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Production Inventory Model for Deteriorating Items with On-Hand Inventory and Time Dependent Demand

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Abstract

Production inventory model plays a dominant role in production scheduling and planning. For the determination of optimal downtime, uptime of production and production quantity, it is required to minimize the expected total cost. The total cost of production is dependent on demand, production rate and rate of decay in deteriorating items. In this paper, we develop and analyze the production inventory model for deteriorating items by assuming that the demand is a function of both on-hand inventory and time. It is also assumed that the lifetime of commodity is random and follows a Weibull distribution. The sensitivity of the model is analyzed with respect to the parameters and costs. A case study is carried out to determine production schedules in a pickle manufacturing industry. This model also includes other production-level inventory models as particular cases for specific values of the parameters.

Keywords: Production cycle, Weibull decay, On-hand inventory and time, Production scheduling.

1. Introduction

Recently much emphasis is given to study the control and maintenance of production inventories of the deteriorating items. The deterioration of inventory on stocks during the storage period constituents an important factor. The deterioration in general may be considered because of various effects on the stock, some of which are damage, spoilage, obsolescence, decay, decreasing usefulness and many more. For example, in manufacturing industries like drugs, pharmaceuticals, food products, radioactive substances, the item deteriorates over a period. Nahmias [1], Raafat [2], Goyal and Giri [3] reviewed inventory models for deteriorating items. Cohen [4], Aggarwal [5], Dave and Shah [6], Pal [7], Kalpakham and Sapna [8], Giri and Chaudhari [9] developed the inventory models with exponential lifetime. Tadikamalla [10] developed inventory model with Gamma distribution for deterioration. Covert and Philip [11], Philip [12], Goel and Aggarwal [13], Hwang [14] and Venkatasubbaiah et.al [15] discussed inventory models with Weibull distribution for the lifetime of the commodity. Nirupamadevi et.al [16], [17] studied the inventory models with the assumption that the lifetime of the commodity follows a mixture of Weibull distribution. Lakshminarayana et.al [18] suggested inventory models for deteriorating items with exponential, Weibull and mixture of Weibull lifetime distributions having seconds' sale.

In all these papers, they assumed that the replenishment is instantaneous but in production processes, the replenishment is finite. Srinivasa Rao et.al [19] developed a production inventory model with generalized Pareto lifetime and time dependent demand. Mahapatra and Maity [20], Halim, Giri and Chaudhuri [21] studied the production inventory models for deteriorating items with fuzzy deterioration rate. In these models, they assumed that the demand is dependent on stock or on-hand inventory. Ouyang et.al [22] studied the inventory models with stock dependent demand. Bhowmick et.al [23] suggested a continuous deterministic inventory system for deteriorating items with inventory-level dependent time varying demand. Jie Min et.al. [24] proposed a perishable inventory model with a stock dependent selling rate. They also considered the demand rate is dependent on the negative inventory level during the stock out period. Lee and Hsu [25] developed a two-warehouse inventory model for deteriorating items with time-dependent demand. Manna and Chiang [26] developed two deterministic economic production quantity models for Weibull-distribution deteriorating items with demand rate as a ramp type function of time. Tripathy and Mishra [27] studied an inventory model for weibull deteriorating items with price dependent demand and time-varying holding cost. In all these papers, they have considered that the demand is a function of either stock dependent or time dependent. However, in many manufacturing processes of deteriorating items, the demand is a function of both on-

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hand inventory and time. For example, in Seafood processing units, the rate of deterioration is variable and time dependent. Hence, in this paper, we develop and analyze an inventory model for deteriorating items with the assumption that the lifetime of commodity is random and follows a Weibull distribution. The Weibull rates of decay include increasing/ decreasing/ constant rates of decay. We also assume that the demand is a linear function of on hand inventory and power pattern demand. This model is a general one as it includes several of the earlier models as particular cases for specific values of parameters. We have developed two models by considering with and without shortages where as with shortages model is discussed in detail.

2. Assumptions and Notations

The production inventory model for deteriorating items is developed under the following assumptions and notations:

2.1. Assumptions

i) The production inventory system involves only one type of item.

ii) The life time of commodity is random and follows a three parameter Weibull distribution with probability density function of the form

$$f(t)=\alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}},\ \alpha,\beta>0, t>\gamma;$$

(Johnson et.al,1995) [28]

where, α (alpha) is the shape parameter, β (beta) is the scale parameter and γ (gamma) is the location parameter. The Weibull distribution for deterioration is assumed since in many deteriorating items, the rate of deterioration is a variable depending on time having increasing / decreasing / constant rates of decay. It is reasonable to assume that the deterioration starts only after certain period of life, which is equivalent to γ, hence, the instantaneous rate of deterioration is

$$h(t)=\frac{f(t)}{1-F(t)}=\alpha\beta(t-\gamma)^{\beta-1},\ t>\gamma$$

where, F(t) is the cumulative density function of the Weibull distribution.

This Weibull distribution includes exponential distribution as particular case when $\beta=1, \gamma=0$ and truncated exponential distribution when $\beta=1$.

iii) There is no repair or replacement of the deteriorated item during the production cycle and the deteriorated item is thrown as a scrap.

iv) The rate of production is governed by supply and is finite say (R) .The production rate is greater than demand rate and system is in steady state during production.

v) The rate of demand is a function of quantity as equation(1)

$$D(t)=\left\{\tau+\varphi_1 I(t)+\varphi_2\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\},\quad 0<t\leq T,\quad 0\leq\varphi_1<1, 0\leq\varphi_2<1,\ \tau>0 \tag{1}$$

where, r is demand rate, n is pattern index; φ_1, φ_2 (phi) and τ (tau) are positive constants which can be varied depending on the demand rate. This assumption is considered taking the linear relationship between production and demand. If $\varphi_1=0, \varphi_2=0$, the rate of demand becomes constant and $D(t)=\tau$. If $\varphi_1=0$ and $\tau=0$, the demand rate becomes power pattern demand. If $\varphi_2=0$, the demand rate becomes stock dependent demand.

vi) In shortages model, shortages are allowed and fully backlogged. During the shortages period, the backlogging rate is the surplus available after fulfilling the on hand demand and there is a penalty (π) (pi) for not meeting the demand.

2.2. Notations

The notations employed in this paper are as given below:

A - Setup cost

C - Unit cost

h - Holding cost of a unit per unit time

π - Shortage cost

K(t_1, t_3, T) - The total cost of the system per unit time with shortages model

Q - Total quantity of items produced per unit time

R - Rate of Production of items per unit time

I(t) - On hand inventory at time t

γ - The time point at which deterioration starts

t_1 - The time point at which production stopped

t_2 - The time point at which shortages occur

t_3 - The time point at which the production re commences

T - Production cycle time

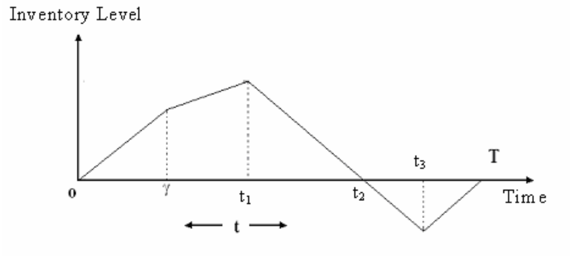
3. Mathematical Modeling

Here, we have considered a production inventory system for deteriorating items, which is assumed to follow the pattern as, described. The production starts when inventory is zero and the produced items meet the demand and deterioration. The production is stopped, when stock reaches to a maximum inventory level and allowed to reach zero gradually due to the demand and deterioration. Shortages are allowed and backlogged until some time interval and at the same time production starts to clear the backlogging and the regular demand until stock becomes zero.

Consider a production-level inventory model, in which shortages are allowed. The production starts at time t = 0, when the stock is zero and reaches to a maximum inventory level at time t = t_1. The time interval is divided into two non-overlapping intervals (0, γ) and (γ, t_1).

During the interval $(0, \gamma)$, the produced items partly meet the demand and during interval (γ, t_1), the produced items are partly consumed due to the demand and deterioration and excess items are stored. The production is stopped at time $t = t_1$ and the stock level is allowed to reduce gradually due to the demand and deterioration and at time $t = t_2$, the inventory becomes zero. Shortages are permitted until time $t = t_3$ and production starts at this instant of time. During the time period (t_3, T), all the backlogged shortages are cleared in addition to fulfilling the on hand demand and the cycle repeats thereafter. The above inventory model is represented in Fig.1.

Fig.1 The inventory system - with shortages.



Let I(t) denote the inventory level of the system at time t ($0 \le t \le T$).

The differential equations describing the instantaneous state of I(t) in the interval (0, T) are given by

$$\frac{d}{dt}I(t) = R - \left\{\tau + \varphi_1 I(t) + \varphi_2 \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}, \qquad 0 \le t \le \gamma \tag{1}$$

$$\frac{d}{dt}I(t) + \alpha\beta(t-\gamma)^{\beta-1} I(t) = R - \left\{\tau + \varphi_1 I(t) + \varphi_2 \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}, \qquad \gamma \le t \le t_1 \tag{2}$$

$$\frac{d}{dt}I(t) + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -\left\{\tau + \varphi_1 I(t) + \varphi_2 \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}, \qquad t_1 \le t \le t_2 \tag{3}$$

$$\frac{d}{dt}I(t) = -\tau, \qquad t_2 \le t \le t_3 \tag{4}$$

$$\frac{d}{dt}I(t) = R - \tau, \qquad t_3 \le t \le T \tag{5}$$

with the boundary conditions $I(0) = 0$, $I(t_2) = 0$ and $I(T) = 0$

By solving the equations (1), (2), (3), (4), and (5) and using boundary conditions, we obtain the instantaneous state of inventory at any given time t, during the interval $(0, \gamma)$ is

$$I(t) = e^{-\varphi_1 t}\int_0^t \left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\varphi_1 u} du, \qquad 0 \le t \le \gamma \tag{6}$$

The instantaneous state of inventory at any time t, during the interval (γ, t_1) is

$$I(t) = e^{-\{\alpha(t-\gamma)^\beta + \phi_1 t\}}\left[\int_\gamma^t \left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\alpha(u-\gamma)^\beta + \varphi_1 u} du + \int_0^\gamma \left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\varphi_1 u} du\right], \qquad \gamma \le t \le t_1 \tag{7}$$

The instantaneous state of inventory at any time t, during the interval (t_1, t_2) is

$$I(t) = e^{-\{\alpha(t-\gamma)^\beta + \phi_1 t\}}\left[\int_t^{t_2} \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\} e^{\alpha(u-\gamma)^\beta + \varphi_1 u} du\right], \qquad t_1 \le t \le t_2 \tag{8}$$

The instantaneous state of inventory at any time t, during the interval (t_2, t_3) is

$$I(t) = \tau(t_2 - t),\ t_2 \le t \le t_3 \tag{9}$$

The instantaneous state of inventory at any time t, during the interval (t_2, T) is

$$I(t) = (R - \tau)(t - T),\ t_3 \le t \le T, \tag{10}$$

From the equations (9) and (10), we get

$$t_2 = \frac{Rt_3 - (R-\tau)T}{\tau}, \tag{11}$$

The total production in the cycle time T is

$$Q = R\,t_1 + R\,(T - t_3), \tag{12}$$

The Stock loss due to deterioration in the interval (0, T) is given by

$$L(t) = R\,t_1 - \int_0^T \left\{ \tau + \varphi_1 I(t) + \varphi_2 \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right\} dt, \tag{13}$$

Backlogged demand at time t is

$$B(t) = \int_{t_2}^{t_3} \left\{ \tau + \varphi_1 I(t) + \varphi_2 \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right\} dt, \tag{14}$$

The total cost per unit time K (t_1, t_3, T) is the sum of the set up cost per unit time, purchasing cost per unit time, holding cost per unit time and shortage cost per unit time.

Therefore,

$$K(t_1, t_3, T) = \frac{A}{T} + \frac{C}{T}Q + \frac{h}{T}\left[\int_0^{\gamma} I(t)dt + \int_{\gamma}^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt\right], \tag{15}$$

By substituting the values of I(t) and Q from the equations (6) to (10) and (12) in equation (15), we get

$$K(t_1, t_3, T) = \frac{A}{T} + \frac{C}{T}\left[Rt_1 + R(T - t_3)\right] + \frac{h}{T}\left[\int_0^{\gamma} e^{-\varphi_1 t} \int_0^{t}\left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\varphi_1 u} du\, dt\right.$$

$$+ \int_{\gamma}^{t_1} e^{-\{\alpha(t-\gamma)^{\beta} + \phi_1 t\}} \left[\int_{\gamma}^{t}\left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\alpha(u-\gamma)^{\beta} + \varphi_1 u} du + \int_0^{\gamma}\left[R - \left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right] e^{\varphi_1 u} du\right] dt$$

$$\left. + \int_{t_1}^{t_2} e^{-\{\alpha(t-\gamma)^{\beta} + \phi_1 t\}} \left[\int_t^{t_2}\left\{\tau + \varphi_2 \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\} e^{\alpha(u-\gamma)^{\beta} + \varphi_1 u} du\right] dt\right] +, \frac{\pi}{T}\left[\int_{t_2}^{t_3} -\tau(t_2 - t)dt + \int_{t_3}^{T} -(R - \tau)(t - T)dt\right] \tag{16}$$

Using the truncated Taylor's series expansion for exponential function and on integrating and simplifying the equation (16), we get

$$K(t_1, t_3, T) = \frac{A}{T} + \frac{C}{T}\left[Rt_1 + R(T - t_3)\right] + \frac{h}{T}\left[\frac{R-\tau}{\varphi_1}\left\{\gamma + \frac{e^{-\varphi_1 \gamma} - 1}{\varphi_1}\right\} - \frac{n\varphi_2 r}{T^{\frac{1}{n}}}\left\{\frac{\gamma^{\frac{1}{n}+1}}{1+n} - \frac{\varphi_1 \gamma^{\frac{1}{n}+2}}{1+2n}\right.\right.$$

$$\left. + \frac{\varphi_1 \gamma^{\frac{1}{n}+2}}{(1+n)(1+2n)} - \frac{\varphi_1 \gamma^{\frac{1}{n}+3}}{(1+n)(1+2n)}\right\} + (R-\tau)\left[\frac{t_1^2}{2} - \frac{\gamma^2}{2} + \frac{\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\varphi_1}{6}\left(t_1^3 - \gamma^3\right)\right.$$

$$- \frac{\alpha}{(\beta+1)}\left\{t_1(t_1-\gamma)^{\beta+1} - \frac{(t_1-\gamma)^{\beta+2}}{\beta+2}\right\} - \frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2} - \frac{\alpha\varphi_1}{2}\left\{t_1^2\frac{(t_1-\gamma)^{\beta+1}}{\beta+1} - \frac{2}{(\beta+1)(\beta+2)}\right.$$

$$\Big\{ t_1(t_1-\gamma)^{\beta+2} - \frac{(t_1-\gamma)^{\beta+3}}{\beta+3} \Big\}\Big\} - \frac{\varphi_1}{3}\left(t_1^3-\gamma^3\right) - \frac{\alpha\varphi_1}{\beta+1}\Big\{ \frac{t_1(t_1-\gamma)^{\beta+2}}{\beta+2} - \frac{(t_1-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)} \Big\}$$

$$-\frac{\varphi_1^2}{8}\left(t_1^4-\gamma^4\right)\Big] - \frac{\varphi_2 r}{nT^{\frac{1}{n}}}\Big[\frac{n^2}{1+n}\left(t_1^{\frac{1}{n}+1}-\gamma^{\frac{1}{n}+1}\right)+\alpha\Big\{\frac{n}{1+\beta n}\Big\{\frac{n}{1+n\beta+n}\left(t_1^{\frac{1}{n}+\beta+1}-\gamma^{\frac{1}{n}+\beta+1}\right)$$

$$-\gamma^{\frac{1}{n}+\beta}(t_1-\gamma)\Big\} - \frac{n\beta\gamma}{1-n+\beta n}\Big(\frac{n}{1+n\beta}\left(t_1^{\frac{1}{n}+\beta}-\gamma^{\frac{1}{n}+\beta}\right)-\gamma^{\frac{1}{n}-1+\beta}(t_1-\gamma)\Big)\Big\}+\frac{\varphi_1 n^2}{(1+n)(1+2n)}$$

$$\left(t_1^{\frac{1}{n}+2}-\gamma^{\frac{1}{n}+2}\right)-\alpha n\Big\{\frac{n}{1+n\beta+n}\left(t_1^{\frac{1}{n}+\beta+1}-\gamma^{\frac{1}{n}+\beta+1}\right)-\frac{n\beta\gamma}{1+n\beta}\left(t_1^{\frac{1}{n}+\beta}-\gamma^{\frac{1}{n}+\beta}\right)\Big\}$$

$$-\alpha^2\Big\{\frac{n}{1+\beta n}\Big\{\frac{n}{1+2\beta n+n}\left(t_1^{\frac{1}{n}+2\beta+1}-\gamma^{\frac{1}{n}+2\beta+1}\right)-\frac{n\beta\gamma}{1+2\beta n}\left(t_1^{\frac{1}{n}+2\beta}-\gamma^{\frac{1}{n}+2\beta}\right)-\frac{\gamma^{\frac{1}{n}+\beta}}{\beta+1}(t_1-\gamma)^{\beta+1}\Big\}$$

$$-\frac{n\beta\gamma}{1-n+\beta n}\Big\{\frac{n}{1+2\beta n}\left(t_1^{\frac{1}{n}+2\beta}-\gamma^{\frac{1}{n}+2\beta}\right)-\frac{n\beta\gamma}{1+2\beta n-n}\left(t_1^{\frac{1}{n}+2\beta-1}-\gamma^{\frac{1}{n}+2\beta-1}\right)-\frac{\gamma^{\frac{1}{n}-1+\beta}}{\beta+1}(t_1-\gamma)^{\beta+1}\Big\}\Big\}$$

$$-\frac{\alpha\varphi_1 n}{1+n}\Big\{\frac{n}{1+2n+\beta n}\left(t_1^{\frac{1}{n}+2+\beta}-\gamma^{\frac{1}{n}+2+\beta}\right)-\frac{n\beta\gamma}{1+\beta n+n}\left(t_1^{\frac{1}{n}+\beta+1}-\gamma^{\frac{1}{n}+\beta+1}\right)\Big\}-\frac{\varphi_1 n^2}{1+2n}\left(t_1^{\frac{1}{n}+2}-\gamma^{\frac{1}{n}+2}\right)$$

$$-\alpha\varphi_1\Big\{\frac{n}{1+\beta n}\Big\{\frac{n}{1+2n+\beta n}\left(t_1^{\frac{1}{n}+2+\beta}-\gamma^{\frac{1}{n}+2+\beta}\right)-\frac{\gamma^{\frac{1}{n}+\beta}}{2}\left(t_1^2-\gamma^2\right)\Big\}-\frac{n\beta\gamma}{1-n+\beta n}\Big\{\frac{n}{1+\beta n+n}$$

$$\left(t_1^{\frac{1}{n}+\beta+1}-\gamma^{\frac{1}{n}+\beta+1}\right)-\frac{\gamma^{\frac{1}{n}-1+\beta}}{2}\left(t_1^2-\gamma^2\right)\Big\}\Big\}-\frac{\varphi_1^2 n^2}{(1+n)(1+3n)}\left(t_1^{\frac{1}{n}+3}-\gamma^{\frac{1}{n}+3}\right)\Big]+\tau\Big\{t_2(t_2-t_1)-\frac{\left(t_2^2-t_1^2\right)}{2}$$

$$+\frac{\alpha}{\beta+1}\Big\{(t_2-\gamma)^{\beta+1}(t_2-t_1)-\frac{(t_2-\gamma)^{\beta+2}}{\beta+2}+\frac{(t_1-\gamma)^{\beta+2}}{\beta+2}\Big\}+\frac{\varphi_1 t_2^2}{2}(t_2-t_1)-\frac{\varphi_1}{6}\left(t_2^3-t_1^3\right)\Big\}$$

$$+\frac{\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{nt_2^{\frac{1}{n}}(t_2-t_1)-\frac{n^2}{1+n}\left(t_2^{\frac{1}{n}+1}-t_1^{\frac{1}{n}+1}\right)+\alpha\Big\{\frac{n}{n\beta+1}\Big\{t_2^{\frac{1}{n}+\beta}(t_2-t_1)-\frac{n}{1+\beta n+n}\left(t_2^{\frac{1}{n}+\beta+1}-t_1^{\frac{1}{n}+\beta+1}\right)\Big\}$$

$$-\frac{\beta\gamma n}{1-n+\beta n}\Big\{t_2^{\frac{1}{n}-1+\beta}(t_2-t_1)-\frac{n}{1+\beta n}\left(t_2^{\frac{1}{n}+\beta}-t_1^{\frac{1}{n}+\beta}\right)\Big\}\Big\}+\frac{n\varphi_1}{1+n}\Big\{t_2^{\frac{1}{n}+1}(t_2-t_1)-\frac{n}{1+2n}\left(t_2^{\frac{1}{n}+2}-t_1^{\frac{1}{n}+2}\right)\Big\}\Big\}$$

$$-\alpha\tau\Big\{\frac{t_2}{\beta+1}\Big\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\Big\}-\frac{1}{\beta+1}\Big\{t_2(t_2-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+1}-\frac{1}{\beta+2}\Big\{(t_2-\gamma)^{\beta+2}$$

$$-(t_1-\gamma)^{\beta+2}\Big\}+\frac{\alpha}{(\beta+1)^2}\Big\{(t_2-\gamma)^{\beta+1}\Big\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\Big\}-\frac{(t_2-\gamma)^{2(\beta+1)}}{2}+\frac{(t_1-\gamma)^{2(\beta+1)}}{2}\Big\}$$

$$+\frac{\varphi_1}{2}\Big\{\frac{-t_2^2}{\beta+1}(t_1-\gamma)^{\beta+1}\Big\}-\Big\{-\frac{t_1^2}{\beta+1}(t_1-\gamma)^{\beta+1}-\frac{2}{(\beta+1)(\beta+2)}\Big\{t_2(t_2-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+2}$$

$$-\frac{(t_2-\gamma)^{\beta+3}}{\beta+3}+\frac{(t_1-\gamma)^{\beta+3}}{\beta+3}\Big\}\Big\}\Big\}\Big\}-\frac{\alpha\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{n\Big\{\frac{t_2^{\frac{1}{n}}}{\beta+1}\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}-\Big\{\frac{n}{1+n\beta+n}$$

$$\Big\{t_2^{\frac{1}{n}+\beta+1}-t_1^{\frac{1}{n}+\beta+1}\Big\}-\frac{n\beta\gamma}{1+n\beta}\Big\{t_2^{\frac{1}{n}+\beta}-t_1^{\frac{1}{n}+\beta}\Big\}\Big\}\Big\}\Big\}+\alpha\Big\{\frac{n}{n\beta+1}\Big\{\frac{t_2^{\frac{1}{n}+\beta}}{\beta+1}\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}$$

$$-\Big\{\frac{n}{1+2\beta n+n}\Big(t_2^{\frac{1}{n}+2\beta+1}-t_1^{\frac{1}{n}+2\beta+1}\Big)-\frac{n\beta\gamma}{1+2\beta n}\Big(t_2^{\frac{1}{n}+2\beta}-t_1^{\frac{1}{n}+2\beta}\Big)\Big\}\Big\}-\frac{\beta\gamma n}{1-n+\beta n}\Big\{\frac{t_2^{\frac{1}{n}-1+\beta}}{\beta+1}$$

$$\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}-\Big\{\frac{n}{1+2\beta n}\Big(t_2^{\frac{1}{n}+2\beta}-t_1^{\frac{1}{n}+2\beta}\Big)-\frac{n\beta\gamma}{1+2\beta n-n}\Big(t_2^{\frac{1}{n}+2\beta-1}-t_1^{\frac{1}{n}+2\beta-1}\Big)\Big\}\Big\}\Big\}\Big\}$$

$$+\frac{n\varphi_1}{1+n}\Big\{\frac{t_2^{\frac{1}{n}+1}}{\beta+1}\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}-\Big\{\frac{n}{1+2n+\beta n}\Big(t_2^{\frac{1}{n}+2+\beta}-t_1^{\frac{1}{n}+2+\beta}\Big)$$

$$-\frac{n\beta\gamma}{1+\beta n+n}\Big(t_2^{\frac{1}{n}+\beta+1}-t_1^{\frac{1}{n}+\beta+1}\Big)\Big\}\Big\}\Big\}-\varphi_1\tau\Big\{\frac{t_2}{2}(t_2^2-t_1^2)-\Big(\frac{t_2^3}{3}-\frac{t_1^3}{3}\Big)+\frac{\alpha}{\beta+1}\Big\{\frac{(t_2-\gamma)^{\beta+1}}{2}(t_2^2-t_1^2)$$

$$-\Big\{\frac{1}{\beta+2}\Big\{t_2(t_2-\gamma)^{\beta+2}-t_1(t_1-\gamma)^{\beta+2}\Big\}-\frac{1}{(\beta+2)(\beta+3)}\{(t_2-\gamma)^{\beta+3}-(t_1-\gamma)^{\beta+3}\}\Big\}\Big\}$$

$$+\frac{\varphi_1}{2}\Big(\frac{t_2^2}{2}(t_2^2-t_1^2)-\Big(\frac{t_2^4-t_1^4}{4}\Big)\Big)\Big\}-\frac{\varphi_1\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{n\frac{t_2^{\frac{1}{n}}}{2}(t_2^2-t_1^2)-\frac{n}{1+2n}\Big(t_2^{\frac{1}{n}+2}-t_1^{\frac{1}{n}+2}\Big)\Big\}+\alpha\Big\{\frac{n}{n\beta+1}$$

$$\Big\{\frac{t_2^{\frac{1}{n}+\beta}}{2}(t_2^2-t_1^2)-\frac{n}{1+\beta n+2n}\Big(t_2^{\frac{1}{n}+\beta+2}-t_1^{\frac{1}{n}+\beta+2}\Big)\Big\}-\frac{\beta\gamma n}{1-n+\beta n}\Big\{\frac{t_2^{\frac{1}{n}-1+\beta}}{2}(t_2^2-t_1^2)-\frac{n}{1+\beta n+n}$$

$$\Big(t_2^{\frac{1}{n}+\beta+1}-t_1^{\frac{1}{n}+\beta+1}\Big)\Big\}\Big\}+\frac{n\varphi_1}{1+n}\Big\{\frac{t_2^{\frac{1}{n}}}{2}(t_2^2-t_1^2)-\frac{n}{1+3n}\Big(t_2^{\frac{1}{n}+3}-t_1^{\frac{1}{n}+3}\Big)\Big\}\Big\}+\frac{\pi}{T}\Big[\tau\Big\{\frac{t_3^2}{2}+\frac{1}{2}\Big\{\frac{R^2t_3^2}{\tau^2}$$

$$+\frac{(R-\tau)^2T^2}{\tau^2}-\frac{2Rt_3}{\tau^2}(R-\tau)\Big\}-\frac{Rt_3{}^2}{\tau}+\frac{(R-\tau)Tt_3}{\tau}\Big\}+(R-\tau)\Big\{\frac{T^2}{2}+\frac{t_3{}^2}{2}-t_3T\Big\}\Big],\qquad(17)$$

The total cost per unit time is to be minimized for obtaining the optimal production scheduling policies. For a given t_3, $K(t_1,t_3,T)$ is a convex function of t_1, we obtain the necessary condition, which minimizes $K(t_1,t_3,T)$ with respect to t_1 is

$$\frac{\partial K(t_1,t_3,T)}{\partial t_1}=0.$$

This implies

$$\frac{\partial K(t_1,t_3,T)}{\partial t_1}=\frac{C}{T}R+\frac{h}{T}\Bigg[(R-\tau)\Bigg\{t_1+\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{\varphi_1 t_1^2}{2}-\frac{\alpha}{\beta+1}\Big\{(t_1-\gamma)^{\beta}(2t_1-\gamma+t_1\beta)-(t_1-\gamma)^{\beta+1}\Big\}$$

$$-\frac{\alpha^2(t_1-\gamma)^{2\beta+1}}{\beta+1}-\frac{\alpha\varphi_1}{2}\Bigg\{\frac{(t_1-\gamma)^{\beta}}{\beta+1}\left(3t_1^2-2t_1\gamma+t_1^2\beta\right)-\frac{2}{(\beta+1)(\beta+2)}\Big\{(t_1-\gamma)^{\beta+1}$$

$$(3t_1-\gamma+t_1\beta)-(t_1-\gamma)^{\beta+2}\Big\}\Bigg\}-\varphi_1 t_1^2-\frac{\alpha\varphi_1}{\beta+1}\Bigg\{\frac{(t_1-\gamma)^{\beta+1}}{\beta+2}(3t_1-\gamma+t_1\beta)-\frac{(t_1-\gamma)^{\beta+2}}{\beta+2}\Bigg\}-\frac{\varphi_1^2 t_1^3}{2}\Bigg]$$

$$-\frac{\varphi_2 r}{nT^{\frac{1}{n}}}\Bigg[nt_1^{\frac{1}{n}}+\alpha\Bigg\{\frac{n}{1+\beta n}\Big\{t_1^{\frac{1}{n}+\beta}-\gamma^{\frac{1}{n}+\beta}\Big\}-\frac{n\beta\gamma}{1-n+\beta n}\Big\{t_1^{\frac{1}{n}+\beta-1}-\gamma^{\frac{1}{n}+\beta-1}\Big\}\Bigg\}+\frac{\varphi_1 n}{1+n}t_1^{\frac{1}{n}+1}$$

$$-\alpha n\Big\{t_1^{\frac{1}{n}+\beta}-\beta\gamma t_1^{\frac{1}{n}+\beta-1}\Big\}-\alpha^2\Bigg\{\frac{n}{1+\beta n}\Big\{t_1^{\frac{1}{n}+2\beta}-\beta\gamma t_1^{\frac{1}{n}+2\beta-1}-\gamma^{\frac{1}{n}+\beta}(t_1-\gamma)^{\beta}\Big\}-\frac{n\beta\gamma}{1-n+\beta n}\Big\{t_1^{\frac{1}{n}+2\beta-1}$$

$$-\beta\gamma t_1^{\frac{1}{n}+2\beta-2}-\gamma^{\frac{1}{n}+\beta-1}(t_1-\gamma)^{\beta}\Big\}\Bigg\}-\frac{\alpha\varphi_1 n}{1+n}\Big\{t_1^{\frac{1}{n}+\beta+1}-\beta\gamma t_1^{\frac{1}{n}+\beta}\Big\}-\varphi_1 n t_1^{\frac{1}{n}+1}-\alpha\varphi_1\Bigg\{\frac{n}{1+\beta n}\Big\{t_1^{\frac{1}{n}+\beta+1}$$

$$-\gamma^{\frac{1}{n}+\beta}t_1\Big\}-\frac{n\beta\gamma}{1-n+\beta n}\Big\{t_1^{\frac{1}{n}+\beta}-\gamma^{\frac{1}{n}+\beta-1}\Big\}\Bigg\}-\frac{\varphi_1^2}{1+n}t_1^{\frac{1}{n}+2}\Bigg\}+\tau\Bigg\{-t_2+t_1+\frac{\alpha}{\beta+1}\Big\{(t_1-\gamma)^{\beta+1}$$

$$-(t_2-\gamma)^{\beta+1}\Big\}-\frac{\varphi_1 t_2^2}{2}+\frac{\varphi_1 t_1^2}{2}\Bigg\}+\frac{\varphi_2 r}{nT^{\frac{1}{n}}}\Bigg\{nt_1^{\frac{1}{n}}-nt_2^{\frac{1}{n}}+\alpha\Bigg\{\frac{n}{n\beta+1}\Big\{t_1^{\frac{1}{n}+\beta}-t_2^{\frac{1}{n}+\beta}\Big\}-\frac{\beta\gamma n}{1-n+\beta n}$$

$$\Big\{t_1^{\frac{1}{n}-1+\beta}-t_2^{\frac{1}{n}-1+\beta}\Big\}\Bigg\}+\frac{n\varphi_1}{1+n}\Big\{t_1^{\frac{1}{n}+1}-t_2^{\frac{1}{n}+1}\Big\}\Bigg\}-\tau\alpha\Bigg\{-t_2(t_1-\gamma)^{\beta}-\frac{1}{\beta+1}\Big\{-(t_1-\gamma)^{\beta}(2t_1-\gamma+t_1\beta)$$

$$+(t_1-\gamma)^{\beta+1}\Big\}+\frac{\alpha}{(\beta+1)}\Big\{(t_1-\gamma)^{2\beta+1}-(t_2-\gamma)^{\beta+1}(t_1-\gamma)^{\beta}\Big\}+\frac{\varphi_1}{2}\Bigg\{-t_2^2(t_1-\gamma)^{\beta}-\Bigg\{-\frac{1}{\beta+1}(t_1-\gamma)^{\beta}$$

$$\left(3t_1^2-2t_1\gamma+t_1^2\beta\right)-\frac{2}{(\beta+1)(\beta+2)}\Big\{-(t_1-\gamma)^{\beta+1}(3t_1-\gamma+t_1\beta)+(t_1-\gamma)^{\beta+2}\Big\}\Bigg\}\Bigg\}\Bigg\}-\frac{\alpha\varphi_2 r}{nT^{\frac{1}{n}}}\Bigg\{n\Big\{t_1^{\frac{1}{n}+2\beta}$$

$$-t_2^{\frac{1}{n}}(t_1-\gamma)^{\beta}-\beta\gamma t_1^{\frac{1}{n}+2\beta-1}\Big\}+\alpha\Bigg\{\frac{n}{n\beta+1}\Big\{-t_2^{\frac{1}{n}+\beta}(t_1-\gamma)^{\beta}+t_1^{\frac{1}{n}+2\beta}+\beta\gamma t_1^{\frac{1}{n}+2\beta-1}\Big\}-\frac{\beta\gamma n}{1-n+\beta n}$$

$$\Big\{-t_2^{\frac{1}{n}+\beta-1}(t_1-\gamma)^{\beta}+t_1^{\frac{1}{n}+2\beta-1}-\beta\gamma+t_1^{\frac{1}{n}+2\beta-2}\Big\}\Bigg\}+\frac{n\varphi_1}{1+n}\Big\{-t_2^{\frac{1}{n}+1}(t_1-\gamma)^{\beta}+t_1^{\frac{1}{n}+\beta+1}-\beta\gamma t_1^{\frac{1}{n}+\beta}\Big\}\Bigg\}$$

$$-\varphi_1\tau\Bigg\{-t_2t_1+t_1^2+\frac{\alpha}{\beta+1}\Bigg\{-t_1(t_2-\gamma)^{\beta+1}-\frac{1}{\beta+2}\Big\{-(t_1-\gamma)^{\beta+1}(3t_1-\gamma+t_1\beta)\Big\}-\frac{1}{\beta+2}(t_1-\gamma)^{\beta+2}\Bigg\}$$

$$+\frac{\varphi 1}{2}\Big\{-t_1t_2^2+t_1^3\Big\}\Bigg\}-\frac{\varphi_1\varphi_2 r}{nT^{\frac{1}{n}}}\Bigg\{n\Big\{-t_1t_2^{\frac{1}{n}}+t_1^{\frac{1}{n}+1}\Big\}+\alpha\Bigg\{\frac{n}{n\beta+1}\Big\{-t_1t_2^{\frac{1}{n}+\beta}+t_1^{\frac{1}{n}+\beta+1}\Big\}-\frac{\beta\gamma n}{1-n+\beta n}\Big\{-t_1t_2^{\frac{1}{n}+\beta-1}$$

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$$+t_1^{\frac{1}{n}+\beta}\Big\}\Big\}+\frac{n\varphi_1}{1+n}\Big\{-t_1t_2^{\frac{1}{n}+1}+t_1^{\frac{1}{n}+2}\Big\}\Big\}=0, \qquad (18)$$

The total cost per unit time is to be minimized for obtaining the optimal production scheduling policies. For a given t_1, $K(t_1,t_3,T)$ is a convex function of t_3, we obtain the necessary condition which minimizes $K(t_1, t_3)$ with respect to t_1 is

$$\frac{\partial K(t_1,t_3,T)}{\partial t_3}=0.$$

$$\frac{\partial K(t_1,t_3,T)}{\partial t_3}=\frac{C}{T}(-R)+\frac{h}{T}\Big[\tau\Big\{2xy-t_1y-xy+\frac{\alpha}{\beta+1}\Big\{x(\beta+1)(x-\gamma)^{\beta}y+(x-\gamma)^{\beta+1}y-(\beta+1)(x-\gamma)^{\beta}$$

$$yt_1-(x-\gamma)^{\beta+1}y\Big\}+\frac{3\varphi_1}{2}x^2y-t_1\varphi_1xy-\frac{\varphi_1}{2}x^2y\Big\}+\frac{\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{n(1+n)x^{\frac{1}{n}}y-t_1x^{\frac{1}{n}-1}y-nx^{\frac{1}{n}}y$$

$$+\alpha\Big\{\frac{n}{n\beta+1}\Big\{\Big(\frac{1}{n}+\beta+1\Big)x^{\frac{1}{n}+\beta}y-t_1y\Big(\frac{1}{n}+\beta\Big)x^{\frac{1}{n}+\beta-1}-x^{\frac{1}{n}+\beta}y\Big\}-\frac{\beta\gamma n}{1-n+\beta n}\Big\{y\Big(\frac{1}{n}+\beta\Big)x^{\frac{1}{n}+\beta-1}$$

$$-t_1y\Big(\frac{1}{n}-1+\beta\Big)x^{\frac{1}{n}+\beta-2}-yx^{\frac{1}{n}+\beta-1}\Big\}\Big\}+\frac{n\varphi_1}{1+n}\Big\{y\Big(\frac{1}{n}+2\Big)x^{\frac{1}{n}+1}-t_1y\Big(\frac{1}{n}+1\Big)x^{\frac{1}{n}}-x^{\frac{1}{n}+1}y\Big\}\Big\}$$

$$-\alpha\tau\Big\{\frac{y}{\beta+1}\{(t_1-\gamma)^{\beta+1}+(x-\gamma)^{\beta+1}\}+\frac{\alpha y}{(\beta+1)}\{2(x-\gamma)^{2\beta+1}-(t_1-\gamma)^{\beta+1}(x-\gamma)^{\beta}-(x-\gamma)^{2\beta+1}\}$$

$$+\frac{\varphi_1xy}{(\beta+1)}\{-(t_1-\gamma)^{\beta+1}+(x-\gamma)^{\beta+1}\}\Big\}-\frac{\alpha\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{n\Big\{\frac{1}{\beta+1}\Big\{x^{\frac{1}{n}}y(\beta+1)(x-\gamma)^{\beta+1}+(x-\gamma)^{\beta+1}$$

$$\frac{x^{\frac{1}{n}-1}}{n}y-(t_1-\gamma)^{\beta+1}y\frac{x^{\frac{1}{n}-1}}{n}\Big\}-yx^{\frac{1}{n}+\beta}+y\beta\gamma x^{\frac{1}{n}+\beta-1}\Big\}+\alpha\Big\{\frac{n}{n\beta+1}\Big\{\frac{1}{\beta+1}\Big\{x^{\frac{1}{n}+\beta}(\beta+1)(x-\gamma)^{\beta}y$$

$$+(x-\gamma)^{\beta+1}\Big(\frac{1}{n}+\beta\Big)x^{\frac{1}{n}+\beta-1}y-(t_1-\gamma)^{\beta+1}\Big(\frac{1}{n}+\beta\Big)x^{\frac{1}{n}+\beta-1}y\Big\}-x^{\frac{1}{n}+2\beta}y+\beta\gamma x^{\frac{1}{n}+\beta-1}y\Big\}-\frac{\beta\gamma n}{1-n+\beta n}$$

$$\Big\{\frac{yx^{\frac{1}{n}+\beta-1}}{\beta+1}\Big\{(\beta+1)(x-\gamma)^{\beta}+(x-\gamma)^{\beta+1}\Big(\frac{1}{n}+\beta-1\Big)-(t_1-\gamma)^{\beta+1}\Big(\frac{1}{n}+\beta-1\Big)\frac{1}{x}\Big\}-yx^{\frac{1}{n}+2\beta-1}$$

$$+\beta\gamma yx^{\frac{1}{n}+2\beta-2}\Big\}\Big\}+\frac{n\varphi_1}{1+n}\Big\{\frac{1}{\beta+1}\Big\{yx^{\frac{1}{n}+1}(\beta+1)(x-\gamma)^{\beta}+(x-\gamma)^{\beta+1}\Big(\frac{1}{n}+1\Big)yx^{\frac{1}{n}}-(t_1-\gamma)^{\beta+1}$$

$$\Big(\frac{1}{n}+1\Big)yx^{\frac{1}{n}}\Big\}-yx^{\frac{1}{n}+\beta+1}+\beta\gamma yx^{\frac{1}{n}+\beta}\Big\}\Big\}-\varphi_1\tau\Big\{\frac{3}{2}x^2y-\frac{t_1^2y}{2}-x^2y+\frac{\alpha}{\beta+1}\Big\{\frac{1}{2}\{x^2y(\beta+1)(x-\gamma)^{\beta}$$

$$+2xy(\beta+1)(x-\gamma)^{\beta+1}-t_1^2y(\beta+1)(x-\gamma)^{\beta}\}-x(x-\gamma)^{\beta+1}\Big\}+\frac{\varphi_1}{2}\{2x^3y-t_1^2xy-x^3y\}\Big\}$$

$$-\frac{\varphi_1\varphi_2 r}{nT^{\frac{1}{n}}}\Big\{ny\Big\{\frac{1}{2}\Big(\frac{1}{n}+2\Big)x^{\frac{1}{n}}-\frac{t_1^2}{2n}x^{\frac{1}{n}-1}-x^{\frac{1}{n}+1}\Big\}+\alpha\Big\{\frac{n}{n\beta+1}\Big\{\frac{x^{\frac{1}{n}+\beta}y}{2}\Big\{\Big(\frac{1}{n}+2+\beta\Big)x-\frac{t_1^2}{x}\Big(\frac{1}{n}+\beta\Big)$$

$$-x^{\frac{1}{n}+\beta+1}y\Bigg\}-\frac{\beta\gamma n x^{\frac{1}{n}+\beta}y}{1-n+\beta n}\left\{\frac{1}{2}\left\{\left(\frac{1}{n}+\beta-1\right)-\left(\frac{1}{n}+\beta-1\right)\frac{t_1^2}{x^2}\right\}-1\right\}\Bigg\}+\frac{n\varphi_1 x^{\frac{1}{n}}y}{1+n}\left\{\frac{1}{2}\left\{\left(\frac{1}{n}+3\right)x^2\right.\right.$$

$$\left.\left.-t_1^2\left(\frac{1}{n}+1\right)\right\}-x^2\right\}\Bigg\}\Bigg]+\frac{\pi}{T}\left[\tau\left\{t_3+\frac{R^2t_3}{\tau^2}-\frac{R(R-\tau)T}{\tau^2}-\frac{2Rt_3}{\tau}+\frac{(R-\tau)T}{\tau}\right\}+(R-\tau)(t_3-T)\right]$$

$$\text{where } x=\frac{1}{\tau}\{Rt_3-(R-\tau)T\} \text{ and } y=\frac{R}{\tau}, \qquad (19)$$

Solving the equations (18) and (19), using numerical methods, we obtain the optimal production down time t_1 as t_1^* and t_3 as t_3^*. Substituting t_3^* in equation (11), t_1^* and t_3^* in equation (12), we obtain t_2^* and optimal production quantity Q as Q^* respectively.

4. Case Study

Consider the case of deriving the economic production quantity and other optimal policies for a pickle-manufacturing unit. Here, the product is deteriorating type and has random lifetime and assumed to follow a three-parameter Weibull distribution. Discussions held with the personnel connected with the production and marketing and the records verified to decide the values of various parameters.

Let the inventory system with shortages has the following parameter values:

R= 30 units r = 10 units τ = 12 units h = Rs.10/-
C = Rs.10/- A= Rs.75/-
n = 3 T= 4 months π = Rs.15/-

For the assigned values of deterioration parameters (α,β,γ) = (0.1, 1.0, 0.01) and demand parameters (φ_1,φ_2) = (0.1, 0.1), the optimal values of time (t_1^*), production quantity (Q^*), total system cost (K^*) have been determined. The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table 1.

Table 1. Effect of demand and deterioration parameters on optimal policies – Demand is function of on hand inventory and time - with shortages

PARAMETERS													OPTIMAL POLICIES				
α	β	γ	ϕ_1	ϕ_2	τ	r	R	h	C	π	n	A	t_1^*	t_2^*	t_3^*	Q^*	K^*
0.10													0.173	2.383	3.353	24.60	196.487
0.08													0.137	2.373	3.370	23.77	197.668
0.09	1.0	0.01	0.1	0.1	12	10	30	10	10	15	3	75	0.103	2.363	3.386	22.94	198.706
0.11													0.071	2.356	3.402	22.07	199.627
0.12													0.042	2.345	3.416	21.28	200.402
	1.4												0.205	2.293	3.317	26.64	204.124
	1.8												0.229	2.205	3.282	28.41	210.941
	2.2												0.246	2.125	3.250	29.88	217.104
	2.6												0.259	2.053	3.221	31.14	222.720
		0.00											0.449	2.428	3.371	32.34	175.151
		0.04											0.356	2.413	3.365	29.73	182.911
		0.08											0.264	2.398	3.359	27.15	190.019
		0.12											0.084	2.370	3.348	22.08	202.309
			0.12										0.207	2.455	3.382	24.75	198.721
			0.14										0.239	2.520	3.408	24.93	200.780
			0.16										0.268	2.580	3.432	25.08	202.671
			0.18										0.295	2.637	3.455	25.20	204.398
				0.12									0.175	2.385	3.354	24.63	196.344
				0.14									0.176	2.387	3.355	24.63	196.221
				0.16									0.177	2.390	3.356	24.63	196.143
				0.18									0.178	2.390	3.356	24.66	196.064
					14								0.173	2.383	3.353	24.60	197.737
					16								0.173	2.383	3.353	24.60	198.987
					18								0.173	2.383	3.353	24.60	200.237
					20								0.173	2.383	3.353	24.60	201.487
						12							0.179	2.385	3.354	24.75	196.664
						14							0.185	2.387	3.355	24.90	196.839
						16							0.191	2.390	3.356	25.05	197.041
						18							0.197	2.390	3.356	25.23	197.226
							31						0.137	2.373	3.370	23.77	197.668
							32						0.103	2.363	3.386	22.94	198.706
							33						0.071	2.356	3.402	22.07	199.627
							34						0.042	2.345	3.416	21.28	200.402

								11					0.205	2.293	3.317	26.64	204.124
								12					0.229	2.205	3.282	28.41	210.941
								13					0.246	2.125	3.250	29.88	217.104
								14					0.259	2.053	3.221	31.14	222.720
									7				0.449	2.428	3.371	32.34	175.151
									8				0.356	2.413	3.365	29.73	182.911
									9				0.264	2.398	3.359	27.15	190.019
									11				0.084	2.370	3.348	22.08	202.309
										16			0.207	2.455	3.382	24.75	198.721
										17			0.239	2.520	3.408	24.93	200.780
										18			0.268	2.580	3.432	25.08	202.671
										19			0.295	2.637	3.455	25.20	204.398
											3.5		0.175	2.385	3.354	24.63	196.344
											4.0		0.176	2.387	3.355	24.63	196.221
											4.5		0.177	2.390	3.356	24.63	196.143
											5.0		0.178	2.390	3.356	24.66	196.064
												80	0.173	2.383	3.353	24.60	197.737
												85	0.173	2.383	3.353	24.60	198.987
												90	0.173	2.383	3.353	24.60	200.237
												95	0.173	2.383	3.353	24.60	201.487

The increase in deterioration parameter α has shown decreasing trend in all optimal values of time (t_1^*, t_2^*, t_3^*) and production quantity (Q) viz., (0.137 to 0.042, 2.373 to 2.345, 3.370 to 3.416 months) and (23.77 to 21.28 units) but increase in total cost (K^*, Rs.197.668 to 200.402) respectively. Increase in β results in increase in t_1^* (0.173 to 0.259 months), Q^* (24.60 to 31.14 units) and K^* (Rs.196.487 to 222.720) respectively. Increase in γ results decrease in all optimal policies. Increase in demand parameter (φ_1) and shortage cost (π) has shown increase in all the optimal policies. The increase in production rate (R, 30 to 34 units) and unit cost (C, Rs.7 to 11) has decreasing effect on all optimal values of time (t_1^*, t_2^*, t_3^*), production quantity (Q^*) i.e. (t_1^*, t_2^*, t_3^* = 0.173 to 0.042, 2.383 to 2.345, 3.353 to 3.416, 0.449 to 0.084, 2.428 to 2.370, 3.371 to 3.348 months respectively) and (Q^*, 24.60 to 21.28, 32.34 to 22.08 units) respectively. However, they have shown increasing trend in total cost (K^*, Rs.196.487 to 200.402, Rs.175.151 to 202.309).

5. SENSITIVITY ANALYSIS

A sensitivity analysis has been carried out to explore the effect on the optimal policies by varying the value of each parameter at a time and all parameters together. The results obtained by changing parameters by -15%, -10%. -5%, +5%, +10% and +15% are tabulated in Table 2.

It is noticed that the increase in deterioration parameters (α,β) has increasing trend in optimal production downtime, t_1^* (0.173 to 0.193, 0.173 to 0.177 months) but mixed response in production uptime, t_3^* (3.353 to 3.360, 3.353 to 3.349 months) respectively. The increase in shortage cost, π (from -15% to +15%) has increasing effect on optimal times (t_1^*, 0.087 to 0.246 and t_3^*, 3.281 to 3.414 months), quantity Q^* (24.18 to 24.96 units) and total system cost K^* (Rs.190.6 to 201.268).

The graphical representation of the parameters variation effect on optimal policies is shown in Fig.2

6. CONCLUSION

- This model considers the delayed nature of decay by considering the location parameter in the decay distribution.
- This model includes increasing, decreasing and constant rates of decay which is a more general type of distribution for decay.
- In this model, the demand function considered includes a spectra of demand patterns like time dependent demand, stock dependent demand, constant rate of demand, both time and on-hand inventory dependent demand. Hence, this model can be viewed as a generalized EPQ, which serves several types of demands.
- This model is also can be generalized by assuming that product under consideration follows a general distribution like Pearson type distribution which includes Weibull as a particular case. Since, it is capable of providing optimal production schedules it can be commercialized by developing user-friendly software package which serve as robust model for production scheduling and inventory control. ';

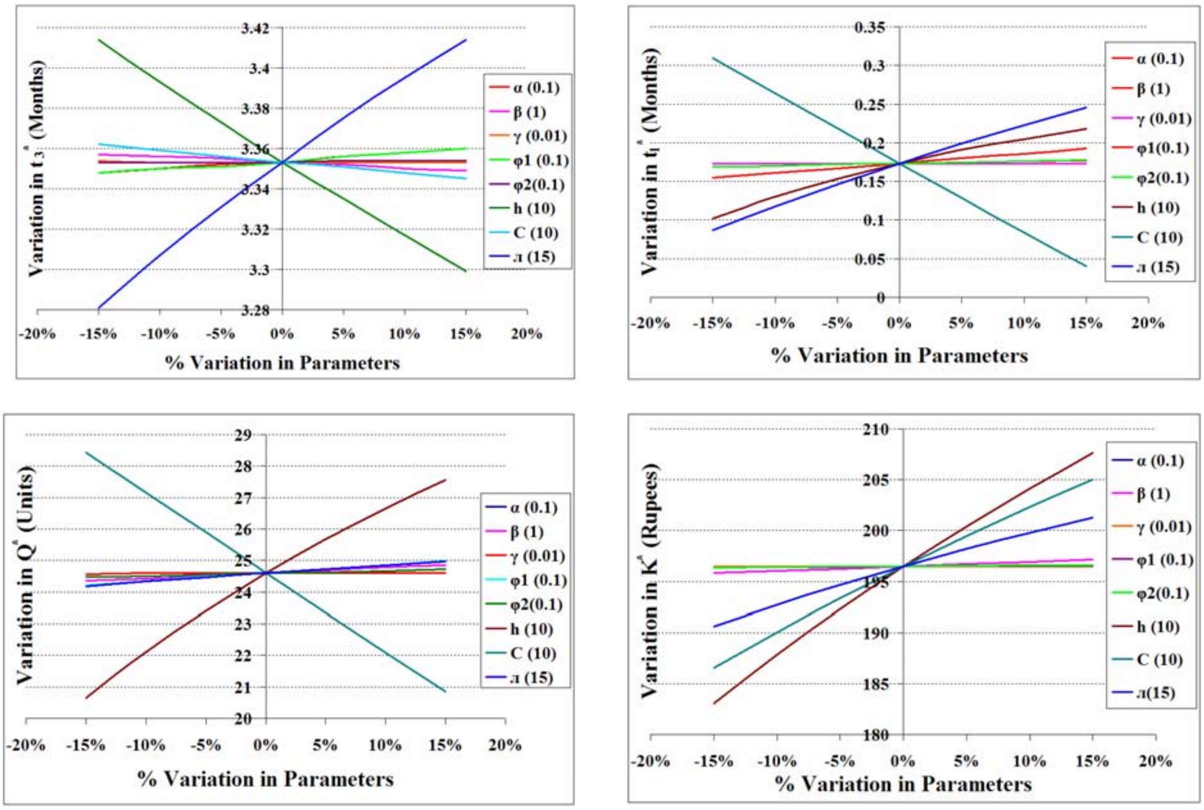
We also reduced the model to without shortages (see Appendix) by formulating the governing equations and obtaining the solutions.

Table-2: Sensitivity analysis of Optimal Policies -Demand is function of on hand inventory and time – with shortages

Parameters	Optimal Policies	- 15 %	- 10 %	- 5 %	0	+ 5 %	+ 10 %	+ 15 %
α (0.1)	t_1^*	0.155	0.161	0.167	0.173	0.180	0.186	0.193
	t_3^*	3.348	3.350	3.352	3.353	3.356	3.358	3.360
	Q^*	24.21	24.33	24.45	24.60	24.72	24.84	24.99
	K^*	196.335	196.389	196.435	196.487	196.502	196.523	201.535
β (1)	t_1^*	0.169	0.170	0.172	0.173	0.175	0.176	0.177
	t_3^*	3.357	3.356	3.355	3.353	3.352	3.350	3.349
	Q^*	24.36	24.42	24.51	24.60	24.69	24.78	24.84
	K^*	195.822	196.029	196.235	196.487	196.695	196.919	197.158
γ (0.01)	t_1^*	0.173	0.173	0.173	0.173	0.173	0.173	0.173
	t_3^*	3.354	3.353	3.353	3.353	3.353	3.353	3.353
	Q^*	24.57	24.60	24.60	24.60	24.60	24.60	24.60
	K^*	196.467	196.484	196.485	196.487	196.489	196.491	196.493
ϕ_1 (0.1)	t_1^*	0.155	0.161	0.167	0.173	0.180	0.186	0.193
	t_3^*	3.348	3.350	3.351	3.353	3.356	3.358	3.360
	Q^*	24.21	24.33	24.48	24.60	24.72	24.84	24.99
	K^*	196.340	196.393	196.422	196.487	196.500	196.519	196.529
ϕ_2 (0.1)	t_1^*	0.169	0.170	0.172	0.173	0.175	0.176	0.178
	t_3^*	3.353	3.353	3.353	3.353	3.354	3.354	3.354
	Q^*	24.48	24.51	24.57	24.60	24.63	24.66	24.72
	K^*	196.342	196.390	196.439	196.487	196.521	196.569	196.617
τ (12)	t_1^*	0.010	0.064	0.119	0.173	0.228	0.283	0.339
	t_3^*	3.435	3.407	3.380	3.353	3.328	3.303	3.279
	Q^*	17.25	19.71	22.17	24.60	27	29.4	31.8
	K^*	174.099	181.903	189.339	196.487	203.264	209.733	215.887
r(10)	t_1^*	0.169	0.17	0.172	0.173	0.175	0.176	0.178
	t_3^*	3.353	3.353	3.353	3.353	3.354	3.354	3.354
	Q^*	24.48	24.51	24.57	24.60	24.63	24.66	24.72
	K^*	196.342	196.390	196.439	196.487	196.521	196.569	196.617
R(30)	t_1^*	0.374	0.299	0.233	0.173	0.12	0.071	0.027
	t_3^*	3.268	3.298	3.327	3.353	3.378	3.402	3.423
	Q^*	28.20	27.02	25.82	24.60	23.373	22.07	20.83
	K^*	189.030	191.972	194.423	196.487	198.184	199.627	200.767
h (10)	t_1^*	0.102	0.130	0.153	0.173	0.191	0.205	0.218
	t_3^*	3.414	3.393	3.373	3.353	3.335	3.317	3.299
	Q^*	20.64	22.11	23.4	24.60	25.68	26.64	27.57
	K^*	183.086	187.832	192.312	196.487	198.184	199.627	200.767
C(10)	t_1^*	0.310	0.264	0.219	0.173	0.129	0.084	0.04
	t_3^*	3.362	3.359	3.356	3.353	3.351	3.348	3.345
	Q^*	28.44	27.15	25.89	24.60	23.34	22.08	20.85
	K^*	186.533	190.019	193.318	196.487	199.453	202.309	205.010
π (15)	t_1^*	0.087	0.118	0.146	0.173	0.199	0.223	0.246
	t_3^*	3.281	3.307	3.331	3.353	3.375	3.395	3.414
	Q^*	24.18	24.33	24.45	24.60	24.72	24.84	24.96
	K^*	190.600	192.704	194.662	196.487	198.193	199.788	201.268
n(3)	t_1^*	0.172	0.172	0.173	0.173	0.174	0.174	0.175
	t_3^*	3.352	3.353	3.353	3.353	3.354	3.354	3.358

	Q^*	24.60	24.57	24.60	24.60	24.60	24.60	24.63
	K^*	196.608	196.576	196.530	196.487	196.432	196.393	196.356
A(75)	t_1^*	0.173	0.173	0.173	0.173	0.173	0.173	0.173
	t_3^*	3.353	3.353	3.353	3.353	3.353	3.353	3.353
	Q^*	24.60	24.60	24.60	24.60	24.60	24.60	24.60
	K^*	193.675	194.612	195.550	196.487	197.425	198.362	199.300
All	t_1^*	0.13	0.144	0.158	0.173	0.19	0.208	.227
	t_3^*	3.347	3.349	3.351	3.353	3.357	3.361	3.367
	Q^*	19.967	21.465	23	24.60	26.239	27.951	29.67
	K^*	143.615	160.179	177.915	196.487	215.954	236.298	257.615

Fig.2.Graphical representation of sensitivity analysis of important parameters –with shortages



Appendix

II (Without Shortages)

In this section, we consider the production level inventory model in which shortages are not allowed. The production starts at time t = 0, when the stock is zero and reaches to a maximum inventory level at time $t = t_1$. The time interval is divided into two non-overlapping intervals $(0, \gamma)$ and (γ, t_1). During the interval $(0, \gamma)$, the produced items partly meet the demand and during interval (γ, t_1), the produced items are partly consumed due to the demand and deterioration and excess items are stored. The production is stopped at time $t = t_1$ and the stock level is allowed to reduce gradually due to the demand and deterioration and at time $t = t_2$, the inventory becomes zero. At this time, the production starts again and the cycle repeats thereafter. The inventory model explained above is shown in Fig.3.

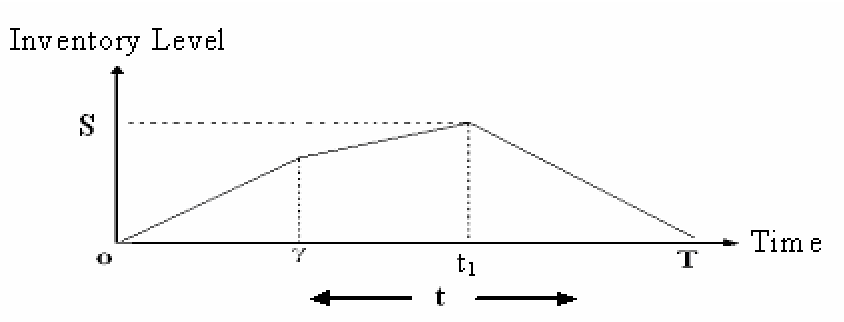


Fig.3 The inventory system – without shortages

Let I (t) denote the inventory level of the system at time t ($0 \le t \le T$), then the differential equations governing the instantaneous state of inventory I(t) at any time t are given by

$$\frac{d}{dt} I(t) = R - \left\{ \tau + \varphi_1 I(t) + \varphi_2 \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\}, \qquad 0 \le t \le \gamma \tag{1}$$

$$\frac{d}{dt} I(t) + \alpha\beta (t-\gamma)^{\beta-1} I(t) = R - \left\{ \tau + \varphi_1 I(t) + \varphi_2 \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\}, \qquad \gamma \le t \le t_1 \tag{2}$$

$$\frac{d}{dt} I(t) + \alpha\beta (t-\gamma)^{\beta-1} I(t) = - \left\{ \tau + \varphi_1 I(t) + \varphi_2 \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\}, \qquad t_1 \le t \le T \tag{3}$$

with the boundary conditions I(0) = 0 and I(T) = 0.

By solving the equations (1), (2) and (3) and using boundary conditions, we obtain the instantaneous state of inventory at any given time t, during the interval (0,γ) is

$$I(t) = e^{-\varphi_1 t} \int_0^t \left[R - \left\{ \tau + \varphi_2 \frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\} \right] e^{\varphi_1 u} du, \qquad 0 \le t \le \gamma \tag{4}$$

The instantaneous state of inventory at any time t, during the interval (γ, t_1) is

$$I(t) = e^{-\{\alpha(t-\gamma)^{\beta} + \phi_1 t\}} \left[\int_\gamma^t \left[R - \left\{ \tau + \varphi_2 \frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\} \right] e^{\alpha(u-\gamma)^{\beta} + \varphi_1 u} du + \int_0^\gamma \left[R - \left\{ \tau + \varphi_2 \frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\} \right] e^{\varphi_1 u} du \right], \qquad \gamma \le t \le t_1 \tag{5}$$

Since the production is stopped after reaching maximum inventory level, at any time t during the interval (t_1, T) is

$$I(t) = e^{-\{\alpha(t-\gamma)^{\beta} + \phi_1 t\}} \left[\int_t^T \left\{ \tau + \varphi_2 \frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right\} e^{\alpha(u-\gamma)^{\beta} + \varphi_1 u} du \right], \qquad t_1 \le t \le T \tag{6}$$

The production quantity during the cycle time (0, T) is the production rate (R) multiplied by time period of production (t_1) and is given by

$$Q = R\, t_1 \tag{7}$$

The total cost per unit time K (t_1, T) is the sum of the set up cost per unit time, purchasing cost per unit time and holding cost per unit time and shortage cost per unit time i.e.,

$$K(t_1, T) = \frac{A}{T} + \frac{C}{T} Q + \frac{h}{T} \left[\int_0^\gamma I(t) dt + \int_\gamma^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right], \tag{8}$$

By substituting the values for I(t) and Q from the equations (4),(5),(6) and (7) in equation (8), we get

$$K(t_1,T) = \frac{A}{T} + \frac{C}{T}Rt_1 + \frac{h}{T}\left[\int_0^{\gamma} e^{-\varphi_1 t}\int_0^t\left[R-\left\{\tau+\varphi_2\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right]e^{\varphi_1 u}du\right.$$

$$+\int_{\gamma}^{t_1} e^{-\{\alpha(t-\gamma)^{\beta}+\phi_1 t\}}\left[\int_{\gamma}^{t}\left[R-\left\{\tau+\varphi_2\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right]e^{\alpha(u-\gamma)^{\beta}+\varphi_1 u}du+\int_0^{\gamma}\left[R-\left\{\tau+\varphi_2\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}\right]e^{\varphi_1 u}du\right]$$

$$\left.+\int_{t_1}^{T} e^{-\{\alpha(t-\gamma)^{\beta}+\phi_1 t\}}\left[\int_t^T\left\{\tau+\varphi_2\frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right\}e^{\alpha(u-\gamma)^{\beta}+\varphi_1 u}du\right]\right], \qquad (9)$$

The above equation was simplified similar fashion as done for shortages model.

By minimizing the total cost per unit time with respect to t_1, we can obtain the optimal production start up time t_1 and the optimal economic production quantity Q. Since $K(t_1,T)$ is a convex function of t_1 for a given T, we obtain the necessary condition, which minimizes $K(t_1,T)$ is $\frac{\partial K(t_1,T)}{\partial t_1} = 0.$

Solving the above non- linear equation of for t_1, by using numerical methods, we can obtain the optimal value of t_1 as t_1^*. Substituting t_1^* in equation (7) we can obtain the optimal production quantity Q as $Q^* = R\, t_1^*$.

The results and the pictorial / graphical representations are presented in Table-3, Table-4, Figure- 3 and Figure-4.

7. Acknowledgements

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Table-3: Effect of demand and deterioration parameters on optimal policies - Demand is function of on hand inventory and time – without shortages

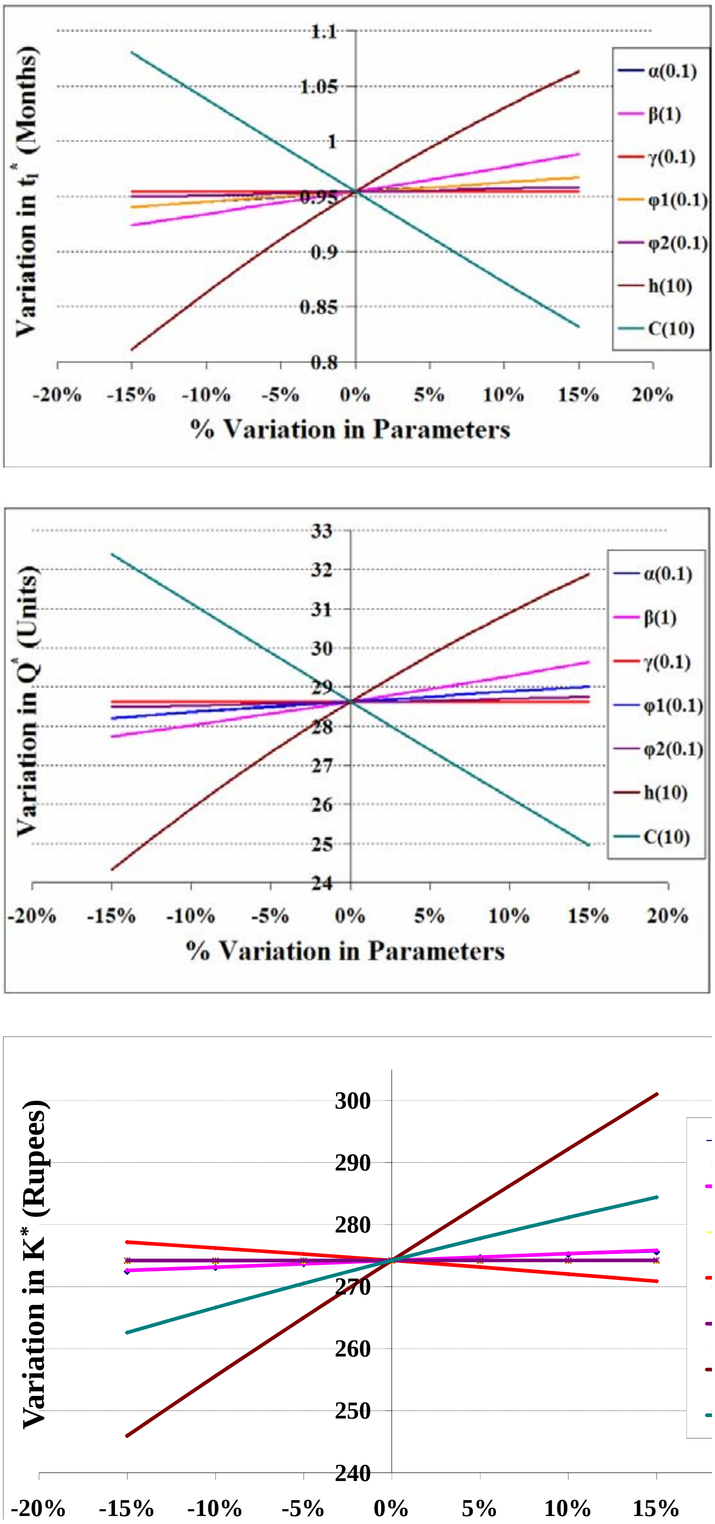
PARAMETERS												OPTIMAL POLICIES		
α	β	γ	ϕ_1	ϕ_2	τ	r	R	h	C	n	A	t_1^*	Q^*	K^*
0.10	1.0	.01	0.10	0.10	12	10	30	10	10	3	75	0.954	28.62	274.210
0.08												0.935	28.05	271.756
0.09												0.945	28.35	273.074
0.11												0.963	28.89	275.166
0.12												0.971	29.13	275.942
	1.4											1.053	31.59	278.026
	1.8											1.186	35.58	276.020
	2.2											1.354	40.62	251.760
	2.6											1.530	45.90	160.750
		0.00										0.953	28.59	274.230
		0.04										0.956	28.68	273.960
		0.08										0.958	28.74	273.650
		0.12										0.960	28.80	273.350
			0.12									0.971	29.13	269.650
			0.14									0.985	29.55	264.380
			0.16									0.996	29.88	258.410
			0.18									1.006	30.18	251.740
				0.12								0.959	28.77	274.240
				0.14								0.965	28.95	274.260
				0.16								0.970	29.10	274.270
				0.18								0.975	29.25	274.290
					14							1.197	35.91	296.350
					16							1.426	42.78	313.100
					18							1.642	49.26	324.940
					20							1.848	55.44	332.250
						12						0.959	28.77	274.240
						14						0.965	28.95	274.260
						16						0.970	29.10	274.270
						18						0.975	29.25	274.290
							31					0.954	28.62	274.210
							32					0.905	28.05	277.600
							33					0.858	27.46	280.760
							34					0.813	26.83	283.710

												0.771	26.21	286.470
								11				1.030	30.90	292.200
								12				1.094	32.82	309.690
								13				1.150	34.50	326.780
								14				1.198	35.94	343.570
									7			1.211	36.33	249.447
									8			1.123	33.69	258.373
									9			1.038	31.14	266.620
									11			0.872	26.16	281.159
										3.5		0.954	28.62	273.951
										4.0		0.953	28.59	273.744
										4.5		0.953	28.59	273.575
										5.0		0.953	28.59	273.434
											80	0.954	28.62	275.460
											85	0.954	28.62	276.710
											90	0.954	28.62	277.960
											95	0.954	28.62	279.210

Tab-4: Sensitivity analysis of Optimal Policies - Demand is function of on hand inventory and time – without shortages

Parameters	Optimal Policies	-15%	-10%	-5%	0	+5%	+10%	+15%
α (0.1)	t_1^*	0.940	0.945	0.949	0.954	0.958	0.963	0.967
	Q^*	28.2	28.35	28.47	28.62	28.74	28.89	29.01
	K^*	272.438	273.074	273.666	274.210	274.711	275.166	275.576
β (1.0)	t_1^*	0.924	0.934	0.944	0.954	0.965	0.976	0.988
	Q^*	27.72	28.02	28.32	28.62	28.95	29.28	29.64
	K^*	272.600	273.126	273.664	274.210	274.758	275.302	275.835
γ (0.1)	t_1^*	0.954	0.954	0.954	0.954	0.954	0.954	0.954
	Q^*	28.62	28.62	28.62	28.62	28.62	28.62	28.62
	K^*	274.223	274.219	274.215	274.210	274.206	274.202	274.198
ϕ_1 (0.1)	t_1^*	0.940	0.945	0.950	0.954	0.958	0.963	0.967
	Q^*	28.20	28.35	28.50	28.62	28.74	28.89	29.01
	K^*	277.161	276.223	275.239	274.210	273.137	272018	270.855
ϕ_2 (0.1)	t_1^*	0.950	0.951	0.953	0.954	0.955	0.957	0.958
	Q^*	28.50	28.53	28.59	28.62	28.65	28.71	28.74
	K^*	274.191	274.198	274.204	274.210	274.217	274.222	274.229
τ (12)	t_1^*	0.723	0.802	0.878	0.954	1.028	1.101	1.173
	Q^*	21.69	24.06	26.34	28.62	30.84	33.03	35.19
	K^*	249.224	258.113	266.437	274.210	281.451	288.173	294.391
r (10)	t_1^*	0.95	0.951	0.953	0.954	0.955	0.957	0.958
	Q^*	28.50	28.53	28.59	28.62	28.65	28.71	28.74
	K^*	274.191	274.198	274.204	274.210	274.217	274.222	274.229
R(30)	t_1^*	1.215	1.12	1.034	0.954	0.881	0.813	0.75
	Q^*	30.983	30.24	29.469	28.62	27.752	26.829	25.875
	K^*	255.597	262.484	268.659	274.210	279.207	283.712	287.778
h(10)	t_1^*	0.811	0.863	0.911	0.954	0.994	1.03	1.063
	Q^*	24.33	25.89	27.33	28.62	29.82	30.9	31.89
	K^*	245.954	255.578	264.986	274.210	283.273	292.197	300.997
C(10)	t_1^*	1.08	1.038	0.996	0.954	0.913	0.872	0.832
	Q^*	32.4	31.14	29.88	28.62	27.39	26.16	24.96
	K^*	262.581	266.620	270.496	274.210	277.763	281.159	284.397
n(3)	t_1^*	0.955	0.954	0.954	0.954	0.954	0.954	0.954
	Q^*	28.65	28.62	28.62	28.62	28.62	28.62	28.62
	K^*	274.506	274.400	274.302	274.210	274.126	274.047	273.974
A(75)	t_1^*	0.954	0.954	0.954	0.954	0.954	0.954	0.954
	Q^*	28.62	28.62	28.62	28.62	28.62	28.62	28.62
	K^*	271.398	272.335	273.273	274.210	275.148	276.085	277.023
All	t_1^*	0.893	0.913	0.933	0.954	0.975	0.996	1.017
	Q^*	22.772	24.651	26.591	28.62	30.712	32.868	35.086
	K^*	199.780	223.449	248.292	274.210	301.078	328.731	356.961

Fig. 4: Graphical representation of sensitivity analysis of important parameters- without shortages



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