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Dynamic Analysis and Control of Automotive Occupant Restraint Systems

Abdulwahab A. Alnaqi[a]* and Ahmet S. Yigit [b]

[a] College of technological Studies, Department of Automotive and Marine Engineering, PAAET, Kuwait

[b] Department of Mechanical Engineering, Kuwait University, Kuwait

Abstract

Although much progress has been made in developing seat belts and mandating their use, the injuries related to seat belts during frontal crashes are still widespread. This paper proposes an approach to control the seat belt restraint system force during a frontal crash to reduce thoracic injuries. A fuzzy logic controller based on moving the attachment point of the seat belt is proposed, and the simulation results with this controller are presented. Also, robustness to parameter variations is investigated. The results show that the proposed controller is very effective in reducing all critical values that lead to possible thoracic injuries during a frontal crash. The controller is also demonstrated to be robust with respect to varying impact conditions and parameters.

Keywords: Seat belt, smart restraint systems, fuzzy logic.

1. Introduction

In today's competitive market, the automotive industry is constantly improving the vehicle design with high speed and varied function. One of the most important pieces of safety equipment in a vehicle is the seat belt. However, as the modern automotive technology has developed and produced luxury vehicles with high performance, the one undesirable result is the high rate of accidents and the increasing severity of injuries when collisions do happen. According to a recent study, the Higher Council of traffic in Kuwait reports that for every 100,000 people 37 people died from car accidents, figure in US is 21 deaths for every 100,000 people. Therefore, the occupant's safety during a collision is very important. Seat belt designed for preventing or reducing the severity of injuries is one of the oldest and most commonly used elements in a vehicle. When used properly, seat belts are about 45 % effective at preventing fatal injuries, and 67 % effective at preventing serious injuries. Despite this safety record, and being mandated by most countries, one should be cautioned though, that the seat belts are also blamed for some serious injuries, themselves. Researchers [1] have studied the seat belt injuries and found that the seat belt injuries include abdominal, chest, or neck bruises and abrasions at the site of the belt contact. It is estimated that 30% of people who came with seat belt injuries suffer from some internal injuries. Seat belt injuries were studies in numerous works in [1- 4].

In order to reduce seat belt injuries different studies dealt with the design of the seat and seat belt [5-7]. Also other studies [8] showed that several vehicle contact points including the steering wheel, door panel, armrest panel, armrest and seat are associated with an increased risk of severe thoracic injury when impacting the occupant.

Realizing the need for improving the performance of these systems, there have been numerous investigations on various aspects of occupant restraint systems used in automotive vehicles in the last decade [9-12]. These studies include the development of four-point harnesses [13, 14], pretensioners and load limiters [15]. Some researchers suggested using active control leading to the development of the so-called "intelligent" restraints. They presented an optimal control of restraint forces in an automobile to reduce the thoracic injury caused by seat belt. Also, authors of [9] studied and investigated ways to reduce occupant injuries that are caused by safety systems like the seat belt and the airbag restraints during a crash. The success of the "intelligent" restraint systems requires a very good understanding of the dynamics phenomena involved during a collision, thus, requiring an adequate model. In this study the effect of seat belt injury is investigated and different approaches to reduce the seat belt injuries through controlling the seat belt force are proposed.

2. Mathematical Model for Thoracic Injury

The dynamics of the human thorax under impact can be described using the mechanical model shown in Figure 1, [16]. In the model, the effective mass of the sternum and a portion of the rib structure and thoracic contents are represented by mass m_1. The remaining portion of the thorax and the part of the total body mass that is coupled to the thorax by the vertebral column are represented by mass m_2. The elasticity and viscous damping of the rib cage and thoracic viscera are

*Corresponding author. eng_abdulwahab1982@yahoo.com

represented by the elements coupling masses m_1 and m_2. The spring and the dashpot model the elastic compliance and the dissipative properties of the thorax, for which the corresponding spring and damping forces, $f(x_2 - x_1)$ and $h(\dot{x}_2 - \dot{x}_1)$ are assumed to be piecewise linear. The action of a seat belt on the thorax is represented by a force $u(t)$ which acts between m_1 and the vehicle. The variables x_v, x_1, x_2 and x_3 represent the displacements with respect to an inertial reference frame (absolute displacements) of the vehicle, mass m_1, mass m_2, and mass m_3, respectively. The parameters for thoracic injury model with a prescribed crash deceleration profile are shown in Table 1.

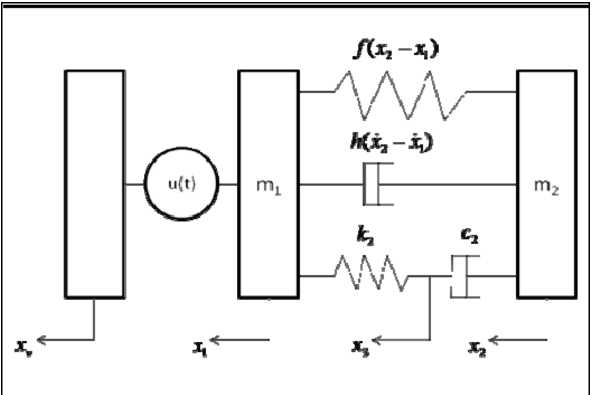

Figure 1. Frontal crash model for thoracic injury

Table 1. The parameters for thoracic injury model

Parameters	Values
m_1	0.3 kg
m_2	18 kg
k_{11}	10522 N/m
k_{12}	7190 N/m
δ_0	0.03 m
c_{11}	403.3 $N.s/m$
c_{12}	2192.1 $N.s/m$
k_2	13153 N/m
c_2	175.4 $N.s/m$
k	10^5 N/m
v_0	48 km/hr

The equations of motion were derived using Newton's second law as follows:

For m_1:

$$m_1\ddot{x}_1 = -u + f(x_2 - x_1) + h(\dot{x}_2 - \dot{x}_1) + k_2(x_3 - x_1) \quad (1)$$

For m_2:

$$m_2\ddot{x}_2 = -f(x_2 - x_1) - h(\dot{x}_2 - \dot{x}_1) - c_2(\dot{x}_2 - \dot{x}_3) \quad (2)$$

It is also required to modify the piece-wise linear functions of the compression deformation and the rate of the chest compression. Using Equations (9) with Equations (7):

For m_3:

$$m_3\ddot{x}_3 = c_2(\dot{x}_2 - \dot{x}_3) - k_2(x_3 - x_1) \quad (3)$$

Then, from Equation (1), (2) and (3) the motion of the vehicle and the occupant is obtained as:

$$m_1\ddot{x}_1 = -u + f(x_2 - x_1) + h(\dot{x}_2 - \dot{x}_1) + k_2(x_3 - x_1)$$
$$m_2\ddot{x}_2 = -f(x_2 - x_1) - h(\dot{x}_2 - \dot{x}_1) - c_2(\dot{x}_2 - \dot{x}_3)$$
$$m_3\ddot{x}_3 = c_2(\dot{x}_2 - \dot{x}_3) - k_2(x_3 - x_1)$$
$$\ddot{x}_v = a(t) \quad (4)$$

This model assumes that the vehicle motion is determined by a prescribed deceleration function. The initial conditions are:

$$x_1(0) = 0, \quad x_2(0) = 0, \quad x_3(0) = 0, \quad x_v(0) = 0$$
$$\dot{x}_1(0) = v_0, \quad \dot{x}_2(0) = v_0, \quad \dot{x}_3(0) = v_0, \quad \dot{x}_v(0) = v_0 \quad (5)$$

where v_0 is the impact velocity and $a(t)$ is the prescribed crash deceleration profile of the vehicle, which is given as follows:

$$a(t) = \begin{cases} -A\sin(\frac{\pi t}{T_p}) & 0 \le t \le T_p \\ 0 & t > 0 \end{cases} \quad (6)$$

where the amplitude of the deceleration profile is given as

$$A = \frac{\pi v_0}{2T_p}$$

The spring force developed in the chest can be defined by the following piecewise linear function of the compression of the chest,

$$f(x_2 - x_1) = \begin{cases} k_{11}(x_2 - x_1) & if \ 0 \le (x_2 - x_1) \le \delta_0 \\ k_{12}(x_2 - x_1) - F_0 & if \ (x_2 - x_1) > \delta_0 \end{cases} \quad (7)$$

where

$$F_0 = (k_{12} - k_{11})\delta_0$$

In addition, the damping force h is given by a piecewise linear function of the rate of the chest compression given as

$$h(\dot{x}_2 - \dot{x}_1) = \begin{cases} c_{11}(\dot{x}_2 - \dot{x}_1) & if \ (\dot{x}_2 - \dot{x}_1) \ge 0 \\ c_{12}(\dot{x}_2 - \dot{x}_1) & if \ (\dot{x}_2 - \dot{x}_1) < 0 \end{cases} \quad (8)$$

The state variables are defined as follows:

$$z_1 = x_1, \quad z_2 = \dot{x}, \quad z_3 = x_2, \quad z_4 = \dot{x}_2, \quad z_5 = x_3, \quad z_6 = \dot{x}_3 \quad (9)$$

Using Equations (9) with Equations (4), the equations of motion can be written in terms of state variables as follows:

$$\dot{z}_1 = z_2 \quad (10)$$

$$\dot{z}_2 = \frac{1}{m_1}(-u + h[z_4 - z_2] + f[z_3 - z_1] + k_2[z_5 - z_1]) \quad (11)$$

$$\dot{z}_3 = z_4 \quad (12)$$

$$\dot{z}_4 = \frac{1}{m_2}(-h[z_4 - z_2] - f[z_3 - z_1] - c_2[z_4 - z_6]) \quad (13)$$

$$\dot{z}_5 = z_6 \quad (14)$$

From equation (3) $m_3\ddot{x}_3 = c_2(\dot{x}_2 - \dot{x}_3) - k_2(x_3 - x_1) = 0$

$$0 = c_2(z_4 - z_6) - k_2(z_5 - z_1)$$

$$z_6 = \frac{-k_2 z_5 + k_2 z_1 + c_2 z_4}{c_2} \quad (15)$$

$$f(z_3 - z_1) = \begin{cases} k_{11}(z_3 - z_1) & if \ 0 \le (z_3 - z_1) \le \delta_0 \\ k_{12}(z_3 - z_1) - F_0 & if \ (z_3 - z_1) > \delta_0 \end{cases} \quad (16)$$

Where

$F_0 = (k_{12} - k_{11})\,\delta_0$

Also using Equations (9) with (8)

$$h(z_4 - z_2) = \begin{cases} c_{11}(z_4 - z_2) & if\ (z_4 - z_2) \geq 0 \\ c_{12}(z_4 - z_2) & if\ (z_4 - z_2) < 0 \end{cases} \qquad (17)$$

In order to find the velocity and vehicle displacement x_v, it is required to integrate the deceleration (Equation (6)) as follows:

$$\ddot{x}_v(t) = a(t) = \begin{cases} -A\sin(\frac{\pi t}{T_p}) & 0 \leq t \leq T_p \\ 0 & t > 0 \end{cases}$$

Thus,

$$x_v(t) = \begin{cases} \frac{1}{2}\frac{v_0 T_P}{\pi}\sin(\frac{\pi t}{T_p}) + \frac{1}{2}v_0 t & 0 \leq t \leq T_p \\ \frac{1}{2}v_0 T_P & t > 0 \end{cases} \qquad (18)$$

The state equations were numerically integrated by using Matlab's ode45 solver. For these simulations a crash velocity of 48 km/h and with impact duration of 0.1 seconds are used to determine the deceleration profile shown in Figure 2, and the simulation was run for 0.12 seconds.

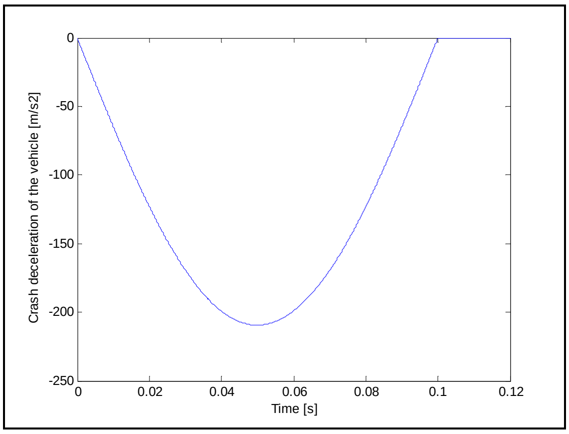

Figure 2. Crash deceleration of the vehicle for the frontal crash with a prescribed deceleration profile ($v_0 = 48\,km/h$)

Extensive studies were conducted in the field of occupant safety to find the injury thresholds that marks the limits for an injury. According to a study of possible prevention of fatal thoracic injuries, the critical value of the chest acceleration is 60g, the critical value of the chest compression is 0.046 m, and the critical value of the chest viscous response can be calculated according which is equal to 0.229 m^2/s. [17]

In order to make sure that the passenger will not have a serious injury, these limits should not be exceeded. Kent et al., (2007) used almost the same value for chest viscous response but the chest acceleration is 80 g and the chest compression is 0.045 m. Figure 3 presents the chest compression$[x_2(t) - x_1(t)]$, the rate of chest compression$[\dot{x}_2(t) - \dot{x}_1(t)]$, the excursion of the occupant$[x_1(t) - x_v(t)]$, seat belt force, the chest viscous response$([\dot{x}_2(t) - \dot{x}_1(t)][x_2(t) - x_1(t)])$, the chest acceleration $\ddot{x}_2(t)$ as a function of time. It was found that the chest compression exceeds the threshold value which is 0.046 m by almost one and a half times (0.11m). Clearly, with this value the occupant will be injured. Also, it is seen from the figure that the maximum value of the chest compression occurred near the end of impact as expected. It was found that the rate of chest compression is within the injury limit (6 m/s). It was found that the occupant will move forward around 5 cm as a result of the large seat belt force. After the impact is finished it is seen that the occupant moves back because there is no seat back force in the model that restricts the motion in this direction. It is seen that the behavior of the response is almost the same as the rate of chest compression with different amplitude. Also it was found that the maximum value of the chest viscous response is within the injury limit (0.229 m^2/s). In addition, it is seen that the maximum value of chest acceleration is within the limit (80 g). Also it can be seen that after the impact the chest acceleration drops to zero.

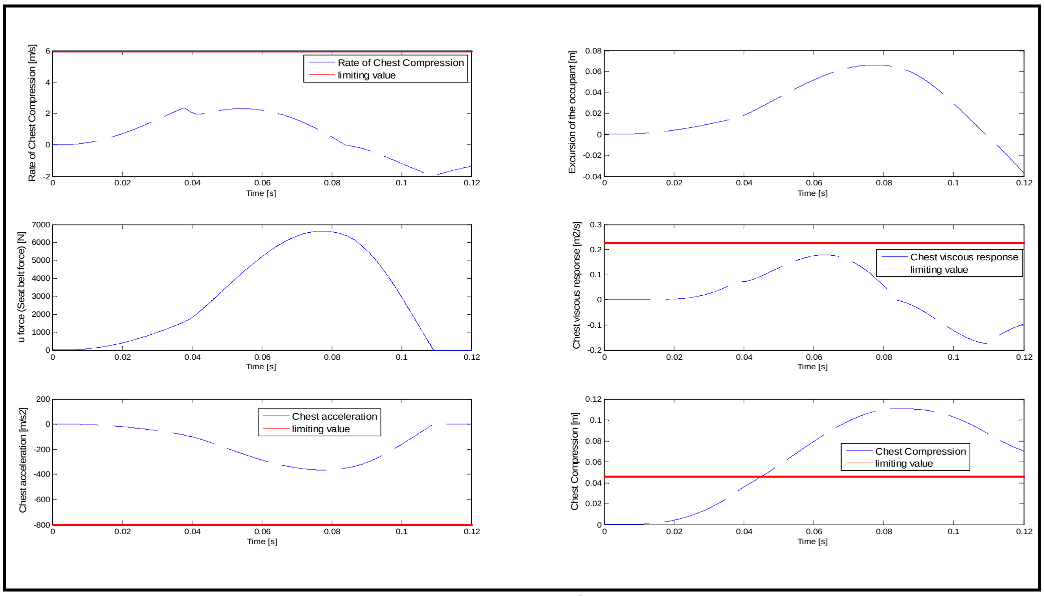

Figure 3. Passive control responses

3. Fuzzy Logic Control

As a benchmark, the performance of an open loop optimal control strategy as well as a feedback controller based on optimal Linear Quadratic Regulator (LQR) approach are investigated. Though very effective in reducing injuries, these controllers were shown to be very sensitive to parameter variations, and modelling uncertainties. As it is well known, to design a control system using a traditional approach such as optimal control, adaptive control or predictive control, it is necessary to have at least a nominal model describing the behavior of the system to be controlled. However, obtaining an adequate and accurate model is very difficult in some cases due to the complex behavior or due to the existence of nonlinearities. This leads to consider other control design approaches which do not rely on any models. One of the successful methods is the fuzzy logic control. In this section the design of a fuzzy controller is explained, and the simulation results when this controller is used in the system are presented. Also, robustness to parameter variations is investigated.

3.1 Design of a fuzzy controller

A fuzzy logic controller can be considered as a control expert system which simulates the occupant thinking. It consists of input and output variables with membership functions, a set of (IF…THEN) rules and an inference system. The controller's inputs are chosen to be the outputs of the process which are x_1 (occupant displacement) and $\dot{x}_1$ (occupant speed) and output is the seat belt force u. It should be noted that these control variables are chosen because they facilitate design of fuzzy rules. In a real implementation, the seat belt force is realized by manipulating the location of the attachment point (i.e., by controlling the length of active portion of the seat belt). Therefore, in effect, the fuzzy controller is trying to control the seat belt force to prevent thoracic injury.

Define five linguistic values denoted as BNE (Big Negative), NE (Negative), ZE (Zero), PO (Positive) and BPO (Big Positive) for each input variables as shown in Figures 4 and 5. The output variable has 9 linguistic values denoted as N_j (j=1 to 9). The index j represents the strength of the linguistic values such that the higher the index, the stronger the linguistic value as shown in Figure 6.

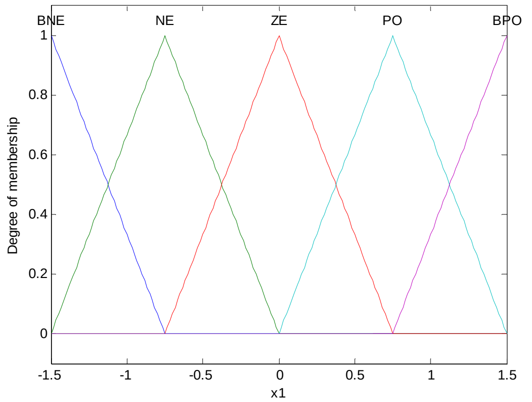

Figure 4. Input x_1 membership function

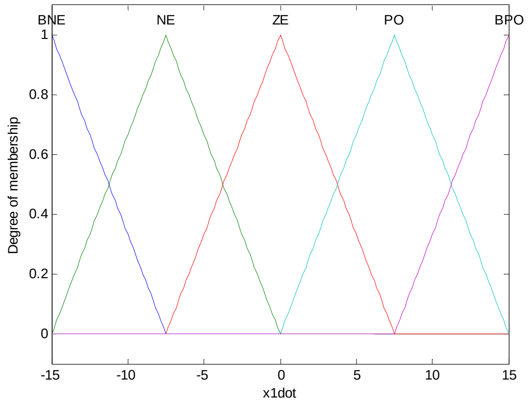

Figure 5. Input $\dot{x}_1$ membership function

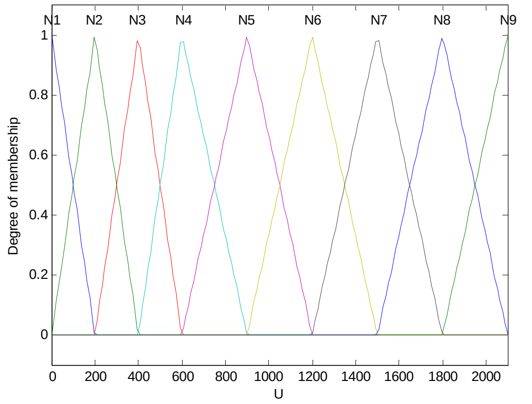

Figure 6. Output u membership function

The Sugeno model (Wang, 1996) is used as the basis of the proposed fuzzy controller. The rule base consists of 25 (IF…THEN) rules derived from occupant knowledge which is presented in Table 3.

Table 3. Rule base system

#	x_1	$\dot{x}_1$	u	#	x_1	$\dot{x}_1$	U
1	BPO	BNE	N7	14	ZE	PO	N7
2	BPO	NE	N7	15	ZE	BPO	N8
3	BPO	ZE	N7	16	NE	BNE	N1
4	BPO	PO	N8	17	NE	NE	N1
5	BPO	BPO	N9	18	NE	ZE	N1
6	PO	BNE	N7	19	NE	PO	N1
7	PO	NE	N8	20	NE	BPO	N1
8	PO	ZE	N6	21	BNE	BNE	N1
9	PO	PO	N7	22	BNE	NE	N1
10	PO	BPO	N8	23	BNE	ZE	N1
11	ZE	BNE	N1	24	BNE	PO	N1
12	ZE	NE	N1	25	BNE	BPO	N1
13	ZE	ZE	N3				

3.2 Simulation results for the nominal case

Matlab is used to implement the fuzzy controller with the dynamic equations in terms of state variables which were defined in the mathematical model section. The simulation time and integration step are chosen as 0.2s and 0.0005s, respectively.

The seat belt force to be applied can be realized by using the concept of releasing and retracting the seat belt or by moving the point of attachment of the seat belt relative to the vehicle (smart seat belt). The motion of the attachment point can be determined by using the control force as follows:

$$X = x_1 - x_v - \frac{u}{k} \tag{19}$$

From Figure 7 which presents the fuzzy control response, it is noticed that the behavior of the chest compression response is almost within the injury threshold limit. On the other hand, it is concluded that using fuzzy control improves the rate of chest compression with respect to magnitude. As can be seen fuzzy control approach improves the excursion of the occupant as compared to the LQR and optimal open loop control with respect to the magnitude (not presented in this paper due to space limitations). Overall behavior is almost the same. From the figure the behavior of the seat belt force is quite different than that of the optimal control and the LQR. It was seen that both the chest viscous response and the chest acceleration response are under the injury thresholds and comparing with the results obtained from the optimal control and the LQR it is seen that fuzzy control greatly improves the response. It is seen From Figure 8 that the behavior of the displacement of the attachment point response is almost the same as the excursion of the occupant with different amplitude.

It was noticed from the simulation results that using fuzzy logic control improves the response with respect to magnitudes as compared to the LQR and open loop optimal control results. This is remarkable considering that fuzzy logic control does not depend on the system model, and therefore, will not suffer from the inaccuracies of in the model. Indeed, it was found that the responses obtained using fuzzy logic control were less sensitive to parameter variations as compared to the responses obtained from the LQR. Therefore, fuzzy logic control has great potential for implementation since all it requires is the measurement of the output variables. In this work the design of actuator dynamic is not included. In a real implementation, the actuator design and dynamics is very important and should be included in the model. Furthermore, in a real implementation, with current instrumentation it is difficult to measure the occupant motion. In this case, the occupant position and velocity can be obtained from the measurement of vehicle motion and the seat belt force. Clearly, this will make the control somewhat model-based, and care should be taken to investigate the effects of modeling errors. However, due to the robust nature of the fuzzy controller, it is expected that the performance degradation due to state estimation will be acceptable.

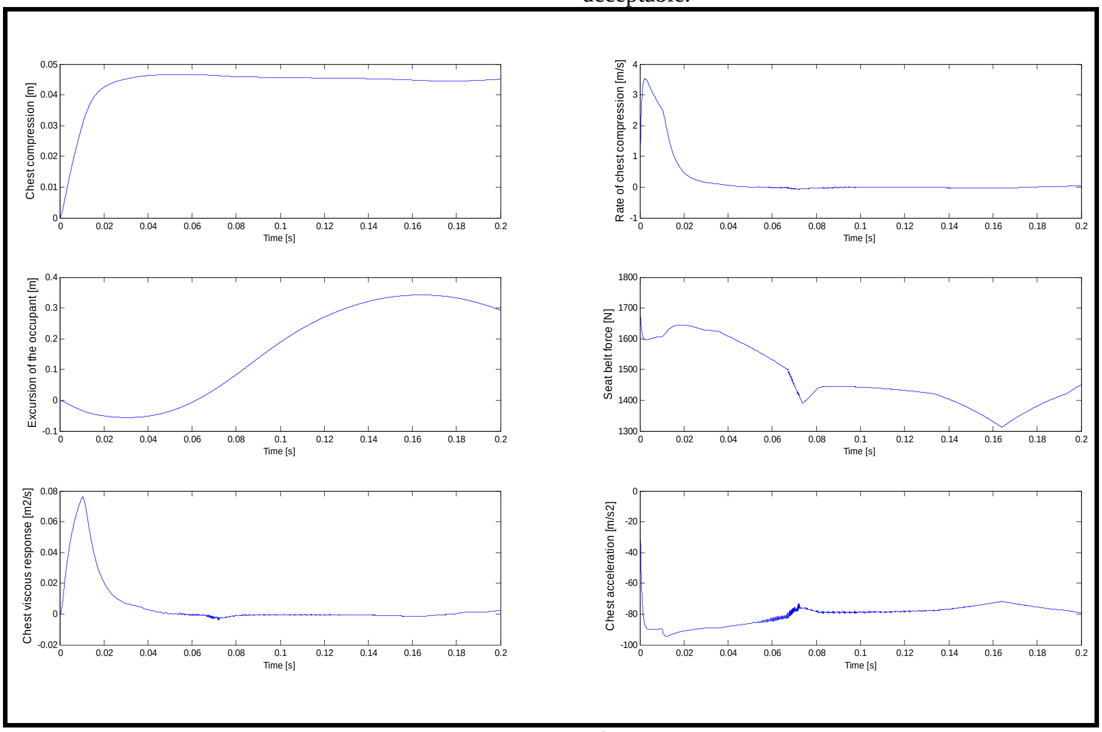

Figure 7. Fuzzy control responses

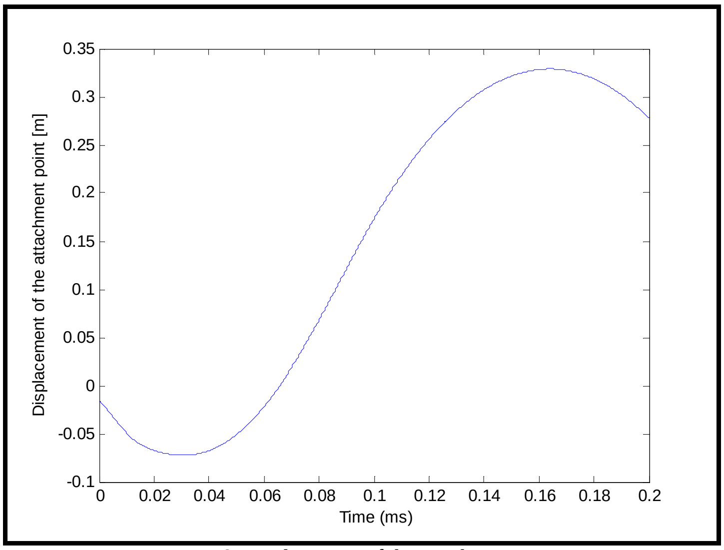

Figure 8. Displacement of the attachment point

4. CONCLUSIONS

A study has been carried out to find ways of reducing thoracic injuries that are caused by seat belt restraint system in a frontal crash. A comprehensive literature review was carried out and a mathematical model with the equation of motion was presented in details. A fuzzy control logic approach was investigated. It was found that using fuzzy logic approach improves the response as compared to the LQR and open loop optimal control. Also it was noticed that the responses obtained using fuzzy logic control is less sensitive to parameter variation. Although this control is non-model based, the implementation may require state estimation by using the model, and this will destroy this attractive property.

Although most of the objectives of the work were achieved for controlling the seat belt system in order to prevent thoracic injury, a number of issues require further work: The frontal crash model was adequate for the purpose of the study. However, for future investigations a more detailed model with the back force of the seat is required to obtain better understanding and more accurate results. Also the number of degrees of freedom could be increased for a more realistic representation.

The design of fuzzy logic controller can also be improved. More sophisticated methods for developing the rule base can be employed [Wang, 1996]. Different combinations with various robust controllers can be applied to improve the robustness (e.g., Neuro-fuzzy logic controllers, fuzzy sliding mode controllers, etc.). In this work, the actuator dynamics is not considered. In a real implementation, the actuator design and dynamics is very important and should be included in the model. One of the available crash simulation software packages can be used to test the performance of the proposed controllers. Experimental work is needed for the frontal crash scenario to validate the results obtained from simulations. The experimental work is needed also for model refinement.

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NOMENCLATURE

A	Amplitude of the crash deceleration half-sine pulse
a	Deceleration of the base (crash deceleration)
c_2	damping coefficient of the dashpot
c_{11}, c_{12}	damping coefficients of the thorax when being compressed and when the shape is being restored, respectively
f	elastic characteristic of the rib cage
g	acceleration due to gravity
h	thoracic damping characteristic
k	coefficient of stiffness of the restraint
k_2	stiffness coefficient of the spring in the series connected spring-dashpot component in the thoracic injury model
k_{11}, k_{12}	coefficients of the rib cage stiffness for small and large strains, respectively
m_1	The effective mass of the sternum and a portion of the rib structure and thoracic contents
m_2	The remaining portion of the thorax and the part of the total body mass that is coupled to the thorax by the vertebral column
T_p	duration of the crash deceleration half-sine pulse
u	control force acting on the object to be protected(a car occupant)
x_1	absolute displacement of the sternum in the two-mass thorax injury model
x_2	absolute displacement of the rear portion of the thorax in the two-mass thorax injury model
x_3	absolute displacement of the point at which the spring and the dashpot connected in series are joined in the two-mass thorax injury model
x_v	absolute displacement of the vehicle
δ_0	threshold deformation of the rib cage at which the stiffness coefficient k_{11} changes to k_{12}
v_0	impact velocity