.15	1.4326	11	3.4388	2.2896	5.15	1.3309	7.0990
41.	1.0	0.05	50	25	5	5	0.5	0.1	0.5	0.8	0.02	0.03	3	2.44	1.50	2.93	6.0073	3	2.4454	1.4787	2.90	5.5483	7.6407
42.	1.0	0.05	50	25	5	5	0.3	0.1	0.5	0.4	0.02	0.03	5	2.10	1.81	3.84	4.9751	5	2.0810	1.7980	3.85	4.5033	9.4832
43.	1.0	0.05	50	25	5	5	0.3	0.1	0.5	0.2	0.02	0.03	7	1.77	2.04	4.45	4.3811	7	1.7604	2.0339	4.45	3.9046	10.8763
44.	1.0	0.05	50	25	5	5	0.3	0.1	0.5	0.1	0.02	0.03	11	1.64	2.27	5.12	3.8778	11	1.6192	2.2627	5.15	3.3960	12.4246
45.	1.0	0.01	50	25	5	5	0.5	0.1	5	0.2	0.02	0.03	13	8.18	1.92	4.78	2.5157	13	8.1724	1.9169	4.80	2.4053	4.3884
46.	1.0	0.01	50	25	5	5	0.6	0.1	5	0.1	0.02	0.03	18	7.71	2.15	5.32	2.3725	18	7.6950	2.1447	5.35	2.2639	4.5774
47.	1.0	0.05	50	25	5	5	0.6	0.1	5	0.2	0.02	0.03	12	4.01	1.85	4.61	6.5125	12	3.9831	1.8372	4.60	6.0100	7.7159
48.	1.0	0.05	50	25	5	5	0.6	0.1	5	0.1	0.02	0.03	17	3.76	2.08	5.20	6.1509	17	3.7267	2.0771	5.20	5.6514	8.1208
49.	1.0	0.01	150	50	2	1	0.4	0.1	0.5	0.4	0.02	0.03	6	2.19	2.03	4.32	4.2405	6	2.2043	2.0336	4.30	4.2331	0.1745
50.	1.0	0.01	150	50	2	1	0.4	0.1	0.5	0.2	0.02	0.03	9	1.98	2.25	4.94	3.5493	9	1.9831	2.2467	4.95	3.5415	0.2198
51.	1.0	0.01	150	50	2	1	0.4	0.1	0.5	0.1	0.02	0.03	13	1.79	2.46	5.50	2.9911	13	1.7926	2.4605	5.50	2.9840	0.2374
52.	1.0	0.05	150	50	2	1	0.5	0.1	0.5	0.8	0.02	0.03	4	1.20	1.76	3.57	13.0558	4	1.2062	1.7507	3.55	13.0318	0.1838
53.	1.0	0.05	150	50	2	1	0.5	0.1	0.5	0.4	0.02	0.03	6	1.03	2.00	4.28	11.4064	6	1.0321	1.9982	4.30	11.3800	0.2314
54.	1.0	0.05	150	50	2	1	0.5	0.1	0.5	0.2	0.02	0.03	9	0.92	2.22	4.91	9.9825	9	0.9290	2.2250	4.90	9.9567	0.2585
55.	1.0	0.05	150	50	2	1	0.5	0.1	0.5	0.1	0.02	0.03	12	0.80	2.42	5.40	8.8167	12	0.8034	2.4303	5.35	8.7957	0.2382
56.	1.0	0.01	150	50	5	5	0.5	0.1	0.5	0.4	0.02	0.03	7	2.36	2.11	4.56	4.6163	7	2.3638	2.1168	4.55	4.5096	2.3114
57.	1.0	0.01	150	50	5	5	0.5	0.1	0.5	0.2	0.02	0.03	10	2.07	2.33	5.13	3.8869	10	2.0707	2.3257	5.15	3.7807	2.7323
58.	1.0	0.01	150	50	5	5	0.6	0.1	0.5	0.1	0.02	0.03	13	1.77	2.52	5.58	3.4476	13	1.7720	2.5200	5.60	3.3426	3.0456
59.	1.0	0.05	150	50	5	5	0.6	0.1	0.5	0.8	0.02	0.03	4	1.17	1.85	3.72	14.6262	4	1.1711	1.8444	3.70	14.1490	3.2626
60.	1.0	0.05	150	50	5	5	0.5	0.1	0.5	0.4	0.02	0.03	6	1.01	2.08	4.39	12.2456	7	1.1167	2.0905	4.50	11.7527	4.0251
61.	1.0	0.05	150	50	5	5	0.5	0.1	0.5	0.2	0.02	0.03	9	0.91	2.29	5.01	10.7404	10	0.9765	2.3124	5.05	10.2451	4.6116
62.	1.0	0.05	150	50	5	5	0.5	0.1	0.5	0.1	0.02	0.03	13	0.82	2.50	5.56	9.5223	13	0.8196	2.5018	5.60	9.0275	5.1962
63.	1.0	0.01	150	50	2	1	0.6	0.1	5	0.8	0.02	0.03	6	4.57	1.59	3.66	6.5753	6	4.5848	1.5935	3.65	6.5541	0.3224
64.	1.0	0.01	150	50	2	1	0.6	0.1	5	0.4	0.02	0.03	10	4.31	1.87	4.50	5.8736	10	4.3180	1.8749	4.50	5.8512	0.3814
65.	1.0	0.01	150	50	2	1	0.6	0.1	5	0.2	0.02	0.03	15	4.06	2.12	5.15	5.3024	15	4.0627	2.1247	5.15	5.2834	0.3583
66.	1.0	0.01	150	50	2	1	0.3	0.1	5	0.1	0.02	0.03	20	3.79	2.33	5.63	4.4447	20	3.7959	2.3407	5.60	4.4291	0.3510
67.	1.0	0.05	150	50	2	1	0.6	0.1	5	0.8	0.02	0.03	6	2.19	1.55	3.59	16.2683	6	2.1967	1.5357	3.60	16.1775	0.5581
68.	1.0	0.05	150	50	2	1	0.3	0.1	5	0.4	0.02	0.03	10	2.02	1.84	4.46	13.0180	10	2.0266	1.8418	4.45	12.9289	0.6844
69.	1.0	0.05	150	50	2	1	0.3	0.1	5	0.2	0.02	0.03	14	1.86	2.06	5.03	11.8297	14	1.8618	2.0598	5.05	11.7532	0.6467
70.	1.0	0.05	150	50	2	1	0.3	0.1	5	0.1	0.02	0.03	20	1.76	2.31	5.61	10.9214	20	1.7605	2.3124	5.60	10.8560	0.5988
71.	1.0	0.01	150	50	5	5	0.4	0.1	5	0.4	0.02	0.03	11	4.38	1.97	4.72	5.8022	11	4.3926	1.9796	4.70	5.6818	2.0751
72.	1.0	0.01	150	50	5	5	0.4	0.1	5	0.2	0.02	0.03	15	4.03	2.18	5.23	5.1946	15	4.0337	2.1806	5.25	5.0778	2.2485
73.	1.0	0.01	150	50	5	5	0.5	0.1	5	0.1	0.02	0.03	21	3.82	2.41	5.77	4.8813	21	3.8152	2.4120	5.80	4.7685	2.3109
74.	1.0	0.05	150	50	5	5	0.6	0.1	5	0.8	0.02	0.03	7	2.25	1.68	3.93	17.1665	7	2.2671	1.6771	3.90	16.6163	3.2051
75.	1.0	0.05	150	50	5	5	0.5	0.1	5	0.4	0.02	0.03	10	2.03	1.91	4.56	15.0487	11	2.0727	1.9375	4.70	14.5029	3.6269
76.	1.0	0.05	150	50	5	5	0.5	0.1	5	0.2	0.02	0.03	15	1.90	2.15	5.20	13.8127	15	1.8990	2.1563	5.20	13.2736	3.9029
77.	1.0	0.05	150	50	5	5	0.5	0.1	5	0.1	0.02	0.03	21	1.79	2.39	5.75	12.8792	21	1.7847	2.3933	5.75	12.3492	4.1152
78.	1.5	0.01	50	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	3.40	2.06	3.53	1.5699	3	3.3936	2.0511	3.55	1.5691	0.0510
79.	1.5	0.01	50	25	2	1	0.6	0.1	0.5	0.2	0.02	0.04	4	3.00	2.29	4.06	1.3562	4	3.0059	2.2867	4.05	1.3549	0.0959
80.	1.5	0.01	50	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	6	2.84	2.52	4.67	1.0477	6	2.8517	2.5254	4.65	1.0451	0.2482
81.	1.5	0.05	50	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	1.64	2.02	3.47	4.1584	3	1.6448	2.0124	3.45	4.1501	0.1996
82.	1.5	0.05	50	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	1.40	2.27	4.03	3.0758	4	1.4116	2.2612	4.00	3.0679	0.2568
83.	1.5	0.05	50	25	2	1	0.4	0.1	0.5	0.1	0.02	0.04	6	1.33	2.50	4.64	2.9502	6	1.3318	2.4992	4.65	2.9403	0.3356
84.	1.5	0.01	50	25	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	3.26	2.25	3.80	1.6657	3	3.2515	2.2437	3.80	1.5686	5.8294
85.	1.5	0.01	50	25	5	5	0.4	0.1	0.5	0.2	0.02	0.04	5	3.25	2.50	4.51	1.4283	5	3.2506	2.4997	4.50	1.3284	6.9943
86.	1.5	0.01	50	25	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	2.79	2.66	4.86	1.2008	7	2.9859	2.7120	5.00	1.1003	8.3694
87.	1.5	0.05	50	25	5	5	0.6	0.1	0.5	0.4	0.02	0.04	3	1.59	2.22	3.76	4.8826	3	1.5763	2.2095	3.75	4.4210	9.4540
88.	1.5	0.05	50	25	5	5	0.3	0.1	0.5	0.2	0.02	0.04	5	1.54	2.48	4.48	3.7728	5	1.5236	2.4739	4.50	3.2942	12.6855
89.	1.5	0.05	50	25	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	1.32	2.65	4.84	3.3970	6	1.3120	2.6524	4.80	2.9159	14.1625
90.	1.5	0.05	50	25	2	1	0.6	0.1	5.0	0.1	0.02	0.04	10	3.47	2.34	4.76	5.2434	10	3.4761	2.3351	4.75	5.1922	0.9765
91.	1.5	0.01	50	25	5	5	0.4	0.1	5.0	0.2	0.02	0.04	8	7.52	2.32	4.59	2.2593	8	7.5071	2.3229	4.60	2.1509	4.7979
92.	1.5	0.01	50	25	5	5	0.4	0.1	5.0	0.1	0.02	0.04	11	2.55	5.11	7.25	2.1304	11	7.2428	2.5600	5.10	2.0228	5.0507
93.	1.5	0.05	50	25	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	3.92	2.08	4.04	6.2683	6	3.9039	2.0686	4.00	5.7667	8.0022
94.	1.5	0.05	50	25	5	5	0.5	0.1	5.0	0.2	0.02	0.04	8	3.68	2.29	4.55	5.9134	8	3.6457	2.2840	4.55	5.4125	8.4706
95.	1.5	0.05	50	25	5	5	0.6	0.1	5.0	0.1	0.02	0.04	11	3.55	2.52	5.07	5.8564	11	3.5145	2.5298	5.05	5.3591	8.4916
96.	1.5	0.01	150	50	2	1	0.6	0.1	0.5	0.2	0.02	0.04	5	1.56	2.64	4.70	3.1579	5	1.5621	2.6399	4.70	3.1553	0.0823
97.	1.5	0.01	150	50	2	1	0.3	0.1	0.5	0.1	0.02	0.04	7	1.45	2.83	5.18	2.3313	7	1.4539	2.8396	5.15	2.3268	0.1930
98.	1.5	0.05	150	50	2	1	0.6	0.1	0.5	0.8	0.02	0.04	3	0.99	2.20	3.73	11.7112	3	0.9978	2.1921	3.70	11.7027	0.0726
99.	1.5	0.05	150	50	2	1	0.4	0.1	0.5	0.4	0.02	0.04	4	0.84	2.43	4.27	8.9925	4	0.8471	2.4304	4.25	8.9821	0.1157
100.	1.5	0.05	150	50	2	1	0.3	0.1	0.5	0.2	0.02	0.04	5	0.72	2.63	4.68	7.2060	5	0.7203	2.6276	4.65	7.1948	0.1554

[c] The values of cost and process parameters are same as Chung and Chen[6].

Table 3 (cont.): Comparison of Results

Cost and process parameters Chung and Chen's results Differential Evolution results % of Reduction in Loss-cost

e.g. No.[c]	δ	λ	V_0	V_1	K_r	K_s	Z_0	Z_1	b	c	σ_0	σ_1	n	h	K_1	K_2	L	n	h	K_1	K_2	L	
101.	1.5	0.05	150	50	2	1	0.3	0.1	0.5	0.1	0.02	0.04	7	0.66	2.82	5.17	6.3645	7	0.6646	2.8250	5.15	6.3498	0.2310
102.	1.5	0.01	150	50	5	5	0.4	0.1	0.5	0.2	0.02	0.04	5	1.54	2.71	4.79	3.0331	6	1.6906	2.7338	5.00	2.9311	3.3629
103.	1.5	0.01	150	50	5	5	0.4	0.1	0.5	0.1	0.02	0.04	7	1.43	2.89	5.26	2.6275	7	1.4485	2.9001	5.20	2.5252	3.8934
104.	1.5	0.05	150	50	5	5	0.6	0.1	0.5	0.4	0.02	0.04	4	0.84	2.50	4.36	11.0358	4	0.8414	2.5011	4.35	10.5584	4.3259
105.	1.5	0.05	150	50	5	5	0.6	0.1	0.5	0.2	0.02	0.04	5	0.72	2.69	4.77	9.9209	5	0.7188	2.6893	4.75	9.4386	4.8615
106.	1.5	0.05	150	50	5	5	0.3	0.1	0.5	0.1	0.02	0.04	7	0.66	2.88	5.25	7.0397	7	0.6622	2.8874	5.20	6.5448	7.0301
107.	1.5	0.01	150	50	2	1	0.5	0.1	5.0	0.4	0.02	0.04	7	3.96	2.33	4.50	5.0818	7	3.9637	2.3312	4.50	5.0642	0.3463
108.	1.5	0.01	150	50	2	1	0.3	0.1	5.0	0.2	0.02	0.04	9	3.71	2.51	4.92	4.3916	9	3.7073	2.5124	4.95	4.3757	0.3621
109.	1.5	0.01	150	50	2	1	0.3	0.1	5.0	0.1	0.02	0.04	12	3.56	2.71	5.38	4.1063	12	3.5687	2.7342	5.35	4.0923	0.3409
110.	1.5	0.05	150	50	2	1	0.5	0.1	5.0	0.4	0.02	0.04	7	1.85	2.30	4.47	12.9502	7	1.8533	2.2923	4.50	12.8793	0.5475
111.	1.5	0.05	150	50	2	1	0.5	0.1	5.0	0.2	0.02	0.04	9	1.73	2.49	4.89	12.1054	9	1.7352	2.4911	4.90	12.0414	0.5287
112.	1.5	0.05	150	50	2	1	0.4	0.1	5.0	0.1	0.02	0.04	12	1.65	2.70	5.35	10.8632	12	1.6557	2.7068	5.35	10.8050	0.5358
113.	1.5	0.01	150	50	5	5	0.6	0.1	5.0	0.4	0.02	0.04	7	3.94	2.39	4.59	5.3874	7	3.9438	2.3889	4.60	5.2726	2.1309
114.	1.5	0.01	150	50	5	5	0.4	0.1	5.0	0.2	0.02	0.04	10	3.77	2.62	5.14	4.6853	10	3.7734	2.6291	5.15	4.5714	2.4310
115.	1.5	0.01	150	50	5	5	0.5	0.1	5.0	0.1	0.02	0.04	12	3.56	2.77	5.44	4.5323	12	3.5641	2.7749	5.45	4.4203	2.4712
116.	1.5	0.05	150	50	5	5	0.6	0.1	5.0	0.8	0.02	0.04	5	1.99	2.13	3.99	15.4402	5	1.9874	2.1226	4.00	14.9111	3.4268
117.	1.5	0.05	150	50	5	5	0.5	0.1	5.0	0.4	0.02	0.04	7	1.85	2.36	4.56	13.6345	7	1.8521	2.3648	4.55	13.1003	3.9180
118.	1.5	0.05	150	50	5	5	0.5	0.1	5.0	0.2	0.02	0.04	9	1.73	2.55	4.97	12.7639	9	1.7348	2.5553	4.95	12.2344	4.1484
119.	1.5	0.05	150	50	5	5	0.6	0.1	5.0	0.1	0.02	0.04	12	1.67	2.75	5.42	12.7929	12	1.6647	2.7485	5.45	12.2699	4.0882
120.	2.0	0.01	50	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	3.48	2.16	3.87	1.4611	3	3.4822	2.1594	3.85	1.4596	0.1027
121.	2.0	0.01	50	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	3.08	2.40	4.41	1.1217	4	3.0800	2.3930	4.40	1.1202	0.1337
122.	2.0	0.01	50	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	5	2.73	2.59	4.82	0.9735	5	2.7355	2.5876	4.80	0.9720	0.1541
123.	2.0	0.05	50	25	2	1	0.5	0.1	0.5	0.4	0.02	0.04	3	1.65	2.13	3.83	3.7313	3	1.6541	2.1253	3.80	3.7213	0.2680
124.	2.0	0.05	50	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	1.44	2.38	4.38	2.8859	4	1.4410	2.3707	4.35	2.8773	0.2980
125.	2.0	0.05	50	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	5	1.27	2.57	4.80	2.5722	5	1.2686	2.5659	4.80	2.5647	0.2916
126.	2.0	0.01	50	25	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	3.38	2.34	4.12	1.5341	3	3.3633	2.3314	4.15	1.4361	6.3881
127.	2.0	0.01	50	25	5	5	0.4	0.1	0.5	0.2	0.02	0.04	4	3.03	2.55	4.62	1.3224	4	3.0261	2.5483	4.60	1.2236	7.4713
128.	2.0	0.01	50	25	5	5	0.5	0.1	0.5	0.1	0.02	0.04	5	2.70	2.73	5.01	1.2140	5	2.6997	2.7276	5.00	1.1147	8.1796
129.	2.0	0.05	50	25	5	5	0.6	0.1	0.5	0.4	0.02	0.04	3	1.63	2.31	4.08	4.6250	3	1.6238	2.3054	4.05	4.1584	10.0886
130.	2.0	0.05	50	25	5	5	0.5	0.1	0.5	0.2	0.02	0.04	4	1.44	2.53	4.59	3.9854	4	1.4287	2.5241	4.60	3.5121	11.8758
131.	2.0	0.05	50	25	5	5	0.5	0.1	0.5	0.1	0.02	0.04	5	1.28	2.71	4.99	3.6644	5	1.2708	2.7127	4.95	3.1881	12.9980
132.	2.0	0.05	50	25	2	1	0.6	0.1	5.0	0.1	0.02	0.04	8	3.40	2.40	4.90	5.1444	8	3.4021	2.3901	4.90	5.0966	0.9292
133.	2.0	0.01	50	25	5	5	0.6	0.1	5.0	0.2	0.02	0.04	7	7.42	2.43	4.84	2.2725	7	7.4081	2.4278	4.85	2.1658	4.6953
134.	2.0	0.01	50	25	5	5	0.3	0.1	5.0	0.1	0.02	0.04	9	7.12	2.65	5.29	2.0284	9	7.1077	2.6478	5.30	1.9220	5.2455
135.	2.0	0.05	50	25	5	5	0.5	0.1	5.0	0.4	0.02	0.04	5	3.78	2.13	4.20	6.0691	5	3.7478	2.1139	4.20	5.5708	8.2104
136.	2.0	0.05	50	25	5	5	0.3	0.1	5.0	0.2	0.02	0.04	7	3.57	2.40	4.81	5.3637	7	3.5394	2.3942	4.80	4.8601	9.3890
137.	2.0	0.05	50	25	5	5	0.3	0.1	5.0	0.1	0.02	0.04	9	2.62	5.26	3.43	5.1463	9	3.3910	2.6201	5.25	4.6440	9.7604
138.	2.0	0.01	150	50	2	1	0.4	0.1	0.5	0.2	0.02	0.04	4	1.46	2.69	4.81	2.6341	4	1.4608	2.6891	4.80	2.6333	0.0304
139.	2.0	0.01	150	50	2	1	0.4	0.1	0.5	0.1	0.02	0.04	6	1.41	2.92	5.38	2.2963	6	1.4170	2.9204	5.35	2.2940	0.1002
140.	2.0	0.05	150	50	2	1	0.5	0.1	0.5	0.4	0.02	0.04	3	0.75	2.48	4.32	9.0347	3	0.7456	2.4684	4.35	9.0296	0.0564
141.	2.0	0.05	150	50	2	1	0.5	0.1	0.5	0.2	0.02	0.04	4	0.67	2.67	4.79	8.0672	4	0.6738	2.6704	4.80	8.0606	0.0818
142.	2.0	0.05	150	50	2	1	0.6	0.1	0.5	0.1	0.02	0.04	6	0.65	2.90	5.36	8.0331	6	0.6538	2.9040	5.35	8.0237	0.1170
143.	2.0	0.01	150	50	5	5	0.6	0.1	0.5	0.2	0.02	0.04	5	1.62	2.81	5.12	3.0774	5	1.6191	2.8042	5.15	2.9773	3.2527
144.	2.0	0.01	150	50	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	1.41	2.97	5.45	2.2925	6	1.4061	2.9752	5.45	2.1916	4.4013
145.	2.0	0.05	150	50	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	0.74	2.55	4.42	9.0710	3	0.7403	2.5489	4.35	8.5921	5.2795
145.	2.0	0.05	150	50	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	0.74	2.55	4.42	9.0710	3	0.7403	2.5489	4.35	8.5921	5.2795
146.	2.0	0.05	150	50	5	5	0.3	0.1	0.5	0.2	0.02	0.04	4	0.66	2.74	4.88	7.3852	5	0.7403	2.7908	5.15	6.8975	6.6037
147.	2.0	0.05	150	50	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	0.64	2.96	5.44	6.6344	6	0.6426	2.9638	5.45	6.1426	7.4129
148.	2.0	0.01	150	50	5	5	0.6	0.1	5.0	0.4	0.02	0.04	6	3.87	2.41	4.71	4.9841	6	3.8698	2.4122	4.70	4.9709	0.2648
149.	2.0	0.01	150	50	5	5	0.6	0.1	5.0	0.2	0.02	0.04	7	3.60	2.55	5.01	4.6404	7	3.6066	2.5572	5.00	4.6283	0.2608
150.	2.0	0.01	150	50	5	5	0.4	0.1	5.0	0.1	0.02	0.04	9	3.48	2.76	5.44	4.1236	9	3.4763	2.7622	5.45	4.1125	0.2692
151.	2.0	0.05	150	50	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	1.80	2.39	4.68	12.4604	6	1.8086	2.3864	4.65	12.3993	0.4904
152.	2.0	0.05	150	50	5	5	0.4	0.1	5.0	0.2	0.02	0.04	7	1.67	2.54	4.98	11.0894	7	1.6718	2.5291	5.00	11.0350	0.4906
153.	2.0	0.05	150	50	5	5	0.4	0.1	5.0	0.1	0.02	0.04	9	1.61	2.74	5.41	10.5993	9	1.6106	2.7373	5.45	10.5489	0.4755
154.	2.0	0.01	150	50	5	5	0.6	0.1	5.0	0.4	0.02	0.04	6	3.85	2.47	4.79	5.1348	6	3.8550	2.4703	4.80	5.0243	2.1520
155.	2.0	0.01	150	50	5	5	0.4	0.1	5.0	0.2	0.02	0.04	8	3.67	2.70	5.28	4.4982	8	3.6661	2.6981	5.30	4.3880	2.4499
156.	2.0	0.01	150	50	5	5	0.4	0.1	5.0	0.1	0.02	0.04	10	3.51	2.89	5.67	4.2590	10	3.5133	2.8990	5.65	4.1500	2.5593
157.	2.0	0.05	150	50	5	5	0.6	0.1	5.0	0.8	0.02	0.04	4	1.88	2.17	4.09	14.7881	4	1.8844	2.1604	4.05	14.2702	3.5021
158.	2.0	0.05	150	50	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	1.81	2.45	4.77	13.1163	6	1.8037	2.4474	4.75	12.5913	4.0027
159.	2.0	0.05	150	50	5	5	0.3	0.1	5.0	0.2	0.02	0.04	8	1.70	2.68	5.26	11.0839	8	1.6966	2.6744	5.30	10.5561	4.7619
160.	2.0	0.05	150	50	5	5	0.3	0.1	5.0	0.1	0.02	0.04	10	1.63	2.88	5.65	10.5750	10	1.6227	2.8784	5.65	10.0507	4.9579

[c] The values of cost and process parameters are same as Chung and Chen[6].

4. Results and Discussion

Differential Evolution algorithm has been applied in the joint economic design of $\bar{X}$ and R charts by utilizing the cost and process parameters of Saniga and Montgomery [5]. Given the cost and risk factors and other process parameters, the present work finds the sample size, the interval between samples and the control limit coefficient for each chart that minimize the expected loss-cost per hour. A large number of designs (160) have been considered and the solutions obtained are compared with the solutions reported by Chung and Chen [6]. In all the cases, the present algorithm has been found to yield lower loss-costs compared to Chung and Chen's algorithm. A maximum cost reduction of 14% has been obtained which shows the effectiveness of the DE. Also, it has been observed that the algorithm could provide the same best solutions even after a number of times the algorithm was run with different initial solutions.

The optimal sample sizes of the joint economic designs obtained by Chung and Chen [6] are found to range from 2 to 30. Hence, the probability integral for the standardized range values, $F_n(w_0)$, for n between 2 and 30 is required for the joint economic designs. Pearson and Hartley [17] published the function $F_n(w_0)$ for the values of n between 2 and 20 which can be used for designs involving n values up to 20. Beyond the sample size of 20, $F_n(w_0)$ values are not published, hence are not readily available Therefore, in the present work a program has been developed to evaluate $F_n(w_0)$. A database for the values of $F_n(w_0)$ has been developed for n between 2 and 33 since it takes lot of time to evaluate the probability integral for different values of n while the DE algorithm is running. The cost function evaluation program is made to use the same database for easy and instant retrieval of the $F_n(w_0)$ values. This saves a lot of time in the cost function evaluations using DE. The values of $F_n(w_0)$ for n between 21 and 33 are presented in table 4 for ready reference.

Finally, it is concluded that the economic designs obtained using Differential evolution, an evolutionary global optimization technique, are much superior in that they provided cost reductions of up to 14% compared to the earlier designs of Chung and Chen [6]. Hence, it is recommended to use evolutionary optimization techniques in the economic design of control charts as it is difficult to obtain closed form solutions by differentiating the loss-cost functions and also the designs are superior to the algorithms used earlier.

Table 4: Probability Integral of the Standardized Range W_0 for Normal Samples (of size n between 21 and 33)

n W_0	21	22	23	24	25	26	27	28	29	30	31	32	33
1.55													
1.60	0.0001												
1.65	0.0001	0.0001											
1.70	0.0002	0.0001	0.0001										
1.75	0.0002	0.0002	0.0001	0.0001									
1.80	0.0004	0.0002	0.0002	0.0001	0.0001								
1.85	0.0005	0.0004	0.0002	0.0002	0.0001	0.0001							
1.90	0.0008	0.0005	0.0004	0.0002	0.0002	0.0001	0.0001						
1.95	0.0012	0.0008	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001				
2.00	0.0016	0.0011	0.0008	0.0006	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001			
2.05	0.0023	0.0016	0.0011	0.0008	0.0006	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	
2.10	0.0031	0.0023	0.0016	0.0012	0.0008	0.0006	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001
2.15	0.0042	0.0031	0.0023	0.0017	0.0012	0.0009	0.0007	0.0005	0.0003	0.0003	0.0002	0.0001	0.0001
2.20	0.0057	0.0042	0.0031	0.0023	0.0017	0.0013	0.0010	0.0007	0.0005	0.0004	0.0003	0.0002	0.0002
2.25	0.0075	0.0056	0.0043	0.0032	0.0024	0.0018	0.0014	0.0010	0.0008	0.0006	0.0004	0.0003	0.0002
2.30	0.0097	0.0074	0.0057	0.0044	0.0033	0.0025	0.0019	0.0015	0.0011	0.0009	0.0007	0.0005	0.0004
2.35	0.0125	0.0097	0.0075	0.0058	0.0045	0.0035	0.0027	0.0021	0.0016	0.0013	0.0010	0.0007	0.0006
2.40	0.0159	0.0125	0.0098	0.0077	0.0061	0.0048	0.0037	0.0029	0.0023	0.0018	0.0014	0.0011	0.0009
2.45	0.0200	0.0160	0.0127	0.0101	0.0080	0.0064	0.0050	0.0040	0.0032	0.0025	0.0020	0.0016	0.0012
2.50	0.0249	0.0201	0.0162	0.0130	0.0105	0.0084	0.0067	0.0054	0.0043	0.0035	0.0028	0.0022	0.0018
2.55	0.0307	0.0251	0.0204	0.0166	0.0135	0.0110	0.0089	0.0072	0.0059	0.0047	0.0038	0.0031	0.0025
2.60	0.0375	0.0309	0.0254	0.0209	0.0172	0.0141	0.0116	0.0095	0.0078	0.0064	0.0052	0.0043	0.0035
2.65	0.0454	0.0378	0.0314	0.0261	0.0217	0.0180	0.0149	0.0124	0.0102	0.0085	0.0070	0.0058	0.0048
2.70	0.0544	0.0457	0.0384	0.0322	0.0270	0.0226	0.0190	0.0159	0.0133	0.0111	0.0093	0.0077	0.0065
2.75	0.0647	0.0549	0.0465	0.0394	0.0333	0.0282	0.0238	0.0201	0.0170	0.0144	0.0121	0.0102	0.0086
2.80	0.0762	0.0652	0.0558	0.0477	0.0407	0.0348	0.0297	0.0253	0.0215	0.0183	0.0156	0.0133	0.0113
2.85	0.0891	0.0769	0.0664	0.0572	0.0493	0.0424	0.0365	0.0314	0.0270	0.0232	0.0199	0.0171	0.0147
2.90	0.1033	0.0900	0.0782	0.0680	0.0591	0.0513	0.0445	0.0386	0.0334	0.0290	0.0251	0.0217	0.0188
2.95	0.1190	0.1044	0.0915	0.0802	0.0702	0.0614	0.0537	0.0469	0.0410	0.0358	0.0312	0.0272	0.0238
3.00	0.1360	0.1203	0.1062	0.0938	0.0827	0.0729	0.0642	0.0566	0.0498	0.0438	0.0385	0.0338	0.0297
3.05	0.1545	0.1375	0.1223	0.1088	0.0966	0.0858	0.0761	0.0675	0.0599	0.0530	0.0470	0.0416	0.0368
3.10	0.1743	0.1562	0.1399	0.1252	0.1120	0.1001	0.0895	0.0799	0.0713	0.0636	0.0567	0.0506	0.0451
3.15	0.1953	0.1762	0.1589	0.1432	0.1289	0.1160	0.1043	0.0938	0.0842	0.0756	0.0679	0.0609	0.0546
3.20	0.2177	0.1976	0.1792	0.1625	0.1472	0.1333	0.1206	0.1091	0.0986	0.0891	0.0805	0.0727	0.0656
3.25	0.2411	0.2202	0.2009	0.1832	0.1670	0.1521	0.1385	0.1260	0.1146	0.1042	0.0946	0.0860	0.0781
3.30	0.2656	0.2439	0.2238	0.2053	0.1881	0.1723	0.1578	0.1444	0.1320	0.1207	0.1103	0.1008	0.0920
3.35	0.2910	0.2687	0.2479	0.2285	0.2106	0.1939	0.1785	0.1642	0.1510	0.1388	0.1276	0.1172	0.1076
3.40	0.3173	0.2944	0.2730	0.2530	0.2343	0.2169	0.2006	0.1855	0.1715	0.1585	0.1464	0.1351	0.1247
3.45	0.3441	0.3209	0.2990	0.2784	0.2591	0.2410	0.2241	0.2082	0.1934	0.1796	0.1667	0.1546	0.1434
3.50	0.3716	0.3480	0.3257	0.3047	0.2849	0.2662	0.2487	0.2322	0.2167	0.2021	0.1885	0.1757	0.1637
3.55	0.3994	0.3757	0.3532	0.3318	0.3116	0.2925	0.2744	0.2573	0.2412	0.2260	0.2116	0.1982	0.1855
3.60	0.4274	0.4037	0.3811	0.3595	0.3390	0.3195	0.3010	0.2834	0.2668	0.2510	0.2361	0.2220	0.2087
3.65	0.4555	0.4319	0.4093	0.3877	0.3670	0.3473	0.3285	0.3105	0.2934	0.2772	0.2618	0.2471	0.2332
3.70	0.4836	0.4602	0.4378	0.4162	0.3954	0.3756	0.3565	0.3383	0.3209	0.3043	0.2885	0.2734	0.2589

Table 4 (cont.): Probability Integral of the Standardized Range W_0 for Normal Samples (of size n between 21 and 33)

n W_0	21	22	23	24	25	26	27	28	29	30	31	32	33
3.75	0.5115	0.4885	0.4662	0.4448	0.4241	0.4042	0.3851	0.3668	0.3491	0.3323	0.3161	0.3006	0.2858
3.85	0.5662	0.5441	0.5227	0.5019	0.4817	0.4621	0.4431	0.4247	0.4070	0.3899	0.3733	0.3574	0.3420
3.90	0.5927	0.5713	0.5504	0.5300	0.5102	0.4909	0.4722	0.4540	0.4363	0.4192	0.4027	0.3866	0.3712
3.95	0.6186	0.5979	0.5776	0.5578	0.5384	0.5195	0.5011	0.4831	0.4657	0.4487	0.4322	0.4162	0.4007
4.00	0.6438	0.6238	0.6042	0.5850	0.5662	0.5477	0.5297	0.5121	0.4949	0.4782	0.4618	0.4459	0.4305
4.05	0.6681	0.6490	0.6301	0.6116	0.5933	0.5754	0.5579	0.5407	0.5239	0.5074	0.4914	0.4757	0.4604
4.10	0.6915	0.6733	0.6552	0.6374	0.6198	0.6025	0.5855	0.5688	0.5524	0.5364	0.5206	0.5052	0.4901
4.15	0.7140	0.6966	0.6794	0.6623	0.6455	0.6289	0.6125	0.5963	0.5804	0.5648	0.5494	0.5344	0.5196
4.20	0.7355	0.7190	0.7026	0.6864	0.6703	0.6544	0.6386	0.6231	0.6077	0.5926	0.5777	0.5631	0.5487
4.25	0.7559	0.7404	0.7249	0.7094	0.6941	0.6789	0.6639	0.6490	0.6343	0.6197	0.6054	0.5912	0.5772
4.30	0.7753	0.7607	0.7461	0.7315	0.7170	0.7026	0.6882	0.6740	0.6599	0.6460	0.6322	0.6185	0.6051
4.35	0.7937	0.7800	0.7662	0.7525	0.7388	0.7251	0.7115	0.6980	0.6846	0.6713	0.6581	0.6450	0.6321
4.40	0.8110	0.7981	0.7853	0.7724	0.7595	0.7466	0.7338	0.7210	0.7083	0.6956	0.6831	0.6706	0.6582
4.45	0.8272	0.8153	0.8032	0.7912	0.7791	0.7670	0.7550	0.7429	0.7309	0.7189	0.7070	0.6952	0.6834
4.50	0.8424	0.8313	0.8201	0.8089	0.7976	0.7863	0.7750	0.7637	0.7524	0.7411	0.7298	0.7186	0.7075
4.55	0.8566	0.8463	0.8360	0.8255	0.8150	0.8045	0.7939	0.7833	0.7727	0.7621	0.7516	0.7410	0.7304
4.60	0.8698	0.8603	0.8507	0.8411	0.8313	0.8215	0.8117	0.8018	0.7919	0.7820	0.7721	0.7622	0.7523
4.65	0.8820	0.8733	0.8645	0.8556	0.8466	0.8375	0.8284	0.8192	0.8100	0.8007	0.7915	0.7822	0.7729
4.70	0.8934	0.8854	0.8772	0.8690	0.8607	0.8524	0.8439	0.8354	0.8269	0.8183	0.8097	0.8011	0.7924
4.75	0.9038	0.8965	0.8890	0.8815	0.8739	0.8662	0.8584	0.8506	0.8427	0.8347	0.8267	0.8187	0.8107
4.80	0.9134	0.9067	0.8999	0.8930	0.8861	0.8790	0.8718	0.8646	0.8574	0.8500	0.8427	0.8352	0.8278
4.85	0.9222	0.9161	0.9099	0.9037	0.8973	0.8908	0.8843	0.8776	0.8710	0.8642	0.8574	0.8506	0.8437
4.90	0.9302	0.9247	0.9191	0.9134	0.9076	0.9017	0.8957	0.8897	0.8836	0.8774	0.8711	0.8649	0.8585
4.95	0.9376	0.9326	0.9275	0.9223	0.9170	0.9117	0.9062	0.9007	0.8951	0.8895	0.8838	0.8780	0.8722
5.00	0.9443	0.9398	0.9352	0.9305	0.9257	0.9208	0.9159	0.9109	0.9058	0.9007	0.8955	0.8902	0.8849
5.05	0.9503	0.9463	0.9421	0.9379	0.9336	0.9292	0.9247	0.9202	0.9156	0.9109	0.9062	0.9014	0.8965
5.10	0.9558	0.9522	0.9484	0.9446	0.9407	0.9368	0.9327	0.9286	0.9245	0.9202	0.9159	0.9116	0.9072
5.15	0.9608	0.9575	0.9542	0.9507	0.9472	0.9437	0.9400	0.9363	0.9326	0.9288	0.9249	0.9209	0.9170
5.20	0.9652	0.9623	0.9593	0.9563	0.9531	0.9499	0.9467	0.9433	0.9399	0.9365	0.9330	0.9295	0.9259
5.25	0.9693	0.9667	0.9640	0.9612	0.9584	0.9556	0.9526	0.9497	0.9466	0.9435	0.9404	0.9372	0.9339
5.30	0.9729	0.9705	0.9682	0.9657	0.9632	0.9607	0.9580	0.9554	0.9526	0.9499	0.9471	0.9442	0.9413
5.35	0.9761	0.9740	0.9719	0.9697	0.9675	0.9652	0.9629	0.9605	0.9581	0.9556	0.9531	0.9505	0.9479
5.40	0.9790	0.9771	0.9753	0.9733	0.9714	0.9693	0.9673	0.9651	0.9630	0.9608	0.9585	0.9562	0.9539
5.45	0.9815	0.9799	0.9783	0.9766	0.9748	0.9730	0.9712	0.9693	0.9673	0.9654	0.9634	0.9613	0.9593
5.50	0.9838	0.9824	0.9809	0.9794	0.9779	0.9763	0.9746	0.9730	0.9713	0.9695	0.9677	0.9659	0.9641
5.55	0.9858	0.9846	0.9833	0.9820	0.9806	0.9792	0.9777	0.9763	0.9748	0.9732	0.9716	0.9700	0.9684
5.60	0.9876	0.9865	0.9854	0.9842	0.9830	0.9818	0.9805	0.9792	0.9779	0.9765	0.9751	0.9737	0.9722
5.65	0.9892	0.9882	0.9873	0.9862	0.9852	0.9841	0.9830	0.9818	0.9806	0.9794	0.9782	0.9769	0.9757
5.70	0.9906	0.9898	0.9889	0.9880	0.9871	0.9861	0.9851	0.9841	0.9831	0.9820	0.9809	0.9798	0.9787
5.75	0.9918	0.9911	0.9903	0.9896	0.9887	0.9879	0.9870	0.9862	0.9852	0.9843	0.9834	0.9824	0.9814
5.80	0.9929	0.9923	0.9916	0.9909	0.9902	0.9895	0.9887	0.9880	0.9872	0.9863	0.9855	0.9847	0.9838
5.85	0.9939	0.9933	0.9927	0.9921	0.9915	0.9909	0.9902	0.9895	0.9888	0.9881	0.9874	0.9867	0.9859
5.90	0.9947	0.9942	0.9937	0.9932	0.9926	0.9921	0.9915	0.9909	0.9903	0.9897	0.9891	0.9884	0.9878
5.95	0.9954	0.9950	0.9946	0.9941	0.9936	0.9932	0.9927	0.9922	0.9916	0.9911	0.9905	0.9900	0.9894

Table 4 (cont.): Probability Integral of the Standardized Range W_0 for Normal Samples (of size n between 21 and 33)

n W_0	21	22	23	24	25	26	27	28	29	30	31	32	33
6.00	0.9961	0.9957	0.9953	0.9949	0.9945	0.9941	0.9937	0.9932	0.9928	0.9923	0.9918	0.9913	0.9908
6.05	0.9966	0.9963	0.9960	0.9956	0.9953	0.9949	0.9945	0.9942	0.9938	0.9933	0.9929	0.9925	0.9921
6.10	0.9971	0.9968	0.9965	0.9962	0.9959	0.9956	0.9953	0.9950	0.9946	0.9943	0.9939	0.9935	0.9932
6.15	0.9975	0.9973	0.9970	0.9968	0.9965	0.9962	0.9960	0.9957	0.9954	0.9951	0.9948	0.9944	0.9941
6.20	0.9979	0.9977	0.9974	0.9972	0.9970	0.9968	0.9965	0.9963	0.9960	0.9958	0.9955	0.9952	0.9949
6.25	0.9982	0.9980	0.9978	0.9976	0.9974	0.9972	0.9970	0.9968	0.9966	0.9964	0.9961	0.9959	0.9957
6.30	0.9984	0.9983	0.9981	0.9980	0.9978	0.9976	0.9975	0.9973	0.9971	0.9969	0.9967	0.9965	0.9963
6.35	0.9987	0.9985	0.9984	0.9983	0.9981	0.9980	0.9978	0.9977	0.9975	0.9973	0.9972	0.9970	0.9968
6.40	0.9989	0.9988	0.9986	0.9985	0.9984	0.9983	0.9982	0.9980	0.9979	0.9977	0.9976	0.9974	0.9973
6.45	0.9990	0.9989	0.9988	0.9987	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9979	0.9978	0.9977
6.50	0.9992	0.9991	0.9990	0.9989	0.9988	0.9988	0.9987	0.9986	0.9985	0.9984	0.9983	0.9981	0.9980
6.55	0.9993	0.9992	0.9992	0.9991	0.9990	0.9989	0.9989	0.9988	0.9987	0.9986	0.9985	0.9984	0.9983
6.60	0.9994	0.9994	0.9993	0.9992	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0.9987	0.9986
6.65	0.9995	0.9995	0.9994	0.9994	0.9993	0.9992	0.9992	0.9991	0.9991	0.9990	0.9989	0.9989	0.9988
6.70	0.9996	0.9995	0.9995	0.9995	0.9994	0.9994	0.9993	0.9993	0.9992	0.9992	0.9991	0.9990	0.9990
6.75	0.9996	0.9996	0.9996	0.9995	0.9995	0.9995	0.9994	0.9994	0.9993	0.9993	0.9992	0.9992	0.9991
6.80	0.9997	0.9997	0.9996	0.9996	0.9996	0.9995	0.9995	0.9995	0.9994	0.9994	0.9994	0.9993	0.9993
6.85	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9996	0.9996	0.9995	0.9995	0.9995	0.9994	0.9994
6.90	0.9998	0.9998	0.9997	0.9997	0.9997	0.9997	0.9997	0.9996	0.9996	0.9996	0.9995	0.9995	0.9995
6.95	0.9998	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9997	0.9996	0.9996	0.9996	0.9996
7.00	0.9999	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9997	0.9997	0.9996
7.05	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997
7.10	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9997
7.15	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
7.20	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998
7.25	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

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